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SPRING COLLEGE IN MATERIALS SCIENCE

ON

"METALLIC MATERIALS"

(11 May - 19 June 1987)

DISLOCATION DYNAMICS

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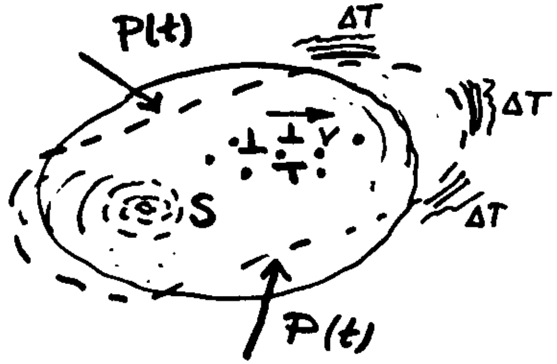
These are preliminary lecture notes, intended only for distribution to participants.

Dislocation Dynamics:

(some topics of a dislocation mean-field theory of plastic deformation and acoustic emission)

by C. E. Bottani – Politecnico di Milano

GOAL



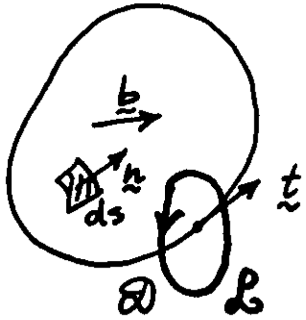
INPUT: EXTERNAL LOADS, ...

RESPONSE: plastic deformation
radiation of acoustic waves (AE)
temperature variations (TE)
.....

MODEL: Dislocation motion, Dislocation-Dislocation and Dislocation-Defects interactions, Dislocation-Elastic field interactions (static and dynamic) in a "crystal".

Burgers circuit

$$\oint_{\mathcal{L}} du_i^e = \oint_{\mathcal{L}} \frac{\partial u_i^e}{\partial x_j} dx_j = -b_i$$



Elastic distortion tensor

$$w_{ji}^e = \left(\frac{\partial u_i^e}{\partial x_j}\right)$$

$$\oint_{\mathcal{L}} w_{ji}^e dx_j = -b_i$$

Elastic strain tensor

$$\varepsilon_{ij}^e \cong \underbrace{\frac{1}{2}\left(\frac{\partial u_i^e}{\partial x_j} + \frac{\partial u_j^e}{\partial x_i}\right)}_{\text{only for discrete loops}} = \underbrace{\frac{1}{2}\left(w_{ji}^e + w_{ij}^e\right)}_{\text{general}}$$

meaningless on D-lines !!!

$\Updownarrow$

$\underset{\approx}{W}^e = \text{grad } \underset{\sim}{u}_e$ (not general)

$\underset{\approx}{\varepsilon}^e = \text{Sym } \underset{\approx}{W}^e$ (general for small strains)

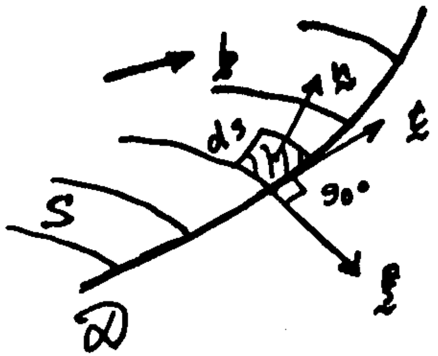
Differential form of the Burgers definition

- Stokes theorem for tensors

$$-b_k = \oint_{L} w^e_{ik}\, dx_i = \iint_S \epsilon_{ilm} \frac{\partial w^e_{mk}}{\partial x_l} n_i \, dS$$

$$\epsilon_{ilm} : \text{Ricci tensor} \begin{cases} +1 & \text{even perm. (123)} \\ -1 & \text{odd perm. (123)} \\ 0 & \text{in all other cases} \end{cases}$$

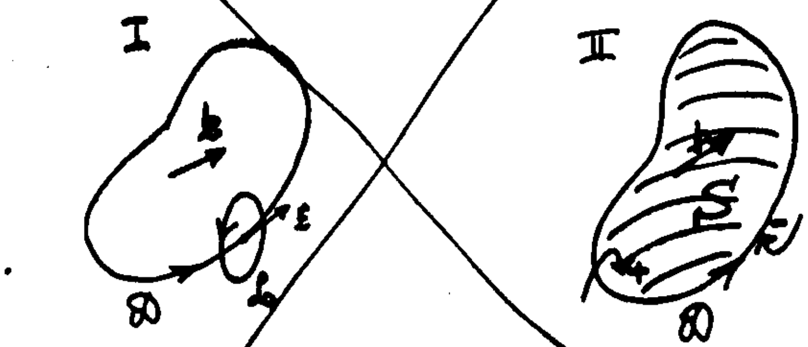
$$b_k = \iint_S t_i b_k \, \delta(\underset{\sim}{\xi})\, n_i \, dS$$



$$\boxed{\begin{gathered} \epsilon_{ilm} \frac{\partial w^e_{mk}}{\partial x_l} = - t_i b_k \, \delta(\underset{\sim}{\xi}) \\ \mathrm{rot}\, \underset{\approx}{w}^e = - \underline{t}\, \underline{b}\, \delta(\underset{\sim}{\xi}) \end{gathered}}$$

The theory of continuously distributed dislocations is the natural theoretical frame to start a quantitative description of yield and plastic flow in terms of a <u>non-linear</u> field theory, playing the role of Navier-Stokes eqs. in fluid dynamics.

Crystal dislocations in the physics of continua



$\underset{\sim}{u}^e$ ambiguous

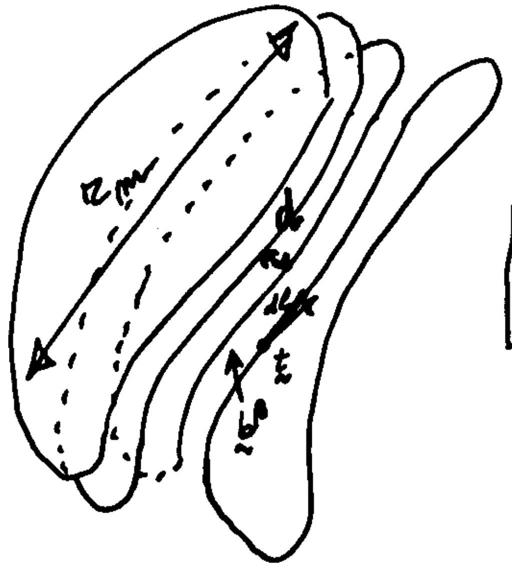
$\mathcal{D}$ singular line

$$\oint_L d\underset{\sim}{u}^e_i = -\underset{\sim}{b}$$

$\underset{\sim}{u}^e$ single valued

$\mathcal{D}$ boundary of a singular surface S

$$\left(\underset{\sim}{u}^{+\Sigma} - \underset{\sim}{u}^{-\Sigma}\right)_{\in S} = \underset{\sim}{b}$$

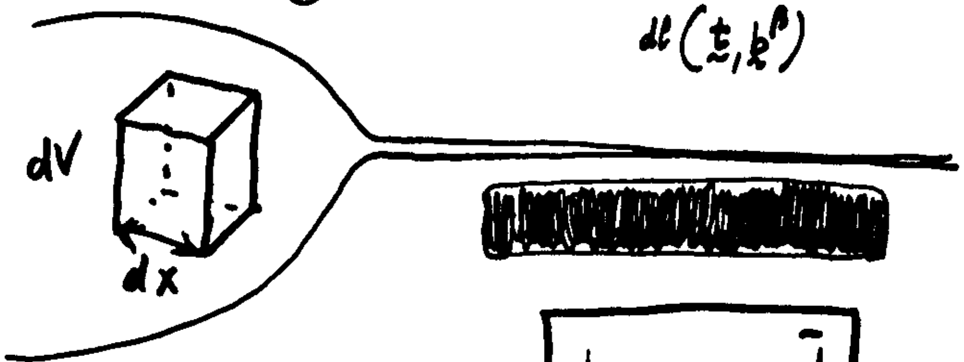


$r_m \gg \bar{d}$

$$\boxed{\bar{d} \ll dx \ll r_m}$$

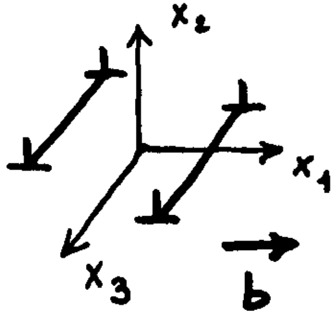
The "segment" case

$dl(\underset{\sim}{t}, \underset{\sim}{b}^{\beta})$

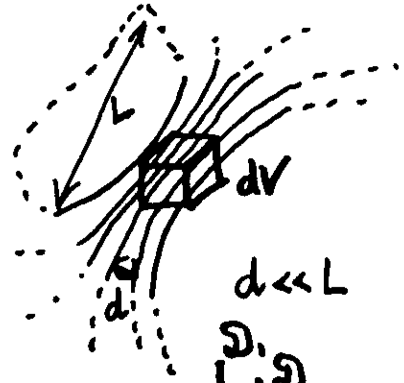


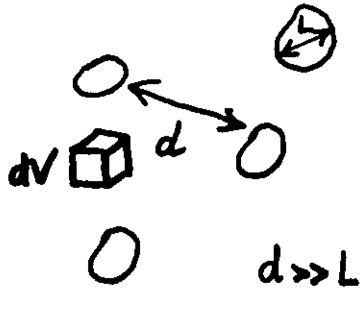
$$\boxed{dx \gg r_m, \bar{d}}$$

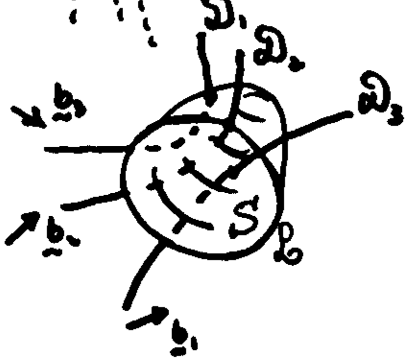
The "loop" case



A "coarse-grained" treatment is necessary_

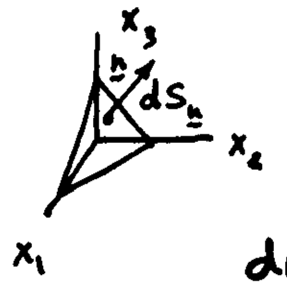






$$b_k^{(S)} = \iint_S \alpha_{ik}\, n_i \, dS$$

$$\frac{db_k}{dS_n} = \alpha_{ik}\, n_i$$



$(n_i \equiv \cos \widehat{n x_i})$

The dislocation density tensor $\underset{\approx}{\alpha}$ _

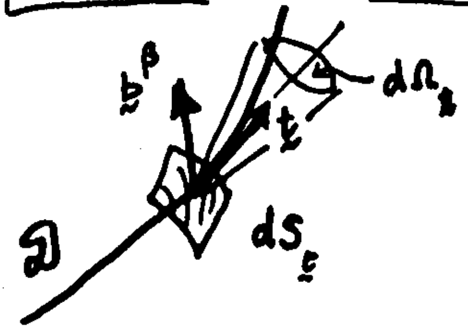
$$\alpha_{31} \quad \begin{cases} \underset{\sim}{n} \equiv \underset{\sim}{u}_3 = -\underset{\sim}{t} \\ \underset{\sim}{b} = b\,\underset{\sim}{u}_1 \end{cases}$$

$$\boxed{\begin{aligned}\varepsilon_{ilm}\frac{\partial w^e_{mk}}{\partial x_l} &= -\alpha_{ik}\\ \epsilon^e_{ik} &= \frac{1}{2}\left(w^e_{ik}+w^e_{ki}\right)\end{aligned}}$$

The material is densely filled by singular lines so that no elastic displacement vector can be defined:

if $w^e_{ik} = \frac{\partial u^e_k}{\partial x_i} \Longrightarrow \alpha_{ik} \equiv 0$!

$$\boxed{\alpha_{ik} = \sum_\beta \int d\Omega_{\mathbf{t}}\; t_i b^\beta_k \rho^\beta(\mathbf{t};\mathbf{x})}$$



$$\frac{dN^\beta(\mathbf{t})}{dS_{\mathbf{x}}} = \rho^\beta(\mathbf{t};\mathbf{x})\,d\Omega$$

$$\boxed{\rho = \sum_\beta \int d\Omega_{\mathbf{t}}\, \rho^\beta(\mathbf{t};\mathbf{x})}$$

N: ρ is not directly connected with the strains !!!

because rot $\mathbf{w}^e = -\boldsymbol{\alpha} \Longrightarrow$ div $\boldsymbol{\alpha} = 0$

$$\boxed{\frac{\partial \alpha_{ik}}{\partial x_i} = 0}$$

$\mathbf{v} \equiv \frac{\partial \mathbf{u}^T}{\partial t}$ velocity of the total geometrical displacement

$$\mathbf{u}^T = \mathbf{x}(\mathbf{X}) - \mathbf{X}$$

If dislocations are in motion

$$\frac{\partial v_i}{\partial x_k} = \frac{\partial w^e_{ki}}{\partial t} + \frac{\partial w^p_{ki}}{\partial t}$$

$$\boxed{\begin{aligned}J_{ik} &\equiv -\frac{\partial w^p_{ki}}{\partial t}\\ \dot{\varepsilon}^p_{ik} &= -\frac{1}{2}\left(J_{ik}+J_{ki}\right)\end{aligned}}$$

Dislocation flux density tensor $\mathbf{J}$
(neglecting disl.-disl. reactions)

$$\boxed{J_{ik} = \varepsilon_{ilm}\sum_\beta \int d\Omega_{\mathbf{t}}\; t_l b^\beta_k V^\beta_m(\mathbf{t};\mathbf{x},t)\,\rho^\beta(\mathbf{t};\mathbf{x},t)}$$

$\updownarrow$

$$\dot{\gamma}^p = \overline{m}\,\rho b v$$

Orowan equation

$$\boxed{\frac{\partial \alpha_{ik}}{\partial t} + \varepsilon_{ilm} \frac{\partial J_{mk}}{\partial x_l} = 0}$$

$$\frac{\partial \underset{\approx}{\alpha}}{\partial t} + \mathrm{rot}\, \underset{\approx}{J} = 0$$

"Conservation of Burgers Vector"

$$\varepsilon_{ipq}\, \varepsilon_{qlm} = \delta_{il}\delta_{pm} - \delta_{im}\delta_{pl}$$

$$\sum_\beta \int d\Omega_{\underset{\sim}{t}} \Big\{ \underbrace{t_i b_k^\beta \frac{\partial \rho^\beta}{\partial t} + t_i b_k^\beta \,\mathrm{div}(\rho^\beta \underset{\sim}{V}^\beta)} - \underbrace{b_k^\beta\, \underset{\sim}{t} \cdot \underset{\sim}{\mathrm{grad}}\, (\rho^\beta V_i^\beta)}_{\text{"peculiar" flux term}} \Big\} = 0$$

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \underset{\sim}{V}) = 0$$

The above equations can be generalize to ~~allow~~ for dislocation reactions and to find a precise relation between dislocation sources and plastic strains.

Kosevich's theory can be easily improved taking into account dislocation production and an exact relationship between dislocation sources and plastic strains (C.E. Bottani, proceedings of ~~1986~~ Varenna school on ~~un~~ ~~linear~~ equ. phase transitions)

(XCIX)

$$\begin{cases} \dfrac{\partial v_i}{\partial x_k} = \dfrac{\partial w_{ik}^{el}}{\partial t} - J_{ik} - S_{ik} & [a] \\[2mm] P_{ik} = \sum_\beta \int d\Omega_{\underset{\sim}{t}}\, P^\beta(\underset{\sim}{t}; \underset{\sim}{x}, t)\, t_i b_k^\beta \\[2mm] P^\beta = \left(\dfrac{dW^\beta}{dt}\right)_{S=1} \end{cases}$$

$$-P_{ik} = \epsilon_{ilm} \frac{\partial S_{mk}}{\partial x_l} \leftrightarrow -\underset{\approx}{P} = \mathrm{rot}\, \underset{\approx}{S}$$

take the curl of [a] and use

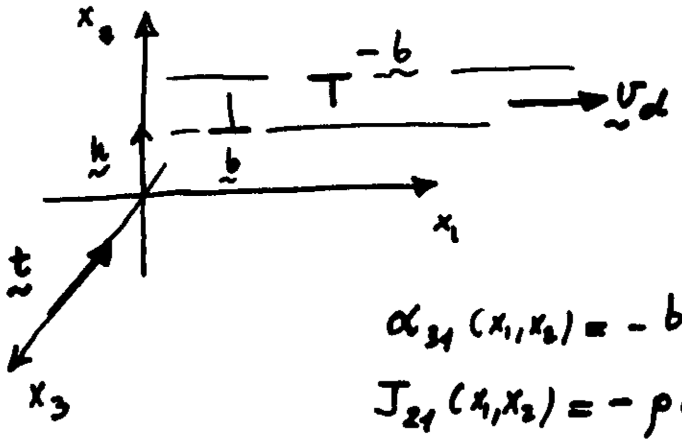
$$\mathrm{rot}\, \dot{\underset{\approx}{w}}^{(el)} = -\dot{\underset{\approx}{\alpha}}$$

$$\boxed{\frac{\partial \underset{\approx}{\alpha}}{\partial t} + \mathrm{rot}\, \underset{\approx}{J} = \underset{\approx}{P}}$$

Burgers vector Balance

$$\boxed{\underset{\approx}{P} = -\mathrm{rot}\, \underset{\approx}{S}} \rightarrow \boxed{\mathrm{DIV}\, \underset{\approx}{P} = 0}$$ N.b.

Plane strain in an isotropic solid



$\alpha_{31}(x_1,x_2) = -b\rho(x_1,x_2)$

$J_{21}(x_1,x_2) = -\rho b v_d(x_1,x_2)$

$P_{31}(x_1,x_2) = P_3(x_1,x_2,\tau,T_0)$

$$\frac{\partial \alpha_{31}}{\partial t} + \frac{\partial j_{21}}{\partial x_1} = P_{31} \qquad \dot{\varepsilon}^p_{ij} = \mathrm{Sym}(\dot{w}^p_{ij})$$

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_1}(\rho v_d) = P_3}$$

$$\frac{\partial w_{21}^{pl}}{\partial t} = -j_{21} + S_{21} = \rho b v_d + S_{21}$$

$$\frac{\partial w_{11}^{pl}}{\partial t} = S_{11}$$

Orowan

$$\frac{\partial S_{21}}{\partial x_1} - \frac{\partial S_{11}}{\partial x_2} = P_{31} \qquad \dot{\varepsilon}_{21}^{pl} = \rho b v_d$$

$$\boxed{\frac{\partial^2 w_{21}^{pl}}{\partial t\,\partial x_1} - \frac{\partial^2 w_{11}^{pl}}{\partial t\,\partial x_2} = P_{31} + \frac{\partial}{\partial x_1}(\rho b v_d)}$$

$$\boxed{\sigma_{ik} = 2\mu w_{ik}^e + \lambda \delta_{ik} w_{kk}^e}$$

Hooke law

$$\boxed{\rho_m \frac{dv_i}{dt} = \frac{\partial \sigma_{ik}}{\partial x_k}}$$

Linear momentum balance

We have a closed system of equations provided α and J are known!

Equation of motion of continuously distributed dislocations described by $\rho^\beta(t;x,t)$.

$$\varepsilon_{ilm} t_l b_k^\beta \sigma_{mk} - B^\beta(t) V_i^\beta(t;x,t) - F_i^\beta(t) = 0$$

$$\sigma_{mk} = \sigma^e_{mk} + \sigma^{dd}_{mk}\{\alpha, J\} + O\left(\left(\frac{V_d}{V_s}\right)^2\right)$$

dislocation inertia is negligible

$\rho(x)\delta(y) = \alpha_{31}$

$$\sigma^{dd}(x) = \frac{-\mu b}{\pi(1-\nu)} \int \frac{\rho(x')\,dx'}{x'-x}$$

Basic Equations

$$\begin{cases} \text{rot}\, \hat{W}^{el} = -\hat{\alpha} \\ \hat{\sigma} = \hat{\hat{C}}\, \hat{W}^{el} \quad \text{(Hooke law)} \\ \rho_m \frac{d\underset{\sim}{v}}{dt} = \text{Div}\, \hat{\sigma} \\ \text{grad}\, \underset{\sim}{v} = \frac{\partial \hat{W}^{el}}{\partial t} + \frac{\partial \hat{W}^{pl}}{\partial t} \end{cases}$$

$$\begin{cases} \text{Div}\, \hat{\alpha} = 0 \\ \frac{\partial \hat{\alpha}}{\partial t} - \text{rot}\left(\frac{\partial W^{pl}}{\partial t}\right) = 0 \end{cases} \quad \text{compatibility conditions}$$

$$\frac{\partial \hat{W}^{pl}}{\partial t} = -\underbrace{\hat{J}}_{\text{flux strain rates}} - \underbrace{\hat{S}}_{\text{sources and reactions strain rates}} = -\hat{R}$$

$$\text{rot}\, \hat{S} = -\underbrace{\hat{\beta}}_{\text{net dislocation }(\beta)\text{ production rates}} \quad ; \quad \text{Div}\, \hat{\beta} = 0$$

$$\text{rot}\left[\hat{W}^{pl}(\underset{\sim}{r},t) + \hat{W}^{pl}(\underset{\sim}{r},0)\right] = \hat{\alpha}(\underset{\sim}{r},t) + \hat{\alpha}(\underset{\sim}{r},0)$$

$\hat{W}^{pl}$ is not a state function

$$\frac{dv_i}{dt} = \frac{\partial v_i}{\partial t} + v_k \frac{\partial v_i}{\partial x_k} \approx \frac{\partial v_i}{\partial t} - v_k R_{ki}$$

Linear theory

$$\frac{\partial \sigma_{ki}}{\partial x_k} = \rho_m \frac{\partial v_i}{\partial t} - \rho_m v_k \left(j_{ki} + S_{ki}\right)$$

$$\underbrace{j \propto V_d}_{}$$

with these terms no electromagnetic analogy is possible

Small deformation, isotropic theory

$$\begin{cases} \rho_m \frac{\partial v_i}{\partial t} = \mu \nabla_k \left(w_{ik}^{el} + w_{ki}^{el}\right) + \lambda \nabla_i w_{kk}^{el} \\ \epsilon_{ilm} \nabla_l w_{mk}^{el} = -\alpha_{ik} \\ \nabla_i v_k = \frac{\partial w_{ik}^{el}}{\partial t} - R_{ik} \end{cases}$$

for the moment

$R_{ik} = j_{ik}$

flux only

The problem can be solved by generalized potentials, $\underset{\sim}{\varphi}$, $\hat{A}$, $\hat{B}$

$$\begin{cases} w^{d}_{ik} = \nabla_i(\varphi_k + \chi_k) - \epsilon_{ilm}\nabla_l A_{mk} + \rho_m \dfrac{\partial B_{ik}}{\partial t} \\ v_i = \dfrac{\partial}{\partial t}(\varphi_i + \chi_i) + \mu \nabla_k B_{ki} \quad ; \; \chi_i \equiv \epsilon_{ikl} A_{kl} \end{cases}$$

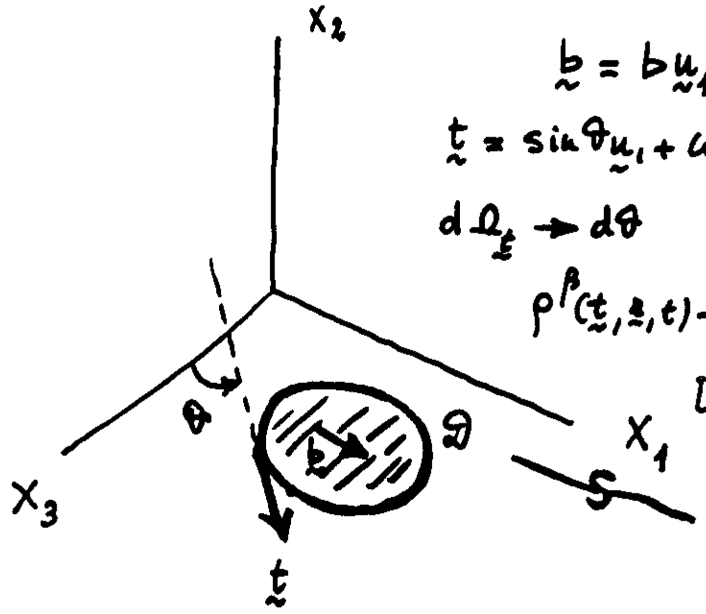
$$\begin{cases} \rho_m \dfrac{\partial^2 \underset{\sim}{\varphi}}{\partial t^2} - \mu \nabla^2 \underset{\sim}{\varphi} - (\mu+\lambda) \underset{\sim}{\nabla}\,\underset{\sim}{\nabla}\cdot\underset{\sim}{\varphi} = -2\mu \underset{\sim}{D} \\ \qquad D_i = \epsilon_{ikl}\alpha_{kl} \\ \qquad (\underset{\sim}{0} \text{ for screw dislocations}) \\ \nabla^2 \hat{A} - c_t^{-2} \dfrac{\partial^2 \hat{A}}{\partial t^2} = -\hat{\alpha}(\underset{\sim}{x},t) \\ \nabla^2 \hat{B} - c_t^{-2} \dfrac{\partial^2 \hat{B}}{\partial t^2} = -\dfrac{\hat{J}(\underset{\sim}{x},t)}{\mu} \end{cases} \qquad \left(c_t^2 \equiv \frac{\mu}{\rho}\right)$$

shear sound speed

with
$$\begin{cases} \mathrm{Rot}\, \hat{B} + \dfrac{1}{\mu}\dfrac{\partial \hat{A}}{\partial t} = 0 \\ \mathrm{Div}\, \hat{A} = 0 \end{cases}$$

$$\begin{cases} \hat{A}(\underset{\sim}{x},t) = \dfrac{1}{4\pi}\displaystyle\int \dfrac{dV'}{R}\hat{\alpha}\left(\underset{\sim}{x}', t - \dfrac{R}{c_t}\right) \\ R = |\underset{\sim}{x} - \underset{\sim}{x}'| \end{cases}$$

A special case : planar loops



$\underset{\sim}{b} = b\,\underset{\sim}{u}_1$

$\underset{\sim}{t} = \sin\vartheta\,\underset{\sim}{u}_1 + \cos\vartheta\,\underset{\sim}{u}_3$

$d\Omega_t \to d\vartheta$

$\rho^{\beta}(\underset{\sim}{t},\underset{\sim}{x},t) \to \rho(\vartheta,\underset{\sim}{x},t)$

$[\rho(\vartheta+\pi) = \rho(\vartheta,$

$$\alpha_{11} = \int_0^{\pi} d\vartheta \sin\vartheta\, b\,\rho(\vartheta,x,t)$$

$$\alpha_{31} = \int_0^{\pi} d\vartheta \cos\vartheta\, b\,\rho(\vartheta,\underset{\sim}{x},t)$$

The simplest case is when $\rho(\vartheta,\underset{\sim}{x},t) = \rho(x,t)\frac{1}{2} \Rightarrow \alpha_{31} \equiv 0$

the Screw case : $\underset{\sim}{t} \equiv \underset{\sim}{u}_1$

$$\begin{cases} \alpha_{11} = b\rho(x_2,x_3,t) \\ j_{21} = -b\rho\overline{V}_3 \\ j_{31} = -b\rho\overline{V}_2 \end{cases}$$

straight screw dislocation

$$\frac{\partial W_{31}^{el}}{\partial x_2} - \frac{\partial W_{21}^{el}}{\partial x_3} = -\alpha_{11}$$

$$\sigma_{31} = \mu W_{31}^{el}$$

$$\sigma_{21} = \mu W_{21}^{el}$$

$$\frac{\partial \sigma_{21}}{\partial x_2} + \frac{\partial \sigma_{31}}{\partial x_3} = \rho_m \frac{\partial v_1}{\partial t}$$

$$\frac{\partial W_{21}^{el}}{\partial t} - J_{21} - S_{21} = \frac{\partial v_1}{\partial x_2}$$

$$\frac{\partial W_{31}^{el}}{\partial t} - J_{31} - S_{31} = \frac{\partial v_1}{\partial x_3}$$

$$\frac{\partial S_{31}}{\partial x_2} - \frac{\partial S_{21}}{\partial x_3} = -\beta_{11} \qquad \beta_{11} = b(\dot{\rho})_s$$

$$\boxed{\frac{\partial \rho}{\partial t} = -\operatorname{div}(\rho \overline{\underset{\sim}{V}}) + (\dot{\rho})_s}$$

$$\boxed{\frac{\partial^2 v_1}{\partial t^2} - c_t^2 \nabla^2 v_1 = c_t^2 \{(\operatorname{div} \hat{S})_1 + (\operatorname{div} \vec{J})_1\}}$$

$$W_{21}^{el} = -\frac{\partial \varphi}{\partial x_3} + \frac{\partial B_{21}}{\partial t} \rho_m \qquad \begin{cases} \varphi \\ \underset{\sim}{B} = \begin{pmatrix} B_{21} \\ B_{31} \end{pmatrix} \end{cases}$$

$$W_{31}^{el} = +\frac{\partial \varphi}{\partial x_2} + \frac{\partial B_{31}}{\partial t} \rho_m$$

$$v_1 = \mu \operatorname{div} \underset{\sim}{B} \qquad (\operatorname{rot} \underset{\sim}{B})_1 = -\frac{1}{\mu} \frac{\partial \varphi}{\partial t}$$

$$\begin{cases} \nabla^2 \varphi - c_t^{-2} \dfrac{\partial^2 \varphi}{\partial t^2} = -\alpha_{11} \\ \nabla^2 B_{i1} - c_t^{-2} \dfrac{\partial^2 B_{i1}}{\partial t^2} = -\dfrac{1}{\mu} (J_{i1} + S_{i1}) \quad (i = 1, 2) \end{cases}$$

$$\text{if} \quad \overline{V} \ll c_t$$

$$\varphi(x_2, x_3, t) \approx \frac{1}{4\pi} \int \alpha_{11}(\underset{\sim}{r}') \frac{dV'}{R} +$$

$$+ \frac{1}{4\pi} \left\{ \frac{1}{2 c_t^2} \frac{\partial^2}{\partial t^2} \int \alpha_{11}(\underset{\sim}{r}') R \, dV' \right\} + \cdots$$

$$B_{i1} = \frac{1}{4\pi\mu} \int J_{i1}(\underset{\sim}{r}') \frac{dV'}{R} + \left(\text{analog with } \underset{\sim}{S} \right)$$

Lagrangian density

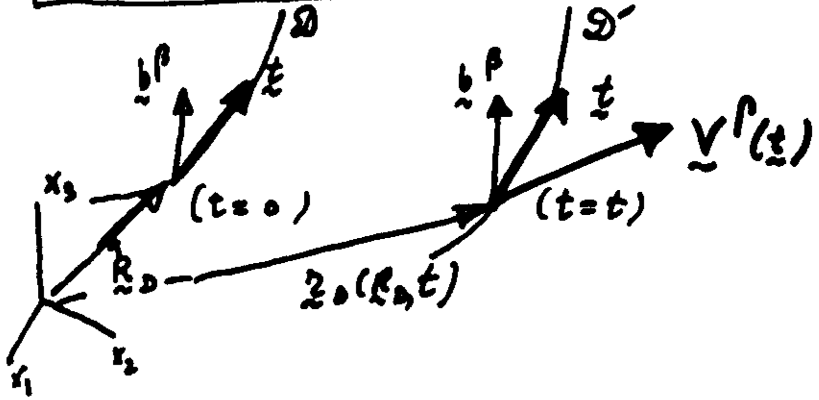
$$\boxed{\mathcal{L} = \frac{1}{2}\left(\sigma_{ik}\epsilon_{ik} - \rho_m v^2\right) - \alpha^0_{ik}\varphi_{ik} + J^0_{ik}\psi_{ik}}$$

$\left(\underset{\approx}{\alpha}^0 \Rightarrow \cancel{\mathrm{Rot}}\ \underset{\approx}{\beta} = \underset{\approx}{\alpha}^0 ,\ \underset{\approx}{J}^0 = \mathrm{sym}\ \underset{\approx}{J} \right)$

$$\begin{cases} \underset{\approx}{\sigma} = \mathrm{curl}\ \underset{\approx}{\varphi} + \dfrac{\partial \underset{\approx}{\psi}}{\partial t} \\ \underset{\sim}{v} = \dfrac{1}{\rho_m}\, \mathrm{div}\ \underset{\approx}{\psi} \end{cases}$$

(for a single segment)

$$\boxed{\epsilon_{ikl}\, t_k\, \sigma_{lp}\left(r_0(R_0,t)\right) b^\beta_p + \rho_m \cancel{(\underset{\sim}{v}\cdot \underset{\sim}{b}^\beta)(\underset{\sim}{t}\times \underset{\sim}{V})_i} = 0}$$



if

$$\bar{h} \ll dx \ll \lambda_{ef} \ll \bar{r}_{exp}$$

Equation of motion of continuously distributed dislocations (in the "segment" case)

elastic | inelastic

$$\boxed{m^*_{ik}(\lambda)\, a_k = \epsilon_{ilm}\, t_l\, b^\beta_k\, \sigma^T_{mk}} \boxed{- B^\beta(\underset{\sim}{t})\, V_i\, f(V_i)}$$

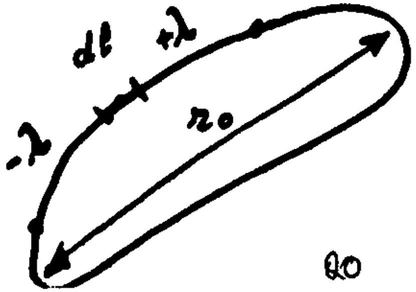
$$\sigma^T_{mk} = \sigma^e_{mk} + \sigma^0_{mk} + \sigma^1_{mk} + \cdots\ O\left(\frac{V^2}{V_s^2}\right)$$

$$m^*_{ik}(\lambda) = \frac{\rho_m b^2}{4\pi}(\delta_{ik} - t_i t_k)(1+\gamma^4 \sin^2\theta)\ln\left(\frac{\lambda}{r_0}\right)$$

$$\sigma^0_{ik} = \frac{\mu}{4\pi}\int \gamma_{iklm}(\underset{\sim}{n})\,\alpha_{lm}(\underset{\sim}{r}',t)\,\frac{dV'}{R^2}$$ quasi-static

$\left(\underset{\sim}{R} \equiv \underset{\sim}{r}' - \underset{\sim}{r} \ ,\ \underset{\sim}{n} \equiv \dfrac{\underset{\sim}{R}}{|\underset{\sim}{R}|} \right)$

$$\sigma^1_{mk} = \frac{\rho_m}{4\pi}\int \beta_{iklm}(\underset{\sim}{n})\,\frac{\partial J_{lm}}{\partial t}\,\frac{dV'}{R}$$



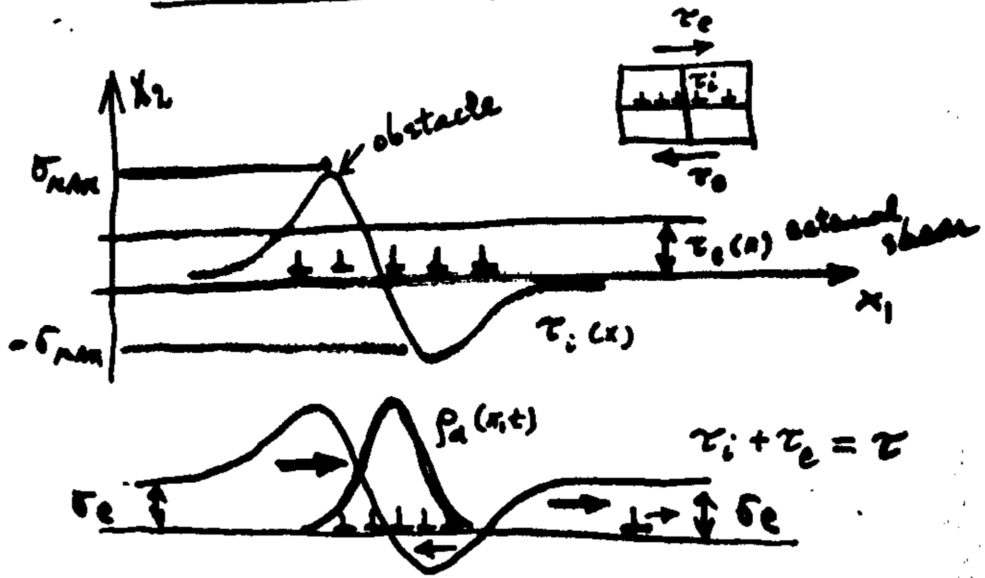
$$\boxed{E_{self} \simeq m^* v_s^2}$$

Confinement - deconfinement transitions in a model of interacting dislocations.

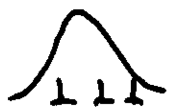
(V. Balakrishnan, C.E. Bottani, Phys. Rev. B, ~~March~~ 33, 7 1986)

$$\underset{\sim}{\alpha} = \begin{cases} \alpha_{31}(x_1,t)\,\delta(x_2) \\ \alpha_{33}(x_1,t)\,\delta(x_2) \end{cases}$$

Interaction of a system of dislocations with a finite internal barrier and an external shear.



→ force on a positive dislocation



trapped dislocations

due to repulsion → $\boxed{\sigma_{critical} < |\sigma_{max}|}$

Problems

1) can a compact distribution of dislocations be sustained by a stress barrier that is finite everywhere?

2) what is the effect of an applied shear on such a distribution?

3) when do bifurcations occur between different kinds of configurations?

4) are the found configurations stable?

Model

(isotropic elastic medium)

$$\begin{cases} \cancel{m^* \dfrac{\partial v}{\partial t}} + Bv = b(\tau_e + \tau_i(x)) - \dfrac{\mu b^2}{2\pi(1-\nu)} \displaystyle\int_L \dfrac{\rho(x',t)}{x'-x}\,dx' \\ \dfrac{\partial \rho}{\partial t} + \dfrac{\partial}{\partial x}(\rho v) = \mathcal{P}[\rho, \tau_e, \tau_0] \end{cases} \qquad (L = [a_1, a_2])$$

if m^* is negligible the problem is much easier. If $\mathcal{P}$ is zero it is solvable analitically for some specific forms of $\tau_i(x)$.

Time independent states

no flow : $v \equiv 0$

$$\boxed{S(x) = \frac{1}{\pi}\int_{a_1}^{a_2} dx' \,(x'-x)^{-1} n(x')}$$

$$S(x) = \sigma - \frac{x}{x^2+\epsilon^2}$$

$N_+, N_- > 0$

$$\begin{cases} \displaystyle\int_{a_1}^{a_2} dx\; n(x) = 2\pi\,(N_+ - N_-) \equiv 2\pi N \\ \displaystyle\int_{a_1}^{a_2} dx\, \frac{S(x)}{\sqrt{(a_2-x)(x-a_1)}} = 0 \end{cases}$$

$\lim_{\rho\to\infty} \bar{s}(\rho) \propto \frac{1}{\rho}$

Muskhelishvili's theorem

$$n(x) = \frac{1}{\pi}\sqrt{(a_2-x)(x-a_1)} \int_{a_1}^{a_2} \frac{S(x')\,dx'}{(x-x')\sqrt{(a_2-x')(x'-a_1)}}$$

Define
$$\begin{cases} \vartheta_1 = \pi - \tan^{-1}\left(\frac{a_1}{\epsilon}\right) \\ \vartheta_2 = \tan^{-1}\left(\frac{a_2}{\epsilon}\right) \\ \varphi = \frac{1}{2}(\vartheta_1 - \vartheta_2) \end{cases}$$

$$n(x) = 2\left[\frac{(a_2-x)(x-a_1)}{(a_1^2+\epsilon^2)^{\frac{1}{2}}(a_2^2+\epsilon^2)^{\frac{1}{2}}}\right]^{\frac{1}{2}}\left(\frac{\epsilon\cos\varphi - x\sin\varphi}{x^2+\epsilon^2}\right)$$

over $L \equiv (a_1 \le x \le a_2)$

$n(x) \equiv 0$ outside

$$\begin{cases} (\sin\vartheta_1 \cdot \sin\vartheta_2)^{1/2} \sin\varphi = \epsilon\sigma \\ 1 - (\sin\vartheta_1 \sin\vartheta_2)^{1/2}\cos\varphi - \frac{1}{2}\sigma(a_1+a_2) = N \end{cases}$$

a) if $N = 0$ only one solution

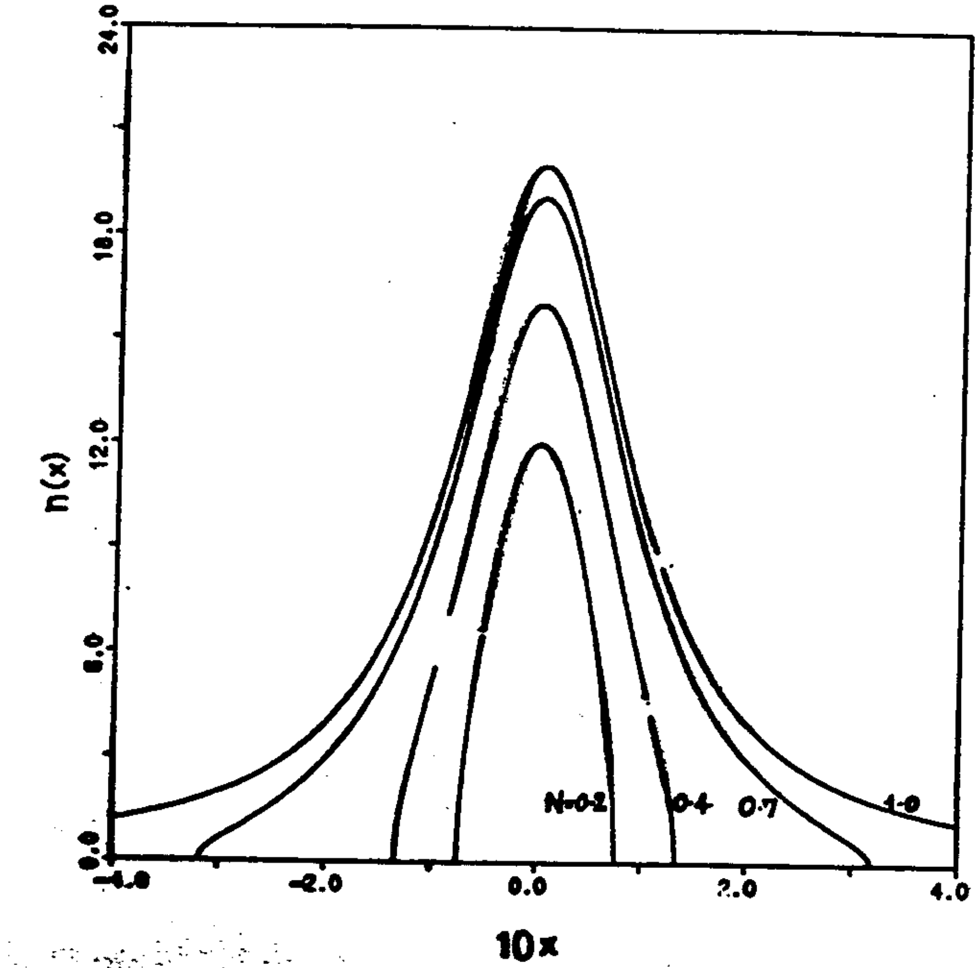
b) if $N > 0$ $\nearrow$ $N^+, N^- = 0$; $\searrow$ $N^+, N^- \neq 0$ but $N^+ > N^-$

a) recombination $\sigma_{cr} = \sigma_{MAX}$

b) $\nearrow$ b.1 breakaway at $\sigma_{cr} = \bar{\sigma}(N,\epsilon) < \sigma_{MAX}$

$\searrow$ b.2 recombination and then breakaway

Fig. 1



No external shear

$\sigma = 0$
The effect of increasing N.
When $N \to 1$
$q_1 q_2 \to \pm\infty$

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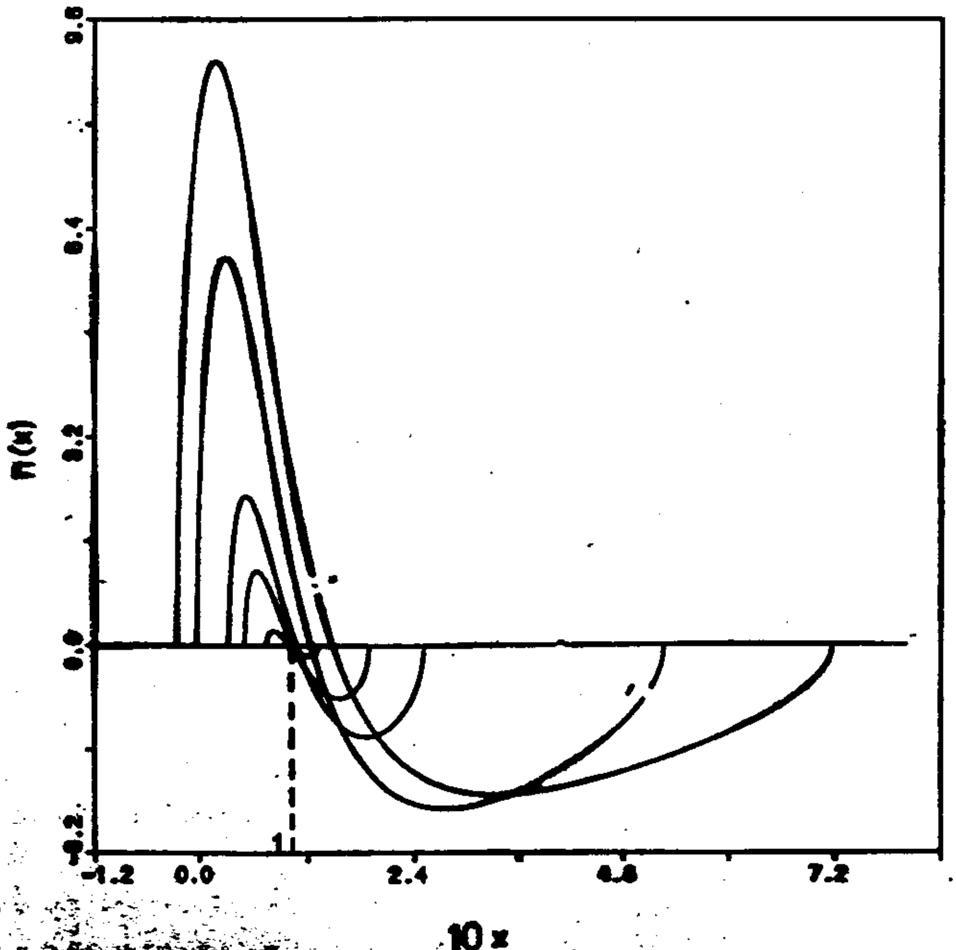
Fig. 2



$N^+ = N^- \to N = 0$
The effect of increasing σ

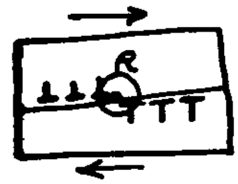


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Fig. 3

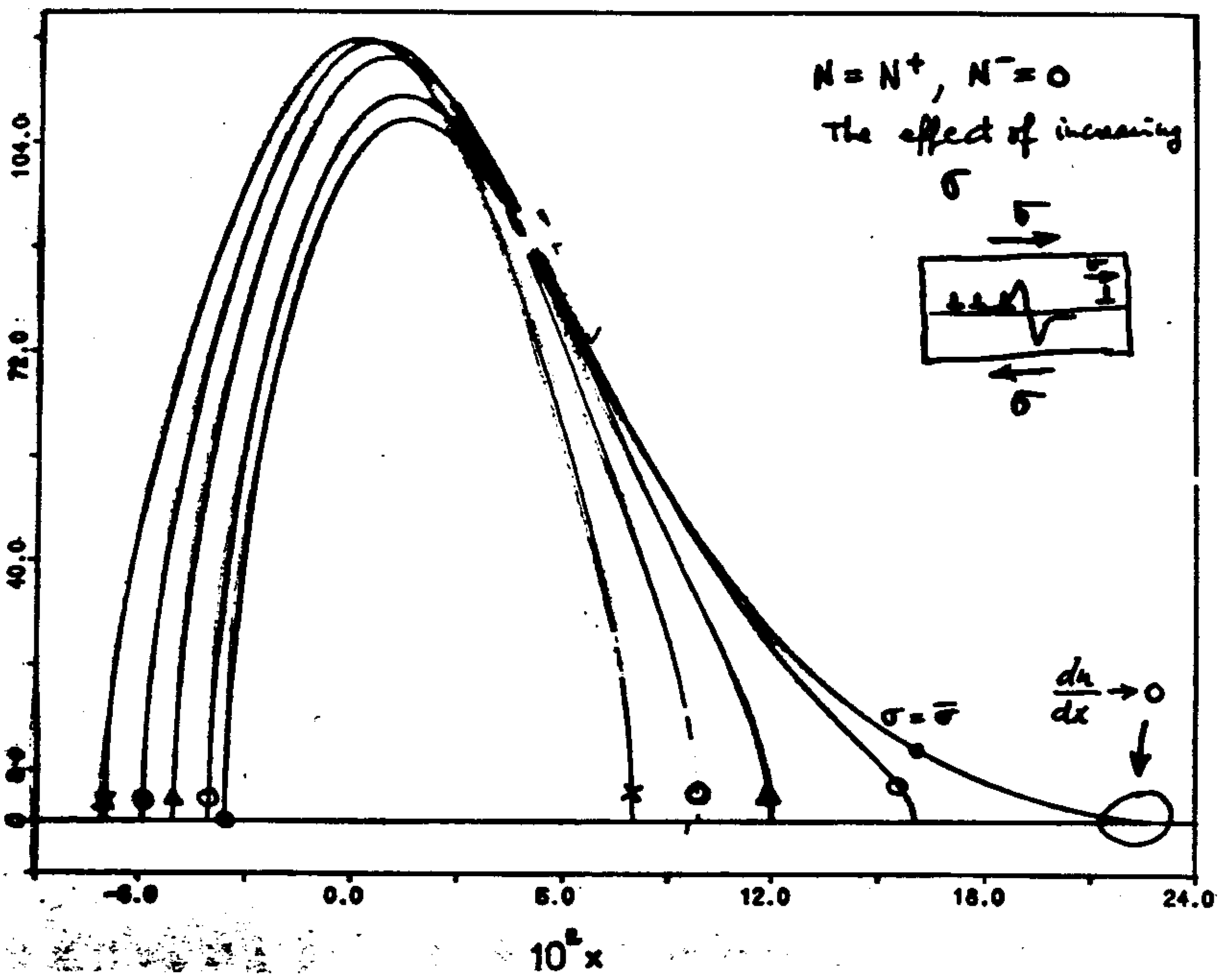
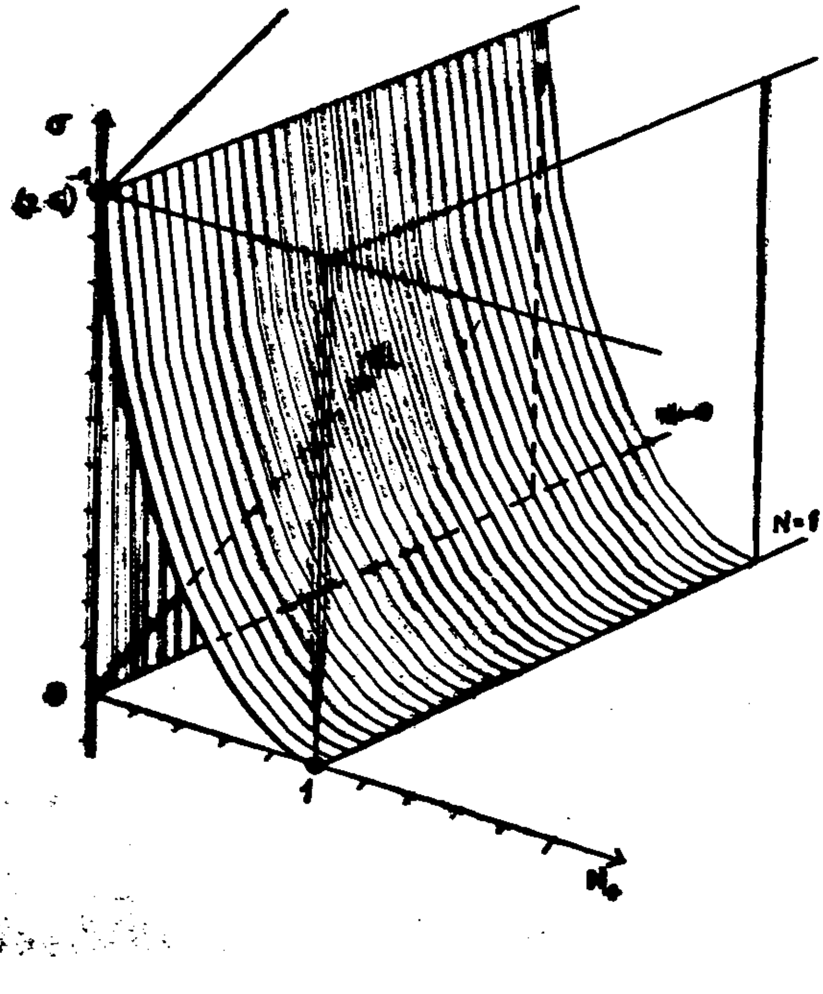


Fig. 4

The "phase" diagram



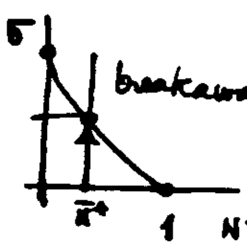


fig. 9

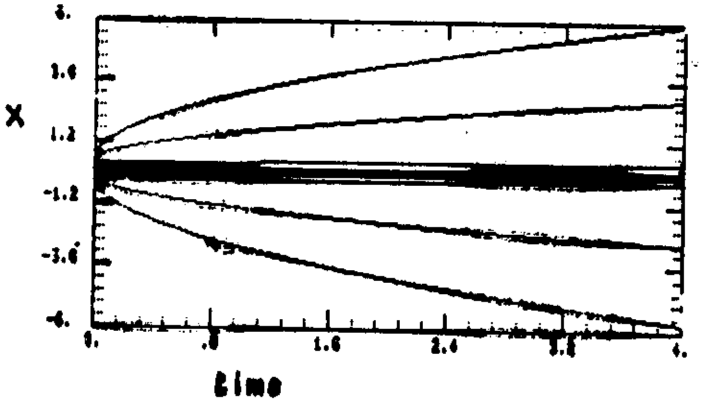
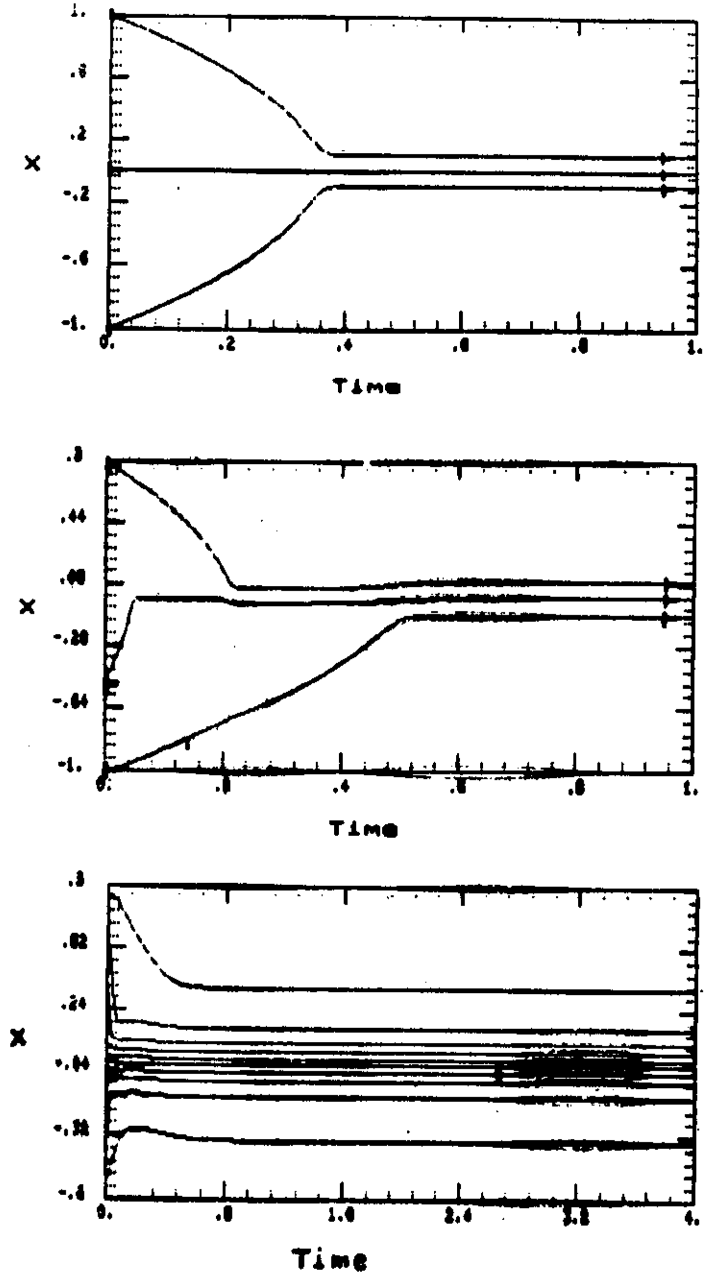


fig. 8

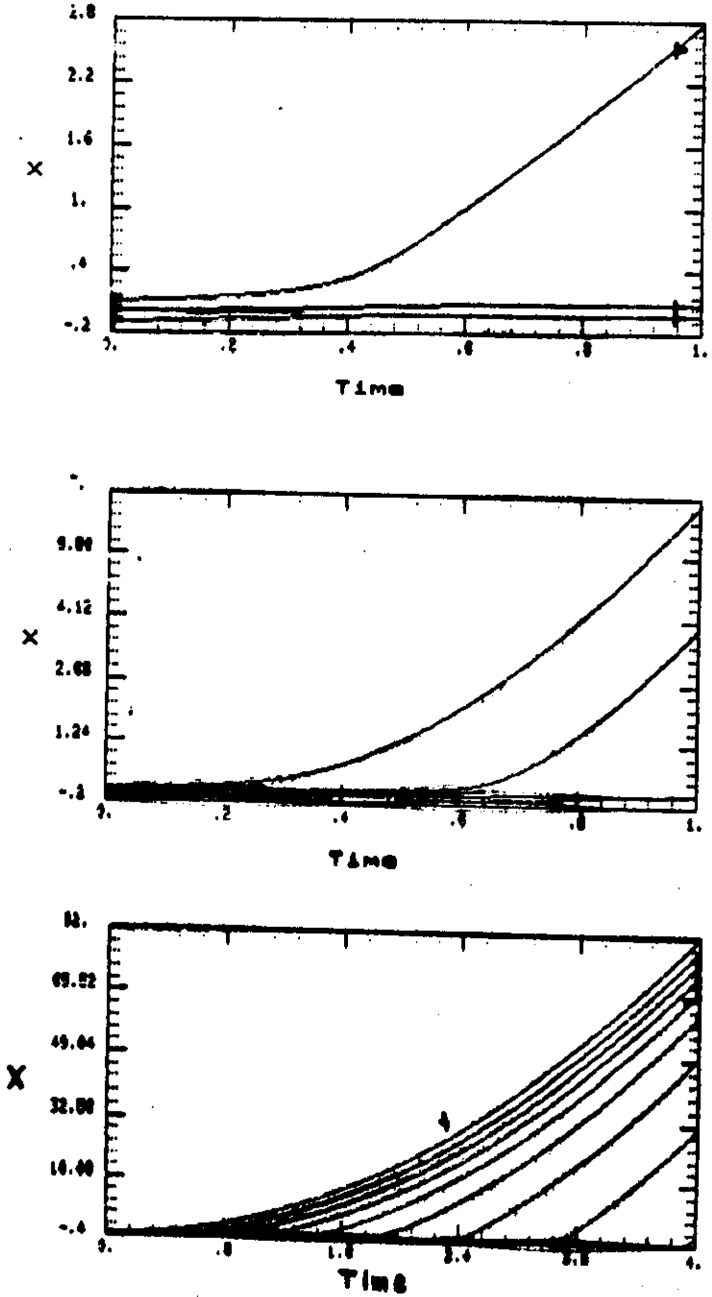


(a)

(b)

(c)

fig. 10



(a)

(b)

(c)

fig. 11