INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) - P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONE: 2240-1
CABLE: CENTRATOM - TELEX 460392-I

SMR/208 - 45

SPRING COLLEGE IN MATERIALS SCIENCE

ON

"METALLIC MATERIALS"

(11 May - 19 June 1987)

FRACTALS AND FRACTURE OF METALS

C.W. LUNG
Institute of Metal Research
Academia Sinica
Wenhua Road
Shenyang
People's Republic of China

FRACTALS AND FRACTURE OF METALS

1. ELEMENTARY INTRODUCTION TO FRACTALS
2. EXAMPLES OF FRACTALS
   (1) CANTOR DUSTS
   (2) KOCH CURVE
   (3) SIERPINSKI CARPET AND SPONGE
3. APPLICATIONS TO FRACTURE OF METALS
   (1) LENGTH-AREA-VOLUME RELATIONS: FRACTAL ANALYSIS IN FRACTOGRAPHY.
   (2) THE TRUE AREA OF A FRACTURE SURFACE: FRACTAL EFFECTS OF THE FRACTURE OF METALS
   (3) PERCOLATION THEORY: FRACTAL MODEL OF FRACTURE

References:

1. B. B. Mandelbrot, The Fractal Geometry of Nature, (1982).

2. L. PIETRONERO and E. TOSATTI, Fractals in Physics, 1986, North-Holland.

* * *

1. B. B. Mandelbrot et al., Nature, vol. 308, 19 (1984) 721.

2. C. W. Lung, in [2], p. 1[illegible]1.

3. P. Ray et al. J. Phys. C: Solid State Phys. **18**, L185. (1985)

4. K. Sieradzki, J. Phys. C: Solid State Phys. **18**, L855 (1985)

Many patterns of Nature are so irregular and fragmented, compared with Euclid — [illegible] to denote all of standard geometry.

* * *

It (fractal) describes many of the irregular and fragmented patterns around us, and leads to full-fledged theories, by identifying a family of shapes I call "fractals".

— B.B. Mandelbrot (1983).

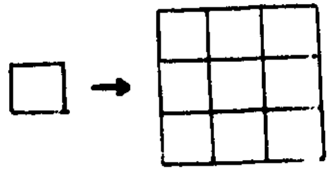
order — irregular — disorder

FRACTALS

Hausdorff dimension

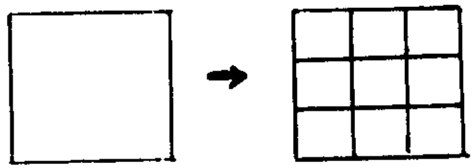
"Growing" analysis :

$3^2 = 9$



$\boxed{D = \lg K / \lg l}$ $\quad (l^D = K)$

"Shrinking" analysis :



$9 \cdot (\frac{1}{3})^2 = 1$

$N \cdot (r)^D = 1$

$$\boxed{D = \lg N / \lg (\tfrac{1}{r})}$$

FRACTAL: a set for which the Hausdorff dimension strictly exceeds the topological dimension

$$\boxed{D > D_T}$$

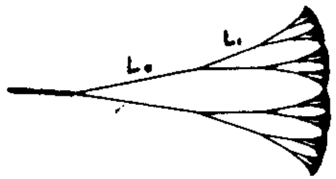
self-similarity: $r > 0, \quad r(x) = (rx_1, \cdots rx_i, \cdots rx_E)$,

self-affinity: $r = (r_1, \cdots r_i, \cdots r_E), \quad r(x) = (r_1x_1, \cdots r_ix_i, \cdots r_Ex_E)$.

Scaling : invariant under certain transformation of scale.

FRACTAL UMBRELLA TREES

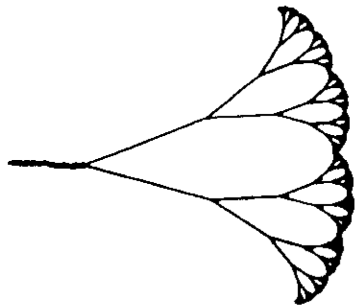
THE TREES HAVE THE SAME ANGLE θ BETWEEN THE BRANCHES THROUGHOUT
D RANGES FROM 1 TO 2, AND FOR EACH D, θ TAKES THE SMALLEST VALUE
THAT IS COMPATIBLE WITH SELF-AVOIDANCE [illegible]



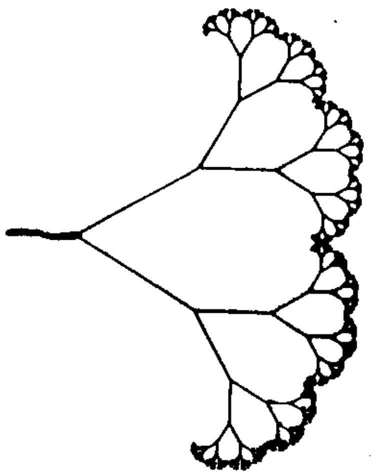
D_1

$N = 2$

$r_1 = L_1 / L_0$



D_2

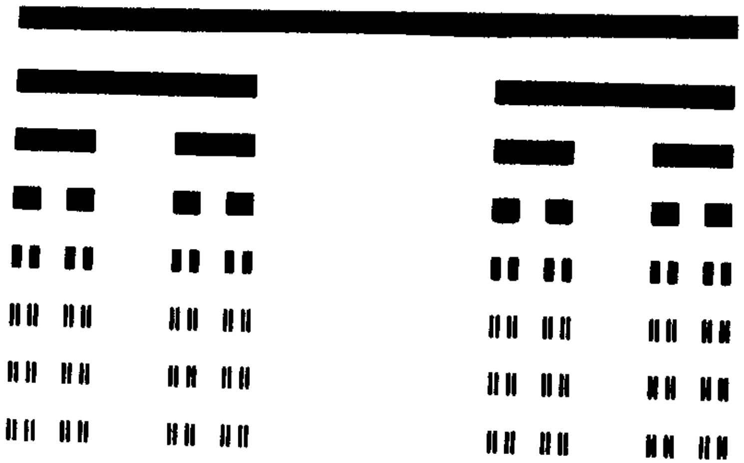


D_3

$1 < D_1 < D_2 < D_3 < 2$ ($\frac{1}{2} < r_1 < r_2 < r_3 <$ [illegible])

$\theta_1 < \theta_2 < \theta_3$

CANTOR DUSTS



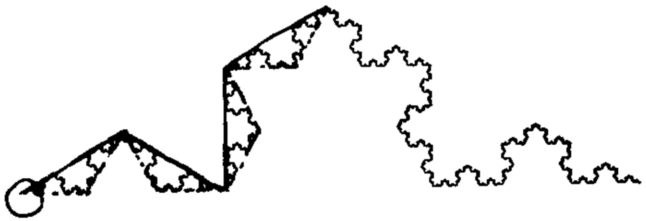
$D = \log 2 / \log 3 = 0.6309$

$(N = 2, \quad r = \frac{1}{3})$

$D_T = 0$

KOCH CURVES

HOW LONG IS THE COAST?



$L(\epsilon) = \epsilon^{1-D}$ $\qquad$ ϵ : yarstick

$N = 2$

$r = 1/\sqrt{3}$

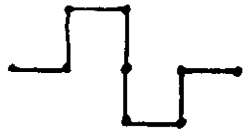
$$D = \log 2/\log\sqrt{3} \sim \log 4/\log 3$$

$180° > \theta > 60°$

θ_{crit}: the critical angle such that the "coastline" is self-avoiding (self-intersection)

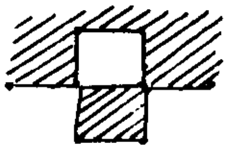
$\sim 60°$

KOCH curve



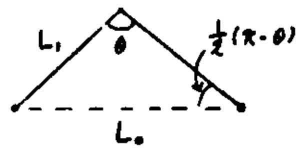
$N = 8$, $r = \frac{1}{4}$

$D = \frac{3}{2}$



$N = 9$, $r = \frac{1}{3}$

$D = 2$



$N = 2$

$r = L_1/L_0 = \dfrac{\sin\frac{1}{2}(\pi-\theta)}{\sin\theta}$

$$d = \frac{\lg 2}{\lg\sin\theta - \lg\sin\frac{1}{2}(\pi-\theta)}$$

$\theta = 120°$ $\qquad$ $d = \dfrac{\lg 4}{\lg 3} \approx 1.2618$

$\theta = 90°$ $\qquad$ $d = 2$

The length of Koch curve,

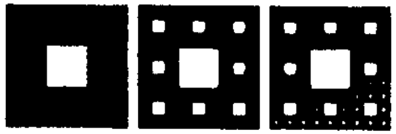
$L(r\epsilon) = rN\,L(\epsilon)$, $\quad L(1) = 1$

This equation has a solution of the form,

$$\boxed{L(\epsilon) = \epsilon^{1-D}}$$

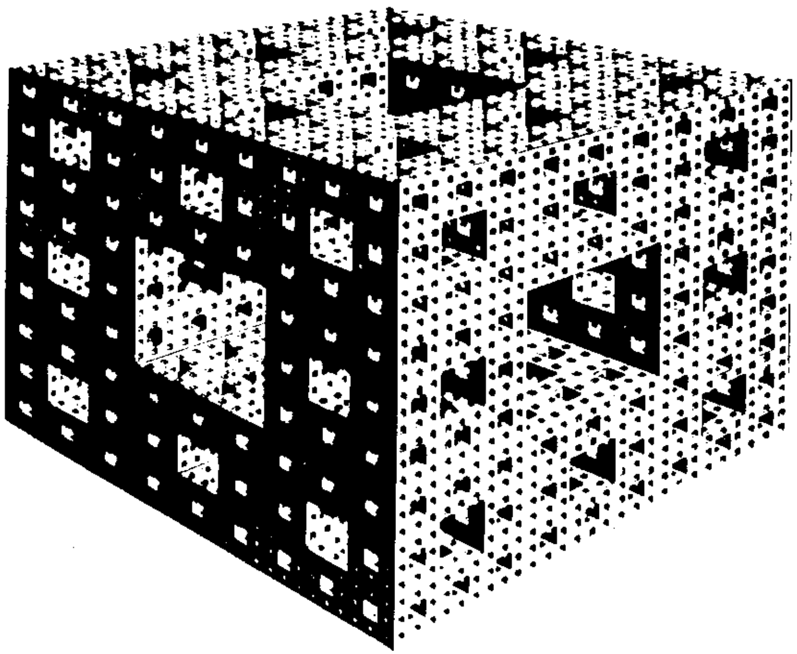
if D satisfies, $\quad D = \lg N/\lg(\frac{1}{r})$

SIERPINSKI CARPET



N=8, r=1/3, D~1.8928.

SIERPINSKI SPONGE



$D \simeq 2.7265$

$N=8\times2+4=20, \quad r=\frac{1}{3}$

SCALING FRACTALS

Length - Area - Volume Relations

I. Standard dimensional analysis

Circle	Square
length = $2\pi R$	length = 4ℓ
area = πR^2	area = ℓ^2

$$(\text{length}) = 2\pi^{\frac{1}{2}}(\text{area})^{\frac{1}{2}}; \quad (\text{length}) = 4\,(\text{area})^{\frac{1}{2}}$$

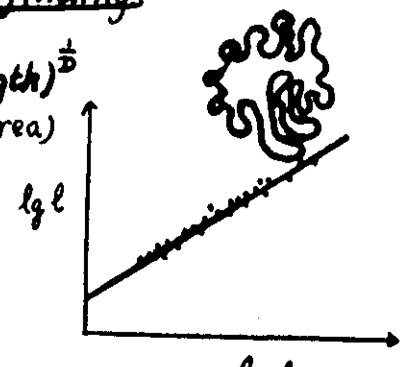
$$(\text{length}) \propto (\text{area})^{\frac{1}{2}}$$

II. Paradoxical dimensional findings

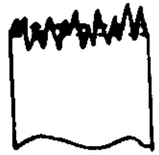
$$(\text{area})^{\frac{1}{2}} \propto (\text{length})^{\frac{1}{D}}$$

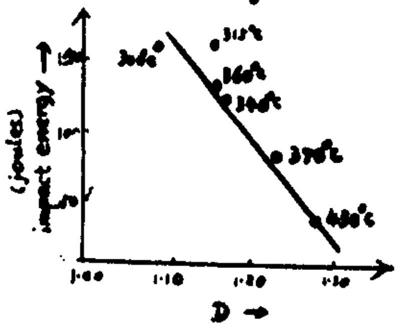
$$\frac{1}{D}\lg(\text{length}) = \text{const.} + \frac{1}{2}\lg(\text{area})$$

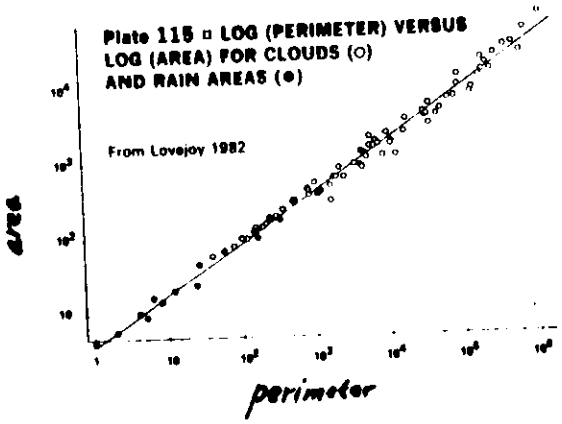
$$D = 2m$$

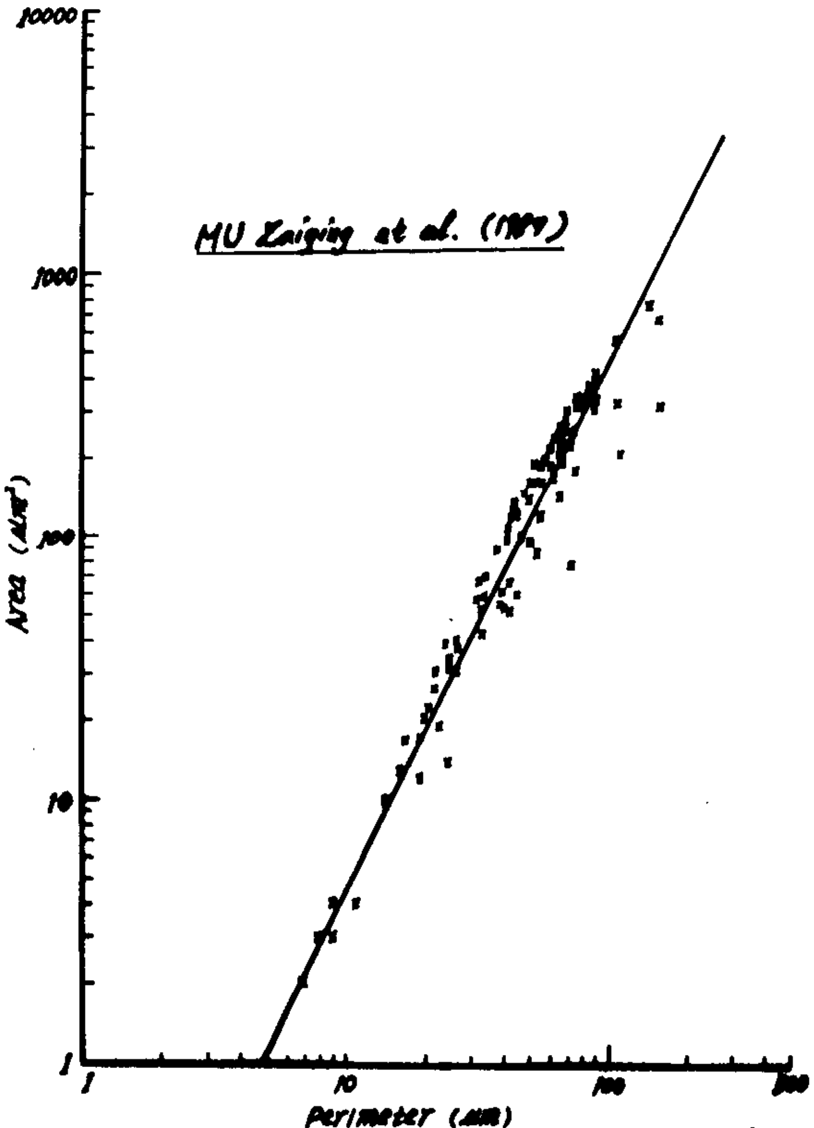


III. Metal fractures









Fractal area–perimeter relationship for slit islands. [illegible]
[illegible] Strength Steel K_{IC} = [illegible] at [illegible]
MU Zaiqin et al. (1989) to be published.
(穆在勤 等)

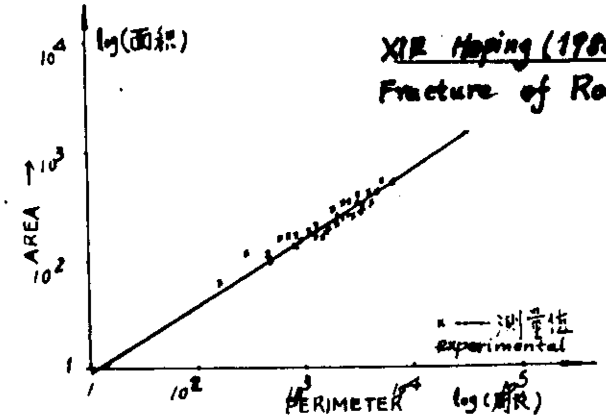


图8

14 16

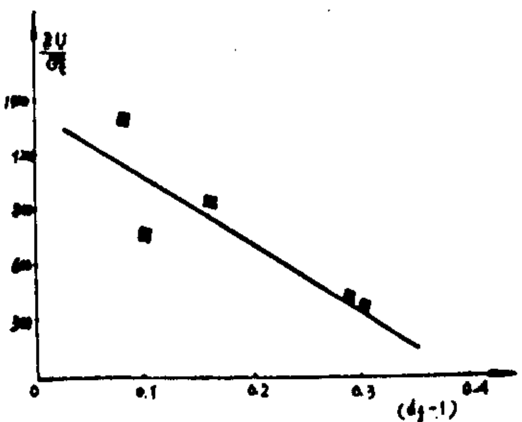


图9

Fractals and the Fracture of Cracked Metals

I. fracture surfaces: «Fractal in Physics» 1985. Trieste. (C. W. Lung)

1, rough.

2, irregular

3. extremely crinkly down to the limits of their microstructural size range.

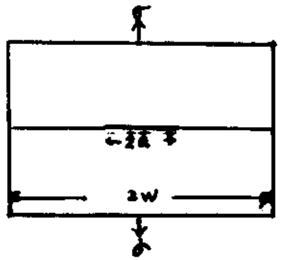
$\ell^{d} = K$, $d = \ln K / \ln \ell$

$\square \rightarrow \boxplus$ $\quad 3^2 = 9 \quad 2 = \lg 9 / \lg 3$

$\square \rightarrow \begin{smallmatrix}\square\square\square\\\square\square\square\\\square\square\square\end{smallmatrix} \quad \dfrac{1^2}{(\frac{1}{3})^2} = 9$

$2 = \dfrac{\lg 9}{\lg \frac{1}{(\frac{1}{3})}}$ $\quad N \cdot r^d = 1$ $\quad N = \dfrac{\lg N}{\lg(\frac{1}{r})}$

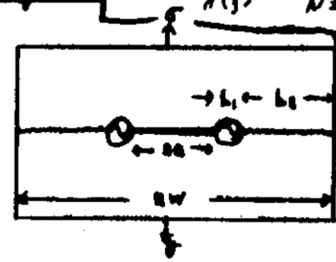
II. The critical crack extension force



(a)

Ideal brittle fracture in glass

$G_c = 2\gamma_s$



(b)

Elastic plastic fracture in metals

$G_c = 2\,(L(\epsilon)/L_0(\epsilon))\,\gamma_s$

$G_c = 2(w-a)^{-1}[L_1(\epsilon_1) + L_2(\epsilon_2)]\gamma_s + \gamma$

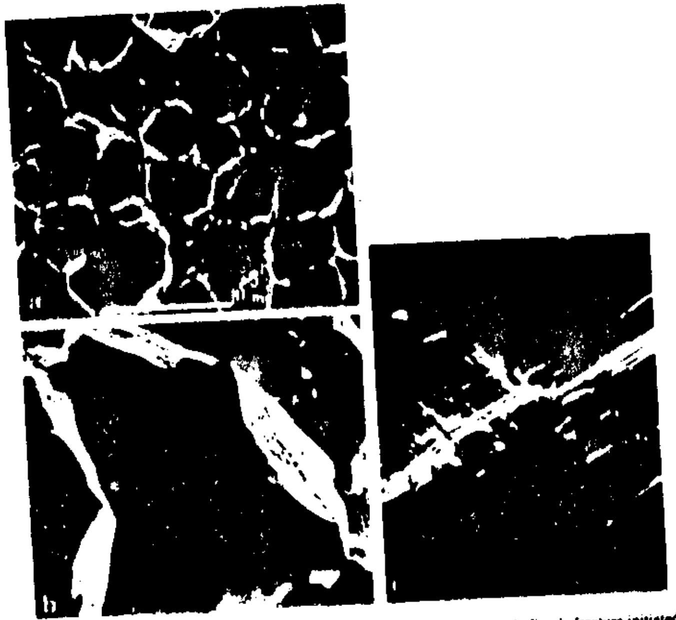
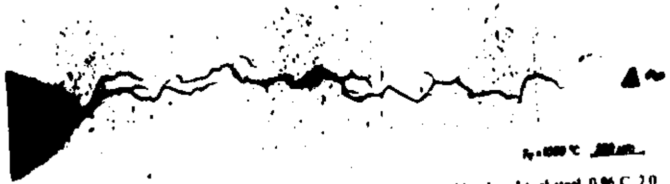


Fig. 43. Scanning electron microscopy of fracture mechanism in steel: (a) tough dimple fracture initiated by pore formation at inclusions; Ni, Cr, Mo-steel, 0.25 wt% C, 2500 ×; (b) brittle intercrystalline fracture at former austenite grain boundaries; Fe 0.6 wt% C, quenched from 1280°C, 150 ×; (c) fatigue-crack tip in austenitic steel propagating along (111) slip bands; 40 at% Ni, 6 at% Al, 5000 ×.

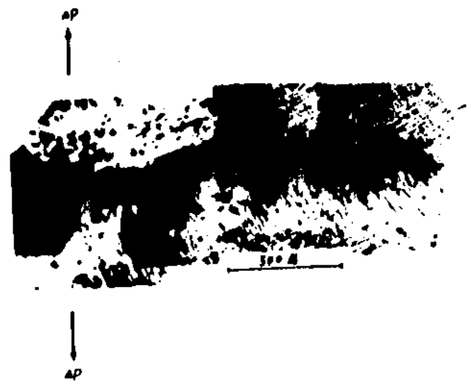
Fig. 43a: courtesy J. ALBRECHT, BBC, Baden; fig. 43b: H. BERNS, Ruhr-University; fig. 43c: K. H. ZUM GAHR, Siegen University.

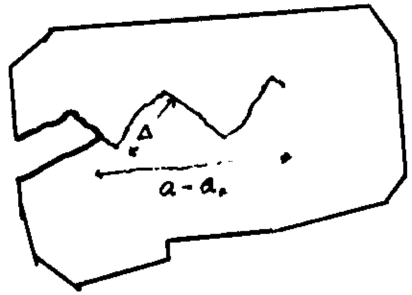


$\Delta \sim 100\ \mu m$

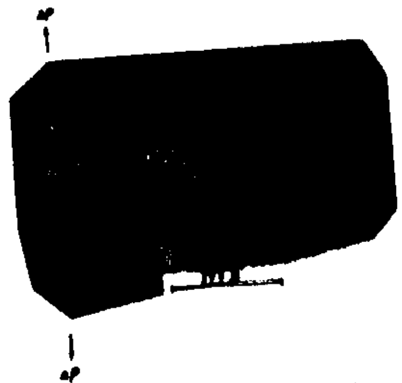
Fig. 44. Brittle inter- and transcrystalline fracture in notched specimen of hardened tool steel, 0.96 C, 2.0 Mn, 0.3 Cr wt%, quenched from 1000°C.

References: p. 1136.



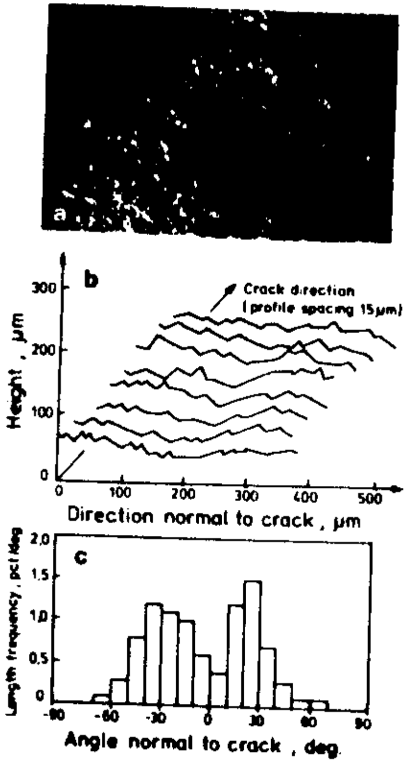


$N\Delta > (a - a_0)$



$\Delta \sim 300\ \mu m$

Al single crystal.



Δ ~ 10 μm

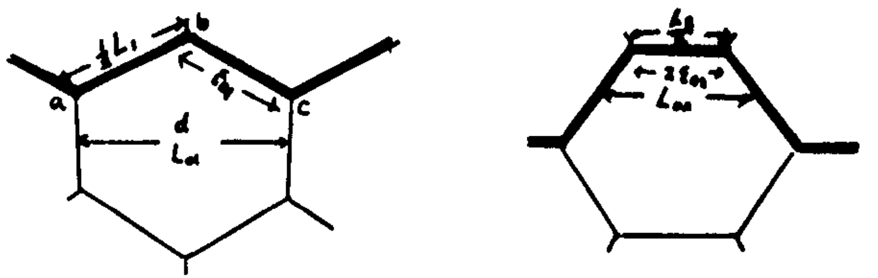
Fig. 9. Fracture-surface analysis by instrumented stereoscopy: (a) scanning electron micrograph evaluated by stereometry; (b) line profiles; (c) distribution of profile line length as a function of tilt angle of surface profiles perpendicular to crack direction. (After Bauer et al. [1982].)

Fig. 9. Hole growth at precipitates in ductile fracture. Between the large holes, shear bands develop consisting of large numbers of much smaller holes. (After Cox and Low [1974]).

III. A Fractal Model

1. New refined zigzag cracks formed on the passage of Large zigzag cracks
2. G_c depends on $L(\varepsilon)$, the smallest zigzag passages.

the grain size (if it is the smallest)



Intergranular brittle fracture

$$D = \log N \Big/ \log\left(\frac{1}{r}\right)$$

$$N = L_i/\varepsilon_{0i}, \quad r = \varepsilon_{0i}/L_{0i}$$

(a) $N = 2$, $r = 1/1.732$, $D = 1.26$

(b) $N = 4$, $r = 1/3$, $D = 1.26$

$$d = L_{01} = 1.73\,\varepsilon_{01} = 3.46\,\varepsilon_{02}$$

$$L_1 = 2\varepsilon_{01}, \quad L_{01} = 1.73\,\varepsilon_{01}, \quad L_2 = 4\varepsilon_{02}, \quad L_{02} = 3\varepsilon_{02}$$

$$G_c = 2\gamma_s\left(L_i/L_{0i}\right) = 2\gamma_s\left(L_{0i}/\varepsilon_{0i}\right)^{D-1}$$

(a) $G_c = 1.73^{0.26} \times 2\gamma_s$

(b) $G_c = 3^{0.26} \times 2\gamma_s$

$$L_i(\varepsilon_i) \sim F\,\varepsilon_i^{1-D} \quad (F = L_0^D)$$

$$G_c \cong 2\gamma_s F L_0^{-1} \varepsilon_i^{1-D} \approx 2\gamma_s d^{-0.26} \qquad (1.73^{0.26} \approx 1.1)$$

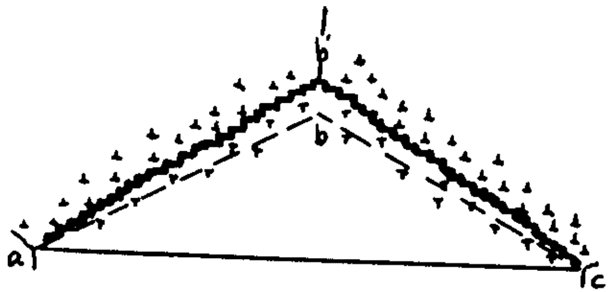
$$(F L_0^{-1} = L_0^{D-1} = 1)$$

$G_c = 2\gamma_s \times 10.96$ for $d = 10^{-4}$ cm

$G_c = 2\gamma_s \times 20$ $d = 10^{-5}$ cm

$G_c = 2\gamma_s \times 36.3$ $d = 10^{-6}$ cm

Ductile fracture



The additional angle formed by plastic deformations in the grain.

$L' = 2\ell$

$L_0 = 2\ell \cos(30^\circ + \theta)$

$N = 2$, $\quad r = [2\cos(30^\circ + \theta)]^{-1}$

$D = \log 2 / \log[2\cos(30^\circ + \theta)] = (1.26 - 2.23)$

$\theta = (\rho b L)/L = \rho b$

$G_c \cong 2\gamma_s \times 8.3 \times 10^4$ (for $d = 10^{-4}$ cm)

$G_c \approx 2\gamma_s \times 1.4 \times 10^6$ (for $d = 10^{-6}$ cm)

($G_c \approx 2\gamma_s d^{-1.23}$, $D = 2.23$)

The grain sizes of almost all the superplastic alloys are very small ($\gtrsim 10^{-4}$ cm) !

FRACTAL MODEL FOR FRACTURE

UNDER HOMOGENEOUS STRESS (K. Sieradzki, 1985)

For 2-D fracture problem the surface energy may be expressed as:

$$\gamma_s = \gamma_s^\circ (L/L_0)^{\bar d - 1} \qquad (1)$$

where L, L_0 ($\approx L_{av}$) the upper and lower limits of the self similar cracks respectively. L_{av} - average crack length. According to Griffith's criterion of fracture

$$\frac{\delta}{\delta L_{av}}\left[\frac{\pi\sigma^2 L_{av}^2}{E} + 2L_{av}\gamma_s^\circ (L/L_0)^{\bar d - 1}\right] = 0 \qquad (2)$$

is the applied stress. E is the elastic modulus of the system. Near the percolation threshold coherence length and the modulus may be expressed by

$$L_{av} \approx L_0 (p - p_c)^{-\nu} \qquad (3)$$

$$E \approx E_0 (p - p_c)^{\tau} \qquad (4)$$

Evaluating eq.(2) and employing (3) and (4)

$$\sigma_f \approx \left(\frac{d}{2\pi L_0}\right)^{1/2} k_c^\circ (p - p_c)^{[\tau + \nu(2 - \bar d)]/2} \qquad (5)$$

where $k_c^\circ = (2E^\circ\gamma_s^\circ)^{1/2}$. Near the percolation threshold $\bar d \approx 2$, so that

$$\sigma_f \sim (p - p_c)^{\tau/2} \qquad (6)$$

This implies that the rigidity exponent in the simulation performed by Ray and Chakrabarte might have been close to 2.

In many actual fracture processes, microcracks combine to be a main long crack and then, the final fracture process is the propagation of the main crack. It is well known that the macrocrack propagates step by step. The local material just ahead of the crack tip consists of many microcracks, voids etc. It seems reasonable to assume that crack propagation is due to the local fracture of the porous structure just ahead of the crack tip.

$$\sigma(r) \approx \frac{k_{ef}}{\sqrt{2\pi r}}$$

$$r_f = \frac{1}{2\pi}\left(\frac{k_{ef}}{\sigma_f}\right)^2$$

r_f is the distance of the crack propagation step.

Expressing k_{ef} as a function of dislocation parameters and σ_f as above, we may obtain the r_f in more detail form.

We remember that (Lung and Gao, 1985)

$$G_c^p \approx 2\gamma_p = \frac{W_i}{L r_f}$$

The critical crack extension force may be obtained.