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SPRING COLLEGE IN MATERIALS SCIENCE
ON
"METALLIC MATERIALS"
(11 May - 19 June 1987)

FATIGUE

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FIVE LECTURES ON FATIGUE

L.M. Brown, June '87

1. WHAT IS FATIGUE? Distinction between high cycle and low cycle fatigue. Fatigue crack growth, characterisation in terms of stress intensity. Cyclic deformation of initially crack-free materials. Stress-control and strain control. Endurance limit. Coffin-Manson 'law'.

2. UNDERSTANDING CRACK GROWTH. The Paris 'law', and its explanations. Threshold stress-intensity and explanations. Origin of the Coffin-Manson 'law'. Problems: What is the mechanism of the endurance limit? Where do the cracks come from?

3. CYCLIC PLASTICITY. Persistent slip bands in single crystals. Characteristic dislocation structures. Empirical relationships between these structures and endurance limits. Behaviour of polycrystals.

4. EXPLANATIONS FOR THE ORIGIN OF THE STRUCTURES. Laird and Kuhlmann-Wilsdorf; Neumann's experiments; Mughrabi's views. A simple tentative exposition. The structures viewed as dissipative structures.

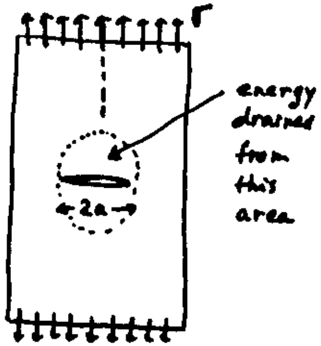
5 ORIGIN OF CRACKS. Surface stress near persistent slip bands. Crack growth in stage I. Can we estimate fatigue life in simple cases?

'Fatigue' is the process whereby ductile materials fracture under variable loading, often at loads much less than the steady load which is required to break them.

As most often experienced, fatigue failure occurs by the progressive propagation of a crack. Each cycle extends the crack irreversibly by a small amount, and the life of the load-bearing component ends when the crack starts to grow catastrophically. However, even in carefully polished single crystals, alternating plastic flow can produce a crack by mechanisms which are still controversial. Many cycles of alternating load are required to do this. When a crack is present, failure tends to occur within a few cycles – so-called 'low cycle' fatigue. When the crack has to be nucleated, many cycles are required – 'high cycle' fatigue.

approximate treatment.
(see p4 for exact formulae)

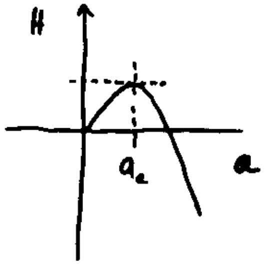
GRIFFITH CRACK



Elastic energy drained from area $\sim \pi a^2$.

Total energy H (incl. grips)

$$H \sim -\frac{\sigma^2}{2E_Y}\cdot \pi a^2 + 4\gamma a.$$



For crack to propagate,

$$\frac{\partial H}{\partial a} = 0 = -\frac{\pi\sigma^2}{E_Y} + 4\gamma$$

∴ critical stress for catastrophic growth, σ_c

$$\sigma_c = \sqrt{\frac{4\gamma E_Y}{\pi a}} \qquad 2.1$$

and Griffith crack length

$$a_c = \sqrt{\frac{4\gamma E_Y}{\pi \sigma_c^2}} \qquad 2.2$$

Modern terminology:

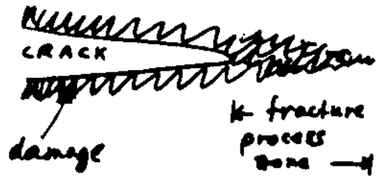
$$K_I = \text{stress intensity} = \sigma\sqrt{\pi a} \qquad 2.3$$

$$G = \text{energy release rate per unit crack advance} = -\frac{1}{2}\frac{\partial \epsilon_{stored}}{\partial a} = \frac{K_I^2}{2E_Y} \qquad 2.4$$

condition for fracture, $G = 2\gamma$ (brittle) 2.5

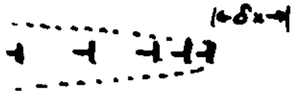
$G = 2\gamma_{eff.}$ (ductile) 2.6

NOTE that substitution of γ_{eff} for γ is quite a subtle step, first taken by Orowan. It



requires that as the crack propagates, the fracture process zone remains unchanged in form, so work to fracture is constant. But the process is irreversible.

RELATION TO DISLOCATIONS: In a pile-up of n dislocations, the effective stress on the leading dislocation is $n\sigma$



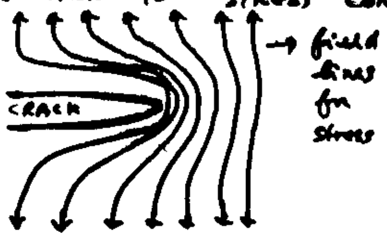
∴ work done if pile-up advances by δx is

$$\delta\epsilon_{el} = n\sigma b\,\delta x = \frac{\sigma\cdot 2a}{4 b E_Y}\sigma b\,\delta x$$

$$\sim \frac{K_I^2}{2E_Y}\delta x \qquad 3.1$$

Thus the energy release rate per unit crack advance may be thought of as the 'crack extension force' - $G = K_I^2/2E_Y$ is the force on the leading dislocation in the pile-up, if the crack is modelled by dislocations.

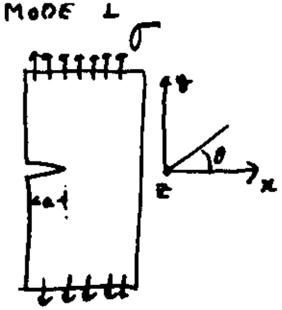
RELATION TO STRESS CONCENTRATIONS



$$\sigma_{max}\ (\text{at crack tip}) = \sigma\, 2\left(\frac{a}{\rho}\right)^{1/2}$$

$$= K_I\frac{2}{\sqrt{\pi\rho}} \qquad 3.2$$

4. MODE I



$K_I = \sigma\sqrt{\pi a}$, ...

$$\sigma_{xx} = \frac{K_1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

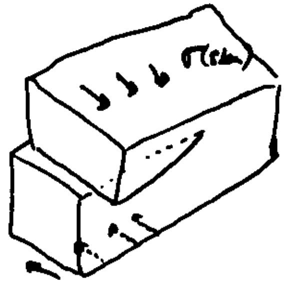
$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_{zz} = \nu\left(\sigma_{xx} + \sigma_{yy}\right)$$

mode II

$K_{II} = \sigma\sqrt{\pi a}$

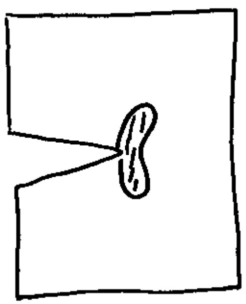


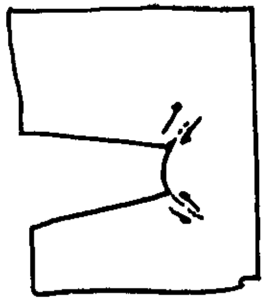
mode III

$K_{III} = \sigma\sqrt{\pi a}$

- in general, $\sigma(r,\theta) \sim \frac{K}{\sqrt{2\pi r}} f(\theta)$ 4.1

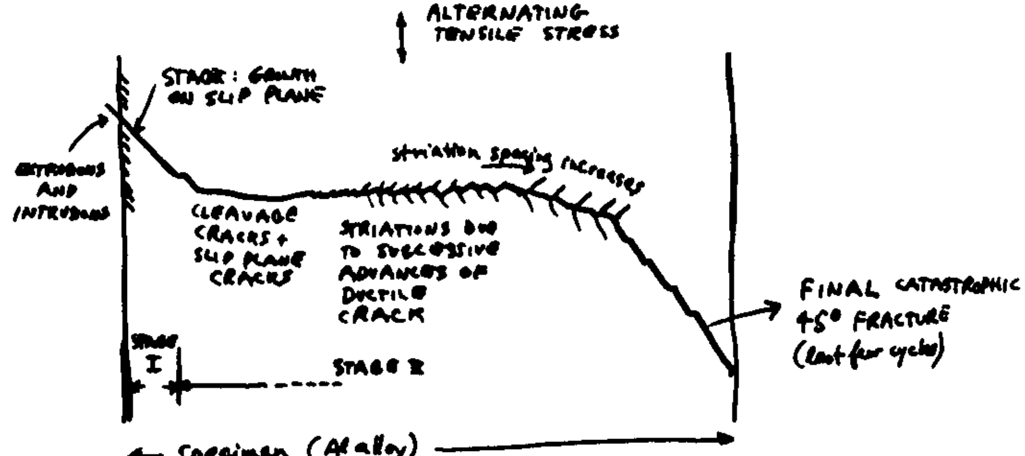
- Solutions for various geometries tabulated: Compendium of Stress Intensity Factors, (D.P. Rooke & D.J. Cartwright, 1976 London HMSO)

- 



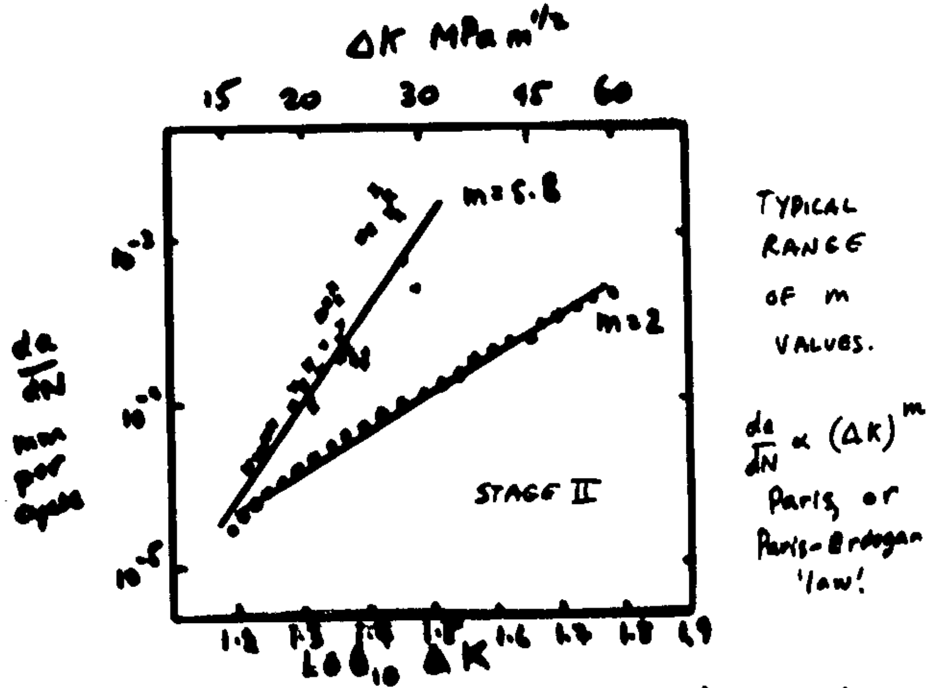
calculation & experiment show plastic zone to act like a 'plastic hinge'.

5. SUMMARY OF RESULTS ON CRACK GROWTH



cycled at constant stress

P.J.E. FORSYTH, Acta Met 1 (1963) 703.



TYPICAL RANGE OF m VALUES.

$\frac{da}{dN} \propto (\Delta K)^m$

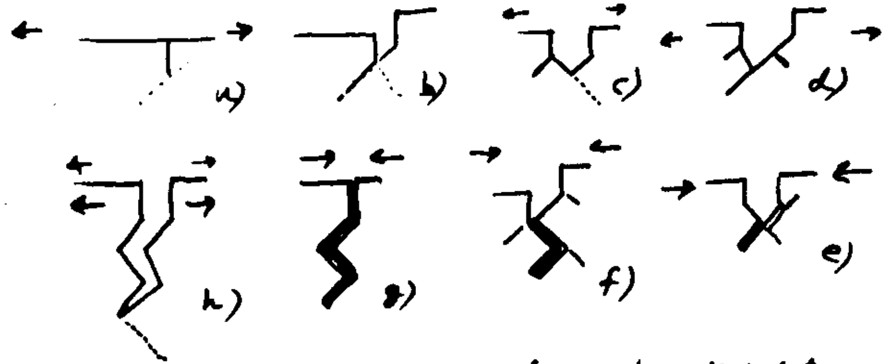
Paris, or Paris-Erdogan 'law'.

- embrittled steel - fast growth with cleavage
- unembrittled - show ductile growth per cycle.

Lindley, Richards & Ritchie : Metallurgia and Metal Forming 43 (1976) 268

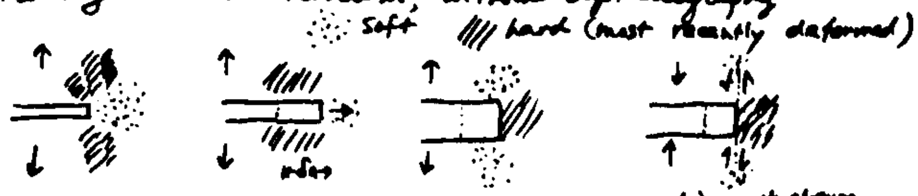
- crack is subcritical i.e. below Griffith length so in the forward cycle, the tip blunts or extends only a limited amount. But the new surface so formed cannot be restored on the reverse cycle, so the growth is irreversible. (Laird & Smith Phil Mag 7 (1962) 847)

- Neumann's mechanism (Ch 24, Physical Met., eds Cahn & Haasen)



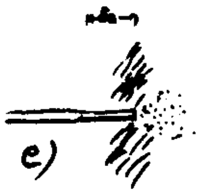
Relies on hardening of slip lines to distribute flow to neighbouring lines on alternative system, together with irreversible surface formation. Beautifully confirmed by observation

- More general mechanism, without crystallography



c) crack extension stops, due to work hardening

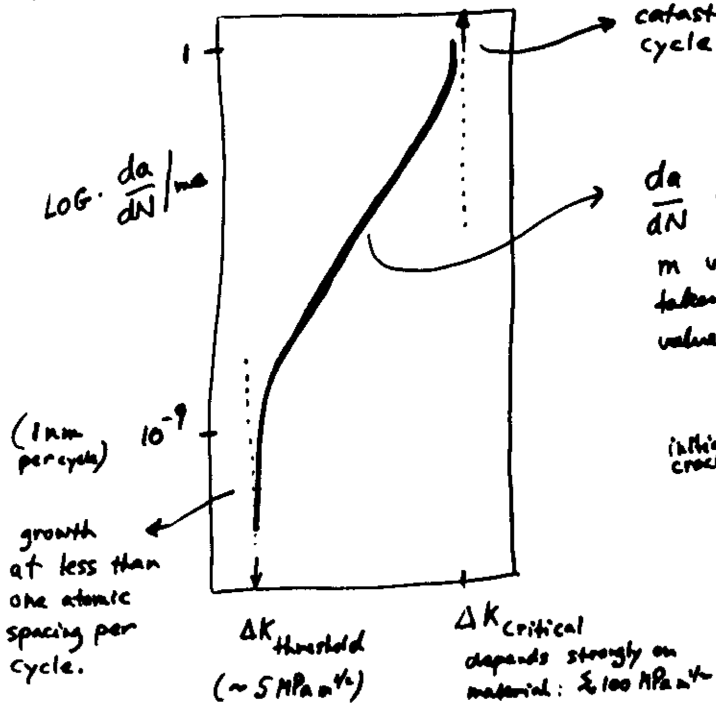
d) crack closure commences, by flow through 'soft' material.



ready to begin at a), now longer

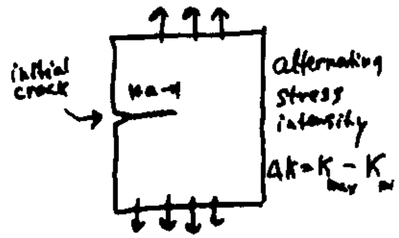
RATCHET MECHANISMS REQUIRE SUBTLE CONDITIONS OF WORK-HARDENING TO PRODUCE IRREVERSIBLE FLOW AND CRACK EXTENSION, at subcritical crack lengths

7. Summary:



$\frac{da}{dN} \propto (\Delta K)^m$ Paris 'law.'

m variable, sometimes taken to be 4, but minimum value for ductile growth ≈ 2.



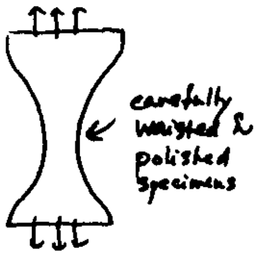
Around threshold, there are large effects of environment, mean stress, microstructure

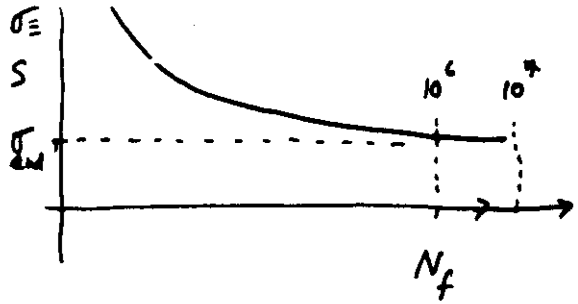
Paris 'law' holds independently of environment. But microstructure, particularly toughness, has an effect on the 'm' value, which must be determined by tests on engineering materials under service conditions, to be of use in practical problems.

Nevertheless the Paris 'law' is of great engineering importance to help decide whether visible cracks in machinery are dangerous.

8. Wöhler or S/N curves.

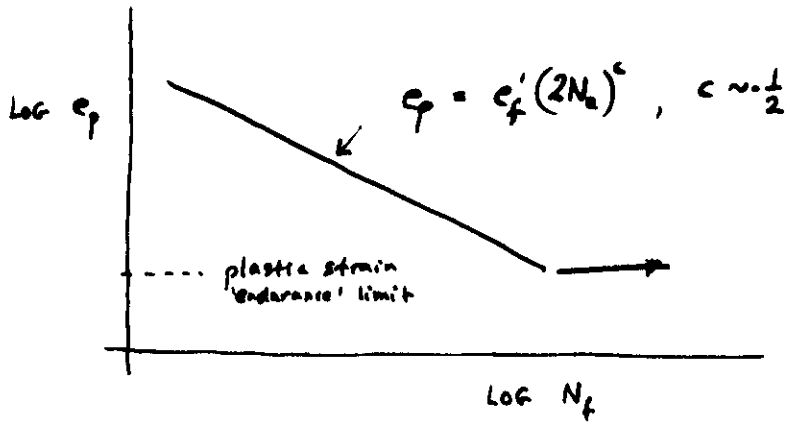
(August Wöhler - German engineer - experiments in 19th C. on railway carriage axles)





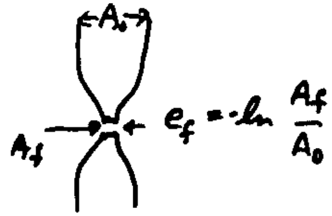
- some metals (low carbon steel) show a well-defined fatigue limit
- others show an endurance limit, stress which permits 10^6 or 10^7 cycles.
- recent work, to be discussed, suggests that most metals have an fatigue limit, but it is hard to measure from an S/N curve because of its asymptotic approach at large N.

Coffin-Manson 'law'. Plastic strain amplitude e_p applied to crack-free specimen. Thorough discussion in book by Klesnil & Lukaš, Fatigue of Metallic Materials Elsevier 1992



$$e_p = e_f'(2N_f)^c, \quad c \sim -\frac{1}{2}.$$

9. • The value of e_f is of the same order as the fracture strain in a unidirectional test, with $2N_f = 1$ i.e.



$$e_f = -\ln \frac{A_f}{A_0}$$

one defines 'ductility' in this way for unidirectional test

In general, $e_f' \sim \frac{1}{2} e_f$. But this correspondence suggests that the failure mechanism in fatigue is just the cyclic progression of the failure mechanism in a unidirectional test, probably by the propagation of the 'crack' produced by fibrous ductile fracture

- The value of c is variable. At large no. of cycles, it may increase, particularly in single crystals. Then, near the plastic strain endurance limit, it goes to zero.

- The Coffin-Manson law is of very great practical utility for the design of bellows, hinges, ... objects where plastic strain is imposed.

2 EXPLANATION of the Paris 'Law'.

If we suppose that the crack extension per cycle depends only on the stress intensity (that is, we can scale for longer cracks by dividing the current applied stress by the square-root of the crack length) then the method of dimensions gives us the crack-growth law, independent of any detailed physical mechanism. We must have

$$\frac{da}{dN} \propto (\Delta K)^2 / E_Y^2 \qquad \text{.. 10.1}$$

(Traditionally, the Young modulus E_Y is used). It is now fairly widely accepted that the 'ideal' Paris exponent is $m=2$, increases from this value being due to steeper slopes either at the threshold end, or the catastrophic failure end, where other effects are entering.

EXPLANATION OF the threshold stress intensity.

If the material were brittle, so surfaces were reversibly made and unmade, we would find a threshold stress intensity below which the crack could not open

$$\Delta K_{Griffith} = \sqrt{4\gamma E_Y} \qquad \text{see 2.1}$$

Inserting values: (in $MPa\, m^{1/2}$)

	copper	nickel	aluminium
$\Delta K_{Griffith}$	1	1.3	0.6
$\Delta K_{threshold}$	1.8	3.6	1.2

We see that the observed values are larger by a factor of 2 to 3 than the 'thermodynamic minimum' Griffith values. This is due to 'crack closure', the presence of irreversible work from asperities impinging on the opposite sides of the crack faces when they close. (see, for example, C.J. Beevers and R.L. Carlson in 'Fatigue Crack Growth' ed R.A. Smith, Pergamon 1984)

NB. In this discussion, I have chosen the minimum values of ΔK_{th}, found when the ratio of K_{max}/K_{min} is nearly unity, and crack-closure effects are minimised. A detailed discussion of this effect is beyond the scope of these lectures.

EXPLANATION OF THE COFFIN-MANSON LAW.

If we consider the type of mechanism proposed by Laird and Smith (see p6, bottom) we see that crack extension can be thought of as due to lateral contraction of the plastic zone.

$$\delta a = e_p r_p \qquad 11.1$$

But $\sigma \sim K/\sqrt{2\pi r}$, so if σ_f is the flow stress of the fully work-hardened zone,

$$r_p \sim \sigma^2 a / \sigma_f^2 \qquad \text{.. 11.2}$$

But there will be a work-hardening law,

$$\sigma \propto e_p^{\,q}, \quad q \sim 1/2 \qquad \text{.. 11.3}$$

Combining 11.1, 11.2, 11.3,

$$\delta a \propto e_p^{(1+2q)}\, a, \text{ i.e.}$$

$$\frac{\partial a}{\partial N} = \text{const.}\; e_p^{(1+2q)}\, a$$

Integrating, we find [illegible] e.g. 2. Lankford (1983) Fat. Eng. Mat. and Structures 5, 13–26.

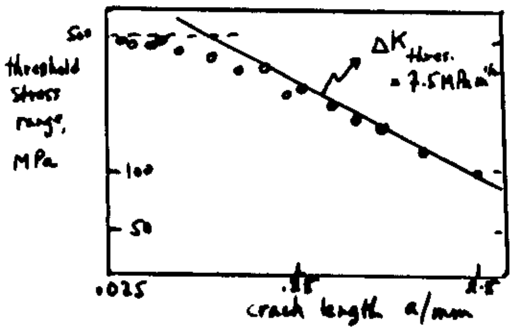
$$\ln \frac{a_f}{a_0} = \text{const.} \quad N_f \, \epsilon_p^{(1+2q)} = \text{const.} \qquad \ldots 12.1$$

This is the Coffin–Manson law with $c = -1/(1+2q)$ (see pp. 8, 9). If parabolic work-hardening prevails, $q = 1/2$, and $c = -1/2$. (This argument is an improved -Stage- version of that produced by Tomkins: see J.F. Knott in Fatigue Crack Growth, ed R.A. Smith, Pergamon 1984)

SUCCESS AND FAILURE OF FRACTURE MECHANICS AS IT IS CONVENTIONALLY APPLIED TO FATIGUE

- It is a useful tool where cracks are visible.
- It cannot explain behaviour of very short cracks, nor mechanical properties which are associated with the endurance limit.

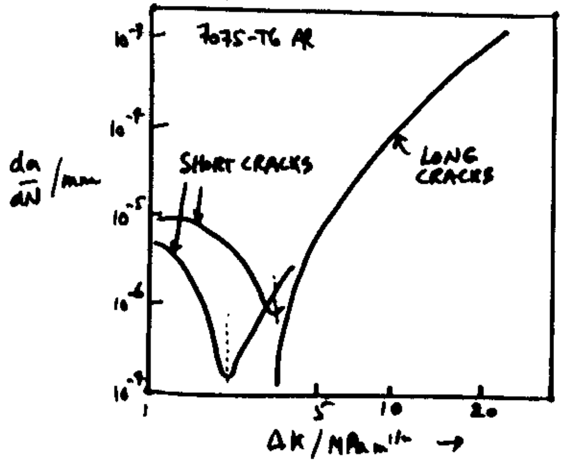
e.g. 1



after Kitagawa and Takahashi 2nd Conf on Mech. Behaviour of Materials, Boston 1976

Below a certain crack length, the threshold stress range is controlled by the Wöhler curve.

NOTE that the measurement of ΔK_{thres} for longer cracks will grossly underestimate the growth rate of shorter cracks, of the order of .1 mm.



- as ΔK is reduced, long cracks slow up to the threshold, when they stop.
- but short cracks grow, often rapidly, slowing up when they are nearly equal to the grain size in length. Then they may accelerate away

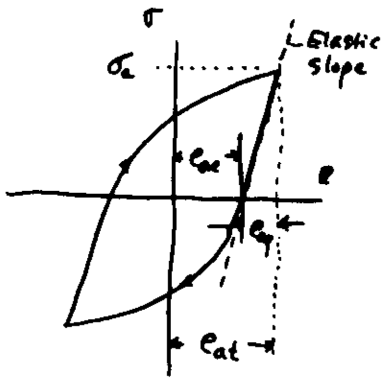
Note that short cracks can grow, even if they are less than the Griffith length for ideally brittle fracture!

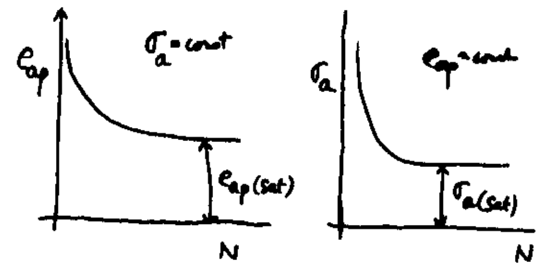
There is another problem: the mechanism of the endurance limit. Experimental data suggest that the endurance limit is a real physical property, independent of the existence or not of cracks at the start of a fatigue test. The next lectures are devoted to the questions:

What controls the fatigue strength (endurance limit) of metals?

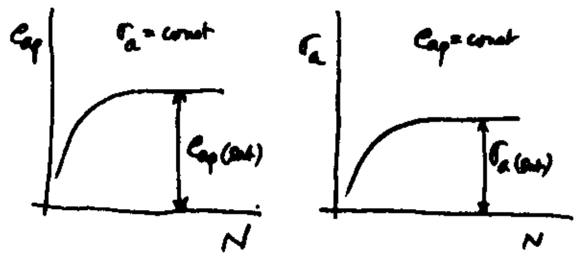
How are cracks produced by cyclic strain on an otherwise smooth polished surface?

In modern testing machines, it is possible to control either stress or strain throughout the test. The stress may be permitted to find its value as either the plastic strain amplitude e_{ap} is fixed, or the total strain amplitude, e_{at} is fixed. Or the strain may find its value if the stress amplitude σ_a is fixed. It is usually found that after a number of cycles ranging from a few hundred to a few thousand, the material under test enters an unchanging a saturation state. Both softening and hardening may be observed, and it is the fundamental postulate that there is a cyclic stress-strain curve which may be obtained whether stress or strain control is utilised.

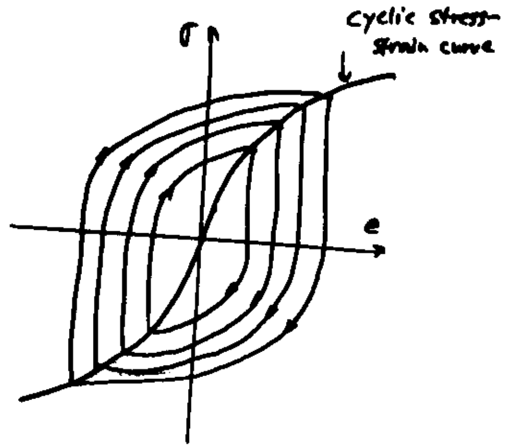




cyclic softening



cyclic hardening



after 'Fatigue of Metallic Materials' by Klesnil & Lukáš (Elsevier 1980)

This postulate is normally well borne out in practice, unless failure intervenes. In general, if the ratio of ultimate tensile stress σ_{UTS} to the initial flow stress is higher than 1.4, the material will harden under cyclic loading. If it is less than 1.2, the material will soften.

Materials with relatively easy cross-slip approach saturation in relatively few cycles (less than 3% of the cycles to fracture) whereas planar-slip materials (α-brass, Fe-Si alloy, austenitic steels) take longer (30%-40% of the cycles to fracture).

Not only does saturation affect the stress-strain curves, it affects all other physical properties (except those whose slow development leads eventually to fracture!). Resistivity, dislocation density etc. all saturate. Saturation is the result of dynamic equilibrium between hardening and softening processes.

In precipitation-hardened alloys, for example, constant cutting and re-cutting of precipitates eventually breaks them up and the alloy softens. This is why aluminium alloys are particularly weak