



INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION



INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) U.P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONE: 2240-1
CABLE: CENTRATOM - TELEX 400392-1

SMR/208 - 51

SPRING COLLEGE IN MATERIALS SCIENCE
ON
"METALLIC MATERIALS"
(11 May - 19 June 1987)

FATIGUE
(Part II)

L.M. BROWN
Cavendish Laboratory
Department of Physics
University of Cambridge
Madingley Road
Cambridge CB3 OHE
U.K.

These are preliminary lecture notes, intended only for distribution to participants.

16 One of the main virtues of the cyclic stress-strain curve is that it enables fatigue life curves at constant stress (Wöhler) or at constant plastic strain amplitude (Coffin-Manson) to be transformed into one another. The extent to which this may be done can be found in the book by Kleinig and Lukas, already cited p.15.

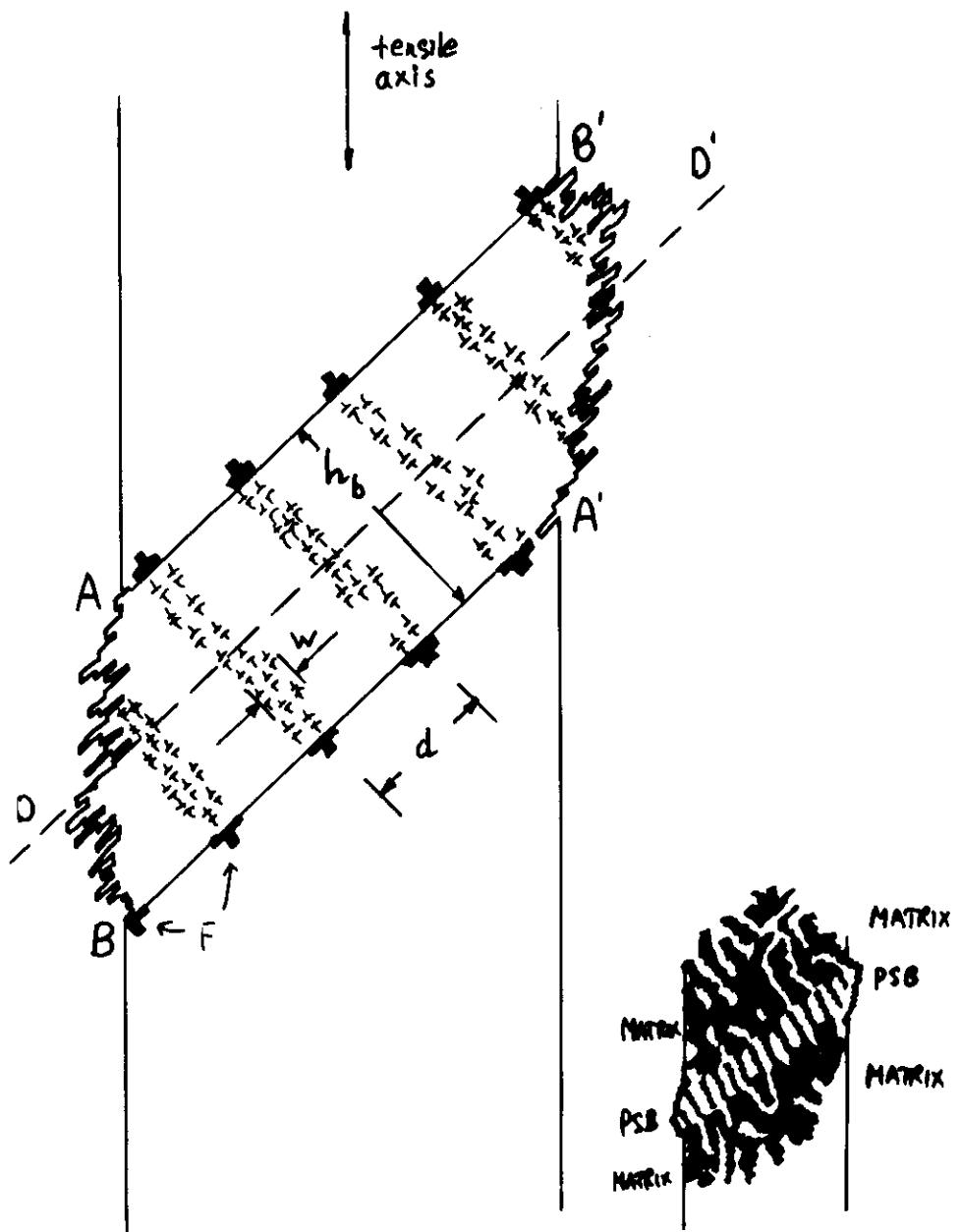
PERSISTENT SLIP PHENOMENON

[Ewing J.A. and Humphrey J.C.W. (1953) The Fracture of metals under repeated alternating stresses Proc Roy Soc A200, p241
Forsyth, P.J.E. (1957) Proc Roy Soc A312, p198
(1961) with D.A. Ryder Metallurgia 62 p117
Thompson N., Wadsworth & Longfellow, (1956) Phil Mag 1, p113]

It has been appreciated for many years that cyclic straining is accompanied by bands of intense slip which appear on the surface and deepen into cracks. It is still believed by many that this is a 'surface' phenomenon, but electron microscopy showed conclusively that persistent slip bands are associated with a planar dislocation structure, running parallel to the slip planes, over complete grains or indeed over a single crystal of macroscopic dimensions.

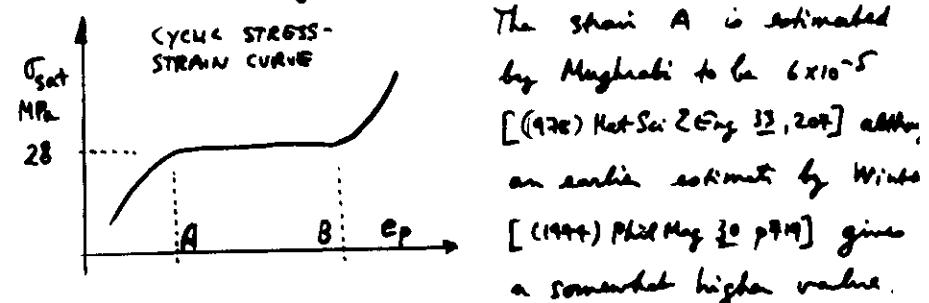
Page 17 shows an idealised view of a persistent slip band. The line DD' is parallel to the Burgers vector direction. The band nucleates in a matrix of dense clumps of dipoles, with clear wave-like regions running between them. This is sketched on the lower right-hand diagram of p.17. The diagrams are appropriate for single-crystals, for which the clearest evidence exists.

3)



3) A wealth of information is now known about PSBs in metals. We shall now try to summarise it briefly:

i) CONDITIONS FOR NUCLEATION: Under stress control, PSBs require a stress amplitude of about 32 MPa to appear: crystals cycled at lower than this amplitude are 'safe' whereas crystals cycled at more than this amplitude develop PSBs and fail in about 10^5 cycles. [Robert W.N. Phil Mag 20 (1959) 675]. Under strain control, provided the plastic strain lies between about 10^{-4} and 10^{-2} the bands form and the cyclic stress-strain curve is flat.



It is difficult to estimate precisely, but all workers believe this strain exists: it is the nucleation strain amplitude for PSBs. The strain B for Cu is $\sim 7.5 \times 10^{-3}$. The plateau stress shown as 28 MPa in the diagram has been variously estimated to lie between 28 MPa and 33 MPa possibly due to different testing conditions, strain rates, etc. PSBs can be nucleated in work-hardened crystals, [Winter 1978 Phil Mag 37 457] in dispersion-hardened

cryptals [Atkinson, Brown, Kiey, Stobbs, Winter, Woods 1973, 3rd Int Conf. on Strength of Metals and Alloys]. They may be found in many different crystal structures: their existence is a dislocation phenomenon, not highly dependent on crystallography.

(i) TWO PHASE MODEL: In the plateau region AB shown on p18 the volume fraction of persistent slip band in the crystal varies linearly from zero to unity (WINTER's law: 1974 Phil Mag 30 719]. This means that through this strain range of nearly two orders of magnitude, the plastic strain is ~~carried~~ carried by the persistent slip bands, whose structure therefore determine the cyclic plasticity. A neat way to summarise the relationships is to think of the thermodynamic equilibrium between a liquid and its vapour: the saturation stress play the role of the vapour pressure; the imposed plastic strain amplitude density. This model is lent credibility because of the way the PSB's (liquid) condense out of disordered matrix (vapour). The model clearly shows that the endurance limit for copper is to be interpreted as $\sigma_{st} = 28 \text{ MPa}$ and the endurance plastic strain amplitude as the strain at A: the matrix strain just bearable without PSBs, $\sim 6 \times 10^{-5}$ in Cu. (A tabulation of these values for several metals is given by ...)

10

It must be borne in mind, however, that the PSB - MATRIX relationship, unlike the liquid-vapour relationship, is not reversible. The volume fraction of bands may be made to increase by increasing the plastic strain amplitude during the test, but cannot be made to decrease. It is, of course, also possible to think of the PSB's as a kind of Luders band in the cyclic stress-strain curve.

(ii) DETAILED OBSERVATIONS OF STRUCTURE: The structure consists of edge dislocation dipoles, all either of primary Burgers vector or derived from that by conversion into Frank dipoles or loops. [Antognola & Winter Phil Mag 33 (1976) 87; ESSMANN & MEGHLI Phil Mag A 10 (1971) 73] Another clear indication that secondary dislocations are not required is the observation of these structures in magnesium oriented for single slip: such dislocations are clearly absent [Kivalis & Brown, Acta Met ((1976) 24 1113)]. An most interesting observation is that the dislocation dipoles are biased towards vacancy type: they have their half-planes pointing outwards, as shown on Fig 17. (Antognola, Brown & Winter Phil Mag 39 (1976) 549]. All authors are agreed that within the walls the dipole density is about $5 \times 10^{15} \text{ m}^{-2}$.

²¹ Between the walls, screw dislocations are observed. The most beautiful observations of these have been made by Meghribi in neutron-irradiated Cu (to stabilize the dislocations). [Meghribi, 1979 Proceedings of 5th International Conference on Strength of Metals and Alloys, ed. Haasen, Gerold, Kestens, Pergamon Press. See also Meghribi, Ackerson and Hess, 1979, ASTM special technical publication 675] The density of screw dislocations seems to be about 10^{13} m^{-2} , and no two of them of opposite sign are more closely spaced than about 50 nm.

With reference to p17, the ratio of w/d is about 1/10.

The length d is very temperature dependent. The table reproduces values from an earlier paper [Brown 1980 Proceedings of Int. Conf. on Dislocation Modelling of Physical Systems, Gainesville, Fla. Eds. Ashby, Bullough, Hartley, Hirth, Pergamon Press]

Material	$\sigma_{\text{Sat}}/\text{MPa}$	Homologous temp. T/T_m of observation	wall spacing $d/\mu\text{m}$	$\frac{\sigma_{\text{Sat}}(1-\nu)}{\mu b \ln(d/w)}$
Ca^+	25.2	.22	1.3±.08	1.3
	48.4	.06	0.72±.05	1.7
	73.5	.003	0.95±.03	1.7
Mg^{++}	2	.33	8	2
	7.4	.08	2	2
Ni^{++}	55	.17	1.3	2.2

⁺ Basinski; Korbel & Basinski, 1980 Acta Met 28 19,
⁺⁺ Kradje and Brown 1976 Acta Met 26 1117

¹²⁾ A striking feature of this data is that the saturation stress is inversely proportional to the wall spacing, a relationship which must be interpreted as like a Frank-Read source ^o:

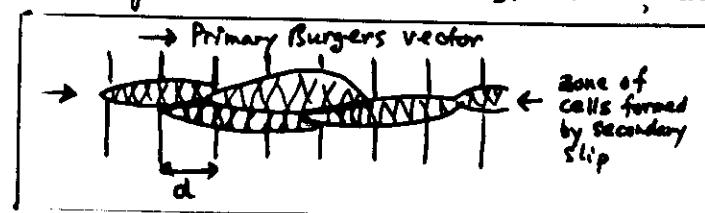
$$\sigma_{\text{Sat}} = \alpha \frac{\mu b \ln(d/w)}{2\pi(1-\nu) d}$$

^{22.1}

The value of α , which should be unity for a simple Frank-Read value, is nearer 2.

iv) APPEARANCE OF SECONDARY SLIP AFTER LARGE N :

By cycling in vacuo, and thus prolonging ^{return 2 Rpp} the life of the specimen, Wang and Meghribi in a beautiful study [Mat. Sci. Eng. 65 (1984) ²¹⁹] have been able to show that characteristic secondary dislocations appear within the PSB, harden it, and cause it to spread. It acquires a 'sandwich' structure, as sketched below.



This behavior has been noted before [e.g., for

example, Brown 1977 Metal Science p315]. Amazing pictures can be found in Scoble and Weissman 1973 Crystal Lattice Defects § 123.

v) FINAL COMMENTARY ON THIS SECTION: DO WE REALLY BELIEVE THAT FOR A WIDE CATEGORY OF METALS, THE SATURATION STRESS EQUALS THE ENDURANCE LIMIT?

Yes!

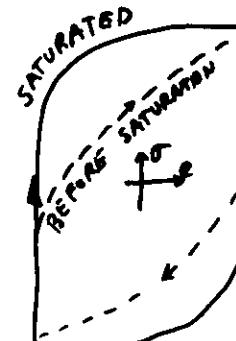
Some evidence is collected in Winter's (1974) paper (cited p19). Other evidence is presented in the book by Kleonil and Lukas (cited p15). They correlate the saturation stress observed in polycrystals over a strain range from 10^{-5} to 10^{-2} with the fatigue limit measured directly from Wohler curves.

They get good agreement for Al, Cu, Cu-Zn, carbon steels, austenitic steels and alloyed steels. See pp 161 and 162 of their book.

This means that the dislocation structures of the persistent slip bands actually control the fatigue strength of metals, and answers the first question on p.13. To quote Laird's very clear statement: "A true fatigue-stress limit and a true fatigue strain-limit (connected directly through the cyclic stress-strain curve) exist for crystalline materials: these limits are based on the stress and strain requirements to form persistent slip bands" [Nat. Sci. Eng. 22 (1976) 23]. I believe it was first realised by the Cambridge group (Atkinson et al, ref top p19) but not so clearly stated.

(8)

24) APPEARANCE OF HYSTERESIS LOOPS: One gains much insight into the behaviour of the various structures from the hysteresis loops. Before saturation, the stress-strain loops are pointed, often showing very large instantaneous work-hardening rates. However, after saturation and the nucleation of persistent slip bands, the loops become very flat.



The very high hardening rates before saturation can equal one-third of the elastic shear modulus, that is 100x greater than work-hardening in simple tension. The explanation for this, first proposed by Pedersen and Winter [Acta Met 30 (1982) 711] is that the dipole clumps in the matrix, where flow occurs before saturation, are indeforably. Their large volume fraction must be strained elastically, so the overall work-hardening rate of the composite is very high. When the PSBs form, the volume fraction of dipole clumps drops to about 1/10 (i.e. W/d, p17). And the platelike walls or rungs of the ladder are much better accommodated elastically than the equiaxed clumps of the matrix, so the hardening rate in the saturated state is very low.

(a)

25) BEHAVIOUR OF POLYCRYSTALS: For many years, it was widely believed that PSBs would not be found in polycrystals. This is because the PSB produces only primary slip, which cannot produce a general stage change, and so cannot produce compatible deformation within each grain of a polycrystal. However a beautiful study by Winter, Pedersen and Rasmussen [Acta Met 29 (1981) 735] showed clearly that at the very low plastic strain amplitudes of the plateau region, PSBs do form both in bulk and in surface grains of a polycrystal. The volume fraction of these structures, and the wall spacing, are in accord with what might be found under similar conditions in a single crystal.

At higher strain amplitudes, i.e. at the higher strain end of what would correspond to the plateau in the single crystal, other structures denoted 'labyrinth' structures are found. These correspond to structures observed in single crystals oriented for multiple slip [Jin and Winter, 32 (1984) 1173 Acta Met].
On the basis of these observations, it is possible to predict the cyclic stress-strain curve of polycrystals from that of single crystals. The important idea

) here is the use of the Sachs model [Pedersen, Rasmussen and Winter, Acta Met 30 (1982) p57]. This is because the grains deform on one system each, as envisaged in the Sachs model which ignores the requirement of compatible grain deformation. Reasonable agreement is obtained. Although there are differences in detail, Mughabgi and Wang [1980, Defects and Fracture, Proc. Int. Symp. Tczew, Poland, Sijthoff and Nordhoff] come to much the same conclusion.

By what factor should the resolved shear stress of the single crystal at the plateau be multiplied to produce the tensile endurance limit for the polycrystal? The least safe grain will have a Schmid factor of $\frac{1}{2}$; all grains will be plastic with a Taylor factor of $\frac{1}{3}$. If one looks at the early data quoted in Winter's 1974 paper (cited p19) the empirical factor is nearer 3 than 2. Doubtless this is because failure requires PSBs and cracks to link up in several grains: one or two especially dangerous grains in a polycrystalline aggregate cannot cause failure.

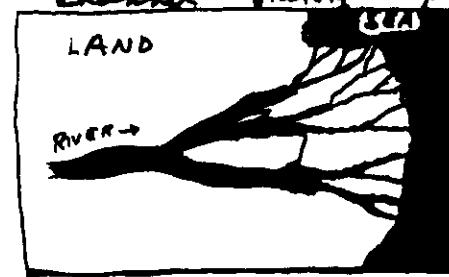
27) EXPLANATIONS FOR THE ORIGIN OF THE STRUCTURES:

(A series of paragraphs now follow, some points made in them are my own opinion which is hotly contested by the other workers in the field!)

1. In my view, it is helpful to think of these structures as 'dissipative structures' in the sense used by Prigogine and his school. [Ref: Pashin, Defects and Microstructure of Nonequilibrium Systems, Ed. D. Walgraef, NATO ASI Series, Mijlster 1987]. Dislocations are rare in thermal equilibrium, so the structures develop far from equilibrium. They 'condense' out of the 'disordered' phase of the matrix, although the matrix structure itself is not disordered. The structures are reminiscent of ridges on the sea-bottom under wave-motion, a point made by Wieder (Fundamentals of Deformation and Fracture, Ed. Billby, Miller, Willis, C.R.P. 1985). They are evidently an extreme example of dislocation structures formed by work-hardening, creep, etc.

2. Wieder, in a paper not widely noticed (Phil Mag A37 (1978) p457), shows micrographs which suggest that the ^{clumps of the} matrix structure are likely to be softer on the inside, and harder on the outside, like a soft-boiled egg. In his view, the wall structure condenses from the matrix structure by a process of 'clumping'

Once this idea is grasped, it is possible to look at many of the published micrographs with increased understanding. (Incidentally, I think the idea applies to unidirectional work-hardening too) Kahlmann-Wilsdorf and Laird (Nat. Sci. Lett. 2 Fig. 27 (1977) 137) seem broadly in agreement: Wieder attributes the idea partially to them. We must imagine screw dislocations pushing dipoles into the clumps as they force their way down the dislocation-free channels. Hardening consists in part of widening the clumps. And then, when the stress rises, the soft center of the clump gives way to produce 2 new clumps. The whole process is much akin to delta formation by sedimentation at the rivermouth. We can imagine a channel silted up as silt is deposited, so that the channel becomes clogged and the riverpath must bifurcate to continue flowing. Finally a dissipative structure is formed with readily recognisable channels and obstacles.



Wieder points out that any theory of the wall structure must show how it evolves from the matrix structure. And a further point must be made: any theory must show how the number density of obstacles increases from very few at low

29) number of cycles to 10,000 or so at or near saturation. The obstacles must multiply!

3. In a series of papers, Kuhlmann-Wilsdorf and Laird address several problems. (Paper 18 in series gives reference Metallurgy 31 (1979) 231) They point out the unlimited path of screw dislocations moving between the walls, and attribute the strain to this. This view is accepted by all authors. They then go on to analyse the hysteresis loops in detail, dividing the resistance to flow into an internal stress and a friction stress. More recent work is fully consistent with their view that in the cyclic hardening stage, a large part of the peak stress derives from back-stresses associated with the dipole clumps of the matrix. And according to Pedersen and Winter (Acta Met 30 (1982) 711) the back-stress entails a 'source-shortening' or friction stress which therefore rises automatically with it, again consistent with Kuhlmann-Wilsdorf and Laird. One might add that these Takemoto stresses were observed first by Nagelski; on the basis of the curvatures of dislocations pinned by reaction irradiation. However, when the wall structure nucleates from the matrix structure, the volume fraction

30) of obstacle falls drastically, and the role of these internal stresses must diminish.

There is an important difference between Kuhlmann-Wilsdorf & Laird's view of the dipole clumps and those of other workers: they

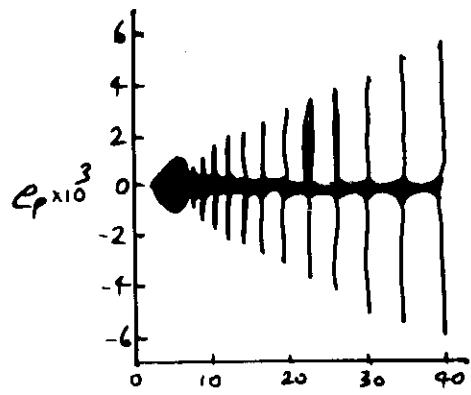
$\begin{array}{ccccccc} L & L & L & L & L \\ T & T & T & T & T \\ L & L & L & L & L \\ T & T & T & T & T \\ L & L & L & L & L \\ T & T & T & T & T \\ L & L & L & L & L \end{array}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\begin{array}{ccc} L & L & L \\ T & T & T \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array}$
TAYLOR LATTICE	DIPOLE ARRAY	

regard them as distributed in a dipole array Taylor lattice, rather than as in a dipole array. The latter distribution is more consistent with the weak-beam observations of Antonopoulos and Winter (Phil Mag 36 (1976) 87) and indeed, in the Taylor lattice there is no distinction between vacancy and interstitial dipoles! - whereas the weak-beam work clearly shows this distinction. One may submit that within the walls the dipoles are separated by something like twenty times their width. This causes Kuhlmann-Wilsdorf and Laird to underestimate the strength of the walls & dipole clumps.

4. All authors believe that the endurance strain amplitude, below which PSOs do not

31) form, is the hysteretic but reversible strain which can be tolerated by the matrix structure. This is accomplished partly by 'flipping' of dipoles and partly by the motion of free screw dislocations. The dipole 'flipping' mechanism, originally suggested by Feltner (Phil Mag 13 (1968) 1329 and 13 (1968) 1219) thus plays an important role. This strain is much less than would be accomplished if all dipoles 'flipped' - as they would in the Taylor lattice.

5. We turn now to the remarkable observations of Neumann, recently reviewed by him in the AIAA book (ref p24). Neumann's observation is that in a fatigue test in which the stress level is slowly increased, the plastic strain amplitude does not increase monotonically, but shows 'bursts': alternating increments which end arise from an instability in the structure. He describes a model for this behavior based on the refinement of dipoles, and the notion that a free dislocation can break up a group of dipoles, thus generating more free



(10)

32) dislocations, as in a chain reaction. This produces a strain burst, but the strain decreases as the free dislocations find closely-spaced partners and stop contributing to the strain. This explanation is carried to the level where Neumann predicts the ratio of successive stresses at which bursts occur, and the ratio of strains in successive bursts. The mathematical development of this highly non-linear theory is a marvel of deduction from an economy of assumptions! - but it does contain an arbitrary function, not really physically determined, and the model is without reference to the observed dislocation distribution.

6. Mughabgi, in a series of papers, emphasizes the crucial role of the cross-slip of screw dislocations. This temperature-dependent process can account for the observed temperature dependence of the wall spacing. This very important factor is absent from all theories which emphasize only the dipole cleavage, the internal stresses, and other athermal contributions to the flow-stress. However, it is Mughabgi's view that cross-slip of screw dislocations and trapping of edge dislocations is not enough to achieve saturation, i.e. dynamic equilibrium of dislocations.

33) The assumption is therefore made that there is an athermal loss of very narrow edge dislocation dipoles, and that this is necessary to achieve equilibrium. The evidence for this mechanism is indirect: it may of course occur, but in my view it is not necessary for equilibrium. The dislocation density in a dipole array will achieve equilibrium when the number of free dislocations entering it equals the number leaving it: this condition seems not to require athermal annihilation.

7. The papers influenced by the Prigogine school utilize the already existing mathematical models, and try to identify dislocation processes in the various reaction terms.

The paper by Walgraef, Schiller and Aifantis (p 257, ref. p27). is an example. These authors consider the interchange between fast (mobile) dislocations, of density β_M and slow (pinning) dislocations, of density β_I . They write

$$\begin{aligned} \dot{\beta}_I + I \cdot f_I &= f(\beta_I) - \beta \beta_I + \gamma \beta_M \beta_I^2 \\ \dot{\beta}_M + I \cdot f_M &= \beta \beta_I - \gamma \beta_M \beta_I^2 \end{aligned} \quad \left\{ \text{eq. 1} \right.$$

These equations thus allow for pinning of

mobile dislocations by dislocation pairs (i.e. dipoles) in the non-linear term $\gamma \beta_M \beta_I^2$. They then identify the other terms on the right-hand sides, in particular the production of dislocations by plastic strain by the break-up of dipoles ($f(\beta_I)$) and a function $f(\beta_I)$ which corresponds to the movement of dipoles. The equations thus contain in an abstract way the known processes. And the equations admit of 'Turing' instability (spatial patterning) and 'Hopf' instability (temporal oscillations, or Neumann strain bursts). This is clearly a more important mathematical development.

7. Finally, I have written a paper* which attempts to combine and the ideas of cross-slip controlled annihilation of screw dislocation, and dipole refinement and obstacle multiplication. This very simple theory makes a prediction: the saturation shear and the characteristic strain in a persistent slip band should be proportional to one another in all materials and temperatures so far studied. It is still not known whether this is the case.

* Brown, L.M. 1980 'Discrete Modelling of Physical Systems' p 21.

35) 8. What do we expect of a theory? The main ingredients of a theory are known: the cross-slip of screw dislocations, and the trapping of screw dislocations and of edge dislocations to form dipoles. The Frank-Read source mechanism, which dissipates energy and controls the obstacle spacing is also well understood. From these ingredients must come: the saturation stress (endurance limit), the strain in a persistent slip band, the strain in the matrix, the characteristic wall spacing and some understanding of the temporal and spatial evolution of the structures. Is this too ambitious? Clearly, the only approach likely to be capable of this in the end is the non-linear reaction kinetic approach because it includes both space and time variables. Yet the mathematics must relate directly and simply to observable quantities.

The theory must predict something. This is a first-rate opportunity for a first-rate theorician!

(20)

15. THE INITIATION OF CRACKS.

In polycrystals, cracks are initiated at grain boundaries and at the intersection of persistent slip bands with the surface. For both cases, one observes the nucleation and slow growth of very short cracks, much shorter than the Griffith length at a stress corresponding to the endurance limit. (The observations are entirely based on apper, but are expected to be generally valid.) The fact that cracks which are much shorter than the thermodynamic minimum length grow slowly, and stay sharp, indicates that some sort of special internal stress must be developed by the persistent slip band, and furthermore that the dislocation energies must be taken into account when considering the growth of the crack.

INTERNAL STRESS and SURFACE STRESS of PSBs
We follow here essentially the treatment of Brown and Ogin (1984 Fundamentals of Deformation and Fracture in Polymers, C.U.P.). It is evident that the persistent slip band (p.s.b.) is accompanied by macroscopic internal stresses. If it

(21)

(+) contains a higher concentration of vacancy dipoles than the matrix, it will be strained in tension along the line DD'. This can be seen by representing the real dislocation dipoles by fictional dislocations F whose effect is equivalent to that of the dipoles. (Essentially this is like replacing a sheet of dipoles by a current loop in electromagnetism (ap13)) If the PSB contains a lower concentration of vacancy dipoles than the matrix, it will be in compression, and indeed Antonjukas, Brown and Winter (ref p20) supposed that the stresses would go from compression to tension as the strain cycling proceeds, and the matrix becomes inactive but dislocation dipoles accumulate in the PSB.

These macroscopic stresses do not affect the dislocation motion, because they exert no real shear stress on the primary dislocations.

It is extremely difficult to estimate their magnitude. If no secondary slip occurs in the PSB, then the tensile strain parallel to the band (the fibre strain) must be less than about 10^{-3} , assuming a latent hardening factor of 3 to propagate secondary slip through the highly anisotropic arrangement of primary dislocations. The most probable value

38) of this strain is rather less, and indeed if it changes sign from compression to tension it will be zero at some stage during the fatigue test.

In addition to this steady strain, Ogin, in experiments on AgCl in a multiply slip orientation, noted the existence of an alternating component in one which goes from tension to compression and back again as the cycling proceeds, due to the PSB orientation coinciding exactly with the primary slip plane. So Brown and Ogin write

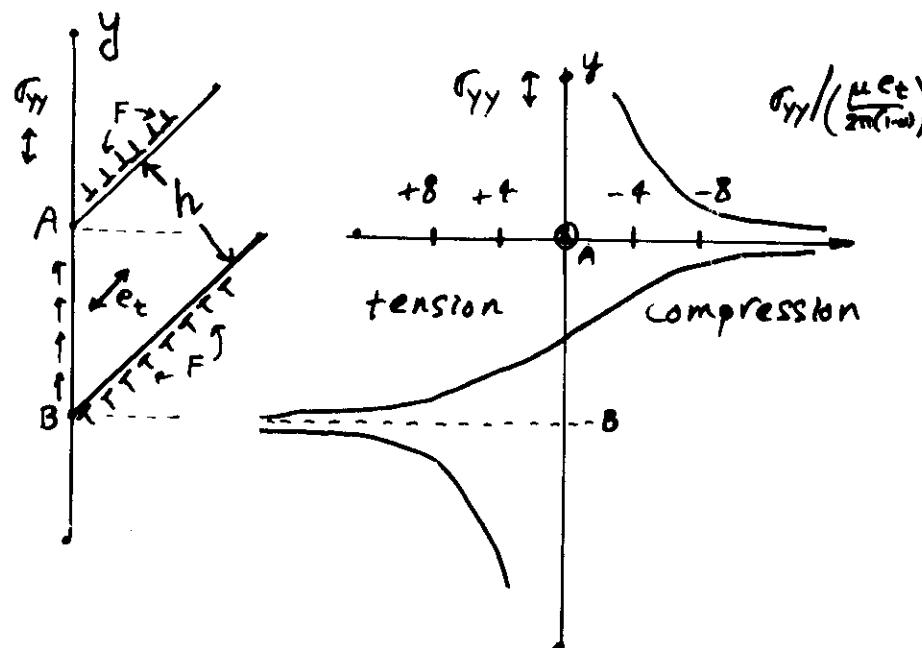
$$e_t = e_{DD'} = e_{\text{const}} + (\phi - \alpha) e_b' \quad 38.1$$

where $e_{\text{const}} \leq 10^{-3}$ is the compression a small DC strain, $(\phi - \alpha)$ is the (normally small) angular deviation between the PSB and the crystallographic shear plane, and e_b' is the plastic strain in the band.

What evidence is there for the existence of these stresses? (1) Ogin's work on stress birefringence in AgCl, which directly shows the alternating component (2) The observation of secondary slip, late in the life of the band, which show that e_t gets large enough to produce plasticity. (3) The bulges or

protrusions at the surface of the PSB, often around $10\mu m$ high in crystals about 1 cm in width, suggesting a strain of $\approx 10^{-3}$.

Now, whatever the value or sign of this fibre stress, σ_{00} or σ_t , it produce a logarithmically infinite surface stress where the PSB meets the external surface of the material. The figure shows the frictional



dislocations of $\pm t$ meeting the surface. The only self-balanced surface stress which can exist in this plane stress problem is σ_{yy} . If the bond is in tension, friction on the surface can be imagined to produce a compressive

- 10) Stress at A and a tensile stress at B, as shown. In the neighbourhood of the points A & B, the stress is given by

$$\sigma_{yy} = \frac{2\mu}{\pi(1-\nu)} \epsilon_t \ln \frac{y+t}{h} \quad \text{for } y > 0$$

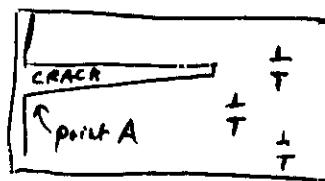
and goes to infinity as $y \rightarrow 0$.

Thus, there is the internal shear distribution which can promote surface cracking. Interestingly, when the cracks run in from A or B, so decohesion occurs, the logarithmic infinity is lost and in the absence of the applied stress the stress distribution become well-behaved, with no singularities. And so we have in essence answered the second question on p 13 : How are cracks produced by cyclic strain on an otherwise smooth polished surface?

SOME DETAILED OBSERVATIONS

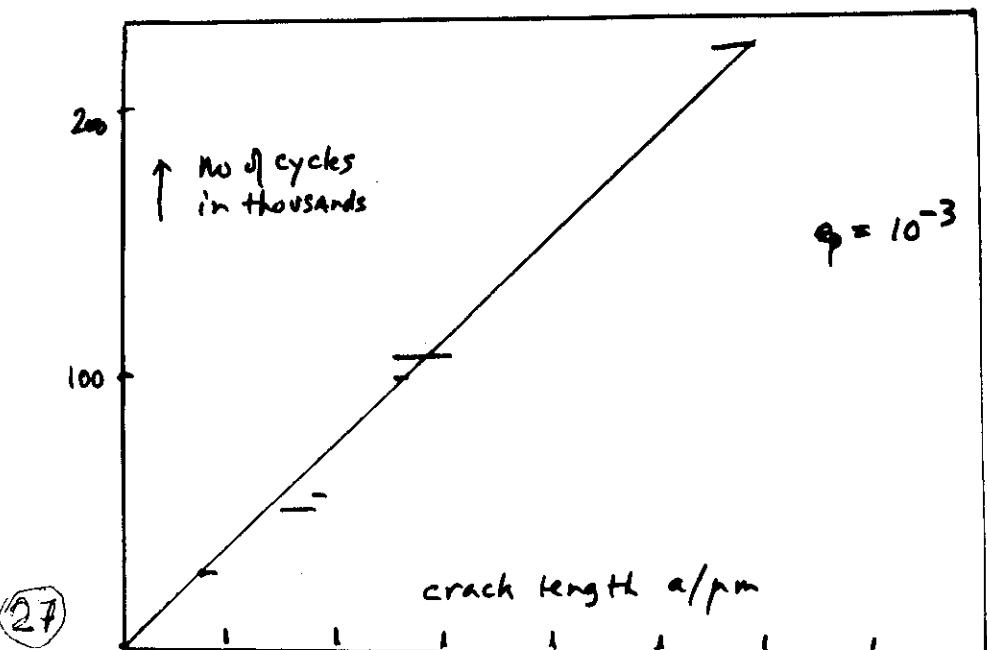
Recently, a wealth of observations have become available. One very pertinent principle must be borne in mind when looking at the results: these very short cracks are always extremely sharp, with no evidence of plastic blunting. Yet copper is a ductile metal. How is this? The answer is simple: at the saturation

41) Stress, the Griffith crack length is about $100\text{ }\mu\text{m}$, or about 60 wall spacings. The cracks under observation are much shorter than this. So it is energetically unfavourable to produce any dislocations or plastic flow of any sort from the tip of the crack. Yet the cracks grow: why is this? They grow by absorbing dislocations from the persistent slip band. Perhaps the easiest mechanism to imagine is that dipoles get swept into the crack, and since the dipoles are mainly vacancy type, the crack extends by one Burgers vector length each time a dipole is swept into it. The energy of the system is lowered because the energy of the dipole is lost. And, of course environmental effects are important.



Very striking observations are reported by Hunsche and Neumann (see ref. Neumann p13, also Acta Met. 1983). They observe that cracks start at the lines of intersection of the PSBs with the surface, early in the test at point A and later in the test at point B. This is most readily explained on the basis of Antropoulos Brown and Winstone's idea that the band

42) stress changes from compression to tension as the cyclic straining proceeds. However a note of caution is needed. The structure of the PSBs late in the test is modified by secondary slips, so the PSBs themselves widen, and the distinction between tension and compression may be lost. A beautiful series of observations by Basiński and Basińska (Acta Met. 1985; see also Fundamentals of Deformation and Fracture, eds Bilby et al. CUP 1984, review no very systematic sites of the band for nucleation. But the studies show very clearly the slow growth of these very short cracks, by about one atomic spacing per cycle.



+3) Evidently, it is this slow growth which controls the number of cycles to failure of defect-free metals. If the Griffith length (controlled, via the / PSB structure by the cross-slip of screw dislocations) is about 100 μm , and the cracks grow by 1 \AA per cycle, the crystal will last about 10^6 cycles with sub-threshold cracks.

I've made end with two further comments:

i) In polycrystals, studies by Nagelski's group (see ref. bottom p19) show that cracks arise from the impingement of PSB's at grain boundaries. Clearly, the bulges or protrusions associated with the bond chain C_2 cannot be easily accommodated at boundaries, so cracking occurs there - particularly, perhaps at points where the boundary meets the external surface - as a result of the local stress. But once this is relieved, slow growth will ensue, and the cycles to failure of the polycrystal may not be very different from that for the single crystal. Very similar observations have been made by Figueredo and Laird (Nat Sci and Eng. 60 (1983) 45).

4) ii) There is the very real possibility of some realistic and elegant theoretical modelling. Because the cracks are shorter than the Griffith length, the complications of plasticity can be ignored. And very powerful path integrals can be used to calculate the energy release rates associated with the surface cracking. In an appendix to the paper by Brown and Srinivasan (pp56) E Shelly shows how the energy release rates can be calculated for points A and B (diagram, p39 and p17). By geometry, the energy release rate at point B is larger than that for A. This may partially explain Hunsche and Neumann's observation that the dangerous cracks start from B. And, in this area, there is - a first-class opportunity for a first class derivation!

45)

TABLE OF CONTENTS

- Fatigue: Definition, p1
 Stress intensity: p2
 Griffith crack p2
 Relation to dislocations p3
 Formulae p4
 Summary of results on crack growth p5
 Stage II (Paris Law) p6
 Wöhler or S-N curves p8
 Coffin-Manson Law p8
 Explanation of the Paris Law p9
 Explanation of Threshold Stress Intensity p10
 Explanation of Coffin-Manson Law p11
 Success & Failure of Fracture Mechanics p12
 Cyclic Plasticity p14
 Persistent Slip Phenomenon p16
 Nucleation p18
 Two phase model p19
 Structural Details p20
 Saturation Stress Effects Endurance Limit p22
 Hysteresis Loops p24
 Behaviour of Polycrystals p25
 Explanations for Dislocation Structures p27
 Winter p27
 Kuhlmann-Wilsdorf & Laird p29

) Neumann	p 31
Mughrabi	p 32
Prigogine school	p 33
Brown	p 39
THE INITIATION OF CRACKS p 36	
Internal Stress and Surface Stress p 36	
Detailed Observations	p 38
Hunsche & Neumann	p 41
Basinski & Basinski	p 42
Mughrabi	p 43
Theory	p 48

L.M.B.
 JUNE 1987.