Coherent interactions of high energy particles with nuclei

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Outline :

- Glauber
- Model independent approach
- C.I. in the additive quark model

- Emphasis on the fragmentation of the projectile
- Nucleus as a tool to study the properties of weakly bound states

⇒ <u>Coherent processes</u> : nucleus remains in its ground state

$$\frac{d\sigma_A}{d\Omega}\Big|_{\text{coherent}} = |\langle i|T_A|i\rangle|^2$$

characterized by $|t| < \frac{1}{|R_A|^2}$

+ for production $|t_{min}| = q_e^2 = \left(\frac{m_b^2 - m_a^2}{2E_{inc}}\right)^2$

⇒ <u>Nucleus is excited but its final state is not observed</u> (incoherent process)

$$\frac{d\sigma_A}{d\Omega}\Big|_{\text{inch}} = \sum_{f\neq i} \langle i|T|f\rangle\langle f|T^\dagger|i\rangle$$

$$= \langle i||T|^2|i\rangle - \langle i|T|i\rangle^2$$

Typically $|t| > \frac{1}{R_A^2}$

+ <u>Glauber approach</u>

Interaction with nuclei described by

$$S_A = \prod_{i=1}^{A} S_i \rightarrow \quad t_A = 1 - \prod_{i=1}^{A}(1-t_i)$$

<u>1 channel case</u>

Elastic amplitude $\quad aA \rightarrow aA$

$$f_A(\vec{q}) \equiv \langle in|t_A|in\rangle = \frac{iK}{2\pi}\int d^2b\, e^{i\vec{q}\cdot\vec{b}}\left(1-\left[1-\frac{\sigma_T}{2}\frac{T(b)}{A}\right]^A\right)$$

$\frac{\sigma_T}{2}\frac{T(b)}{A}$ comes from $\frac{1}{A}\int \rho(\vec{s},z)\,T(\vec{b}-\vec{s})\, d^2s\, dz$

with $T(b)$: F.T. in the transverse plane of the elastic scatt. amplitude on a single nucleon

optical form : $\left[1-\frac{\sigma_T}{2}\frac{T(b)}{A}\right]^A \simeq e^{-\frac{\sigma_T}{2}T(b)}$

$$\Rightarrow \begin{cases} \sigma_{tot}^A = 2\int d^2b\left(1-e^{-\frac{\sigma_T}{2}T(b)}\right) \\ \sigma_{el}^A = \int d^2b\left(1-e^{-\frac{\sigma_T}{2}T(b)}\right)^2 \end{cases}$$

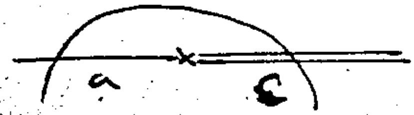
Parameter free expressions Comparison with experiment O.K. for incoming energies $< 30 \div 40$ GeV

To study production one needs to consider at least two channels. Taking into account that the transition amplitude is small with respect to the elastic one, one will write (for coherent prod.)

$$t^A_{ac} = \sum_{i=1}^{A} t^N_{ac} \prod_{j\neq i} (1 - t^N_j)$$

$$\Rightarrow f^A_{ac}(\vec{q}) = \int f^N_{ac}(0)\, e^{i q_e z}\, e^{i \vec{q}_\perp \cdot \vec{b}} \cdot \rho(b,z)\, e^{-T(b;-\infty,z)\frac{\sigma_a}{2}}\, e^{-T(b;z,+\infty)\frac{\sigma_c}{2}}\, d^2b\, dz$$

with $T(b;-\infty,z) \equiv \int_{-\infty}^{z} \rho(b,z')\, dz'$



If $q_e z < 1$ $(E_{inc} > R_A(m_c^2 - m_a^2))$ all points along the z direction interfere constructively. Neglecting $q_e z$ one may write:

$$f^A_{ac}(\vec{q}) = \frac{2 f^N_{ac}(0)}{\sigma_a - \sigma_c} \int d^2b\, e^{i\vec{q}\cdot\vec{b}} \left(e^{-\frac{\sigma_c}{2}T(b)} - e^{-\frac{\sigma_a}{2}T(b)} \right)$$

If $q_c z > 1$ $(E_{inc} < R_A(m_c^2 - m_a^2))$ the coherence along the z direction is destroyed. Summing over all possible final nuclear states one then obtains for the incoherent production:

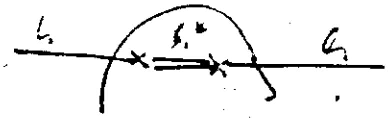
$$\left.\frac{d\sigma^A_{ac}}{dt}\right|_{t=t_{min}} = \left.\frac{d\sigma^N_{ac}}{dt}\right|_{t=t_{min}} \int d^2b\, \frac{e^{-\sigma_c T(b)} - e^{-\sigma_a T(b)}}{\sigma_a - \sigma_c}$$

- Relations obtained by Kölbig and Margolis
- The...

The values of σ_c obtained in this way from experiment are not consistent

1) σ_c small for states with many particles
2) σ_c from coherent production $\neq \sigma_c$ from incoh.

However
Treating perturbatively the off diagonal elements in the T matrix one may write the lowest order correction to the total (and elastic) cross section due to the presence of the inelastic intermediate states



$$\Delta\sigma_{tot} = -4\pi \int d^2b \, T^2(b) \left.\frac{d\sigma^N}{dq^2}\right|_{q^2=0} e^{-\frac{\sigma_{tot}}{2} T(b)}$$

(where one moreover assumes $\sigma_{h^*} = \sigma_h$)
The expression (derived by Karmanov and Kondratyuk) is parameter free and agrees well with experiment up to 300 GeV.

So

- Elastic and total cross section well described by the above formalism
- Production no

- The eigenstate method by Miettinen and Pumplin to describe diffraction in hadron-hadron collisions

Look for the eigenstates of the imaginary part of the scattering amplitude operator

$$\mathrm{Im}\, T|\alpha\rangle = t_\alpha |\alpha\rangle$$

t_α is the probability for the state $|\alpha\rangle$ to interact with the target. t_α is a function of the impact parameter. One may always write

$$|a\rangle = \sum_\alpha a_\alpha |\alpha\rangle$$

Elastic amplitude

$$\langle a|\mathrm{Im}\, T|a\rangle = \sum_\alpha |a_\alpha|^2 t_\alpha = \langle t\rangle$$

$$\Rightarrow \frac{d\sigma_{el}}{d^2b} = \langle t\rangle^2, \quad \frac{d\sigma_{tot}}{d^2b} = 2\langle t\rangle$$

Diffractive cross section:

$$\frac{d\sigma_{diff}}{d^2b} = \sum_\alpha |\langle\alpha|\mathrm{Im}\, T|a\rangle|^2 - \frac{d\sigma_{el}}{d^2b}$$

$$= \sum_\alpha |a_\alpha|^2 t_\alpha^2 - \langle t\rangle^2 = \langle t^2\rangle - \langle t\rangle^2$$

$$\boxed{\left.\frac{d\sigma_{diff}}{d^2q}\right|_{q^2=0} = \frac{1}{16\pi}\left(\langle\sigma_\alpha^2\rangle - \langle\sigma_\alpha\rangle^2\right)}$$

with σ_α the total cross section of the eigenstate $|\alpha\rangle$ with the target.

The relevant observation is that the large value observed for σ_{diff} (~ 6.5 mb when $\sigma_{tot} = 43$ mb for pp at $\sqrt{s} = 53$ GeV) implies a large dispersion for σ_α

Given the large fluctuation of σ_α one may realise that the two channel approach is not adequate for production. One has however to understand why it works for the total (and the elastic) cross section.

+ Multichannel generalization of multiple scattering theory (N.N. Nikolaev)

In a region of space of the order of the nuclear dimension one does not need to consider separately of mass:

$$\int_{-R_A}^{R_A} e^{ik_1 z} e^{-ik_2 z}\, dz = \frac{2 \sin[(k_1-k_2)R_A]}{k_1-k_2}$$

1 and 2 same energy but different mass:

$$k_1^2 + m_1^2 = k_2^2 + m_2^2 \Rightarrow (k_1-k_2)(k_1+k_2) = m_2^2 - m_1^2$$

$$\Rightarrow k_1 - k_2 \simeq \frac{m_2^2 - m_1^2}{2E} \Rightarrow E \geq (m_2^2 - m_1^2) R_A$$

It is more convenient to work with the eigenstates of the T matrix in the forward direction. Each eigenstate is or absorbed or elastically scattered by the interaction. If the mass differences are such that $\boxed{E_{inc} > (m_2^2 - m_1^2) R_A}$ (condition necessary in order to have a coherent process) then there will be no mixing among the eigenstates caused by the propagation along the z direction.

For each eigenstate the interaction with the nucleus is then described by the simple 1 channel multiple scattering theory

$$\langle in | t_\alpha^A | in \rangle = 1 - \exp\left[-\frac{\sigma_\alpha}{2} T(b)\right]$$

$$\Rightarrow \boxed{\sigma_{tot}^A = 2\int d^2b \left(1 - \langle e^{-\frac{\sigma_\alpha}{2} T(b)} \rangle\right)}$$

[Notice that $\frac{k\sigma_T}{4\pi} = \langle a | \mathrm{Im}\, T | a \rangle = \sum_\alpha |a_\alpha|^2 t_\alpha = \frac{k}{4\pi} \sum_\alpha |a_\alpha|^2 \sigma_\alpha$

$\Rightarrow \sigma_T = \langle \sigma_\alpha \rangle$]

The 1 channel approach gives

$$\boxed{\sigma_{tot}^A = 2\int d^2b \left(1 - e^{-\frac{\sigma_T}{2} T(b)}\right)}$$

The difference exact − approximate is then :

$$\Delta\sigma_{tot}^A = 2\int d^2b \left(\langle e^{-\frac{\sigma_\alpha}{2} T(b)} \rangle - e^{-\frac{\langle\sigma_\alpha\rangle}{2} T(b)} \right)$$

Expanding the exponentials one may write

$$\Delta\sigma_{tot}^A = 2\int d^2b\, T^2(b)\, \frac{1}{8}\left[\langle\sigma_\alpha\rangle^2 - \langle\sigma_\alpha^2\rangle\right] e^{-\frac{\langle\sigma_\alpha\rangle}{2} T(b)} + O\left[T^3(b)\langle\sigma_\alpha^3\rangle\right]$$

and since

$$\left.\frac{d\sigma^N_{diff}}{dq^2}\right|_{q^2=0} = \frac{1}{16\pi}\left[\langle\sigma_\alpha^2\rangle - \langle\sigma_\alpha\rangle^2\right]$$

one may write

$$\Delta\sigma_{tot}^A = -4\pi \left.\frac{d\sigma^N_{diff}}{dq^2}\right|_{q^2=0} \int d^2b\, T^2(b)\, e^{-\frac{\langle\sigma_\alpha\rangle}{2} T(b)} , + \ldots$$

The correction to the total (and elastic) cross section is then the one computed by Karmanov and Kondratyuk.

Consider the coherent production:
the exact expression is given by

$$t^A_{ac}(b) = -\sum_\alpha a_\alpha c^*_\alpha \, e^{-\frac{\sigma_\alpha}{2}T(b)}$$

while the two channel approximation gives

$$t^A_{ac}(b) = \frac{2}{\sigma_c-\sigma_a} f^N_{ac}(0)\left[e^{-\frac{\sigma_a}{2}T(b)} - e^{-\frac{\sigma_c}{2}T(b)}\right]$$

If $\sigma_\alpha - \langle\sigma_\alpha\rangle$ is small (so also $\sigma_c \simeq \sigma_a + \varepsilon$) one may write

$$t^A_{ac}(b)\Big|_{\text{approx}} = \frac{2 f^N_{ac}(0)}{\varepsilon}\, e^{-\frac{\sigma_a}{2}T(b)}\,\frac{\varepsilon}{2}T(b) = \boxed{f^N_{ac}(0)T(b)\,e^{-\frac{\sigma_a}{2}T(b)}}$$

for the two channel approximation,
and for the exact expression

$$\begin{aligned}
t^A_{ac}(b)\Big|_{\text{exact}} &= -\sum_\alpha a_\alpha c^*_\alpha\, e^{-\frac{\sigma_\alpha-\langle\sigma_\alpha\rangle+\langle\sigma_\alpha\rangle}{2}T(b)} \\
&= e^{-\frac{\langle\sigma_\alpha\rangle}{2}T(b)}(-)\sum_\alpha a_\alpha c^*_\alpha\left[1+\frac{1}{2}(\langle\sigma_\alpha\rangle-\sigma_\alpha)T(b)\right] \\
&= T(b)\,e^{-\frac{\langle\sigma_\alpha\rangle}{2}T(b)}\sum_\alpha a_\alpha c^*_\alpha \frac{\sigma_\alpha}{2} = \boxed{f^N_{ac}(0)T(b)\,e^{-\frac{\langle\sigma_\alpha\rangle}{2}T(b)}}
\end{aligned}$$

One may then recover the approximate result assuming that the fluctuation of σ_α is small.
But the experimental evidence is that the fluctuations are large. In particular one finds an eigenstate with $\sigma_\alpha \simeq 0$ (passive state)
[in their analysis of diffractive dissociation in pp scattering Miettinen and Pumplin find a probability for the passive state of 5%]

- What is measured in diffractive production in nuclei ?

The differential cross section corresponding to a single inelastic scattering can be written as

$$\frac{d\sigma_{\alpha c}^{A}(\text{inel})}{dq^2} = \frac{1}{4\pi}\int d^2b\, T(b)\left|\sum_\alpha c_\alpha^+ a_\alpha f_\alpha(\vec q)\, e^{-\frac{\sigma_\alpha}{2}T(b)}\right|^2$$

$$\left.\frac{d\sigma_{\alpha c}^{A}(\text{coherent})}{dq^2}\right|_{q^2=0} = \frac{1}{4\pi}\left|\int d^2b\left[\sum_\alpha c_\alpha^+ a_\alpha\left(1-e^{-\frac{\sigma_\alpha}{2}T(b)}\right)\right]\right|^2$$

Introducing the moments $\boxed{M_m = \sum_\alpha c_\alpha^+ a_\alpha(\sigma_\alpha - \sigma_I)^m}$ one may write

$$\sum_\alpha c_\alpha^+ a_\alpha \sigma_\alpha e^{-\frac{\sigma_\alpha}{2}T(b)} = e^{-\frac{\sigma_I}{2}T(b)}\sum_{m=1}^{\infty}\frac{(-1)^{m-1}}{2^m m!}M_m T(b)^{m-1}\left(2m-\sigma_I T(b)\right)$$

$$\sum_\alpha c_\alpha^+ a_\alpha e^{-\frac{\sigma_\alpha}{2}T(b)} = e^{-\frac{\sigma_I}{2}T(b)}\sum_{m=1}^{\infty}\frac{(-1)^m}{2^m m!}M_m T^m(b)$$

One is then measuring the moments M_m

To measure $\boxed{\sigma_I = \sum_\alpha |c_\alpha|^2\sigma_\alpha}$ one needs to measure all the moments M_m

An analysis on the diffractive produced $p\pi$ system in the mass range 1.35 GeV – 1.45 GeV in terms of M_m has been carried on by Nikolaev using the data by Mollet et al the Kolbig-Margolis method gave $\sigma_I = 27 \pm .. mb$

One obtains $M_1 = 3.5$ mb, $M_2 = 40 \pm 15$ mb^2 and $M_3 < 10^{-3}$ mb^3.

Any state $|c\rangle$ normalised to unity, reproducing M_1, M_2 and giving $M_i = 0$ for $i \geq 3$, satisfying the orthogonality relation $M_0 = \sum_k c_k^+ a_k = 0$ will describe the data.

One is not able therefore to determine the cross section σ_c of the produced state.

In the framework of a specific model one may however impose further constraints.

Take the additive quark model:

- only valence quarks
- each quark is active with a probability p_q and passive with probability $1-p_q$

The probability for the passive state is then

$$|c_0|^2 = (1-p_q)^{N_q} \qquad (N_q = \text{n. of quarks})$$

one then may write:

$$|c_1|^2 = N_q (1-p_q)^{N_q-1} p_q$$

Using these relations and the values of a_α given by the analysis by Miettinen and Pumplin Nikolaev finds $p_q \simeq .6 \div .65$ and $\sigma_c > 43$ mb

In the framework of this additive quark model one may easily write other relations

$$\sigma_{Gh}^{aA} = \int d^2b \left\{ \langle a | \left(1 - e^{-\frac{\sigma}{2}T(b)}\right)^2 | a \rangle - \langle a | \left(1 - e^{-\frac{\sigma}{2}T(b)}\right) | a \rangle^2 \right\}$$

$$\doteq \int d^2b \left\{ \prod_{i=1}^{n} \langle e^{-\sigma_q T(b)} \rangle - \prod_{i=1}^{n} \langle e^{-\frac{\sigma_q}{2} T(b)} \rangle^2 \right\}$$

If there are only two states for each quark one has

$$\langle e^{-\sigma_q T(b)} \rangle = (1 - p_q) + p_q \, e^{-\frac{\sigma_{tot}^{qN}}{p_q} T(b)}$$

$$\Rightarrow \langle e^{-\sigma_q T(b)} \rangle \xrightarrow[A \to \infty]{} 1 - p_q$$

$$\Rightarrow \sigma_{Gh}^{aA} \xrightarrow[A \to \infty]{} \int d^2b \left[(1 - p_q)^n - (1 - p_q)^{2n} \right] \theta(R_A - b)$$

$$\boxed{\sigma_{Gh}^{aA} = \pi R_A^2 (1 - p_q)^n \left[1 - (1 - p_q)^n \right]}$$

- Notice that here one has coherent production not only from the edge of the nucleus, but also from its centre with probability proportional to $(1 - p_q)^n$.
- One may also notice that the passive state can have a different origin: it can appear on account of the fact that the quarks in the hadron approach each other closely and the size of the region of the color field decreases.

In principle the ratio $\sigma_{Gh}^{\pi A} / \sigma_{Gh}^{pA}$ should be able to distinguish between the two approaches.

SUMMARY

- One may suitably describe production processes on nuclei at small t with the eigenstate method

- The experimental information can be expressed in a model independent way in terms of the moments M_{mn} both for coherent and for incoherent production

- In the framework of a model the moments M_{mn} can be related to the properties of the hadronic structure