PHIL RATCLIFFE MILANO, JUNE

NUCLEAR SUPERRADIANCE: A THEORY OF THE EMC EFFECT

1. THE EMC EFFECT

i) THE EXPECTATIONS – AF, FERMI MOTION
ii) THE EXPERIMENTAL RESULTS
iii) THE "EXPLANATIONS"

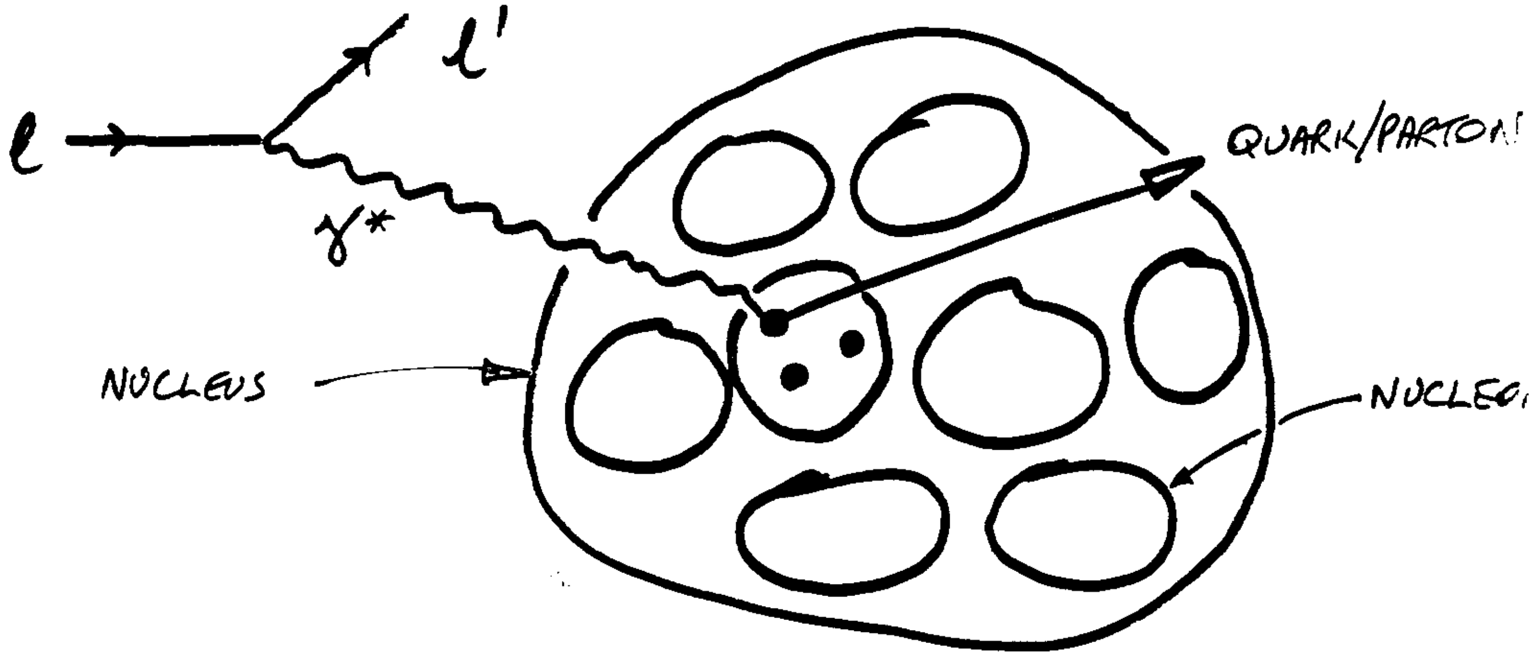
2. NUCLEAR SUPERRADIANCE

i) INTRODUCTION TO SUPERRADIANCE
ii) RECAP OF SHELL MODEL OF NUCLEUS
iii) A SUPERRADIANT SOLUTION

3. NUCLEAR DEEP-INELASTIC SCATTERING

i) THE EMC EFFECT – MASS RESCALING
ii) "ANTI-SHADOWING" – THE RÔLE OF PIONS
iii) "SHADOWING" – SINGLE DIFFRACTION & PAULI BLOCKING.

CONSIDER DEEP-INELASTIC SCATTERING OF A HIGHLY VIRTUAL PHOTON OFF A NUCLEON BOUND IN A NUCLEUS:



IN THE QUARK/PARTON MODEL WE HAVE, SCHEMATICALLY:

$$\underbrace{\sigma(\gamma N)}_{\text{EXPERIMENTAL CROSS-SECTION}} = \underbrace{F_2(x_B)}_{\text{QUARK/PARTON DISTRIBUTION}} \otimes \underbrace{\sigma(\gamma q)}_{\text{POINT-LIKE PHOTON-QUARK CROSS-SECTION}}$$

PHOTON $\quad q^2 = -Q^2 < 0$

NUCLEON $\quad p^2 = M^2$

$$x_B = \frac{Q^2}{2p.q}$$

IN NUCLEON REST-FRAME $\quad p.q = M\nu\,,\quad \nu =$ PHOTON ENERGY

THIS PICTURE IMPLIES THAT, WHILE QUARKS ARE CONFINED INSIDE HADRONS, THEIR INTERACTIONS WITH THE E.M. PROBE OCCUR AS IF THEY WERE FREE OBJECTS.

THIS VIEW IS BORNE OUT BY THE VAST AMOUNT OF DATA ON NUCLEON SCATTERING.

IT FINDS VINDICATION IN THE NOTION OF

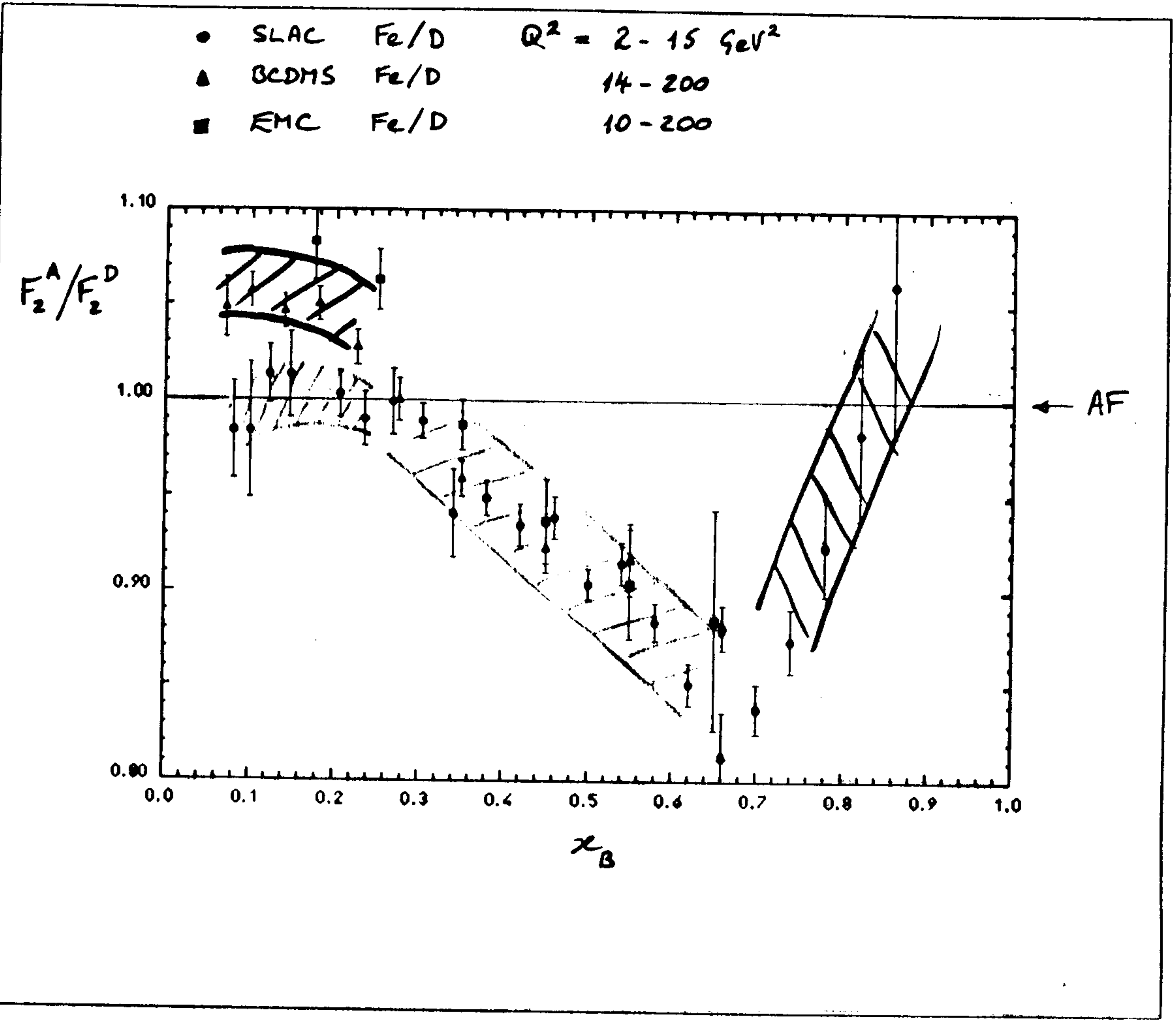
ASYMPTOTIC FREEDOM:

IN QCD (THE UNIVERSALLY ACCEPTED THEORY OF STRONG INTERACTIONS), WHICH HAS THE PECULIARITY OF BEING NON-ABELIAN THE PERTURBATIVE BEHAIOUR OF THE COUPLING α_s IS SUCH THAT AT HIGH ENERGIES

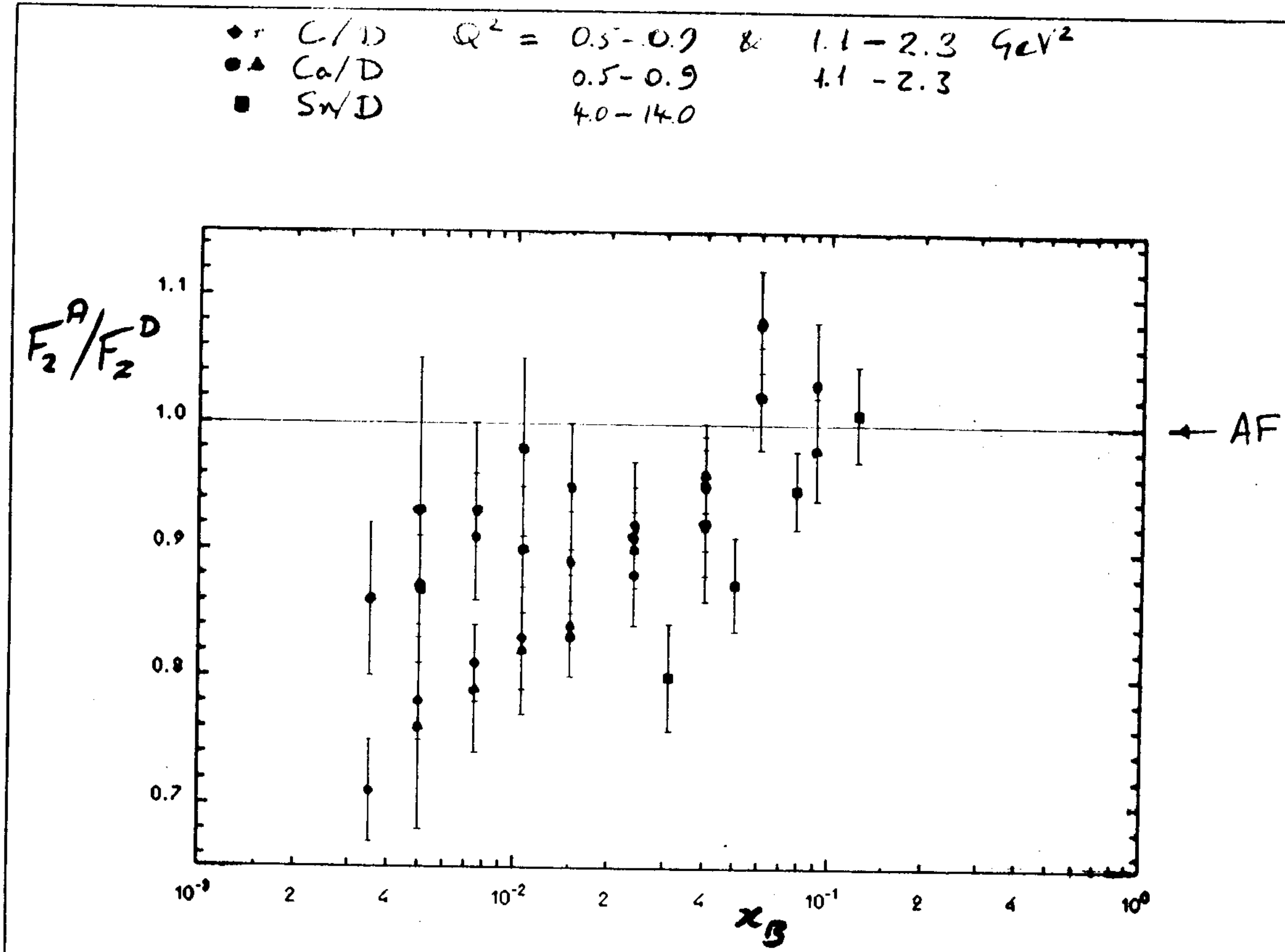
$$\alpha_s \sim \frac{1}{\ln Q^2} \longrightarrow 0$$

IF THE QUARK IS FREE INSIDE THE NUCLEON WHAT DOES IT CARE ABOUT BEING INSIDE A NUCLEUS ?

$$\Longrightarrow \sigma(\gamma A) = A\,\sigma(\gamma N) \Longrightarrow F_2^A(x_B) = F_2^N(x_B)$$



SLAC Fe/D
BCDMS Fe/D
EMC Fe/D
Q² = 2 - 15 GeV²
14 - 200
10 - 200
F₂ᴬ/F₂ᴰ
1.10
1.00
0.90
0.80
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
x_B
AF



C/D Q^2 = 0.5 - 0.9 & 1.1 - 2.3 GeV²
Ca/D 0.5 - 0.9 1.1 - 2.3
Sn/D 4.0 - 14.0
F_2^A/F_2^D
1.1
1.0
0.9
0.8
0.7
← AF
10^{-3}
2
4
10^{-2}
2
4
10^{-1}
2
4
10^{0}
x_B

THUS THE DATA SHOW FOUR DISTINCT REGIONS:

i) LARGE $x_B \sim 1$, $R > 1$.

(THIS <u>IS</u> EXPECTED DUE TO FERMI MOTION SMEARING OF THE DISTRIBUTIONS)

ii) $0.2 \lesssim x_B \lesssim 0.6$, $R < 1$, <u>THE</u> EMC EFFECT

INDEPENDENT OF TARGET (A) AND OF ENERGY (Q^2), I.E., SCALING.

iii) $x_B \sim 0.2$, $R > 1$, "ANTI-SHADOWING".

INDEPENDENT OF TARGET BUT ONLY PRESENT AT LARGE Q^2.

iv) $x_B \lesssim 0.2$, $R < 1$, "SHADOWING"

ONLY MEASURED AT RELATIVELY SMALL Q^2 FOR KINEMATIC REASONS ($x_B = Q^2/2M\nu$), APPEARS TO SCALE THOUGH.

PERSISTS TO HIGHER VALUES OF x_B FOR LARGER A.

RESCALING – Q^2 (CLOSE, ROBERTS & ROSS)

IN PERTURBATIVE QCD THE CALCULATED Q^2 DEPENDENCE OF F_2 IS SUCH THAT IT SOFTENS FOR LARGER Q^2. THUS IF

$$F_2^A(x, Q^2) = F_2^N(x, \xi_A Q^2), \quad \xi_A > 1$$

THEN FOR LARGE x, $F_2^A < F_2^N$

THE RESCALING "MECHANISM" IS A CHANGE OF THE CONFINEMENT RADIUS: $R_N \to R_A$, THEN THE EFFECTIVE WAVELENGTH OF THE PROBE BECOMES $\lambda' = \lambda R_N/R_A$ AND SO $\xi_A = (R_A/R_N)^2$.

IN ADDITION THE INCREASED CONFINEMENT VOLUME ALLOWS EXTRA SOFT-GLUON RADIATION AND FINALLY

$$\xi_A = \left(R_A^2/R_N^2\right)^{\alpha_s(Q_0^2)/\alpha_s(Q^2)}$$

FREE PARAMETER

PIONS, x_B-RESCALING, CLUSTERING

THESE WORK, BUT ARE ALL AD HOC, WITH FREE PARAMETERS TO BE FIT.

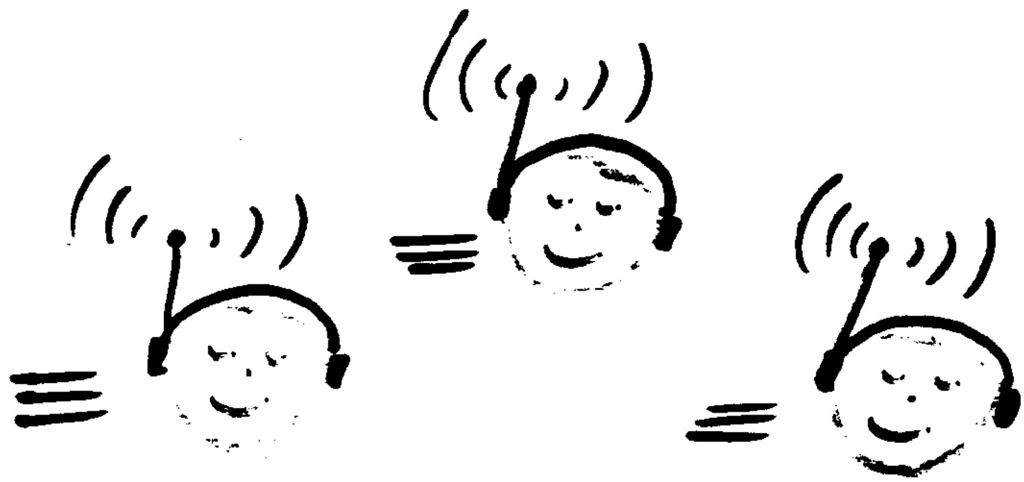
PARTON-FUSION (NIKOLAEV AND ZAKHAROV)

SMALL-x PARTONS OF DIFFERENT NUCLEONS FUSE

2. NUCLEAR SUPERRADIANCE

SUPERRADIANCE: (PREPARATA, FOLGARIA 1989)

THE IDEA OF SUPERRADIANCE, AS APPLIED TO CONDENSED MATTER, IS TO TAKE IN ACCOUNT COLLECTIVE MOTION AS CONTROLL BY A BACKGROUND COHERENT E.M. FIELD, WHICH CAN INDEED DOMINATE THE DYNAMIC OF SUFFICIENTLY LARGE NUMBERS OF SUFFICIENTLY DENSE SUBSYSTEMS.



OSCILLATING CHARGES (DIPOLES) GENERATE AN E.M. FIELD TO WHICH THEY ALSO REACT - IN SUITABLE CONDITIONS THE IN-PHASE OSCILLATING STATE IS STABL I.E., IT HAS A LOWER ENERGY THAN THE VACUUM AND IS THUS THE CORRECT GROUND STATE OF THE COLLECTIVE SYST

THE SHELL MODEL (PHYS. REP. 120 (1985) 1)

SINGLE PARTICLES MOVING INDEPENDENTLY IN A POTENTIAL TYPICALLY OF THE FORM

$$V(r) = \frac{V_0}{1 + \exp\left\{\frac{r - R_V}{a_V}\right\}} \qquad \text{(WOODS-SAXON)}$$

GEOMETRY FIXED, THE PARAMETERS ARE:

$$R_V = r_0 A^{1/3} \quad , \quad r_0 = 1.25 \text{ fm} \quad , \quad a_V = 0.65 \text{ fm}$$

MAGIC NUMBERS $\Longleftrightarrow$ SHELL CLOSURE

BY ANALOGY WITH ATOMIC PHYSICS EACH LEVEL CONTAINS $2(2\ell + 1)$ PARTICLES

$$\Rightarrow \; 2, 8, 20, 34, 40, 58, \ldots$$

IN NUCLEAR PHYSICS THE FIRST THREE ARE SEEN BUT ARE FOLLOWED BY 28, 50, 82, INVOKE A SPIN-ORBIT COUPLING:

$$V_{SO} = -a_{SO}^2 \frac{1}{r} \frac{\partial V}{\partial r} \; \vec{\ell} \cdot \vec{s}$$

SUPERRADIANT SOLUTION (PREPARATA, N. CIM. A, AUGUST)

THE BASIC IDEA IS TO CONSIDER TRANSITIONS $N \longleftrightarrow \Delta$ MEDIATED BY π.

INTRODUCE THE MATTER FIELDS:

$$\psi_N(\vec{x}, t) = N_{st}(\vec{x}, t)\, \chi_s \eta_t$$

$$\psi_\Delta(\vec{x}, t) = \Delta_{ST}(\vec{x}, t)\, \chi_S \eta_T$$

χ, η ARE SPIN, ISOSPIN PAULI SPINORS.

AND THE PION FIELD:

$$\phi_k(\vec{x}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{q}} \frac{1}{\sqrt{2\omega_q}} \left[a_{k\vec{q}}(t)\, e^{-i(\omega_q t - \vec{q}\cdot\vec{x})} + h.c. \right]$$

WHERE $\omega_q = \sqrt{\vec{q}^{\,2} + m_\pi^2}$.

IN TERMS OF THESE WE CAN WRITE THE NUCLEAR HAMILTONIAN AS A SUM OF FOUR PIECES:

$$H_N = H_{kin} + H_\pi + H_{int} + H_{SR}$$

THE LAGRANGIAN IS THEN

$$L(t) = \int_V d^3x \left\{ \psi_N^\dagger \frac{i\partial}{\partial t} \psi_N + \psi_\Delta^\dagger \frac{i\partial}{\partial t} \psi_\Delta \right\}$$

$$+ \sum_{\vec{q},k} \left[\frac{i}{2} \left(a_{\vec{q}k}^\dagger \dot{a}_{\vec{q}k} - a_{\vec{q}k} \dot{a}_{\vec{q}k}^\dagger + \frac{1}{2\omega_q} \dot{a}_{\vec{q}k}^\dagger \dot{a}_{\vec{q}k} \right] \right.$$

$$- H_{kin} - H_{int}$$

NOETHER CONSERVED QUANTITY:

$$N = \int_V d^3x \left[\psi_N^\dagger \psi_N + \psi_\Delta^\dagger \psi_\Delta \right] \equiv A$$ (ATOMIC NO

EULER LAGRANGE EQNS:

$$i \dot{N}_{st} = \left(m_N - \frac{\vec{\nabla}^2}{2m_N} \right) N_{st} - \frac{ig}{\sqrt{4 m_N m_\Delta}} \sqrt{\frac{A}{V}} \sum_{\vec{q}} C_{sS}^a C_{tT}^k \frac{q_a}{\sqrt{2\omega_q}}$$

$$\cdot \left[\alpha_{\vec{q}k} e^{-i(\omega_q t - \vec{q}\cdot\vec{x})} - \alpha_{\vec{q}k}^* e^{i(\omega_q t - \vec{q}\cdot\vec{x})} \right] \Delta_{ST}$$

WHERE

$$a_{\vec{q}k} \rightarrow \sqrt{A}\, \alpha_{\vec{q}k}$$ AND $$C_{sS}^a = \chi_S^{\dagger\, ijk} \epsilon_{ir} (\sigma^a)_j^r \chi_{s,k}$$

+ Δ + π EQNS.

THESE EQUATIONS ARE DRASTICALLY SIMPLIFIED BY LOOKING FOR "MEAN"-FIELD SOLUTIONS IN WHICH THE SPACE DEPENDENCE IS ONLY WEAK. THUS WE ARE INTERESTED IN COHERENCE DOMAINS WHOSE SIZE IS OF THE ORDER OF THE WAVELENGTHS ASSOCIATED WITH THE IMPORTANT MOMENTUM COMPONENTS OF THE π-FIELD.

THUS WE RETAIN ONLY THE RESONATING π-MODE:

$$\omega_q = \sqrt{\vec{q}^{\,2} + m_\pi^2} = m_\Delta - m_N$$

$$\text{i.e.} \quad \omega_q \sim 300 \text{ MeV}$$

IN THIS APPROXIMATION THE COUPLED π-MODES ARE P-WAVES AND SO THE SIZE OF THE COHERENCE DOMAINS CAN BE ESTIMATED FROM THE FIRST ZERO OF THE SPHERICAL BESSEL FUNCTION $J_1(qr)$ WITH $q = 256$ MeV:

$$R_{CD} \simeq \frac{3\pi}{2q} \simeq 3.7 \text{ fm} \longrightarrow \simeq 4.5 \text{ fm}$$

WITH FINITE PION RADIUS

SUCH A DOMAIN CONTAINS ~ 70 NUCLEONS

THE EQUATIONS ARE NOW: $(\tau = \omega_q t)$

$$\dot{n}_{st} = -G\, C^a_{sS}\, C^k_{tT}\, \alpha^*_{ka}\, \delta_{ST}$$

$$\dot{\delta}_{ST} = G\, C^a_{S s}\, C^k_{Tt}\, \alpha_{ka}\, n_{st}$$

$$\dot{\alpha}_{k,a} + \frac{i}{2}\ddot{\alpha}_{k,a} = -\frac{3}{4\pi} G\, C^a_{sS}\, C^k_{tT}\, n^*_{st}\, \delta_{ST}$$

WHERE $G = \frac{\sqrt{2}\,\pi}{3}\, g\, \frac{q}{\sqrt{4 m_N m_\Delta}} \sqrt{\frac{A}{V}}\, \frac{1}{\omega_q^{3/2}}$

$\pi \Delta N$ COUPLING

$$N_{st} = n_{st}\, e^{-i m_N t}\sqrt{\tfrac{A}{V}}\,, \quad \Delta_{ST} = \delta_{ST}\, e^{-i m_\Delta t}\sqrt{\tfrac{A}{V}}\,, \quad \alpha_{k\vec{q}} = \alpha_{k,a}\left(\frac{q_a}{q}\right)$$

FOR SHORT TIMES WE CAN LINEARIZE TO:-

$$\ddot{\alpha}_{k,a} + \frac{i}{2}\dddot{\alpha}_{k,a} + \frac{3}{4\pi}\left(\frac{4}{3}\right)^2 G^2\, \alpha_{k,a} = 0$$

NOW FROM THE Δ DECAY RATE WE CAN ESTIMATE $G \simeq 1$ AND THUS

$$\tilde{g}^2 \equiv \frac{4}{3\pi} G^2 \simeq 0.52 < g_c^2 = 16/27$$

$\Rightarrow$ WEAK SUPERRADIANCE, I.E., NO RUNAWAY SOLUTIONS.

MINIMIZING THE ENERGY OF THE SYSTE[M]
AND SOLVING WE OBTAIN:

$$n^*_{st}\, n_{st} = \cos^2\theta \quad : \quad \theta = 0.44$$

$$\alpha^*_{k,a}\, \alpha_{k,a} = 0.084 \;,\; \omega = 84 \text{ MeV}$$

AND THE ENERGY DIFFERENCE PER NUCL[EON]
BETWEEN THE SUPERRADIANT STATE A[ND]
THE INITIAL STATE ($\cos^2\theta = 1$, ETC.) IS:

$$\frac{\Delta E}{A} \simeq -60 \text{ MeV}$$

INDEPENDENT OF A, THE ATOMIC N[UMBER]

RE-WRITING THE EQUATION FOR $i\dot{n}_{st}$
IN TERMS OF THE DERIVED QUANTITI[ES]
ONE FINDS A SPIN-ORBIT TERM:

$$i\dot{n} \simeq -\frac{\vec{\nabla}^2}{2m_N} n + \left[V_0(\vec{x}) + V_{so}(\vec{x})\, \vec{L}.\vec{s}\right] n$$

WHERE $V_{so} = -a_{so}^2 \frac{1}{r}\frac{\partial V_0}{\partial r}$ AND $a_{so} \simeq 0.8$ fm

3. NUCLEAR DEEP-INELASTIC SCATTERING

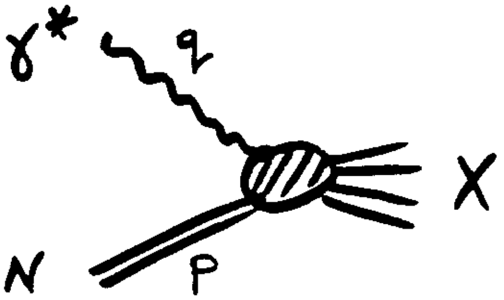
(G. PREPARATA & PGR, MITH 90/9)

LARGE-x_B EFFECT (THE EMC EFFECT)

THERE ARE TWO MAIN INGREDIENTS:

i) $\tilde{m} = m - 60 \text{ MeV}$: EFFECTIVE MASS

ii) FERMI MOTION: $p_F \simeq 264 \text{ MeV}$



IN NUCLEUS REST-FRAME

$$p = (E, \sin\theta\, \rho, 0, \cos\theta\, \rho), \quad q = (\nu, 0, 0, \sqrt{\nu^2+\dots}$$

c.f. $p = (M, 0, 0, 0)$

WHERE NOW: $E^2 = \rho^2 + \tilde{m}^2$ & $p \leq p_F$

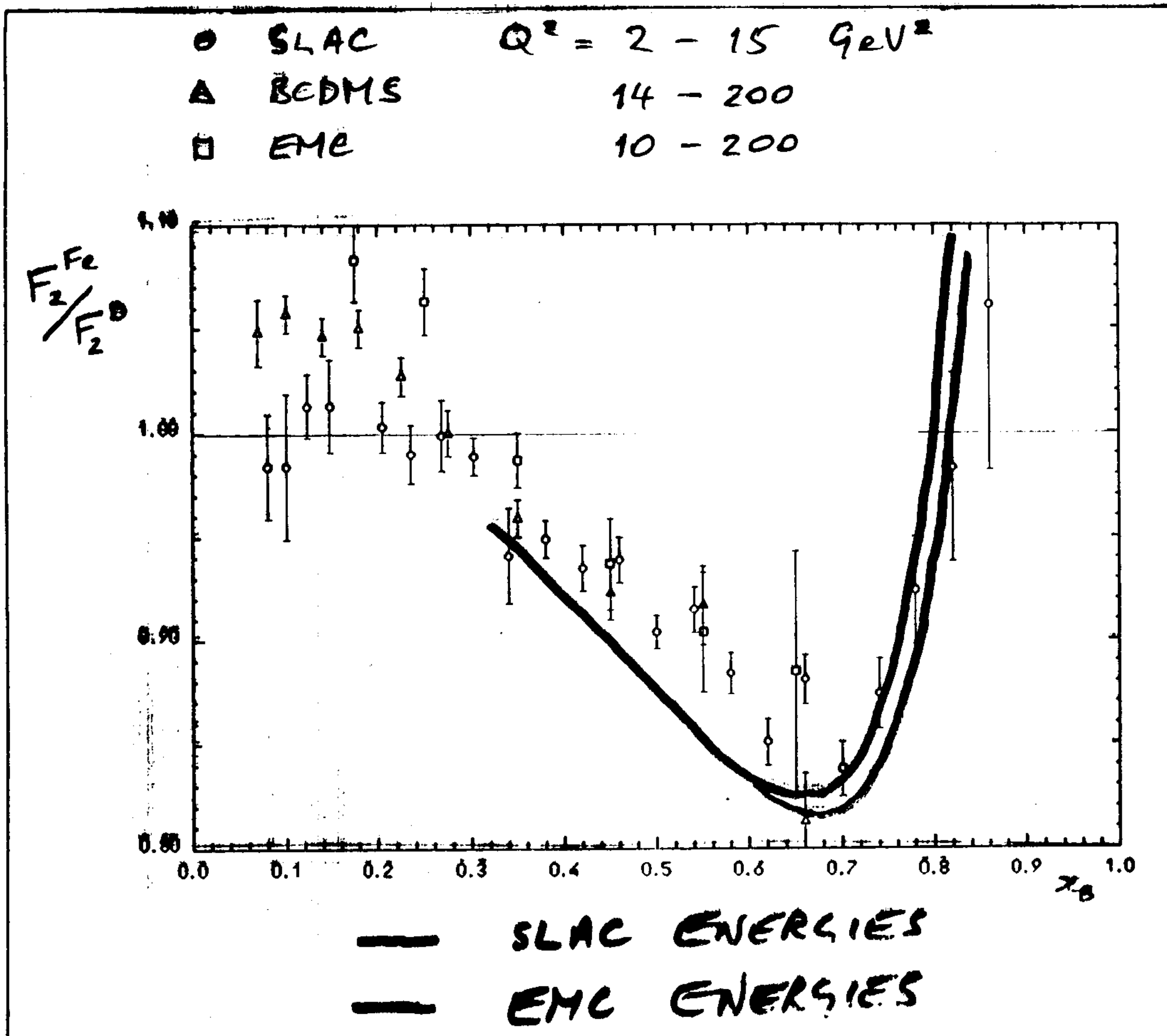
THUS x_B^N FOR THE NUCLEON IS

$$x_B^N \equiv Q^2/2p.q \simeq x_B \cdot \frac{m}{\tilde{m}} \left(1 + \frac{\rho^2}{2\tilde{m}^2} + z\frac{\rho}{\tilde{m}}\right)^{-1}$$

WHERE $z = \cos\theta$ AND $x_B \equiv Q^2/2m\nu$

THUS SINCE $\tilde{m} < m$ WE HAVE $\langle x_B^N \rangle > x_B$

AND $F_2^{A,N}(x_B) \simeq \frac{3}{p_F^3} \int_0^{p_F} \rho^2 d\rho \int_{-1}^{1} \frac{dz}{2} \int_0^1 d\xi\, F_2^D(\xi)\, \delta(\xi - x_B^N)$



SLAC $Q^2 = 2 - 15$ GeV^2
BCDMS 14 - 200
EMC 10 - 200
F_2^{Fe}/F_2^{D}
1.10
1.00
0.90
0.80
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
x_B
SLAC ENERGIES
EMC ENERGIES

"ANTI-SHADOWING"

OUR SUPERRADIANT SOLUTION IMPLIES A π CONTENT - IT EVEN TELLS US HO MUCH:

$$\frac{n_\pi}{A} = \frac{E_\pi/\omega}{A} = \frac{A \times 53 \cdot \frac{1}{84}}{A} \simeq 0.6$$

AND THUS ANALOGOUSLY WE HAVE:

$$F_2^{A,\pi}(x_B) \simeq 0.63 \int_{-1}^{1} \frac{dz}{2} \int_0^1 d\xi\, F_2^\pi(\xi)\, \delta(\xi - x_B^\pi)$$

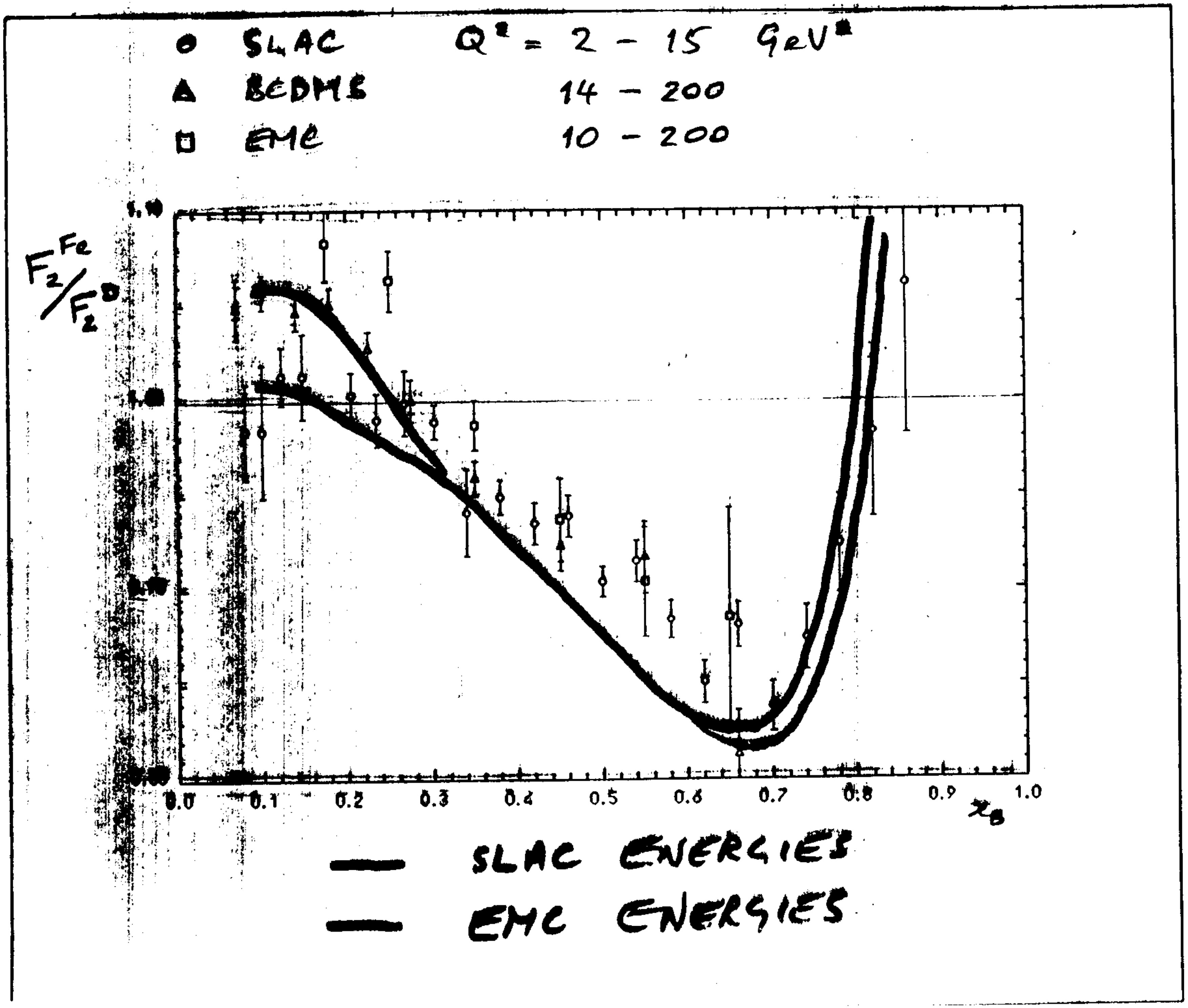
WHERE THE π EFFECTIVE x IS:

$$x_B^\pi \simeq x_B \frac{m}{(\omega + qz)}$$

AND

$$\omega = 84 \text{ MeV} \quad , \quad \omega_0 = \sqrt{q^2 + m_\pi^2} = m_\Delta - m_N$$

THUS SINCE $x_B^\pi < 1$ WE ONLY GET A CONTRIBUTION TO $x_B \lesssim 0.2$.
MOREOVER IF WE IMPOSE $W \gtrsim 1$ GeV
$W^2 = (p+q)^2$ = OUTGOING MASS-SQUARED,
THEN THIS CONTRIBUTION WILL ALSO BE SUPPRESSED FOR SMALL Q^2.



o SLAC Q^2 = 2 - 15 GeV^2
▲ BCDMS 14 - 200
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F_2^{Fe}/F_2^D
1.10
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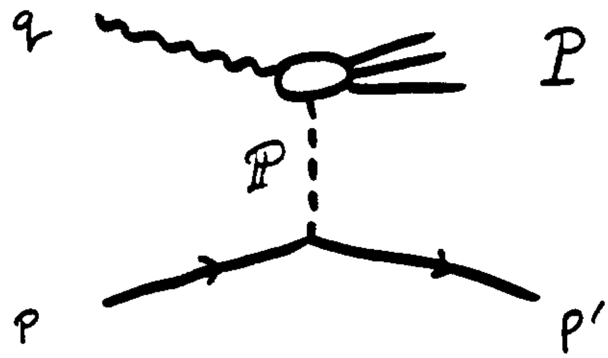
0.4
0.5
0.6
0.7
0.8
0.9
1.0
x_B
SLAC ENERGIES
EMC ENERGIES

"SHADOWING" - VERY SMALL x_B

FROM THE ANALYSIS OF P-P SCATTERIN
(ANGELINI, NITTI, PELLICORO & PREPARATA, PHYS. REV. D41 (1990) 20
WE KNOW THAT THIS REGION, DOMINATE
BY THE POMERON, IS WELL DESCRIBED I
THE SINGLE-DIFFRACTIVE PROCESS:



$$\mathcal{P} = q + p - p'$$

THIS LEADS TO:

$$F_2(x_B) = \int \frac{d^3\vec{p}\,'}{2E'}\, G(t)\left(\cos\theta_t\right)^{2\alpha_{\mathbb{P}}(t)} \left(\mathcal{P}^2\right)^{\alpha_{\mathbb{P}}(0)}$$

WHERE TO A GOOD APPROXIMATION:

$$\cos\theta_t \simeq \frac{1}{2}\left[\frac{1 + x_F(1-x_B)}{1 - x_F(1-x_B)}\right] \qquad x_F = \frac{2E'}{\sqrt{s}}$$

$$\mathcal{P}^2 \simeq Q^2 (1-x_F)(1-x_B)/x_B$$

AND FROM PP ANALYSIS

$$G(t) \simeq G(0)\, e^{(bt+ct^2+dt^3)} \quad ; \; b = 5.81,\ c = 2.71,\ d = 0$$

i) THE RAPID DECREASE FOR LARGE t MEA[NS]
THAT THE OUTGOING PROTON PREFER[S]
THE FORWARD DIRECTION

ii) THE CROSS-SECTION IS STRONGLY PEAKE[D]
FOR $x_F \simeq 1$.

iii) IN A NUCLEUS THIS FORWARD REGION
LIES INSIDE THE FERMI SURFACE A[ND]
IS THEREFORE PAULI BLOCKED.

I.E., THE p' INTEGRAL IS RESTRICTED
TO $p' > p_F$ AND IS THUS SUPPRESSED

iv) RECALL

$$p_F^3 = \frac{3\pi^2}{2}\rho \quad (\rho = \text{NUCLEAR DENSI[TY]})$$

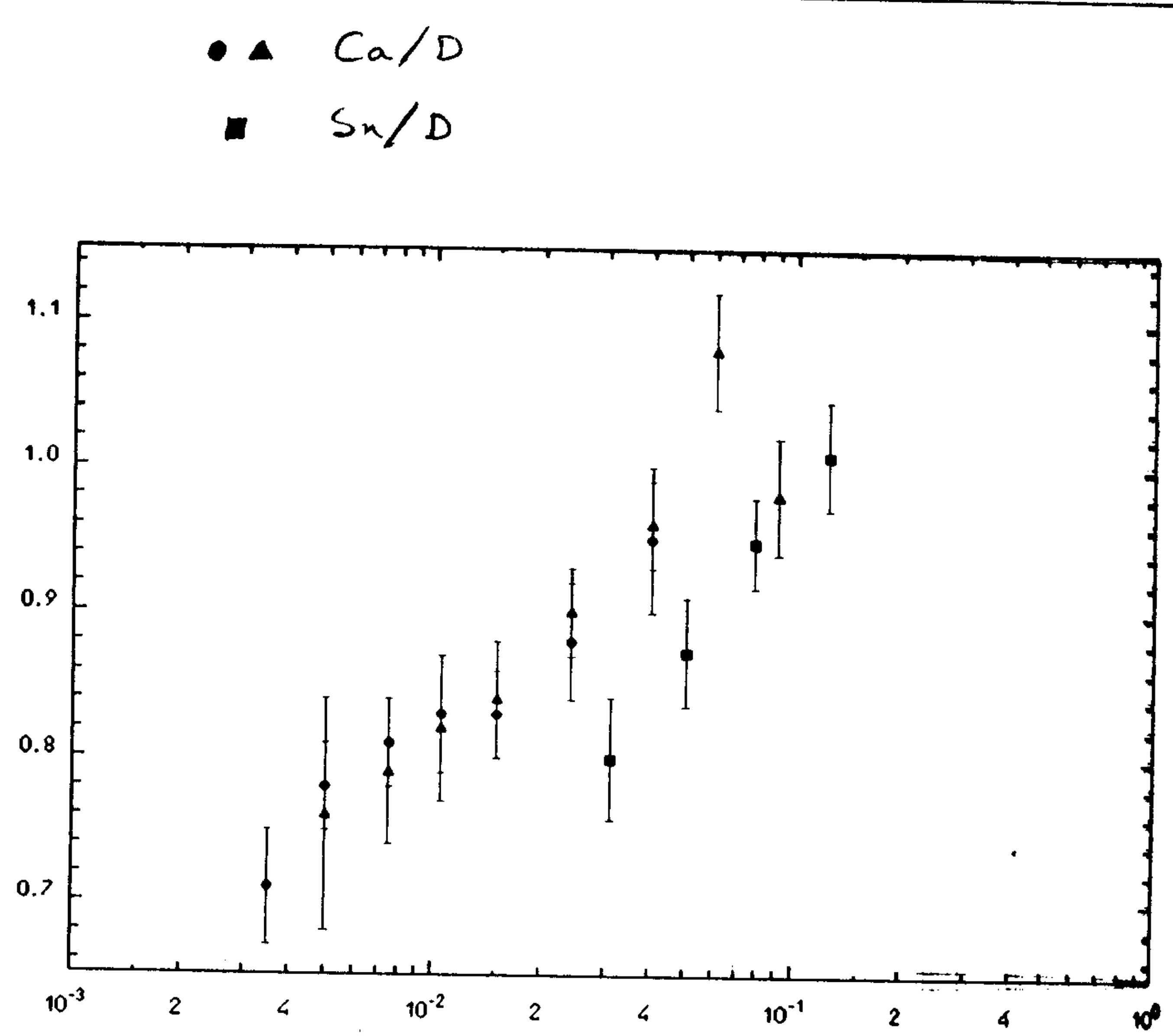
AND NOW TWO EFFECTS LEAD TO A
REDUCTION OF ρ :

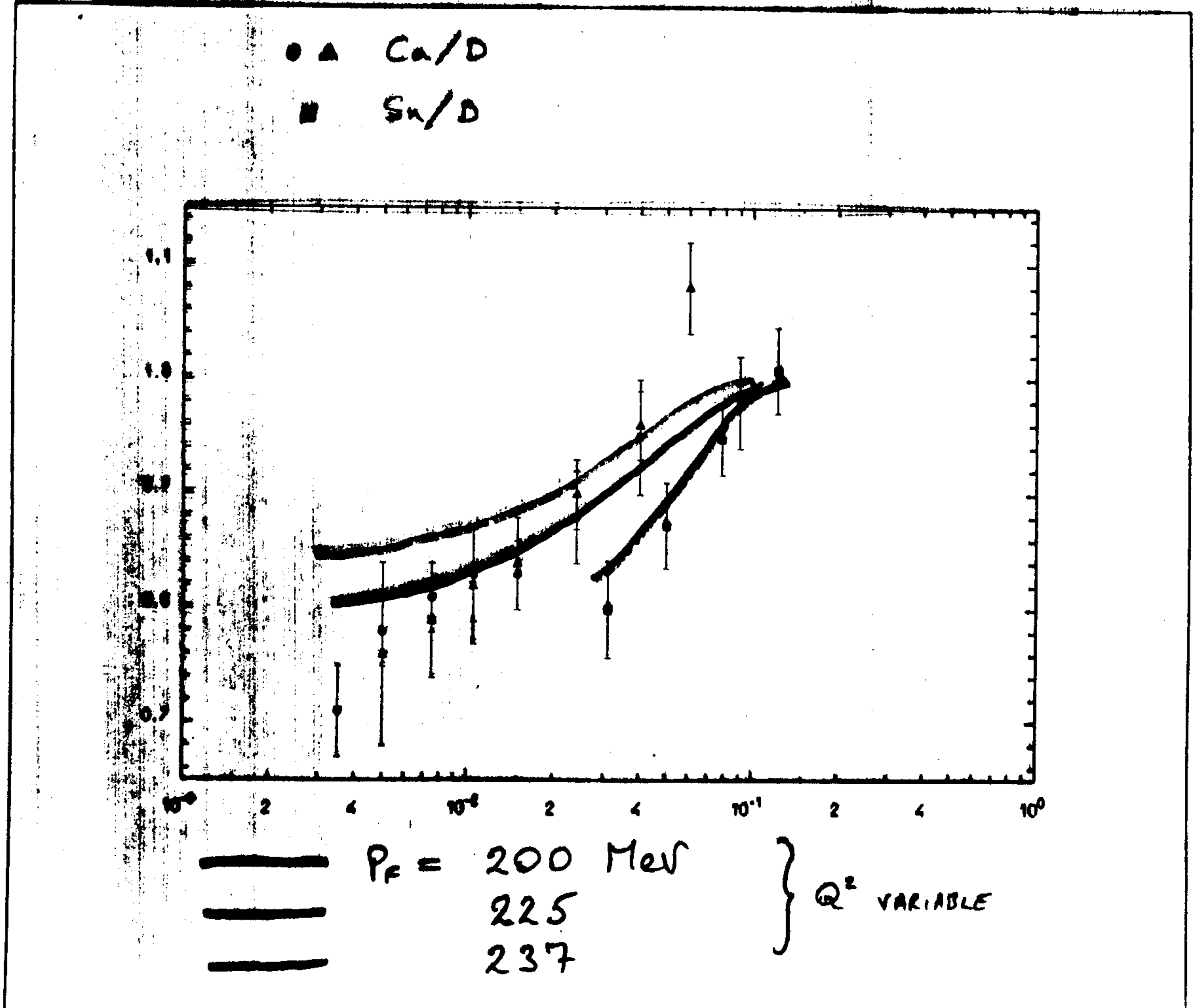
a) $A \rightarrow A\cos^2\vartheta = 0.82\,A \quad (82\%\,N, 18\%\,\Delta)$

b) FINITE NUCLEAR SIZE $\Rightarrow p_F \rightarrow p_F\left(1 + \frac{3.65}{A^{2/3}}\right)^{-1/3}$

E.G.,

	C	Ca	Sn	Au	
$p_F =$	198	225	237	240	MeV


Ca/D
Sn/D
1.1
1.0
0.9
0.8
0.7
10^-3
2
4
10^-2
2
4
10^-1
2
4
10^0



Ca/D
Sn/D
1.1
1.0
0.9
0.8
0.7
10^{-2}
2
4
10^{-1}
2
4
10^{0}
P_F = 200 MeV
225
237
Q^2 VARIABLE

CONCLUSIONS

THE SUPERRADIANT SOLUTIONS OF THE COUPLED EQUATIONS FOR THE N, Δ & π FIELD INSIDE THE NUCLEUS LEADS TO A PICTURE OF THE NUCLEUS IN WHICH THE "COLLECTIVE" SELF-CONSISTENT NUCLEAR FIELD IS GENERATED BY RESONANT OSCILLATION BETWEEN THE TWO STATES N & Δ MEDIATED BY APPROPRIATE MODES OF THE π-FIELD.

$$\frac{n_N}{A} \simeq 0.82 \quad , \quad \frac{n_\Delta}{A} \simeq 0.18 \quad , \quad \frac{n_\pi}{A} \simeq 0.63$$

THE EFFECTIVE MASS OF THE NUCLEON IS LOWERED BY ABOUT 60 MeV, WHICH LEADS TO THE CORRECT SIZE OF x_B - RESCALING REQUIRED TO EXPLAIN THE EMC EFFECT.

THE PION CONTENT EXPLAINS "ANTI-SHADOWING"

AND "SHADOWING" IS DESCRIBED WELL IN A SIMPLE SINGLE-DIFFRACTIVE PICTURE OF SMALL-x_B PROCESSES INTERFERED UPON BY PAULI BLOCKING.