Quark momentum distribution in nucleons.

L. Conci
M.T.

- mom. distr. (and many-body systems)
- mom. distr. and Constituent Quark Models (non relativistic CQM)
- Quark - Parton Model for deep - inelastic lepton - nucleon scattering and Quark mom. distr.
- QCD implications on the Q. P. M. and the constituent Quark mass

Momentum Distribution

* PROBLEM INVESTIGATED IN SEVERAL DOMAINS OF MANY-BODY PHYSICS

1. Atoms, Molecules, Metals* [Compton scattering]
   * first direct verification of Fermi statistics

2. Liquids*, Solids [Neutron scattering]
   * Bose condensate fraction in ^4He

n.b. $\boxed{\text{In all these cases } n(p) \text{ is extracted by using deep inelastic reactions and Impulse Approximation}}$

[large momentum transfer : $qd \simeq 5 \div 20$
where d is the average distance between the constituents]

The momentum wave-function obtained by Fourier transformation (see Podolsky and Pauling 1929 for a complete list of the Fourier transforms of hydrogenic wave-functions) is

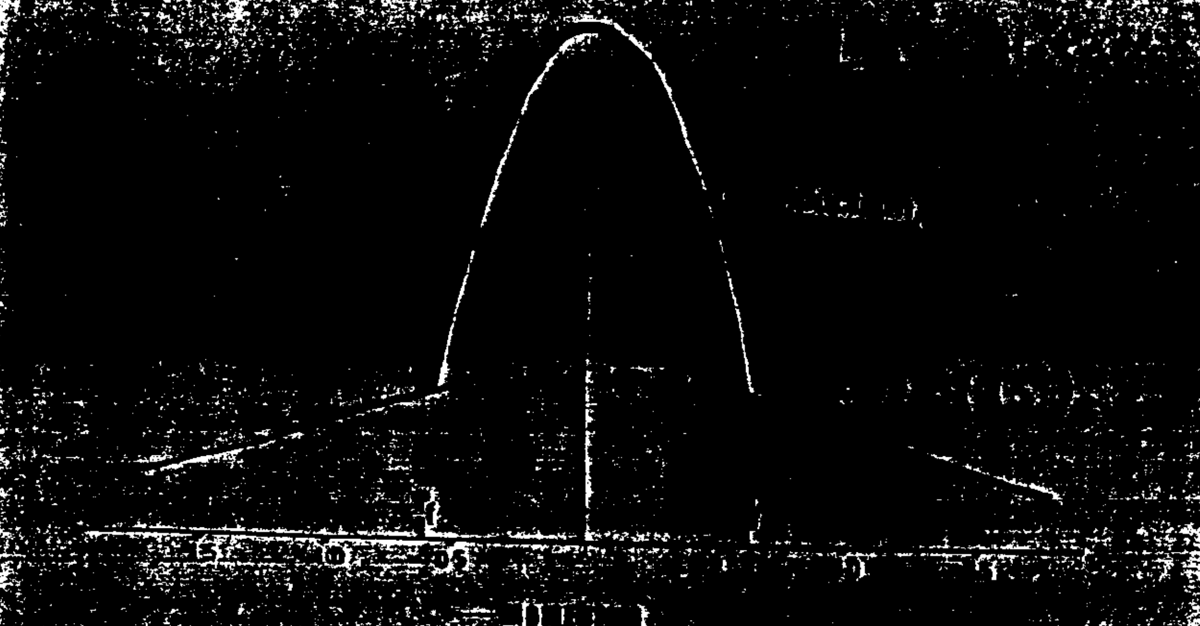
$$\chi(p)=\frac{2}{\pi}\sqrt{(2\gamma)}\frac{1}{(p^2+\gamma^2)^2}$$

giving

$$J(q)=\frac{16}{3\pi}\frac{\gamma^5}{(q^2+\gamma^2)^3}$$

and it is this contribution which accounts for the broad background in fig. 5.

Fig. 5

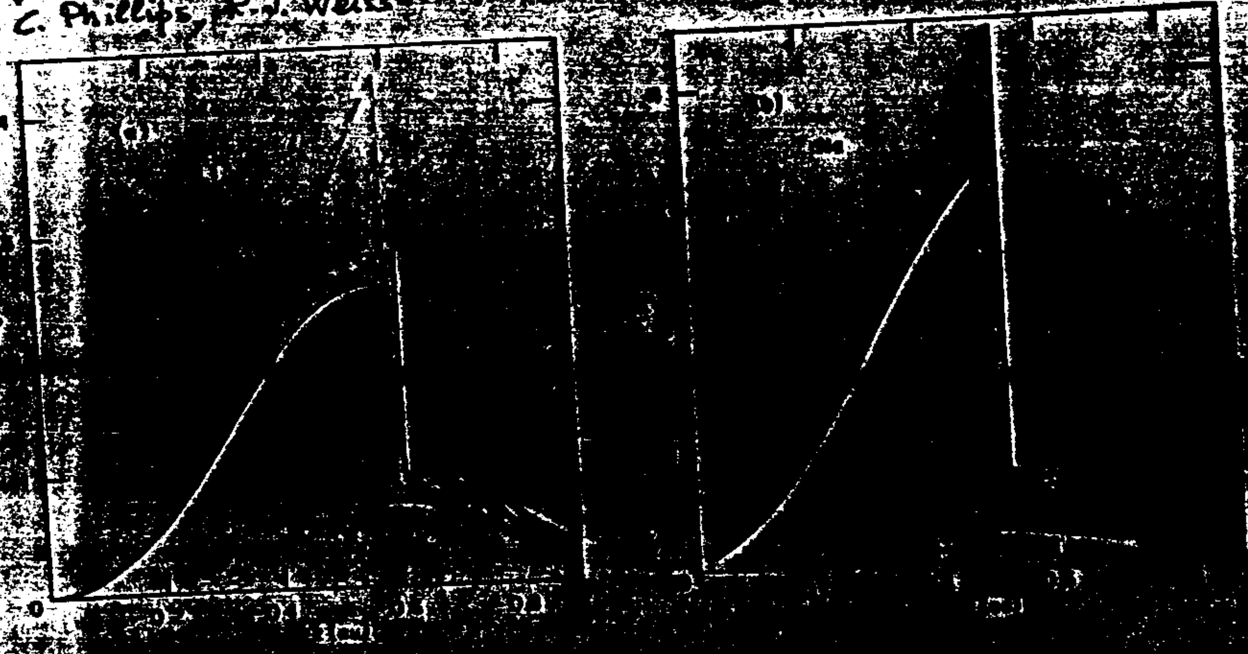


Idealized Compton profile for [illegible]. The conduction [illegible] contribution is an inverted parabola [illegible] cut off at p_F [illegible] superimposed upon the slowly varying [illegible] contribution.

reason for this by the following argument: The average separation of the annihilating positron electron pair is of the order of the Bohr positronium radius (~1 Å) whereas the average distance between valence electrons in light metals is 3–4 Å. Thus the positron electron pair appear partially neutral to the other electrons and electron–electron correlation effects are reduced.

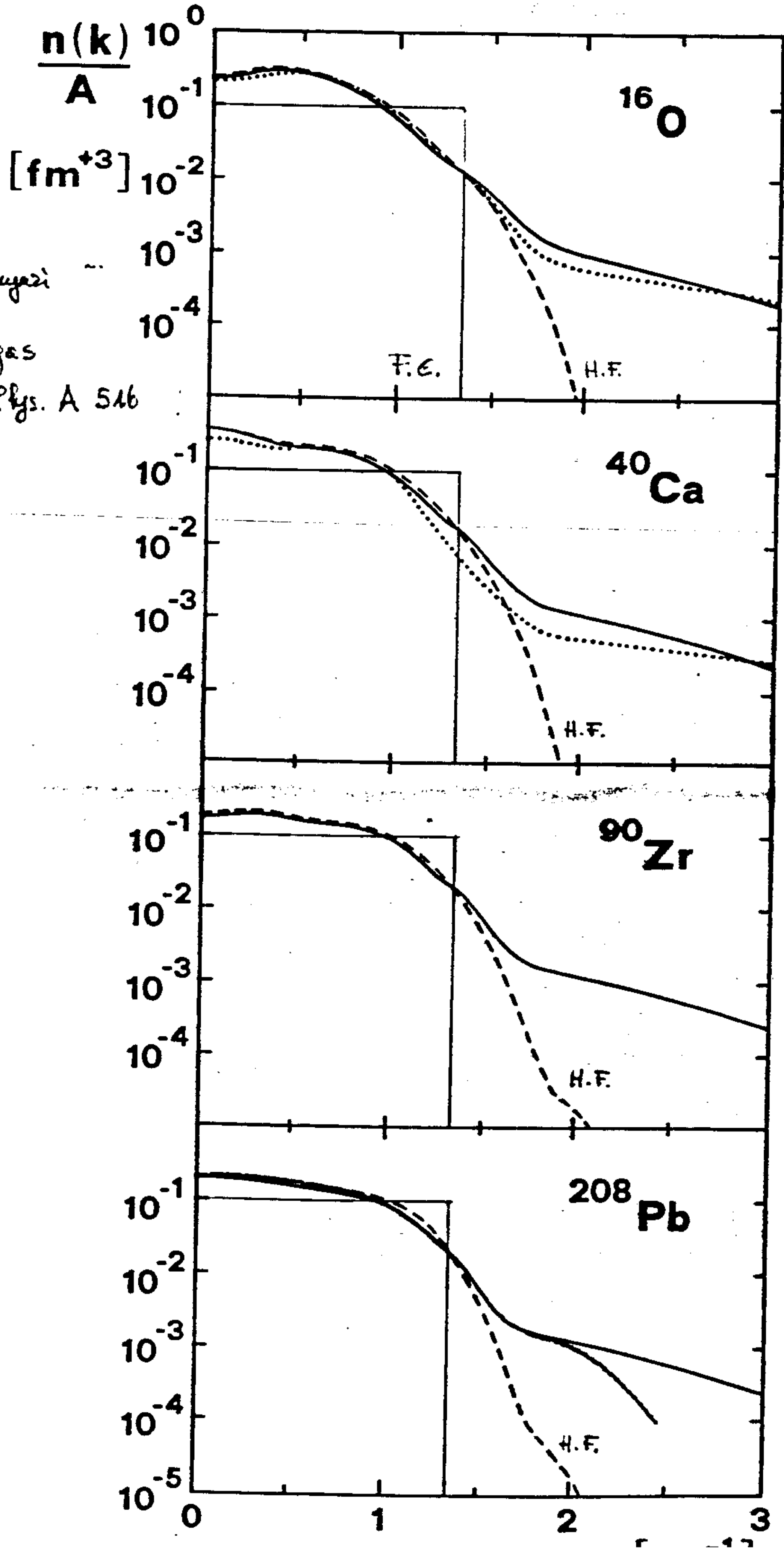
* Phy Rev 171 (1968)
W. C. Phillips, R. J. Weiss

Fig. 10 $4\pi k^2 n(k)$



Momentum density for the valence electron in (1) [illegible] polycrystalline Na, obtained [illegible] Compton line corrected for [illegible] after subtracting background [illegible] (1s² for Li, 2s² 2p⁶ for Na) [illegible] experimental uncertainty which [illegible] principally from [illegible] in determining the slope of the Compton line. The [illegible] (2) is for an interacting free electron [illegible]

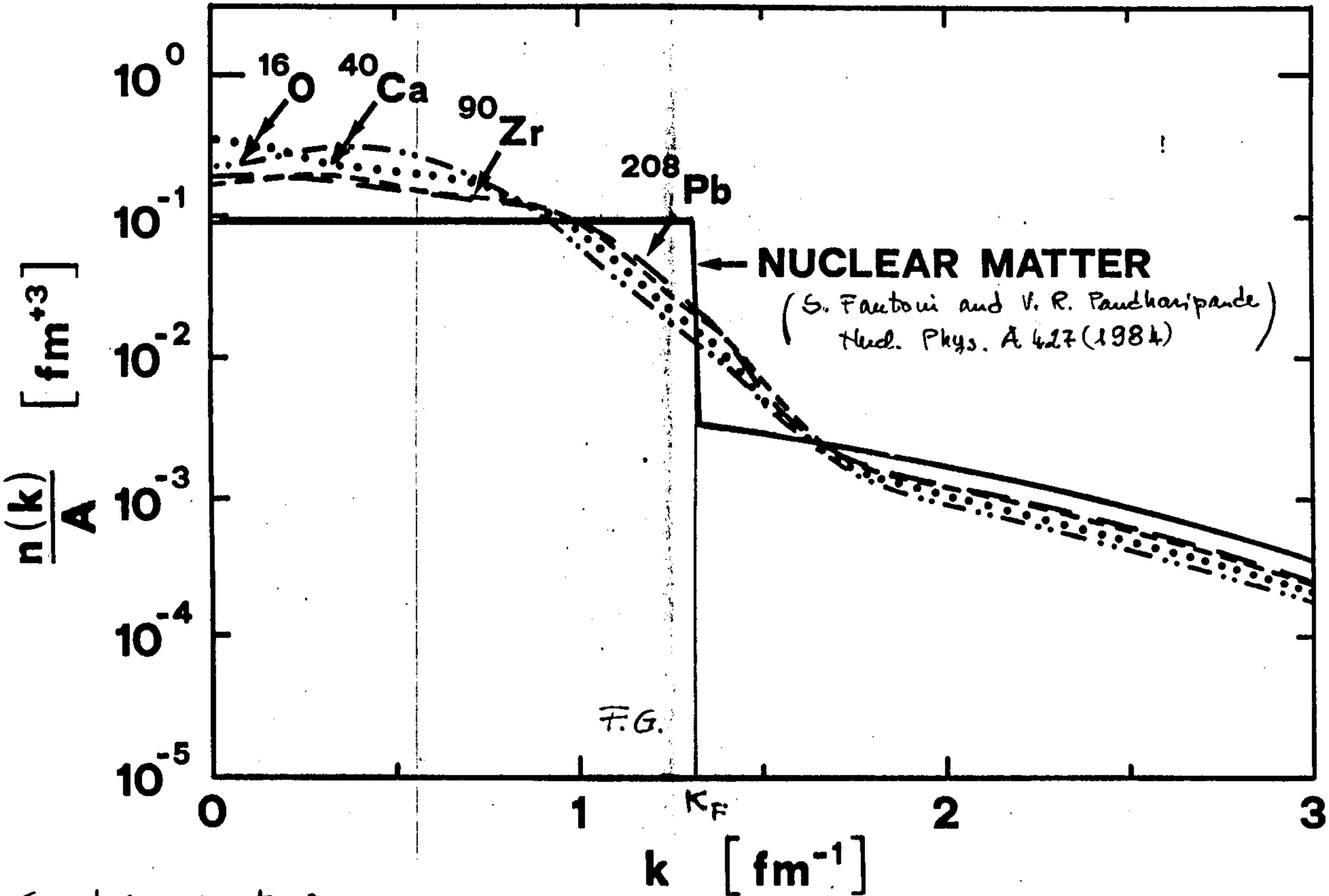
S. Stringari
M.T.
O. Bohigas
Nucl. Phys. A 516



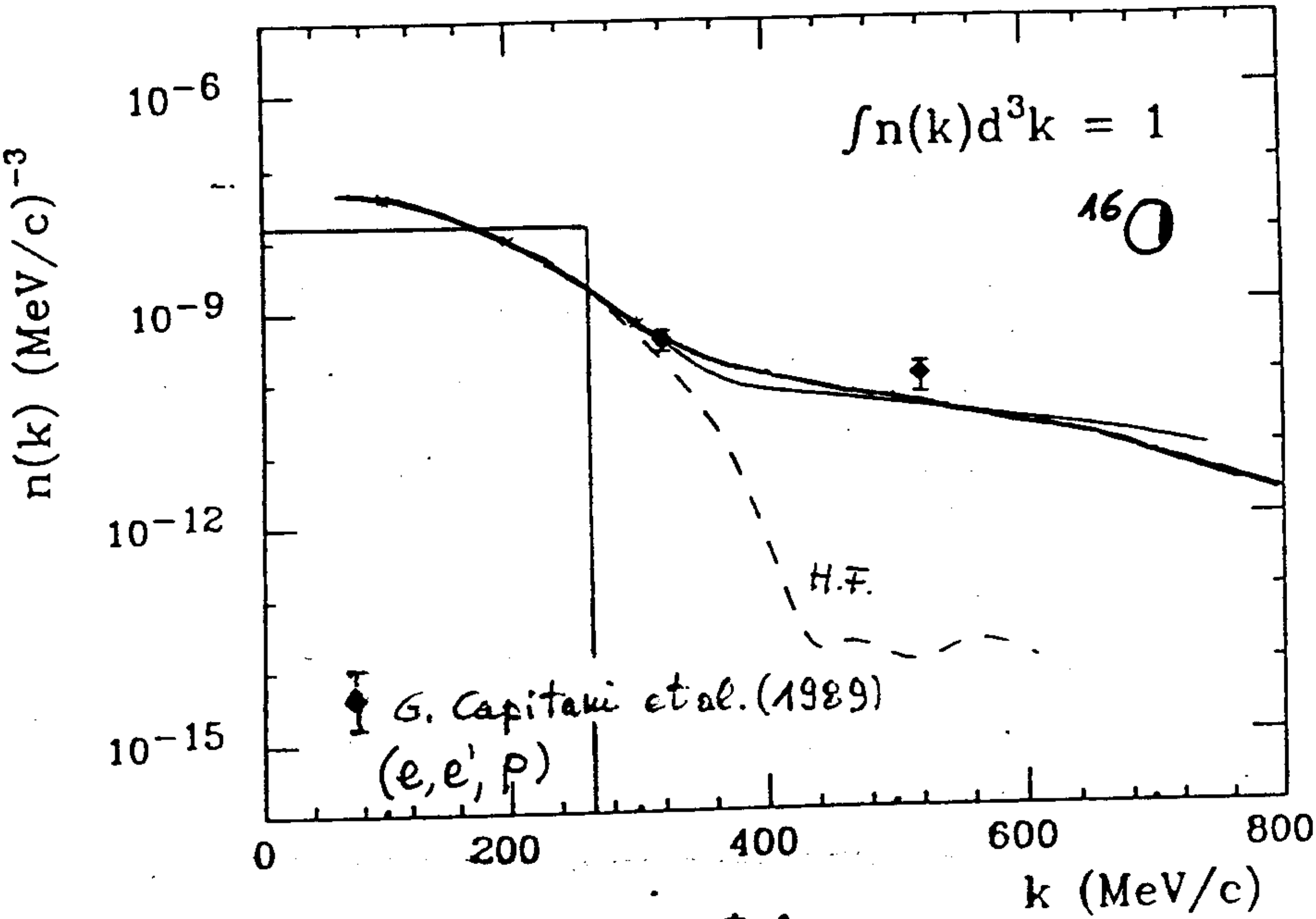
O. Benhar et.
Phys. Lett. 177B
(1986) 135

O. Benhar et a

M. Jaminon
et al.
Nucl. Phys.
A452 (1986) 445



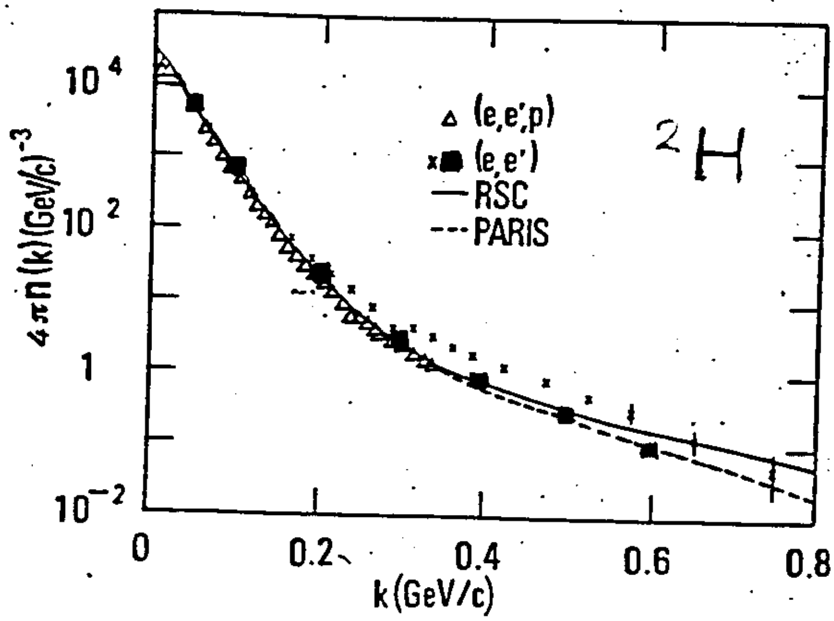
S. Stringari et al
Nucl. Phys. A 516 (1990)

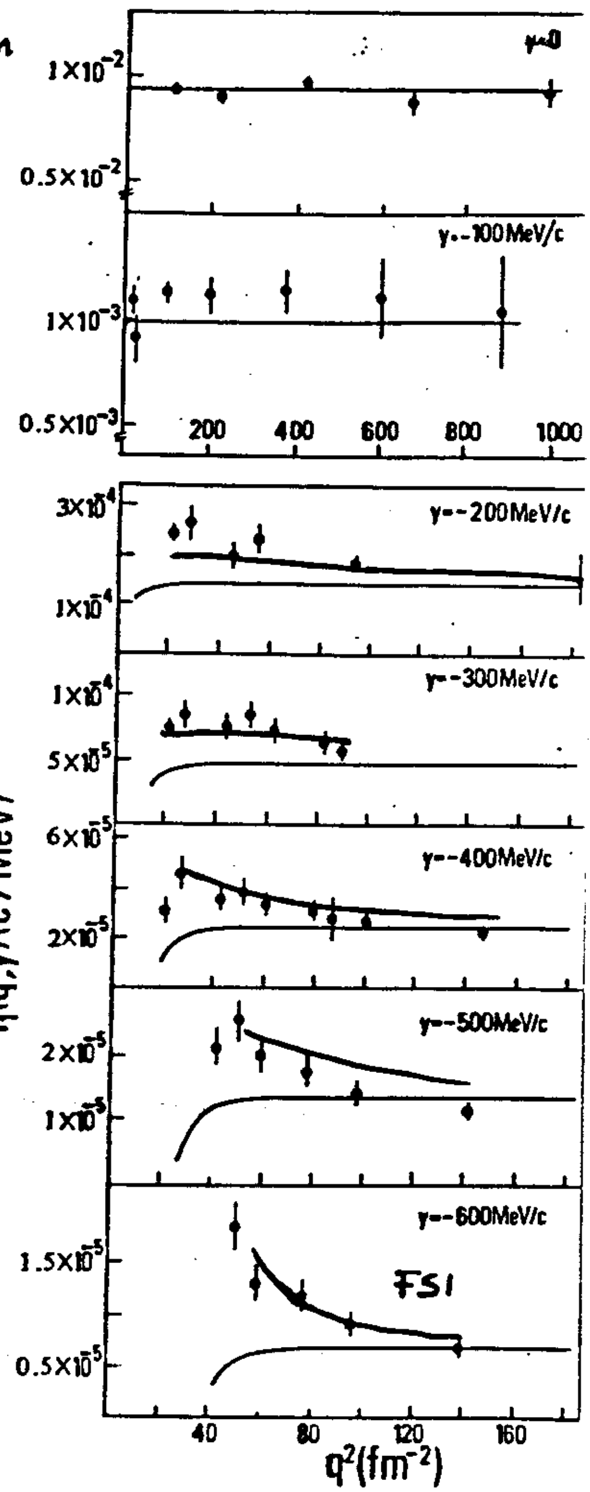


S. Stringari et al
Nucl. Phy. A516 (1990)

O. Benhar et al
Phys. Lett. 177 B (1986)

* Nuclear momentum distribution





C. Ciofi degli Atti, E. Pace, G. Salmè
Phys. Rev. C 36 (1987) 1208

* $n(k) = -\dfrac{1}{2\pi}\dfrac{1}{y}\dfrac{dF}{dy}$, $|y| = k$

* $F(y) = 2\pi \displaystyle\int_{|y|}^{\infty} n(k)\, k\, dk$

* At low q ($q \lesssim 1$ GeV ÷ 2 GeV) [illegible] FSI is questionable.

* At higher values of q [illegible] (?

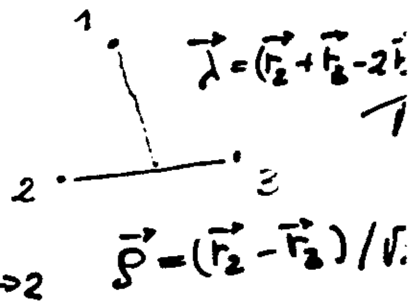
Constituent Quark Model and Mom. dist.

(non rel. CQM)

spin $\frac{1}{2}$

$m_{const} \simeq M/3$ ("effective" mass) $m_u = m_d$

$$\Psi_{3q} = \left[\Phi_{color}\right]_A \otimes \left[\varphi_{flavour} \otimes \chi_{spin} \otimes \varphi_{space}\right]_S$$



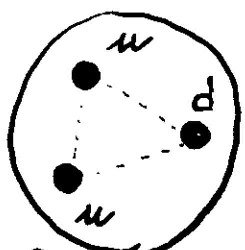
$$H = T_{c.m.} + T + V_{q-q}$$

$$T = \frac{\vec{P}_{\vec{\rho}}^{\,2} + \vec{P}_{\vec{\lambda}}^{\,2}}{2 m_{const}} \; ; \quad T_{CM} = \frac{\vec{P}_{CM}^{\,2}}{2M}$$

V_{q-q} quark-quark two-body interaction

1. harmonic-oscillator Faiman et al. Phys. Rev. 173 (1968) 17

$|p\rangle$



$$V = \frac{1}{2} \sum_{i<j} k (\vec{r}_i - \vec{r}_j)^2 = \frac{3}{2} k \left(\vec{\rho}^{\,2} + \vec{\lambda}^{\,2}\right)$$

$$= \frac{1}{2} m_{const}\, \omega_0^2 \left(\vec{\rho}^{\,2} + \vec{\lambda}\right.$$

$\langle r_n^2 \rangle = 0$

$\langle r_p^2 \rangle = \frac{1}{\alpha_{h.o.}^2}$

$\alpha_{h.o.}^2 = m_{const} \cdot \omega_0$

$\varphi_{space} \equiv \varphi_{h.o.}(\vec{r})$

$\alpha_{h.o.}^2 = 1.35\ fm^{-2}$

C Q M

. De Rújula et al Phys Rev. 12:
• Phys. Rev. D 18 (1978) 4…
• " " D 19 (1979) 2…

2. Isgur - Karl Model

$$V_{q-q} = V_{conf.} + H_{hyp}$$

$$V_{conf} = \sum_{i<j} \Big\{ \underbrace{\frac{1}{2} k (\vec{r}_i - \vec{r}_j)^2}_{\text{h.o.}} + U(|\vec{r}_i - \vec{r}_j|) \Big\}$$

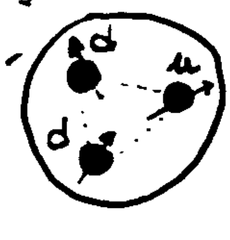
phenom. term removing degeneracies of h.o. states.

$$H_{hyp} = \frac{2\alpha_s}{3 m^2_{const}} \sum_{(i<j)} \Big\{ \underbrace{\frac{8\pi}{3} \vec{S}_i \cdot \vec{S}_j \, \delta(r_{ij})}_{\text{(contact)}} + \underbrace{\frac{1}{r_{ij}^3} \Big[3(\vec{S}_i \cdot \hat{r}_i)(\vec{S}_j \cdot \hat{r}_j) - \vec{S}_i \cdot \vec{S}_j}_{\text{(tensor)}} \ldots$$

n. relat. reduction O.G.E.  diagrams

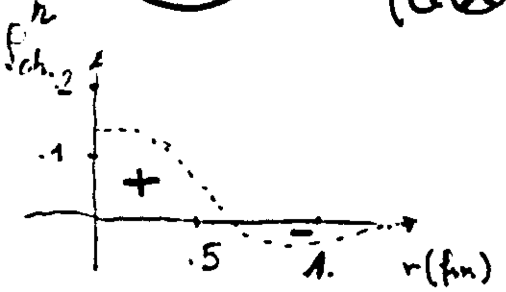
- H_{hyp} is SU(6) breaking and the nucleon is described as a superposition of SU(6) configurations

$|n\rangle$



$(d \otimes d)_{S=1}$ $H_{hyp} \to$ repulsive

$(d \otimes u)_{S=0}$ $H_{hyp} \to$ attractive



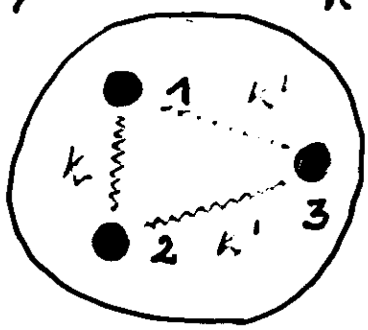
$$\langle r_n^2 \rangle = -9.5 \, 10^{-2} \, fm^2$$

CQM

3. 2 h.o. model

$$V_{q-q} = \frac{1}{2}\, k\,(\vec{r}_1 - \vec{r}_2)^2 + \frac{1}{2}\, k' \left[(\vec{r}_1 - \vec{r}_3)^2 + (\vec{r}_2 - \vec{r}_3)^2 \right]$$

$|\phi\rangle$ $\quad k'/K \ldots ?$



$\alpha^2 = m\omega_{0f} = \left[(2k + k')m\right]^{1/2} = 1.35\, f$

$\beta^2 = m\omega_{02} = (3mk')^{1/2} = 1.99\, fm$

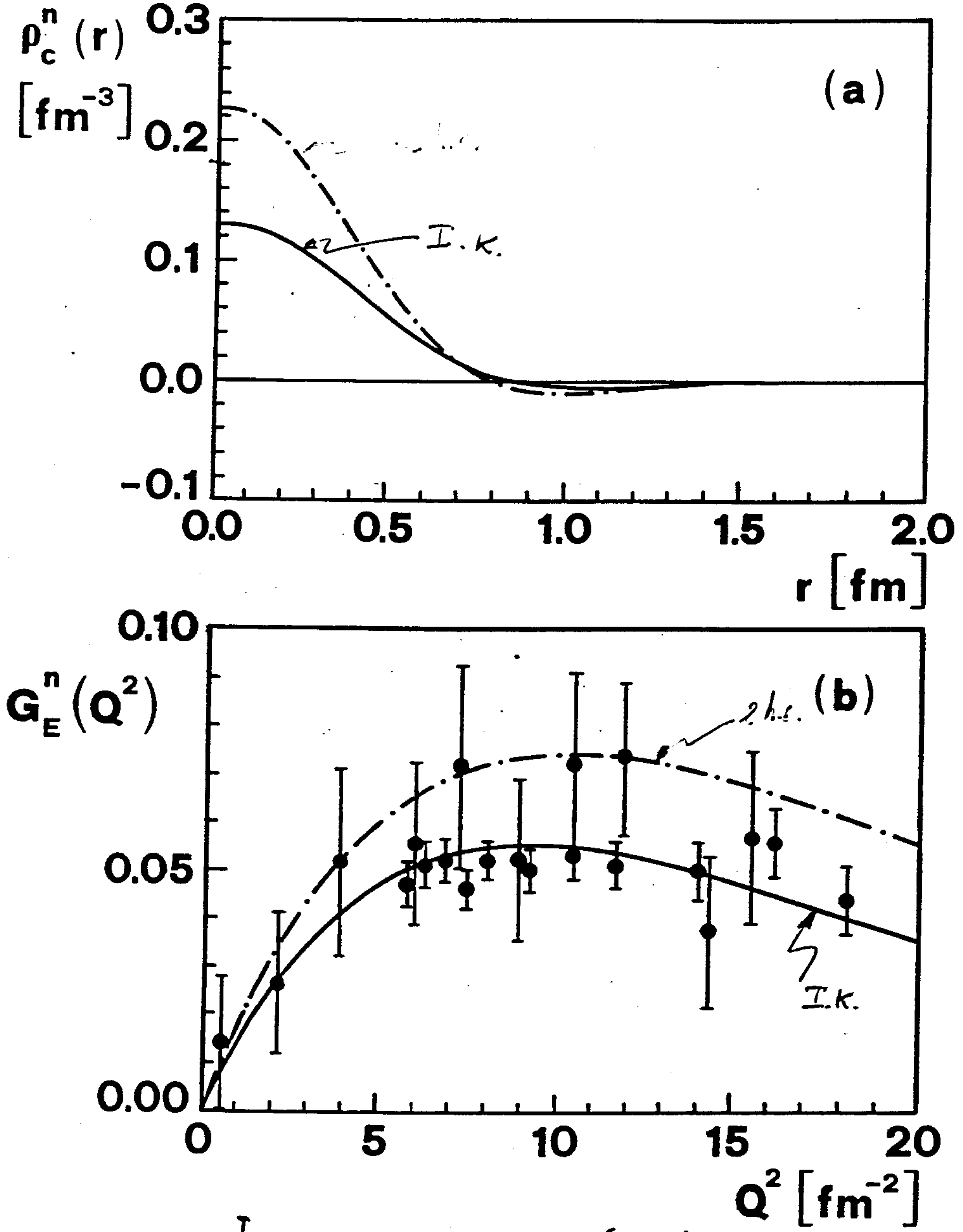
fixed by $\langle r_p^2 \rangle$; $\langle r_n^2 \rangle$

$\phi^{p}_{flavour} = |u(1)\, u(2)\, d(3)\rangle$

$\phi^{n}_{flavour} = |d(1)\, d(2)\, u(3)\rangle$

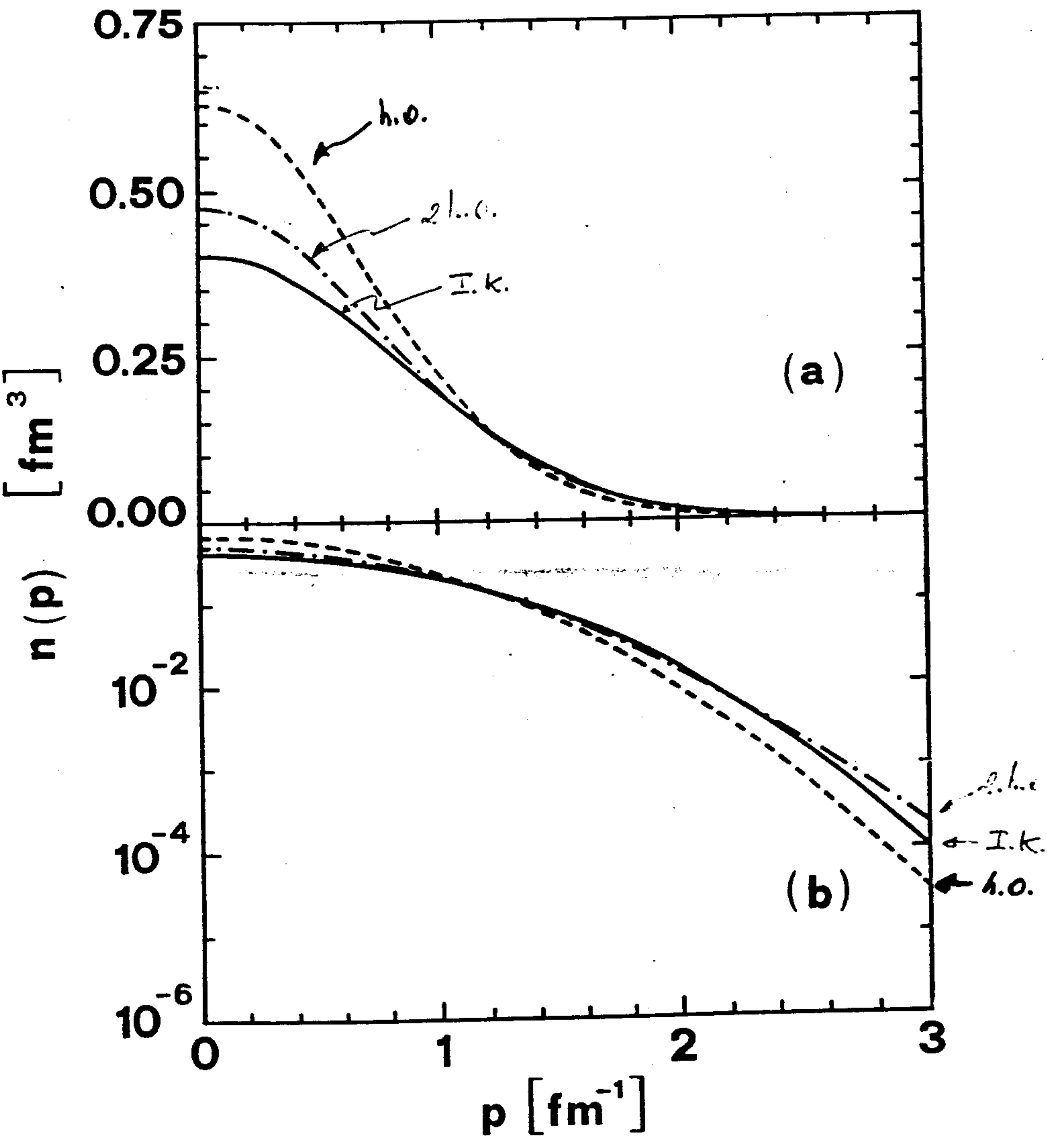
are combinations of $SU(3)_{flavour}$ w. f.

Neutron charge distribution - SU(6) breaking



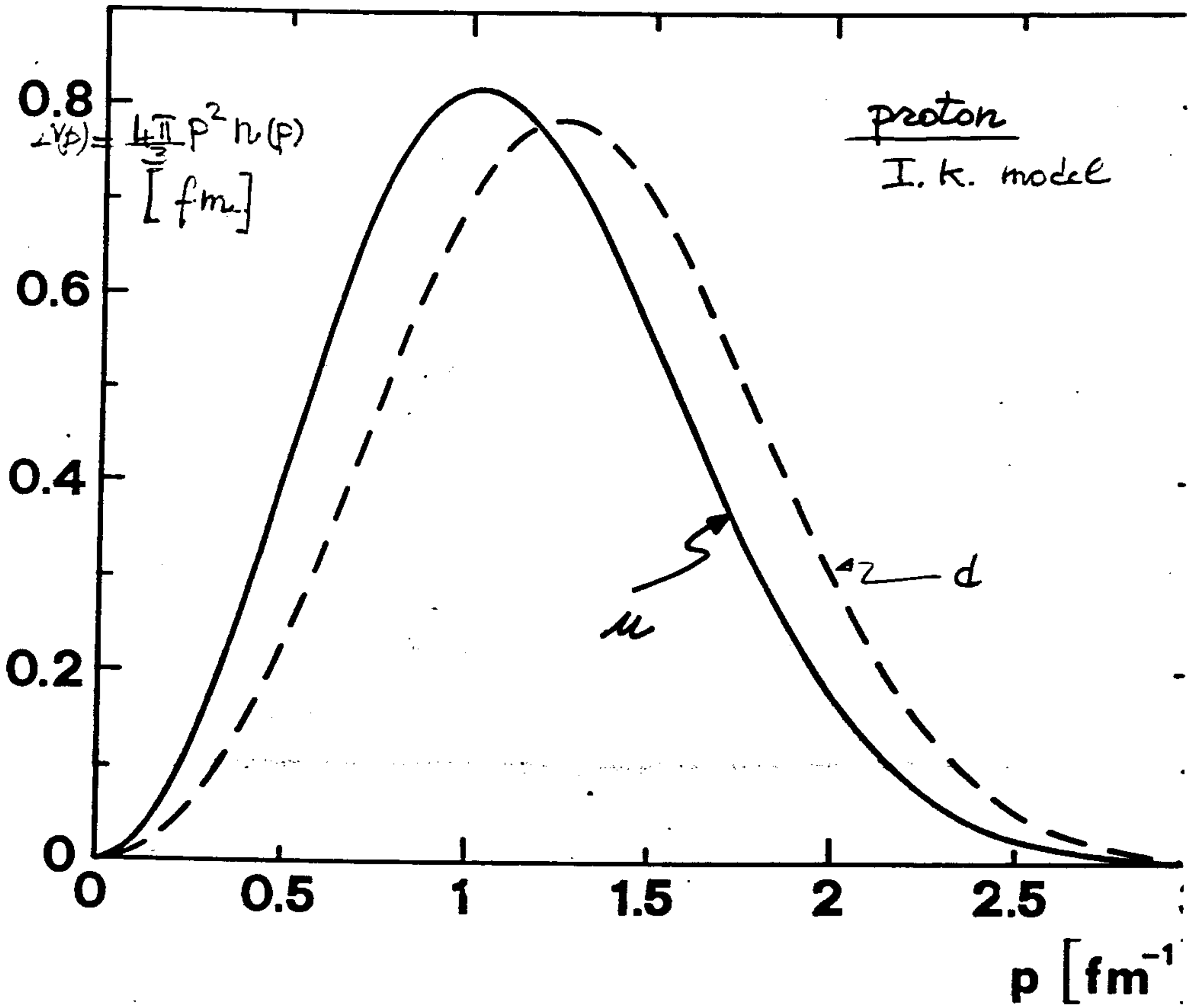
Platchkov S. et al (1989)

Quark Momentum Distribution $\int n(p)\, d\vec{p} = N_u + N_d$



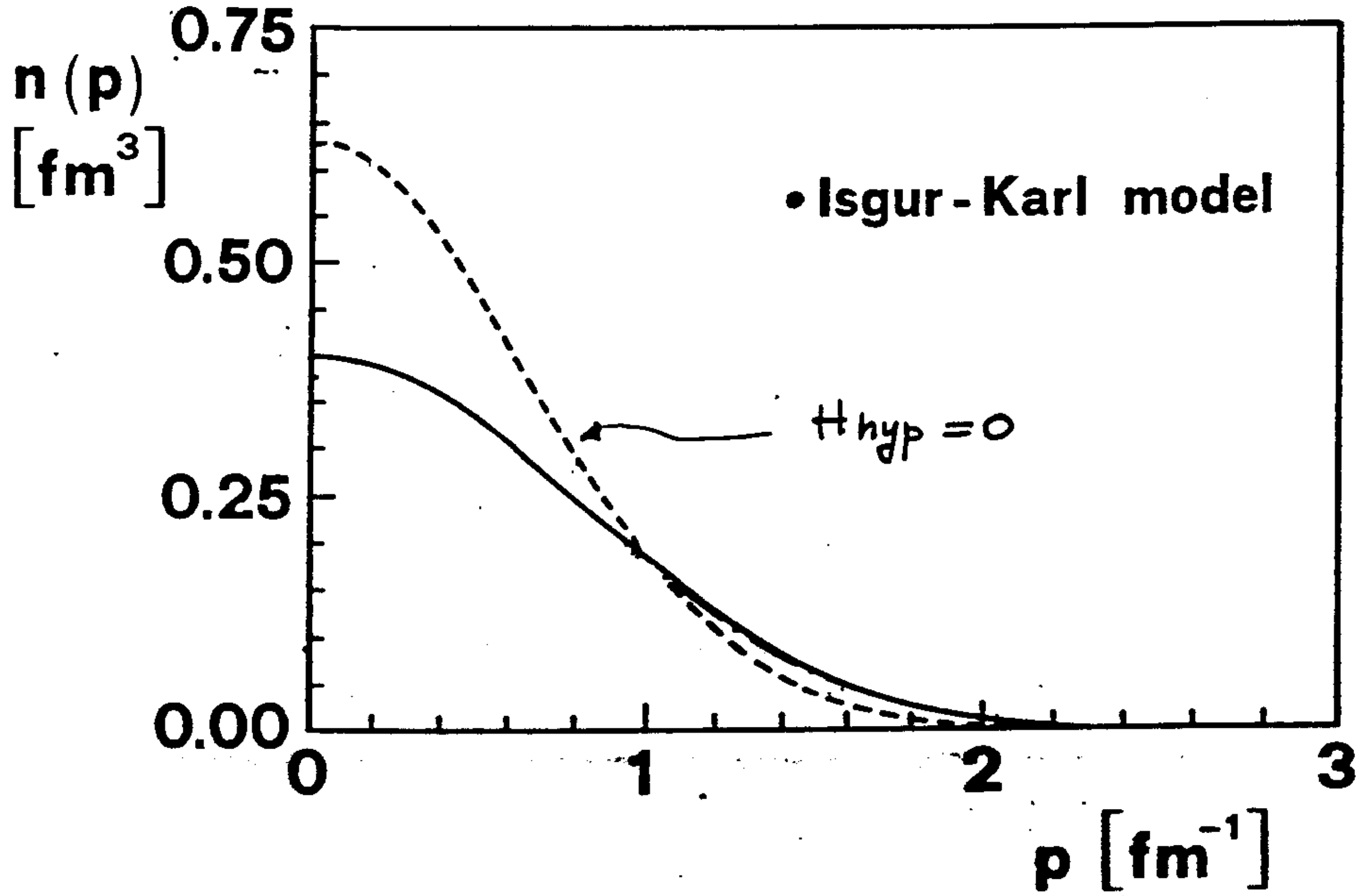
$$n(\vec{p}) = \langle \Psi_{3q} | \sum_{k=1}^{N_q=3} \delta\left(\vec{p} - (\vec{p}_k - \vec{P}_{cm})\right) | \Psi_{3q} \rangle$$

SU(6) breaking effects



$\int \nu(p)\,dp = 1$

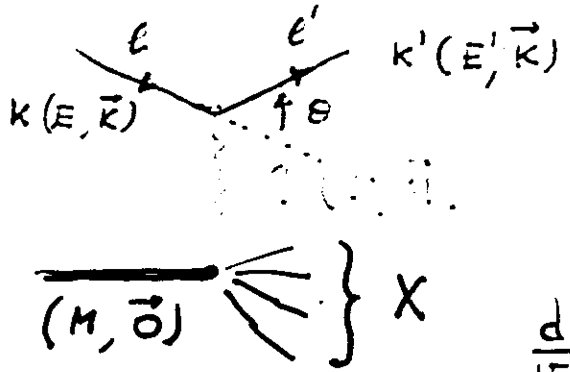
one-dimensional mom. dist.



$$|N\rangle = a_S\,|N\,{}^2S_{1/2}\rangle_S + a'_S\,|N\,{}^2S'_{1/2}\rangle_S + a_M\,|N\,{}^2S_{1/2}\rangle_M + a_D\,|N\,{}^4D_{\ldots}$$

$$|\Psi_{3q}\rangle = (\Phi_{color})_A \otimes |N\rangle$$

Deep inelastic lepton-nucleon scattering – PARTON MODEL (ℓ, ℓ')



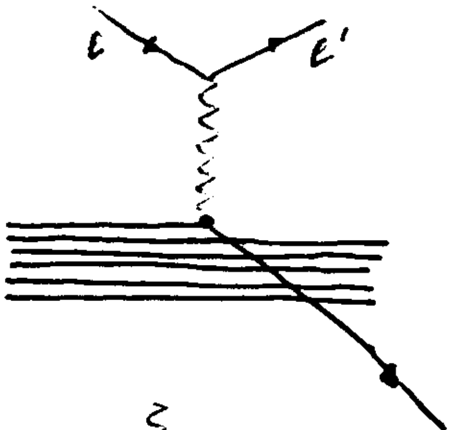
$$\nu \underset{lab}{=} E - E' = \frac{P\cdot q}{M}$$

$$\frac{d\sigma}{dE'd\Omega} = \sigma_M \left[W_2(Q^2,\nu) + 2\, W_1(Q^2,\nu) \tan^2\frac{\theta}{2}\right.$$

- Scaling

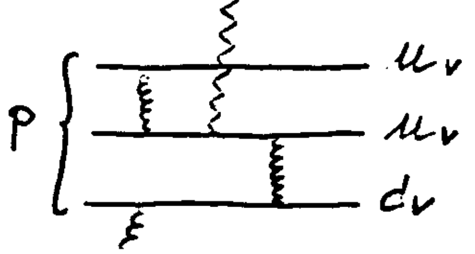
$$\nu\, W_2(Q^2,\nu) \xrightarrow[Q^2\to\infty]{} F_2(x) \qquad x = \frac{Q^2}{2M\nu}\ ; \equiv x_{Bj}$$

$$M\, W_1(Q^2,\nu) \longrightarrow F_1(x) \left[= \frac{1}{2x} F_2(x) \right] \quad \text{Callan-Gross}$$



$x \equiv$ momentum fraction carried by the parton

- $u_v \quad d_v$
- $u\bar{u} \quad d\bar{d} \quad s\bar{s} \ \ldots$
- gluons

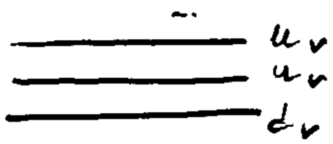


nucleon.

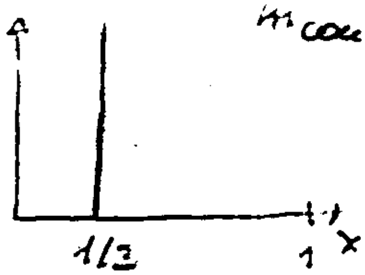
$F_2^{\nu}(x)$

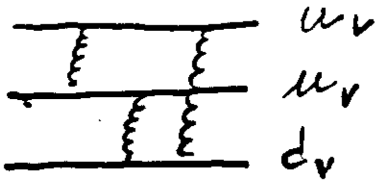
m_v

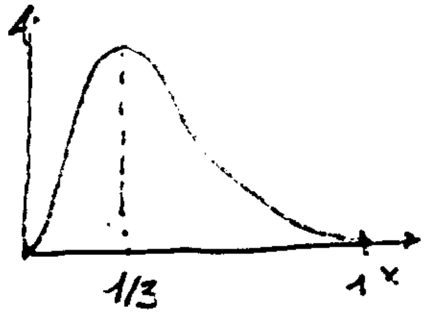
m const

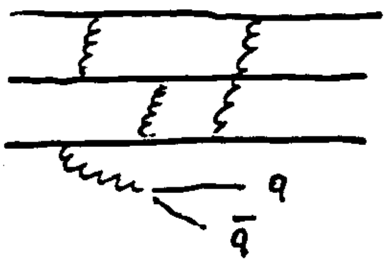


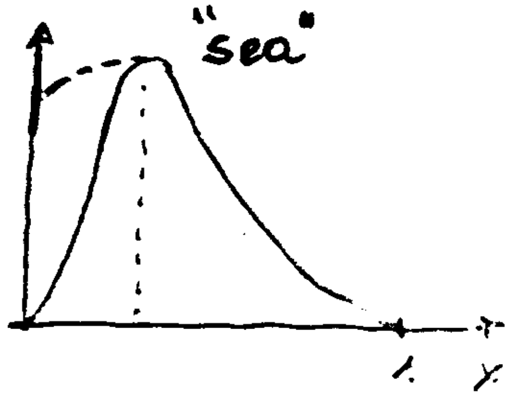
$W_2 \to \delta\left(\nu - \frac{Q^2}{2m_v}\right)$







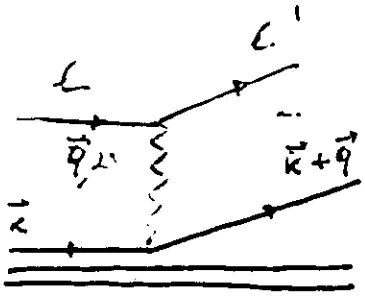




$$\frac{1}{x} F_2^p(x) = \left(\frac{2}{3}\right)^2 \left[u(x) + \bar{u}(x) \right] + \left(\frac{1}{3}\right)^2 \left[d(x) + \bar{d}(x) \right] +$$

$$+ \left(\frac{1}{3}\right)^2 \left[s(x) + \bar{s}(x) \right]$$

Quark - Parton Model and Quark wave function



$$W_2(Q^2, \nu) \sim \int n_q(\vec{k})\, \delta\left(\nu + \sqrt{\vec{k}^2 + m_q^2} - \sqrt{(\vec{k}+\vec{q})^2 + m_q^2}\right) d\vec{k}$$

if $n(\vec{k}) \sim \delta(\vec{k})$ $\quad W_2(Q^2, \nu) \sim \delta\left(\frac{Q^2}{2m_q} - \nu\right)$

$$n_q(\vec{k}) = \langle \Psi_{3q} | \sum_{j=1}^{N_q} \delta\left(\vec{k} - (\vec{p}_j - \vec{P}_{cm})\right) | \Psi_{3q} \rangle$$

$n(k) \Leftrightarrow$ quark w.f. (Hadron w.f.)
without any adjustable parameter.

$$W_2(Q^2, \nu) = 2\pi \frac{m_q}{|\vec{q}|} \sum_j e_j^2 \int_{|k_-(Q^2, x)|}^{k_+(Q^2, \nu)} n_j(k)\, k\, dk$$

$e_j = \frac{2}{3} \quad j \equiv u$

$e_j = -\frac{1}{3} \quad j \equiv d$

$$k_\pm(Q^2, \nu) = \frac{1}{2}\left\{ |\vec{q}| \pm \nu \left(1 + \frac{4m_q^2}{Q^2}\right)^{1/2} \right\}$$

$$k_\pm(Q^2, x) \xrightarrow[Q^2 \to \infty]{} \begin{cases} k_+(x) = \infty \\ k_-(x) = \frac{M}{2}\left(x - \left(\frac{m_q}{M}\right)^2 \frac{1}{x}\right) \end{cases}$$

$$\nu W_2(Q^2, \nu) \longrightarrow F_2(x) = 2\pi m_q \sum_j e_j^2 \int_{|k_-(x)|}^{\infty} n_j(k)\, k\, dk$$

Integral properties of the nucleon structure functions.

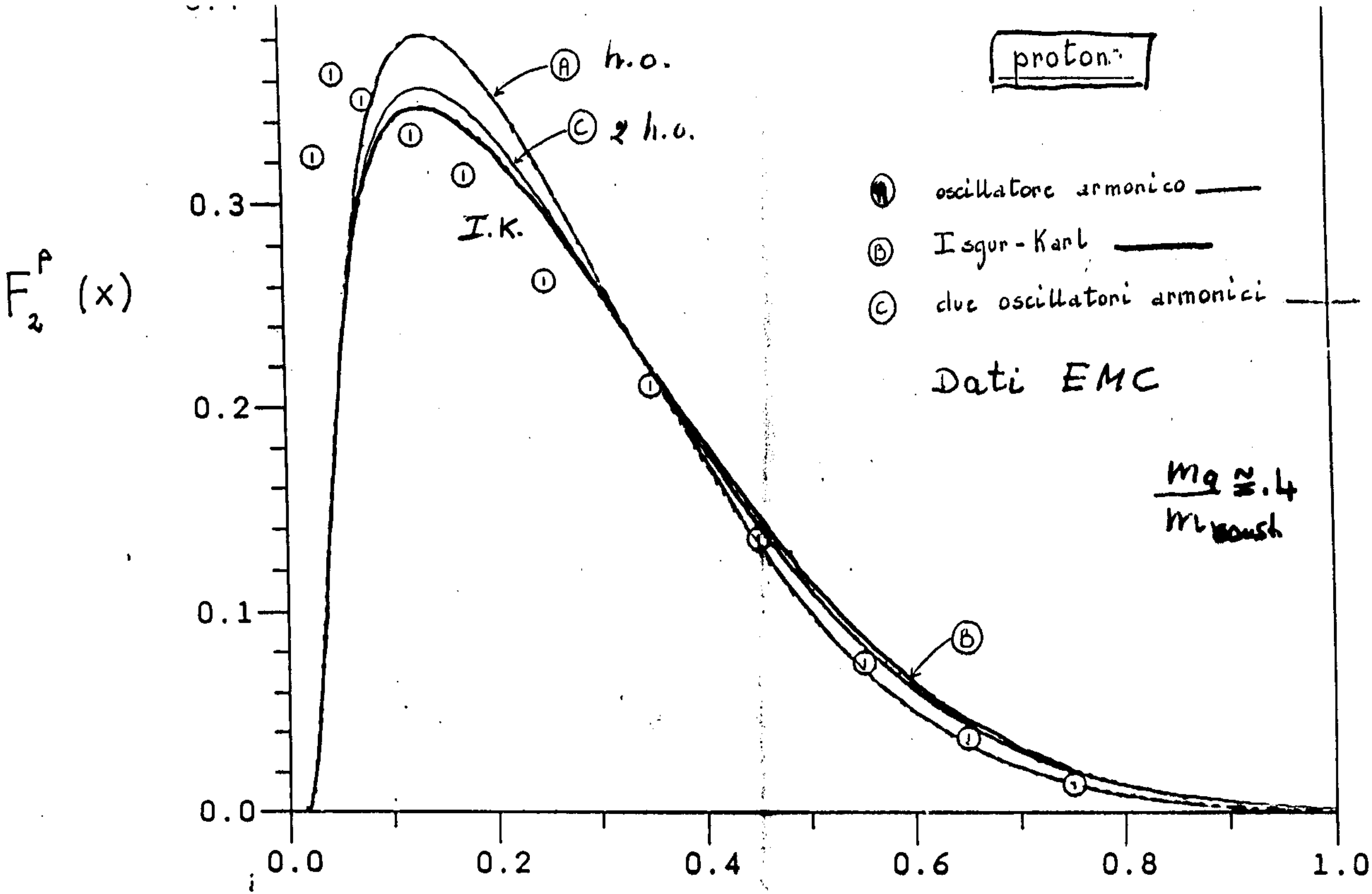
$$\int dx\, F_2^{p}(x) = \frac{4}{9}\underbrace{\int x(u+\bar{u})\,dx}_{\varepsilon_u} + \frac{1}{9}\underbrace{\int x(d+\bar{d})\,dx}_{\varepsilon_d} \qquad \overset{\text{EMC}}{= .135}$$

$$\int dx\, F_2^{n}(x) = \frac{1}{9}\,\varepsilon_u + \frac{4}{9}\,\varepsilon_d \qquad = .080$$

$$\varepsilon_g = 1 - \varepsilon_u - \varepsilon_d \simeq .595 \; !$$

$$\int dx\, F_2^{p}(x) \Rightarrow \frac{m_q}{M_{const.}} \approx .4$$

	ε_u	ε_d	$\int_0^1 F_2^p(x)\,dx$	$\int_0^1 F_2^n(x)\,dx$
$\frac{m_q}{M_{const}} = 1$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{9}$
$\frac{m_q}{M_{const.}} \simeq .4$	.270	.133	.135	.090
EMC	.275 ± 0.011	.116 ± .017	.135 ± .007	.082 ± .009



$F_2^p(x)$
0.0
0.1
0.2
0.3
0.0
0.2
0.4
0.6
0.8
1.0
X
proton
A h.o.
C 2 h.o.
I.K.
B
A oscillatore armonico
B Isgur-Karl
C due oscillatori armonici
Dati EMC

Figura (3.14)

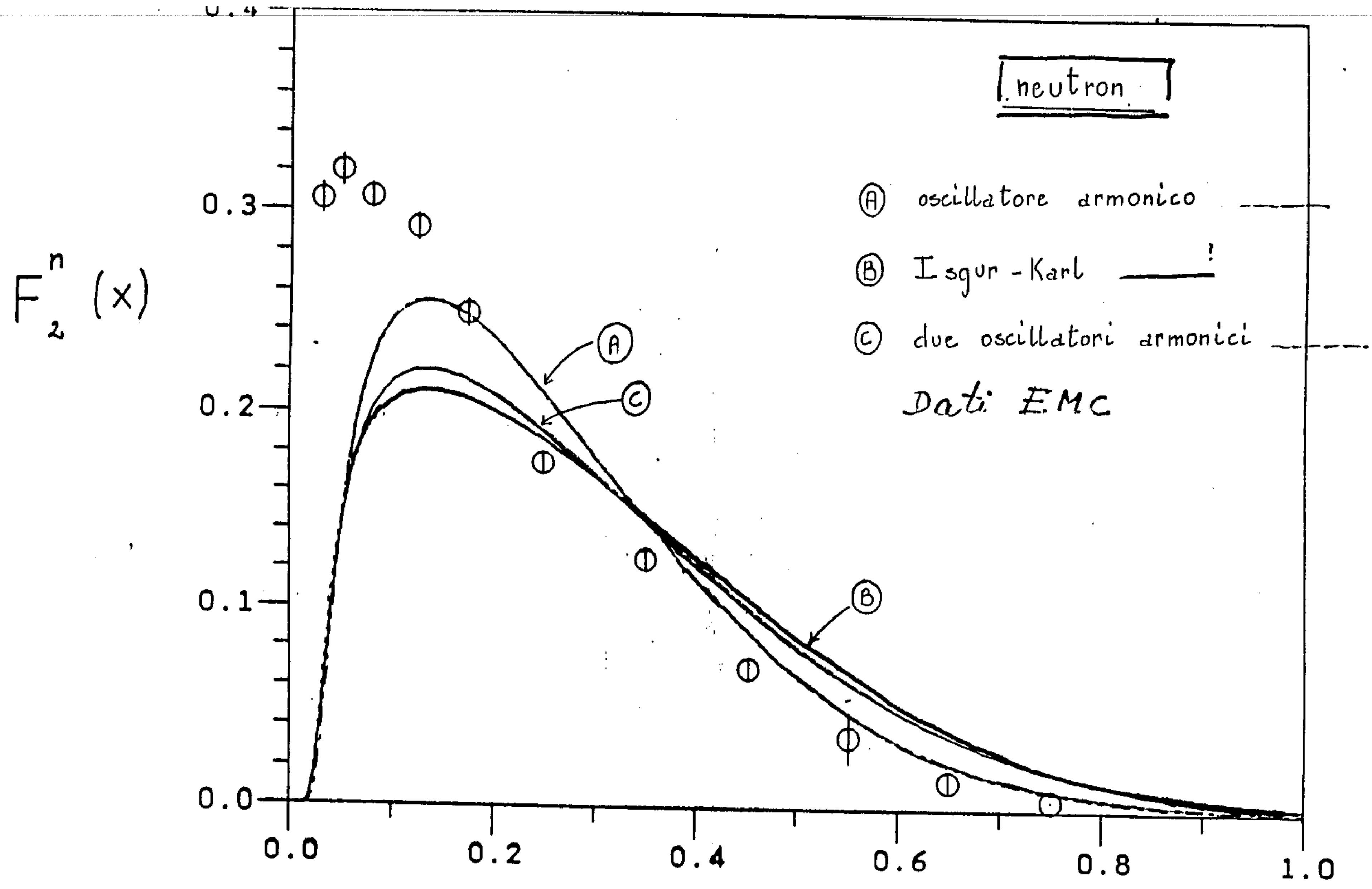
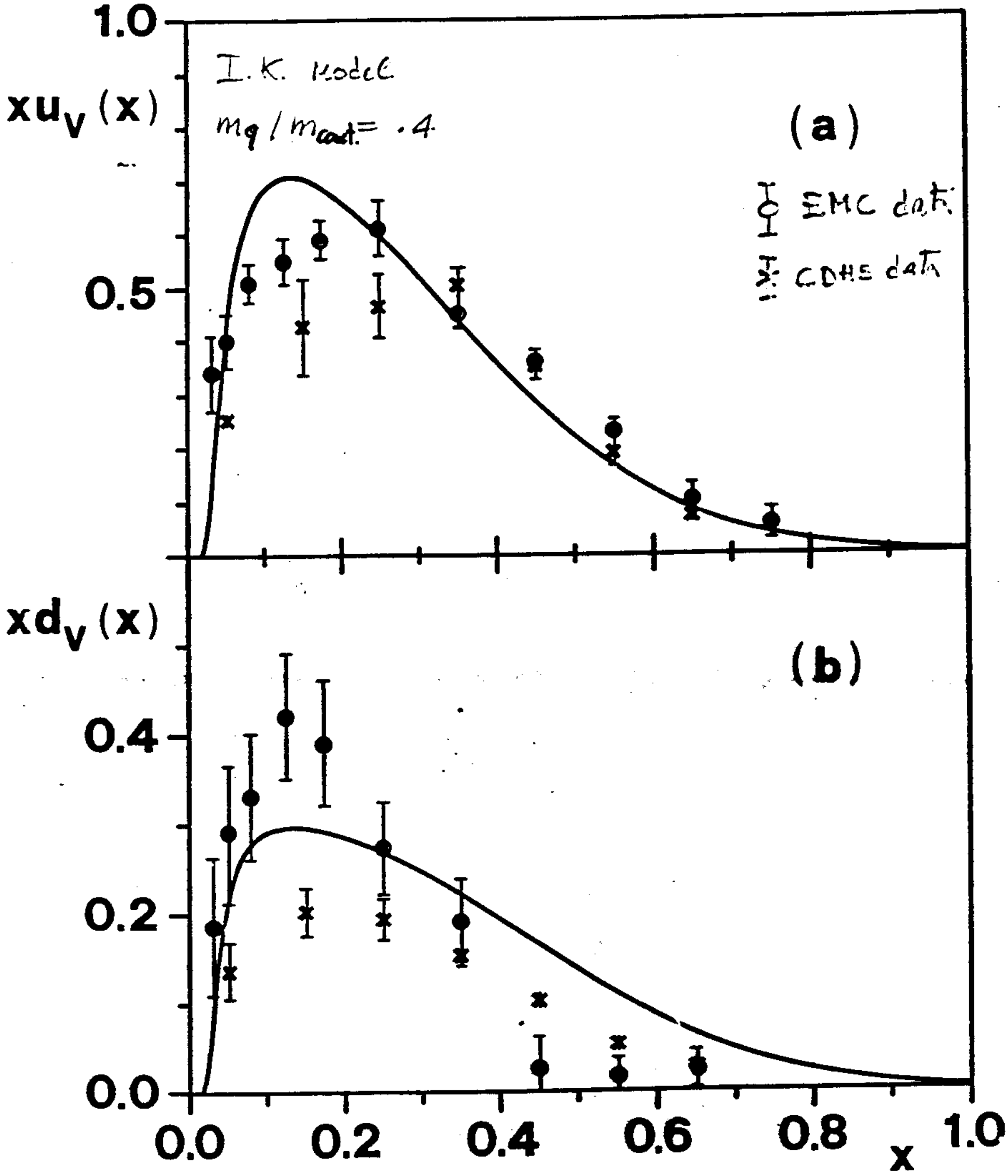

neutron
A oscillatore armonico
B Isgur - Karl
C due oscillatori armonici
Dati EMC
$F_2^n(x)$
0.0
0.1
0.2
0.3
0.4
0.6
0.8
1.0
X
A
B
C

Figura (3.15)

valence quark distribution



QCD scaling violation and Quark mass

- non-singlet contribution

Altarelli - Parisi evolution $q(Q^2, x)$

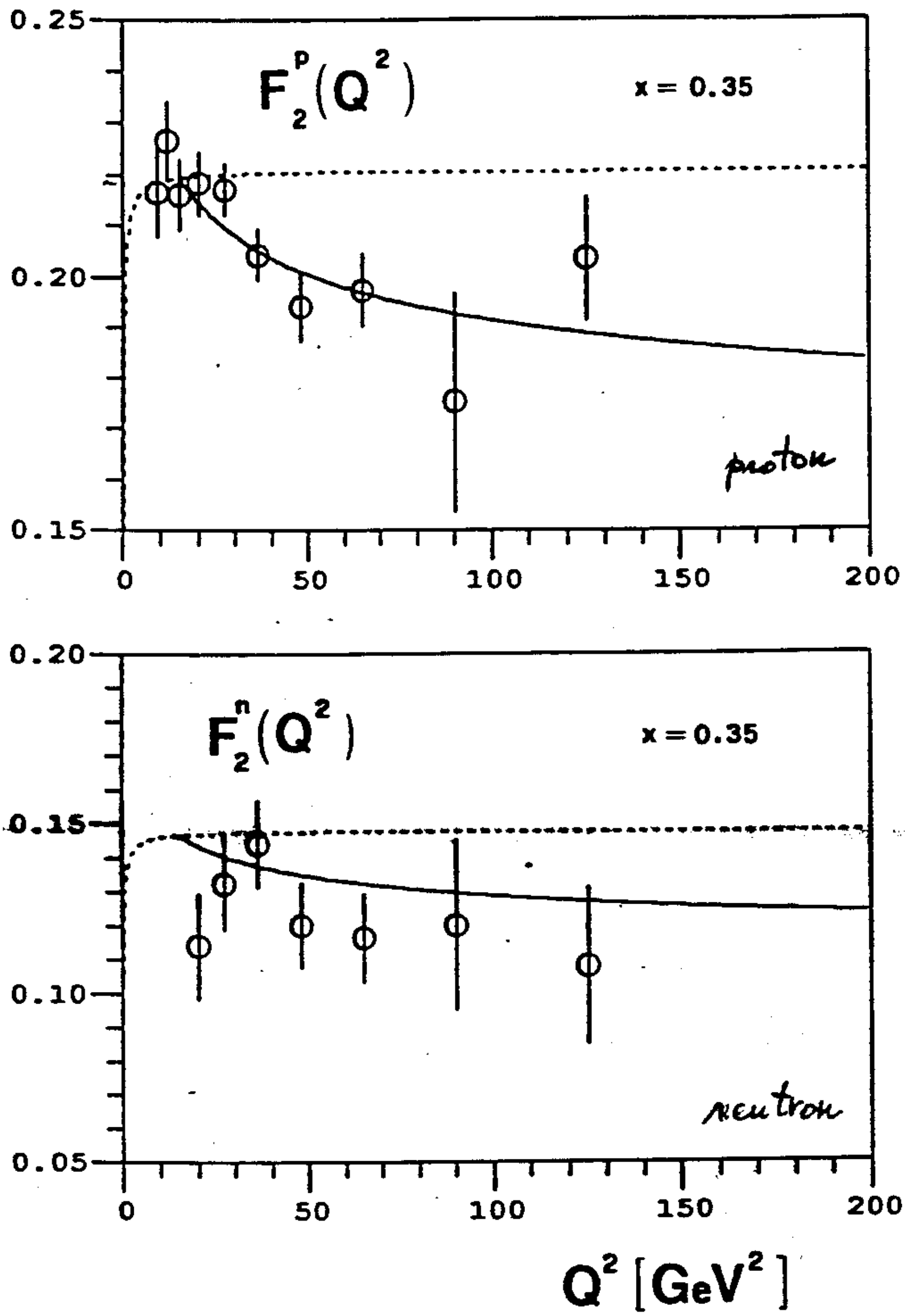
$$\frac{\int x^{n-1} q(Q^2, x)\,dx}{\int x^{n-1} q(Q_0^2, x)\,dx} = \left[\frac{\log(Q^2/\Lambda^2)}{\log(Q_0^2/\Lambda^2)}\right]^{A_n}$$

$n = 2$

$$\downarrow$$

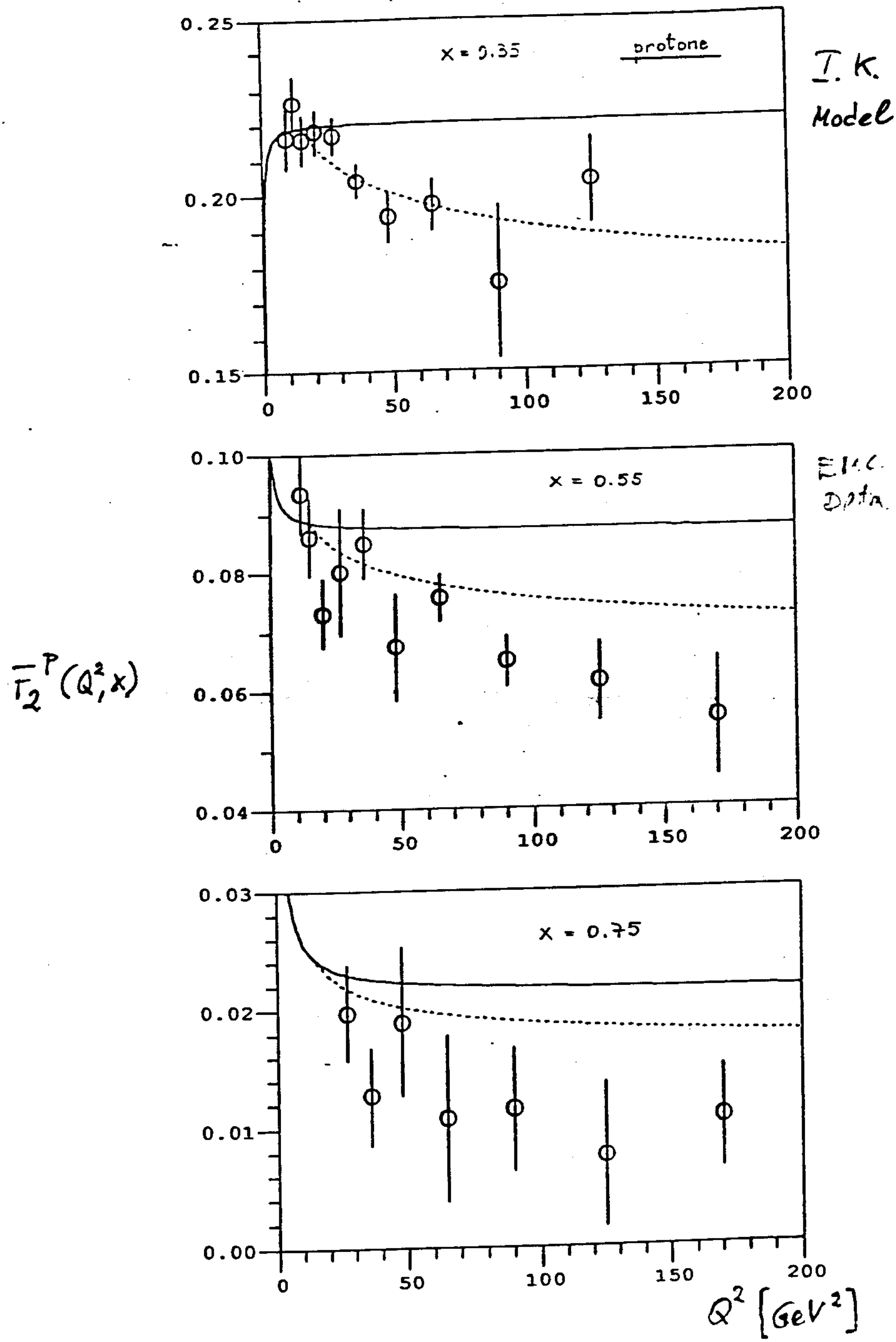
$$\frac{m_q(Q^2)}{m_q(Q_0^2)} = \left[\frac{\log(Q^2/\Lambda^2)}{\log(Q_0^2/\Lambda^2)}\right]^{-\frac{16}{9}\frac{1}{6(33-2n_f)}}$$

$Q_0^2 = 15\ GeV^2$; $m_q(Q_0^2) \cong .4\ m_{const}$



EMC Data

I. K. Model

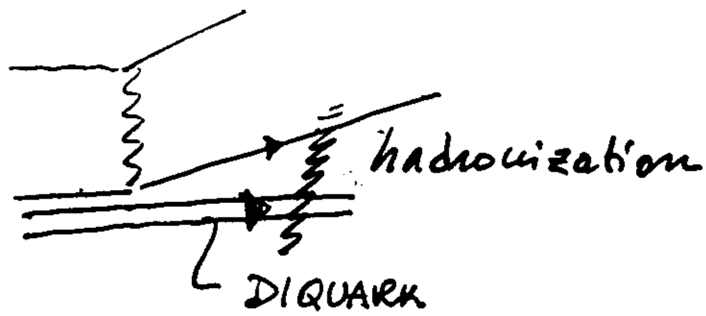


x = 0.35
protone
I. K. Model
0.25
0.20
0.15
0
50
100
150
200
x = 0.55
EMC Data
0.10
0.08
0.06
0.04
x = 0.75
0.03
0.02
0.01
0.00
$F_2^P(Q^2, x)$
Q^2 [GeV^2]

future perspectives

- $n(k)$ in relativistic or/and QCD inspired quark models.

- improving the "impulse approximation" within the Quark Parton Model



- exclusive processes

- Exploring EMC-effect and nucleon properties in the medium