SPINS IN THE PROTON: QCD AND ALL THAT

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1. A short introduction: theory & data.
2. EMC data: the first (and third) moment.
3. Axial couplings and $SU(3)_f$.
4. QCD evolution (to one loop).
5. A toy-model analysis of the quark (and gluon) components of spin.
6. Beyond three flavours and charged leptons: views on the future(s).

Notations

$$\Delta q_i = \int_0^1 dx\,[q_i^+ - q_i^-]$$

$$\Delta g = \int_0^1 dx\,[g_i^+ - g_i^-]$$

Structured "constituent" quarks
(à la Parisi - Petronzio)

$D^{\pm}$, $U^{\pm}$, ~~$S^{\pm}$~~

$$u^{\pm} = D^+ \otimes u_D^{\pm} + D^- \otimes u_D^{\mp} + U^+ \otimes u_U^{\pm} + U^- \otimes u_U^{\mp}$$

$$d^{\pm} = D^+ \otimes d_D^{\pm} + D^- \otimes d_D^{\mp} + U^+ \otimes d_U^{\pm} + U^- \otimes d_U^{\mp}$$

$$s^{\pm} = D^+ \otimes s_D^{\pm} + D^- \otimes s_D^{\mp} + U^+ \otimes s_U^{\pm} + U^- \otimes s_U^{\mp}$$

↘

$$\Delta u = \Delta U \cdot \Delta u_U + \Delta D \cdot \Delta d_U$$

$$\Delta d = \Delta U \cdot \Delta d_U + \Delta D \cdot \Delta d_D$$

$$\Delta s = \Delta U \cdot \Delta s_U + \Delta D\, \Delta s_D$$

$SU(2) \Rightarrow \quad \Delta u_U = \Delta d_D \quad \Delta d_U = \Delta u_D \quad \Delta s_U = \Delta s_D$

↙

$$\Delta u = \Delta U\, \Delta u_U + \Delta D\, \Delta d_U$$

$$\Delta d = \Delta U\, \Delta d_U + \Delta D\, \Delta u_U$$

$$\Delta s = (\Delta U + \Delta D)\, \Delta s_U = \Delta s_U$$

$SU(6)$ w.f. $\quad \Delta U = 4/3 \quad \Delta D = -1/3$

see : H. Fritzsch, Mod. Phys. Lett. A 5 (1990) 625

also : H.J. Lipkin, Phys. Lett. B 237 (1990) 130

A. Abbas, J. Phys. G 16 (1990) L 21

$$A_3 \simeq g_A = \Delta u - \Delta d = (\Delta U - \Delta D)(\Delta u_v - \Delta d_v) =$$

$$= (\Delta U - \Delta D) \cdot \Delta q_{val} = \frac{5}{3} \Delta q_{val}$$

G. Altarelli, W.J. Stirling : Part. World 1 (1989) 40

$$\Delta q_{val} = \frac{3}{5} g_A \simeq 0.755$$

$$A_8 = \Delta u + \Delta d - 2\Delta s = \Delta u_v + \Delta d_v - 2\Delta s_v =$$

$$= \Delta q_{val} + 2(\Delta d_v - \Delta s_v)$$

or

$$A_8 - \frac{3}{5} A_3 = 2(\Delta d_v - \Delta s_v)$$

A_8 measures the $SU(3)_f$ - breaking in the sea :

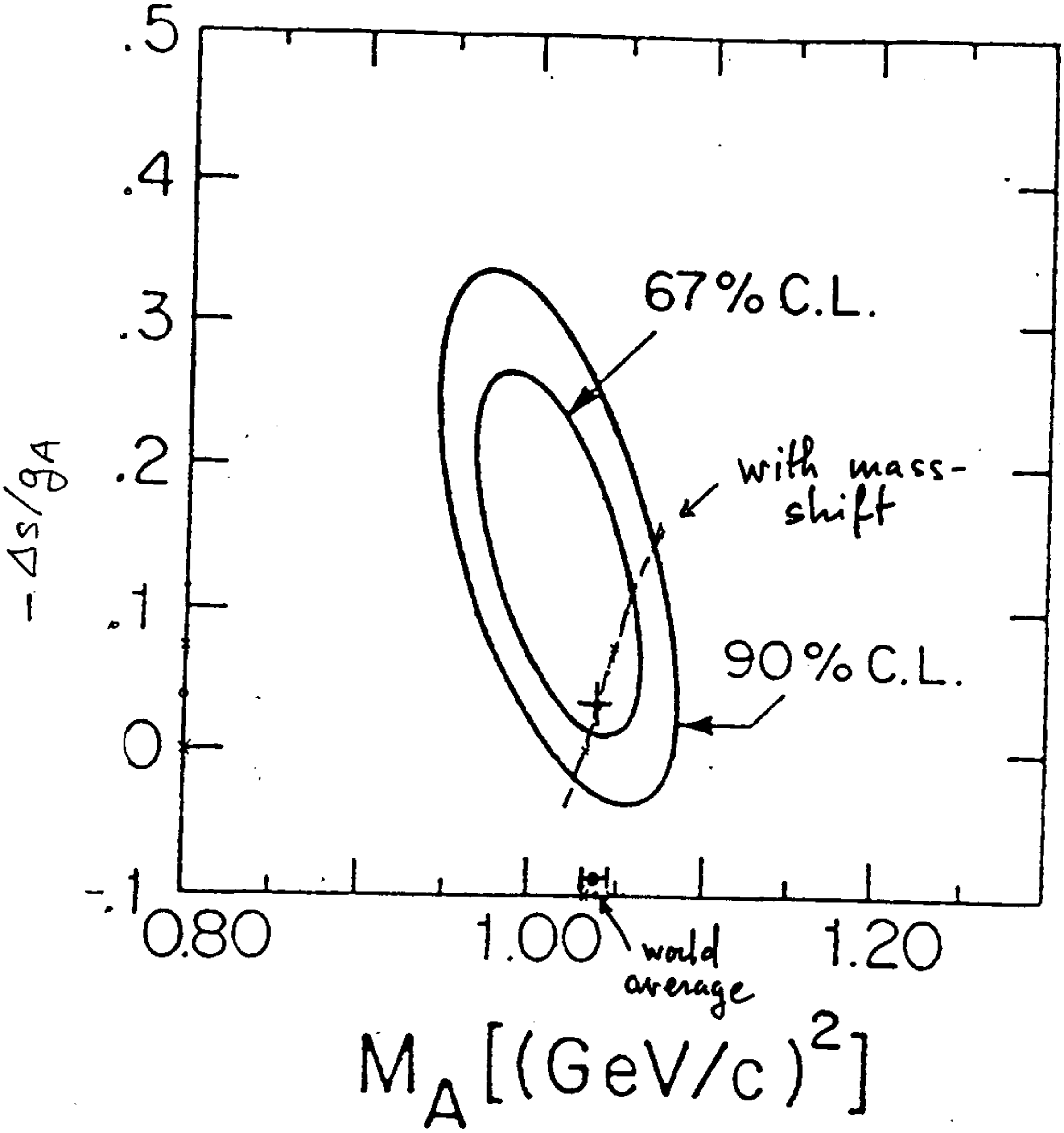
if $\Delta s_v / \Delta d_v = \eta$

$$\Delta q_{sea} = 2\Delta d_v + \Delta s_v = \frac{2+\eta}{2(1-\eta)} \left(A_8 - \frac{3}{5} A_3\right)$$

Δs from $\nu p \to \nu p$?

$$\text{axial n.c.} \propto \frac{\Delta u - \Delta d}{1 - Q^2/M_A^2} - \frac{\Delta s}{1 - Q^2/M_{As}^2} + \frac{\Delta c}{1 - Q^2/M_{Ac}^2}$$

strongly correl.



ıltaneous fit of $d\sigma/dQ^2$ for the neutr

L A Ahrens, et al : Phys Rev D35 (1987) 785

Sum rules (parton model)

J.D. Bjorken : Phys. Rev. D 1 (1970) 1976

J. Ellis, R.L. Jaffe : Phys. Rev. D 9 (1974) 1444

$$\int g_1 \, dx = \frac{1}{2} \sum_i e_i^2 \Delta q_i$$

SLAC experiments

M.J. Alguard, et al.: Phys. Rev. Lett. 37 (1976) 1261; ibid. 41 (1978) 70

G. Baum, et al. : Phys. Rev. Lett. 45 (1980) 2000; ibid. 51 (1983) 1135

$0.2 \lesssim x < 0.7 \qquad 3.5 < Q^2/GeV^2 < 10$

$0.1 \lesssim x < 0.5 \qquad 1 < Q^2/GeV^2 < 4$

$$\int dx \, g_1^p(x) \doteq 0.17 \pm 0.05$$

EMC experiment

J. Ashman, et al.: Phys. Lett. B 206 (1988) 364

$0.01 < x < 0.7 \qquad \langle Q^2 \rangle \simeq 10.7 \, GeV^2$

$$\int dx \, g_1^p(x) = 0.114 \pm 0.012 \, (stat) \pm 0.016 \, (syst)$$

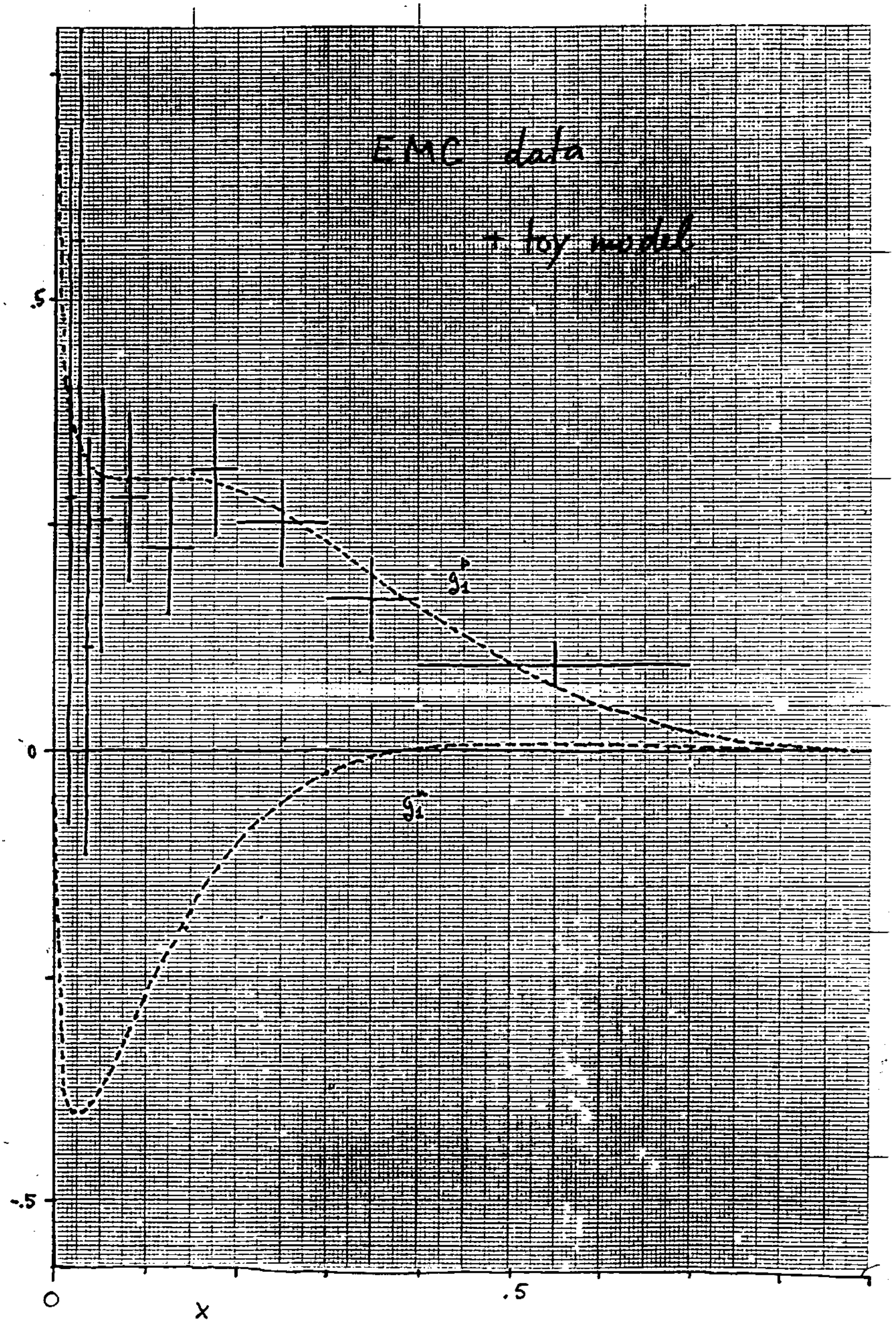
revised by V.W. Hughes, et al.: Phys. Lett. B 212 (1988) 511

$$0.116 \pm 0.009 \, (stat) \pm 0.019 \, (syst)$$

combining SLAC + EMC data

Final EMC data : Nucl. Phys. B 328 (1989) 1

$$0.126 \pm 0.010 \, (stat) \pm 0.015 \, (syst)$$


EMC data
+ toy model
g_1^p
g_1^n
.5
0
-.5
0
.5
x

How "robust" is the EMC result for M_1^p ?

Is the difference between the two figures "hidden" at $x \lesssim 10^{-2}$?

Study fits with

$$g_1^p(x) = x^{-\alpha} G(x) (1-x)^p$$

$p = 2n_s - 1$ (counting rule $\rightarrow n_s = 2$)

$\alpha \rightarrow$ "Leading" Regge-"pole" intercept

$G(x) = A_0 + A_1 x + \ldots \rightarrow$ polynomial

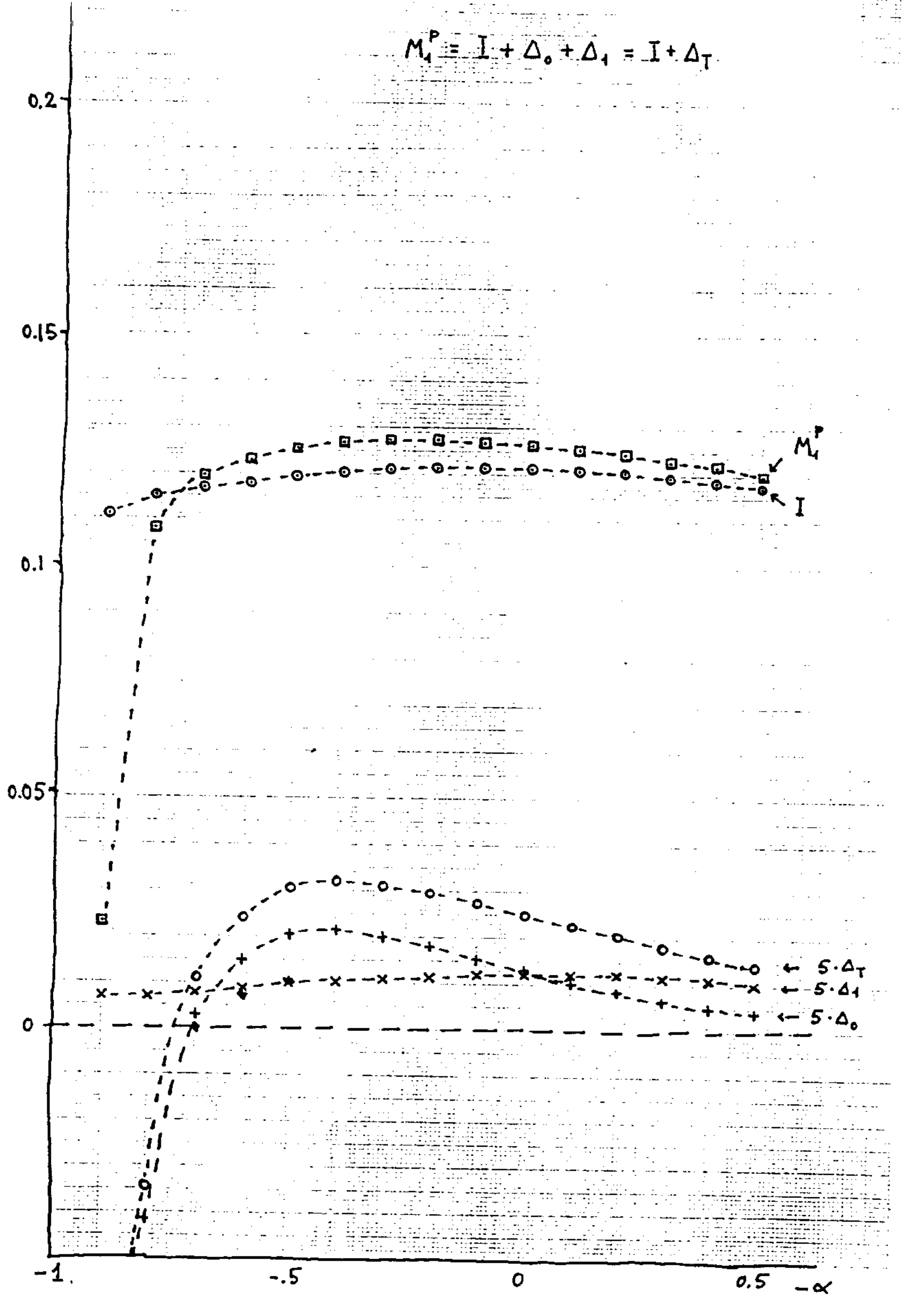
$G(x) \rightarrow$ 1st order

best fit $\alpha \simeq 0 \div 0.2$ $\alpha_\pi \simeq 0$
($\chi^2/NDF \simeq 3/8$) $\alpha_\eta \simeq -0.5$

M_1^p flat for most of the range

$$-0.5 \lesssim \alpha \lesssim 0.5$$

N.B. more singular behaviours as $x \rightarrow 0$ (i.e. for larger values of α) lead to a reduction, not an increase in M_1^p! Increasing the degree of the polynomial G flattens the α-dependence.


$M_1^P = I + \Delta_0 + \Delta_1 = I + \Delta_T$
0.2
0.15
0.1
0.05
0
-1.
-.5
0
0.5
$-\alpha$
M_1^P
I
$5 \cdot \Delta_T$
$5 \cdot \Delta_1$
$5 \cdot \Delta_0$

Axial couplings & $SU(3)_f$-symmetry

using: Ademollo-Gatto th, <u>actual</u> kinematics + Q^2-dep. f.f.

$n \to p e \bar{\nu}$	asy.	1.259 ± 0.004	$F + D$
$\Sigma^{\pm} \to \Lambda e \nu$	rate	0.742 ± 0.021	D
$\Xi^- \to \Xi^0 e \bar{\nu}$	"	$< 2.04 \cdot 10^{3}$	$F - D$
$\Lambda \to p e \bar{\nu}$	asy. + rate	0.710 ± 0.013	$F + \frac{1}{3} D$
$\Sigma^- \to n e \bar{\nu}$	" "	-0.333 ± 0.016	$F - D$
$\Xi^0 \to \Sigma^+ e \bar{\nu}$	rate	< 2.91	$F + D$
$\Xi^- \to \Sigma^0 e \bar{\nu}$	"	$1.280 \pm^{0.202}_{0.209}$	"
$\Xi^- \to \Lambda e \bar{\nu}$	asy. + rate	0.260 ± 0.032	$F - \frac{1}{3} D$

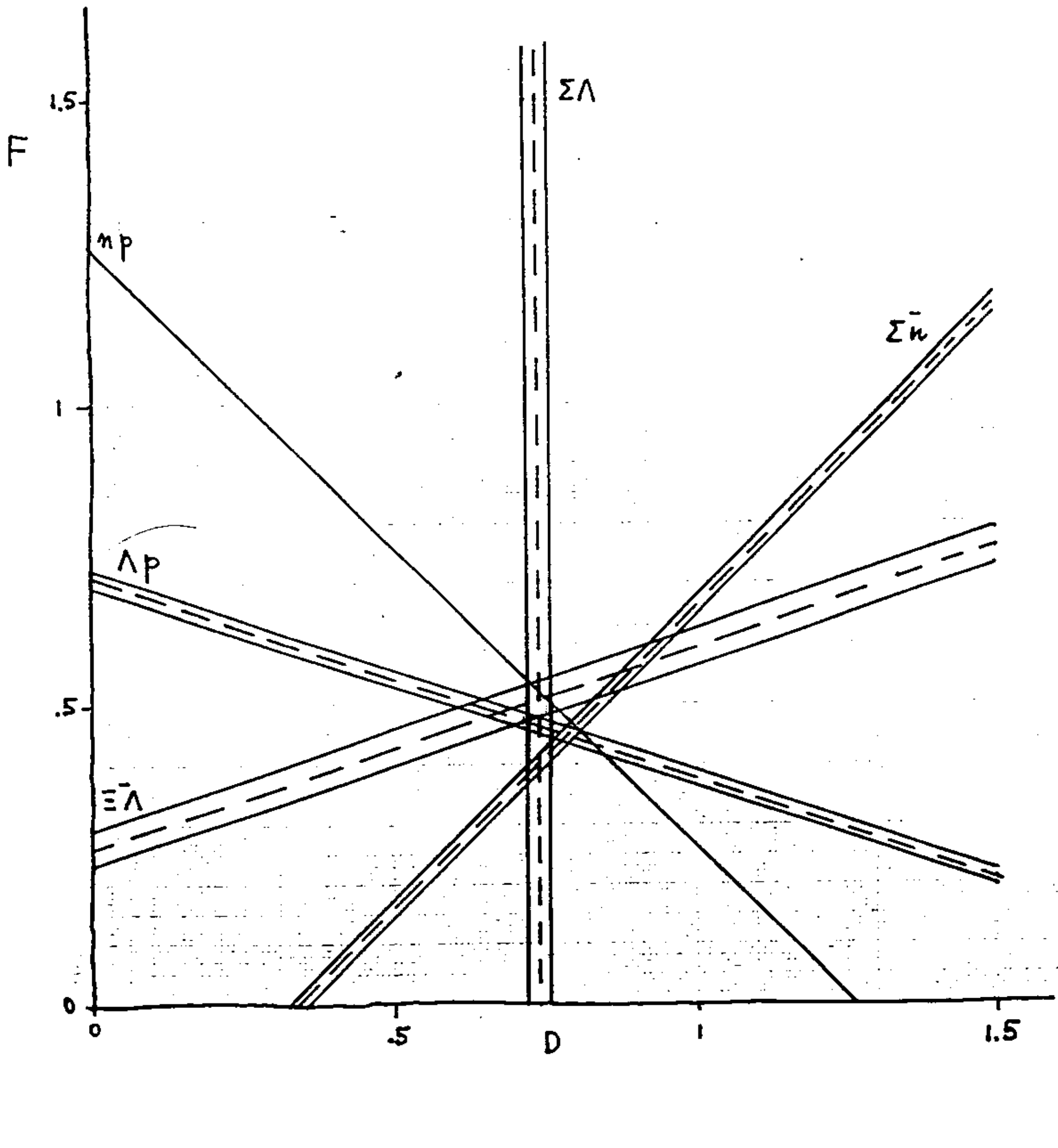
includes <u>all</u> data up to July 1990!

asymmetry and rates are fully consistent with each other if one uses correct kinematics and q^2-dependences of f.f.

R.L. Jaffe, A.V. Manohar: Nucl. Phys. B 337 (1990) 509

P.G. Ratcliffe: Phys. Lett. B 242 (1990) 271

P.M. G.: Nuovo Cimento 103 A (1990) 303


F
1.5
1
.5
0
np
Λp
Ξ̄Λ
ΣΛ
Σ̄n
0
.5
D
1
1.5

Deviations from $SU(3)_f$ accommodated by a mixing picture based on a Rayleigh-Schrödinger perturbation-series formalism

P.M.G. : Nuovo Cimento 104 A (1990) in print

$3g_{np} - 4g_{\Sigma\Lambda}$	0.809 ± 0.085	$+0.113$
$3g_{\Xi\Lambda}$	0.780 ± 0.096	$+0.001$
$3g_{\Lambda p} - 2g_{\Sigma\Lambda}$	0.646 ± 0.057	-0.058
$g_{np} + 2g_{\Sigma n}$	0.593 ± 0.032	-0.013
$\frac{3}{2}(g_{\Sigma n} + g_{\Lambda p})$	0.566 ± 0.031	-0.013
$3g_{\Sigma n} + 2g_{\Sigma\Lambda}$	0.485 ± 0.064	-0.075
$6g_{\Lambda p} - 3g_{np}$	0.483 ± 0.079	-0.229
A_8	?	-0.237

Sum rules

moments

$$M_\nu^{p,n}(Q^2) = \int_0^1 g_1^{p,n}(x, Q^2)\, x^{\nu-1}\, dx$$

Bjorken

$$M_1^p(Q^2) - M_1^n(Q^2) = \tfrac{1}{6} A_3 \left(1 - \frac{\alpha_s}{\pi}\right)$$

Ellis - Jaffe

$$M_1^{p,n}(Q^2) = \left[\frac{1}{36} A_8 \pm \frac{1}{12} A_3 + \frac{1}{9} A_0(Q^2)\right]\left(1 - \frac{\alpha_s}{\pi}\right)$$

QCD corrections to one loop

Using flavour SU(3) symmetry

$$A_8 = 3F - D = 3 g_A \frac{1 - \frac{1}{3}\alpha}{1+\alpha} \qquad \alpha = D/F$$

the Ellis-Jaffe sum rules become

$$M_1^p = \left(1 - \frac{\alpha_s}{\pi}\right)\left[\frac{1}{2}\, \frac{1 - \alpha/9}{1+\alpha}\, g_A + \frac{1}{3} A_0^s\right]$$

$$M_1^n = \left(1 - \frac{\alpha_s}{\pi}\right)\left[\frac{1}{3}\, \frac{1 - \frac{2}{3}\alpha}{1+\alpha}\, g_A + \frac{1}{3} A_0^s\right]$$

OZI-rule plus $\alpha \approx 2/3$ lead to expect

$$M_1^p \simeq 0.19 \quad \text{and} \quad M_1^n \simeq 0$$

QCD evolution at <u>one</u> <u>loop</u> (including the anomaly)

J. Kodaira: Nucl. Phys. B 165 (1980) 129

G. Altarelli, G.G. Ross: Phys. Lett. B 212 (1988) 391

G. Altarelli, B. Lampe: Z. Phys. C 47 (1990) 315

$$A_0(Q^2) = \Delta\Sigma - 3\frac{\alpha_s}{2\pi}\Delta g(Q^2) =$$

$$= \left[\Delta\Sigma - \frac{3\bar{\alpha}_s}{2\pi}\overline{\Delta g}\right]\exp\left(-\frac{2}{3}\frac{\bar{\alpha}_s - \alpha_s}{\pi}\right)$$

$$\Delta\Sigma = \Sigma_i \Delta q_i$$

$$\Delta q_i = \int dx\,[q_i^+ + \bar{q}_i^+ - q_i^- - \bar{q}_i^-] \quad , \quad \Delta g = \int dx\,[g^+ - g^-]$$

$$\Delta\Sigma + 2\Delta g + 2\langle L_z\rangle = 1$$

$$\Delta g = \frac{\bar{\alpha}_s}{\alpha_s}\overline{\Delta g}\exp\left(-\frac{2}{3}\frac{\bar{\alpha}_s - \alpha_s}{\pi}\right) + \frac{2\pi}{\alpha_s}\frac{\Delta\Sigma}{3}\cdot$$

$$\cdot\left[1 - \exp\left(-\frac{2}{3}\frac{\bar{\alpha}_s - \alpha_s}{\pi}\right)\right] \sim c\cdot\log Q^2/\Lambda^2$$

nb $c > 0$ for <u>any</u> $\bar{\alpha}_s$ if we choose the normalization scale so that

$$\Delta\Sigma + 2\,\overline{\Delta g} = 1 \quad , \quad \text{or} \quad \langle L_z\rangle = 0$$

$\chi^2 = 3.11 \quad ND = 8$

$(M_1^p)_{val} = 0.192 \qquad (M_1^n)_{val} = 0$

$(M_1^N)_{sea} = -0.058$

$(M_3^p)_{val} = 0.0130 \qquad (M_3^n)_{val} = 0.0010$

$(M_3^N)_{sea} = -0.0011$

$M_3^p / M_1^p = 0.089 \qquad M_3^n / M_1^n = 0.003$

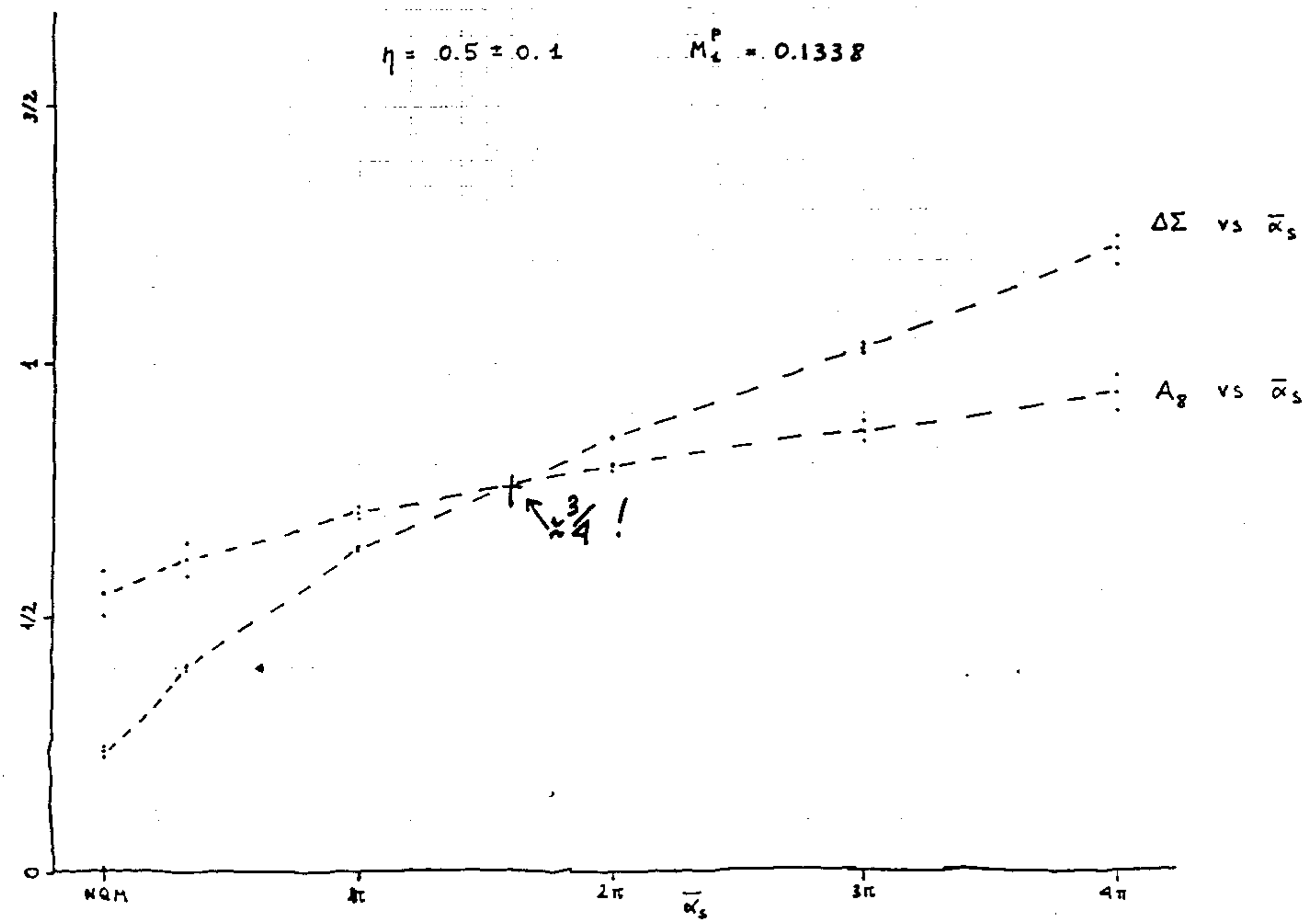
$(M_3^p / M_1^p)/(M_3^n / M_1^n) = 0.032$

Separation of the sea into flavour components : assume constant strange/nonstrange ratio η (as in the non-polarized distributions) + SU(2) symmetry of the sea

$$g_1^N(x)_{sea} = \frac{5+\eta}{9}\,\delta\bar{u}(x)$$

$$\Delta u_{sea} = \Delta d_{sea} = \frac{18}{5+\eta}\left[\int_0^1 g_1^N(x)_{sea}\,dx \Big/ \left(1-\frac{\alpha_s}{\pi}\right) + \frac{\alpha_s}{2\pi}\frac{\Delta g}{3}\right]$$

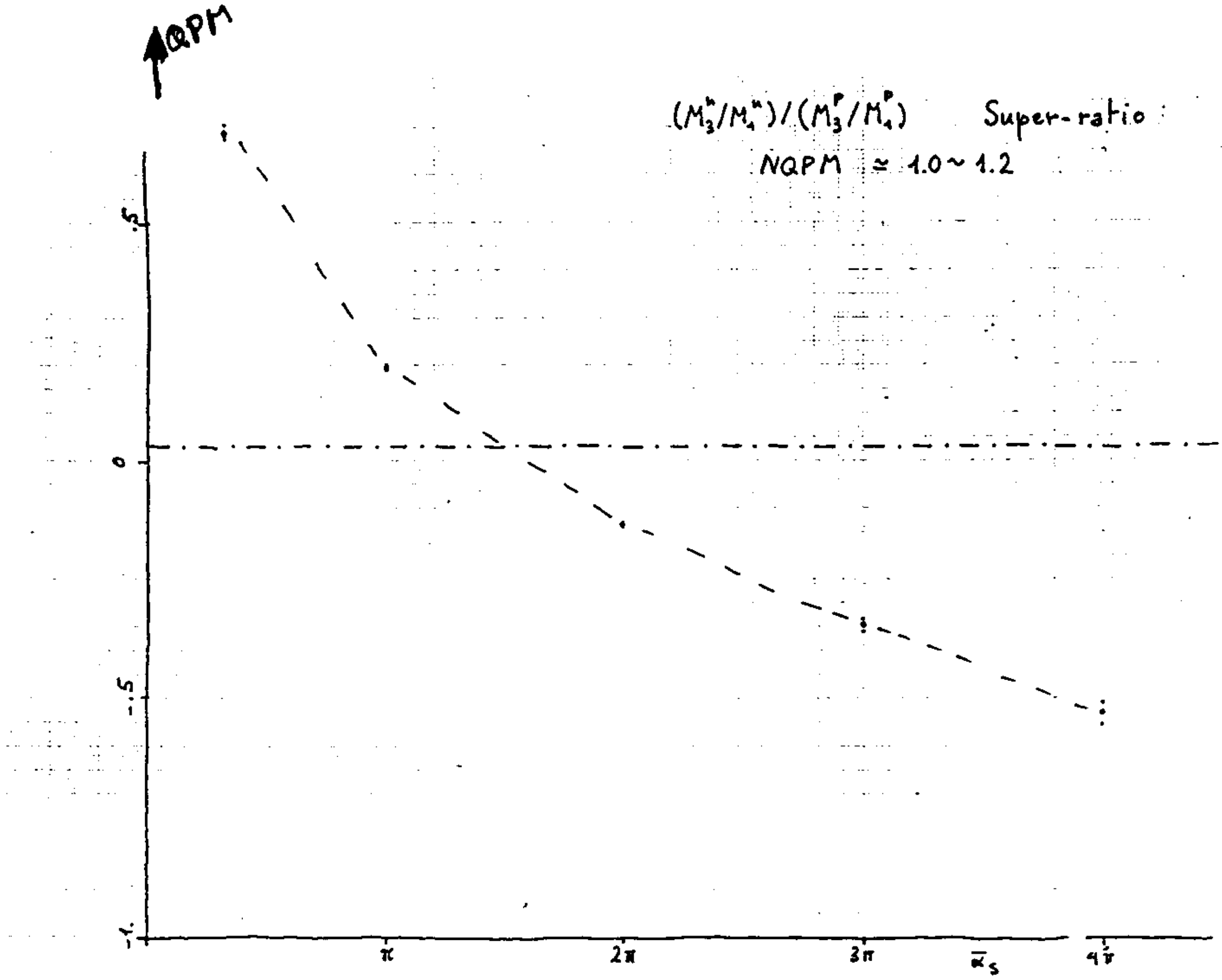
$$\Delta s = \frac{18\eta}{5+\eta}\left[\int_0^1 g_1^N(x)_{sea}\,dx \Big/ \left(1-\frac{\alpha_s}{\pi}\right) + \frac{\alpha_s}{2\pi}\frac{\Delta g}{3}\right]$$



Using the evolution equation we can turn the above expressions into values for $\Delta\Sigma$ and

$$A_8 = \Delta u + \Delta d - 2\Delta s$$

assuming $\langle L_z \rangle = 0$ at the normalization point $\bar{\alpha}_s$, versus $\bar{\alpha}_s$



Is the anomaly contribution essential ?

D. Indumathy, M.V.N. Murthy, V. Ravindran : Mod. Phys. Lett. A 5 (1990) 1125

from an MIT bag model

$$M_3 = \frac{1}{18M^2}\left\{[4\Delta u + \Delta d](\hat{m}_{eff})^2 + \Delta s\,(m^s_{eff})^2\right\}$$

forgetting the anomaly piece in M_1 we would get (with $m^s_{eff}/\hat{m}_{eff} \simeq 3/2$) the "superratio"

$$\frac{M_3^n/M_1^n}{M_3^p/M_1^p} \simeq 1$$

Future of spin measurements:

more precise measurements in the same Q^2 range as SLAC + MIT to test QCD evolution

test of Bjorken sum rule: measurements on ^2H and ^3He

evolution beyond heavy-quark "thresholds"

$$M_1^{p,n} = \left(1 - \frac{\alpha_s}{\pi}\right)\left[\frac{1}{36} A_8 \pm \frac{1}{12} A_3 + \frac{1}{9} A_0\right] \qquad f = 3$$

$$\downarrow$$

$$M_1^{p,n} = \left(1 - \frac{\alpha_s}{\pi}\right)\left[\frac{1}{36} A_8 \pm \frac{1}{12} A_3 - \frac{1}{36} A_{15} + \frac{5}{36} A_0\right] \qquad f = 4$$

$$M_1^{p,n} = \left(1 - \frac{\alpha_s}{\pi}\right)\left[\frac{1}{36} A_8 \pm \frac{1}{12} A_3 - \frac{1}{36} A_{15} + \frac{1}{60} A_{24} + \frac{11}{90} A_0\right] \qquad f = 5$$

$$A_{15} = \Delta\Sigma - 4\Delta c$$

$$A_{24} = \Delta\Sigma + \Delta c - 5\Delta b$$

$$A_0 = \Sigma_i \Delta q_i - f \frac{\alpha_s}{2\pi} \Delta g$$

where A_0 evolves as (Kodaira)

$$A_0(Q^2) = A_0(\bar{Q}^2) \exp\left(-\frac{6f}{33-2f} \frac{\bar{\alpha}_s - \alpha_s}{\pi}\right)$$

Note that the contribution to A_0 for a heavy quark <u>at threshold</u> is defined by the (non-perturbative) quantity Δq_h <u>alone</u>

We have thus

$$A_0(Q^2)_{f=4} = \Delta\Sigma + \Delta c + \Delta\Gamma(Q_c^2) - 4\,\Delta\Gamma(Q^2) =$$

$$= \left\{ [\Delta\Sigma - 3\,\overline{\Delta\Gamma}]\, \exp\left(-\frac{2}{3}\,\frac{\bar{\alpha}_s - \alpha_s(Q_c^2)}{\pi}\right) + \Delta c \right\} \cdot$$

$$\cdot \exp\left(-\frac{24}{25}\,\frac{\alpha_s(Q_c^2) - \alpha_s(Q^2)}{\pi}\right)$$

and

$$A_0(Q^2)_{f=5} = \left(\left[\left\{ [\Delta\Sigma - 3\,\overline{\Delta\Gamma}]\, \exp\left(-\frac{2}{3}\,\frac{\bar{\alpha}_s - \alpha_s(Q_c^2)}{\pi}\right) + \Delta c \right\} \cdot \right.\right.$$

$$\left.\left. \cdot \exp\left(-\frac{24}{25}\,\frac{\alpha_s(Q_c^2) - \alpha_s(Q_b^2)}{\pi}\right) \right] + \Delta b \right) \cdot$$

$$\cdot \exp\left(-\frac{30}{23}\,\frac{\alpha_s(Q_b^2) - \alpha_s(Q^2)}{\pi}\right)$$

where $$\Delta\Gamma = \frac{\alpha_s}{2\pi}\,\Delta g \qquad \text{(Altarelli \& Ross)}$$

Light quarks are contributing to the change in $M_1(Q^2)$ through heavy quark thresholds!

$$M_1^p(Q^2)_{f=4} - M_1^p(Q^2)_{f=3} =$$

$$= \left(1-\frac{\alpha_s}{\pi}\right)\left[\frac{5}{36} A_0(Q^2)_{f=4} - \frac{1}{36} A_{15} - \frac{1}{9} A_0(Q^2)_{f=3}\right] =$$

$$= \left(1-\frac{\alpha_s}{\pi}\right)\left\{\left[\Delta\Sigma - 3\,\overline{\Delta\Gamma}\right]\left[\frac{5}{36}\exp\left(-\frac{2}{3}\frac{\bar{\alpha}_s}{\pi} - \frac{22}{75}\frac{\alpha_s(Q_c^2)}{\pi} + \right.\right.\right.$$

$$\left. + \frac{24}{25}\frac{\alpha_s}{\pi}\right) - \frac{1}{9}\exp\left(-\frac{2}{3}\frac{\bar{\alpha}_s-\alpha_s}{\pi}\right)\Big] + \Delta c\left[\frac{5}{36}\,\cdot\right.$$

$$\left.\left.\cdot \exp\left(-\frac{24}{25}\frac{\alpha_s(Q_c^2)-\alpha_s}{\pi}\right) + \frac{1}{12}\right] - \frac{1}{36}\Delta\Sigma\right\}$$

$$\xrightarrow[\text{NQPM}]{} \frac{2}{9}\Delta c$$

Note that if Δc is small enough, what is observed is rather (if $\bar{\alpha}_s$ is large enough) the quantity

$$-\frac{1}{36}\Delta\Sigma \sim O(10^{-2}) < 0 \;!$$

CONCLUSIONS

1. EMC data can live with QCD: one could even say that they are required by a full QCD treatment of the unitary singlet piece on the r.h.s. of the Ellis-Jaffe sum rule.

2. A toy-model fit can lead to $\Delta\Sigma \simeq O(1)$ and a small Δs (consistent with Preparata-Soffer Regge-model bound), with an exact validity of Bjorken sum rule.

3. More precise data both at intermediate and high Q^2's are needed to test QCD evolution below and through heavy-flavour thresholds.

4. $SU(3)_f$ won't be safely used with new, more precise data (even if its breaking is "lost in the noise" with today's data - Lipkin).

5. To test its breaking we need also to measure better hyperon β-decays (we have only a handful of $\Sigma^+ \to \Lambda e^+ \nu_e$ events!)