FUNCTIONAL METHODS AND CORRELATIONS IN NUCLEAR MATTER

1 Functional Techniques

Useful for obtaining new effective actions and new approximation schemes

Example (used throughout this talk):

Pion Nuclear Physics

Model Lagrangian (simplified scheme)

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\pi + \mathcal{L}_{\pi-N}$$

$$\mathcal{L}_N = \bar{\psi}(i\not{\partial} - M)\psi \quad \text{(or its nonrelativistic limit)}$$

$$\mathcal{L}_\pi = \frac{1}{2}(\partial_\mu \vec{\phi})^2 - \frac{\mu^2}{2}\vec{\phi}^2$$

$$\mathcal{L}_{\pi-N} = i g \bar{\psi} \gamma_5 \vec{\tau} \cdot \vec{\phi} \psi \quad \text{(for instance)}$$

When needed $\mathcal{L} \to \mathcal{L} + \mathcal{L}_{e.m}$

via minimal coupling

1.1. The Effective Action

Generating Functional:

$$Z = \int D[\phi, \psi, \bar{\psi}] \; e^{\, i \int d^4x \, \mathcal{L}[\psi, \bar{\psi}, \phi] + J\phi + \dots}$$

plus other eventual external sources

Z summarizes the whole hyerarchy of the Green's Functions

Note that $\mathcal{L}$ is bilinear in the Fermionic fields $\bar{\psi}, \psi$ ⟹

⟹ Idea: Integrate over $\bar{\psi}, \psi$

[reminds the Hubbard - Stratonovitch transformation and the bosonization of Bleinot]

Identity:

$$Z = \int D[\phi]\, e^{iS_{eff} + J\phi}$$

with

$$S_{eff} = \int d^4x \left\{ \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 \right\} - \sum_{n=2}^{\infty} \frac{1}{n} \Pi_n \phi^n$$

$\sum_{n=2}^{\infty} \frac{1}{n} \Pi_n \phi^n = \frac{1}{2}$  $+ \frac{1}{3}$  $+ \frac{1}{4}$  $+ \ldots$

(• = ϕ ; •——— = Free Fermion Green's Functi

[For future reference:  = Lindhard Function;

 and  analytically given by

R.C., P. Saracco, N.P. A487 (1988) 279.

Properties of S_{eff}:

- Contains pion field only
- The interaction is nonlocal, multilinear and displays one closed fermion loop with any number of vertices
- Renormalization: complicated but possible even inside the nuclear medium: W.M. Alberico, R.C., A. Molinari and P. Saracco, Phys Rev C 38 (1988) 2389

1.2 Mean Field

Nucleonic level $\longrightarrow$ Hartree-Fock Approximation (not discussed here)

Pionic level $\longrightarrow$ R.P.A.

In fact:

Mean Field = Stationary Phase Approximation

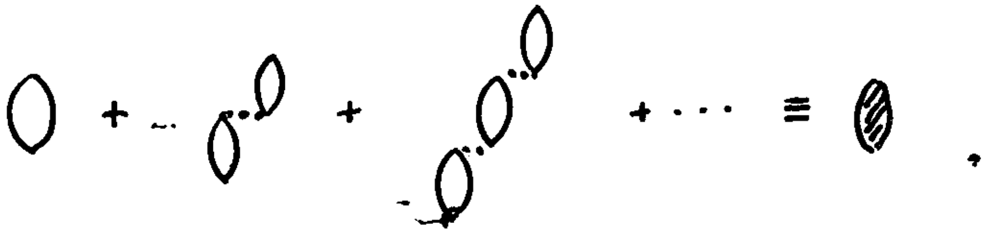
S.P.A equation:

$$\frac{\delta S_{eff}}{\delta \phi(x)} = -J(x)$$

Solution for ϕ in presence of an external source J:

$$\phi = \frac{1}{\Box - \mu^2 - \Pi^{(2)}} J \equiv \text{R.P.A. solution}$$

Mean Field Response Function:

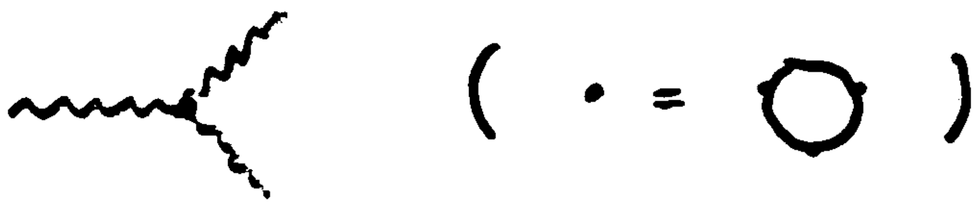


Dressed pionic line : 

General Mean Field Diagrams:

Tree Graphs in 

Examples – (π, 2π) reaction:



(2π, 2π) (whatever it will mean)

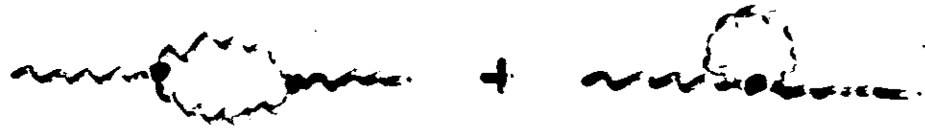
A relevant Point: 4-momenta involved

= those of the external probe $\pm k_F$

⇒ Low momentum Probes Explore low momentum dynamics

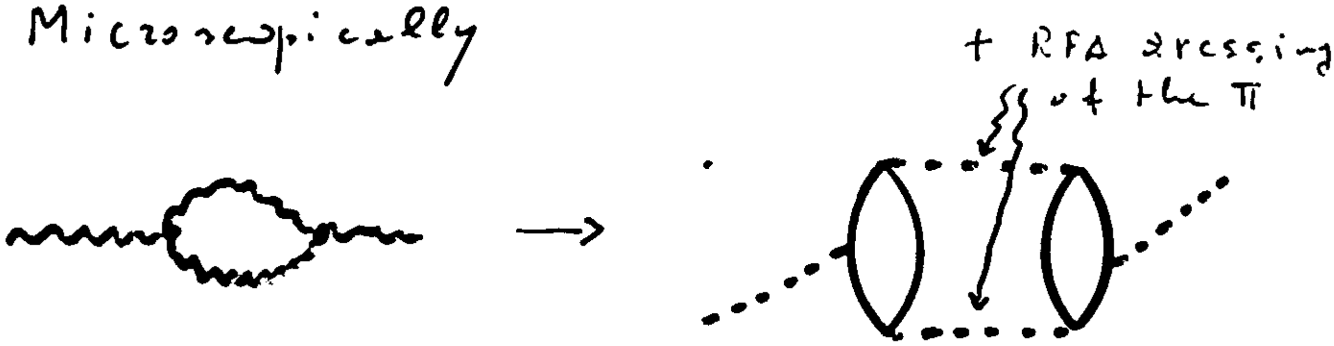
1.5 One Loop Corrections

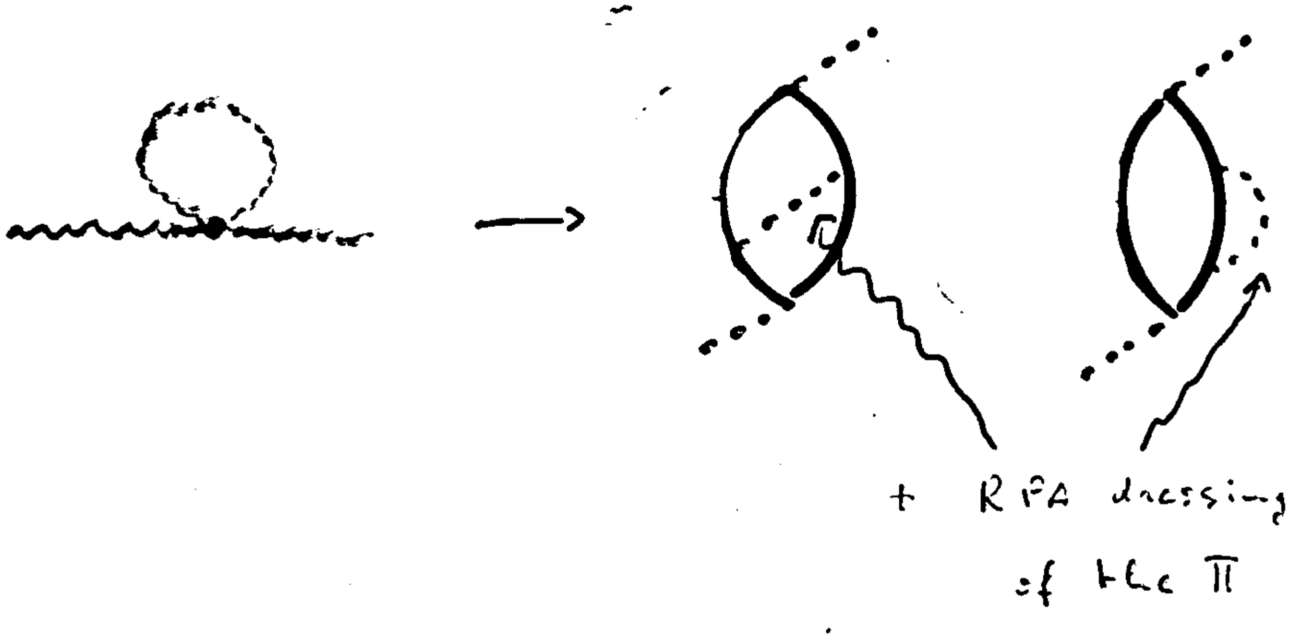
Example on the Response Function

$\Pi^{1\,loop} =$ 

Relevant point (even if seemingly trivial): the loop integrated momentum is not cut by k_F

Microscopically



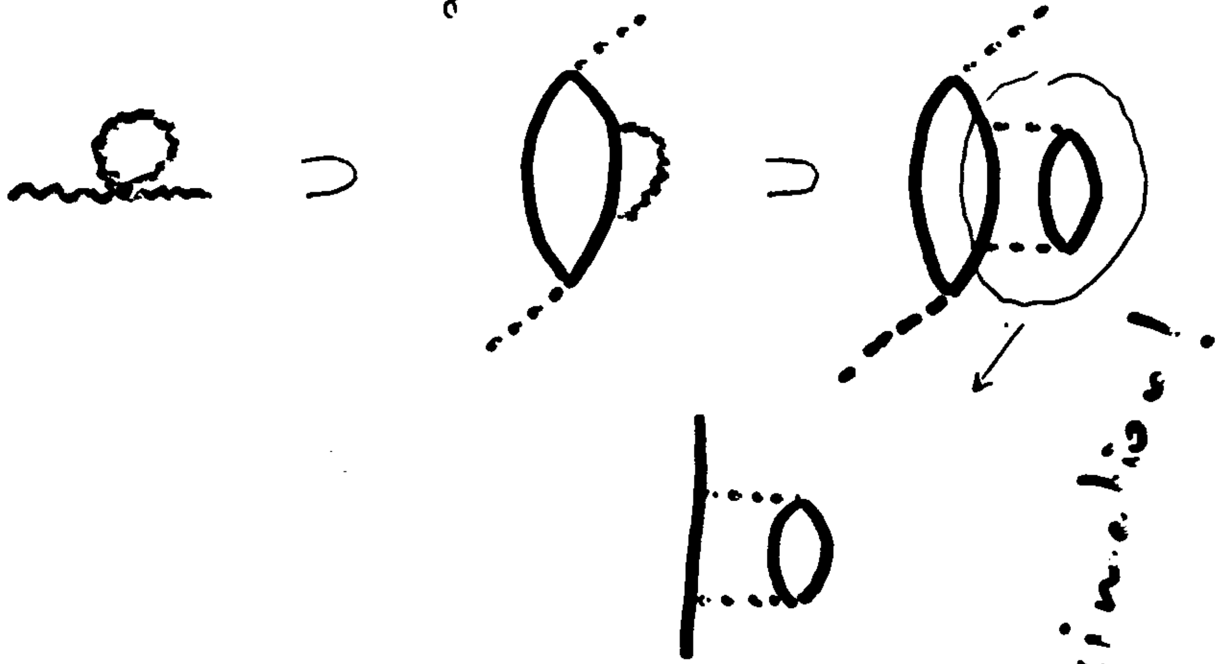


Relevant point:

How does the loop integral converge?

(Renormalization is understood; we are looking to nuclear medium)

A critical diagram



Put

$$V_{opep} = -\frac{f^2}{\mu^2}\,\frac{q^2}{q^2+\mu^2}$$

 $= \Pi^{(1)}(q,\omega) \sim \frac{1}{q^2}$

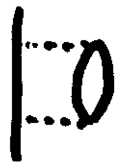
Naive Approximation!

$\Rightarrow$ The loop integral diverge

How can the divergence be cured?

1) Relativistic effects

$$\psi^\dagger(\vec\sigma\cdot\vec\nabla)\cancel{(\vec\tau\cdot\vec\pi)}\psi \longrightarrow \bar\psi\gamma_5\vec\tau\vec\pi\psi$$

now  converges

p and s waves mixed together

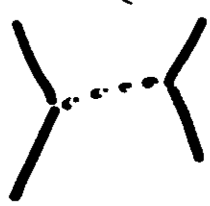
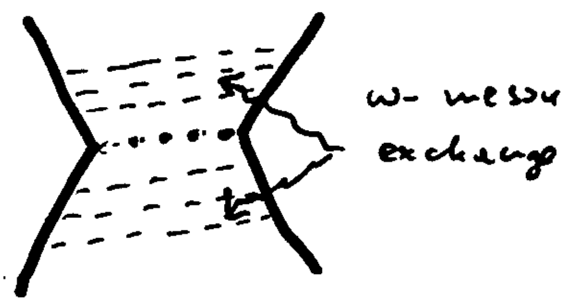
Natural cut-off: The relevant scale is 2M

2) πNN form factor

Typically
$$v(q^2) = \frac{\Lambda^2-\mu^2}{q^2-\mu^2}$$

Typical scale: 1.2 GeV/c

3) Short range correlations

OPEP =  $\longrightarrow$ 

$V_{OPEP} \longrightarrow$ correlated pion exchange

$V_{OPEP} \xrightarrow[q\to\infty]{} 1$: $V^{corr}_{OPEP} \sim \frac{1}{q^2}$ (at least)

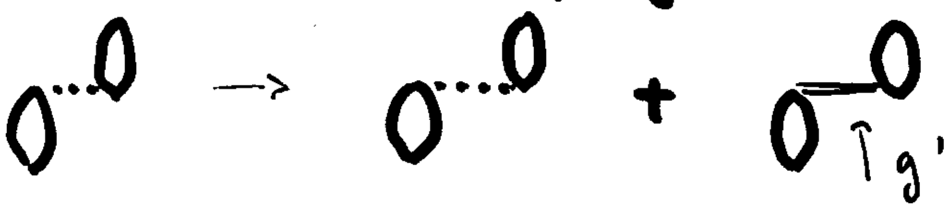
Typical scale: $m_\omega = 770$ MeV

Pion dynamics inside the medium

1) Introduction of the Δ

$$\Pi^{(1)} \longrightarrow \Pi_{ph} + \Pi_{\Delta h}$$

2) Introduction of g'



- Universal choice (for sake of simplicity)

May g' depend upon q and ω?

(Some indications that g' decreases with increasing ω – R.C., P. Saracco Phys Lett B to be published)

3) How to embody short range correlations?

Low energy: $V_{OPEP} \rightarrow \frac{f^2}{\mu^2}\left(g' - \frac{q^2}{q^2+\mu^2}\right)$

High " $V^{cont}_{OPEP} \sim \frac{1}{q^2}$

Idea: $g' \rightarrow g'(q) = 1 + (g'_0 - 1)\frac{m_\omega^2}{m_\omega^2+q^2}$ $\xrightarrow{q\to\infty} 1$, $\xrightarrow{q\to 0} g'_0$

Two parameters:

g'_0 fixes the low energy dynamics

m_ω fixes the high energy dynamics

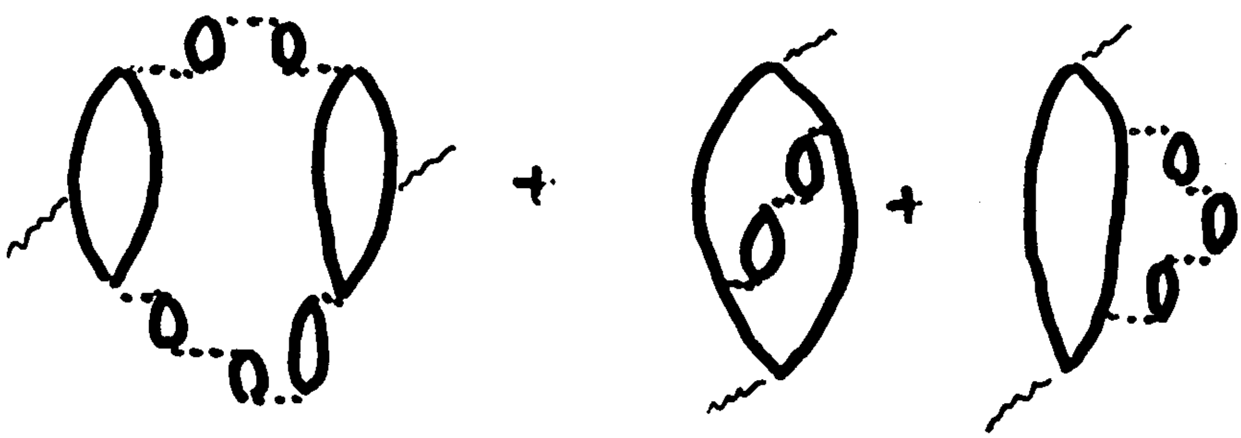
2. Longitudinal e.m. Response Function

1) Mean Field Level

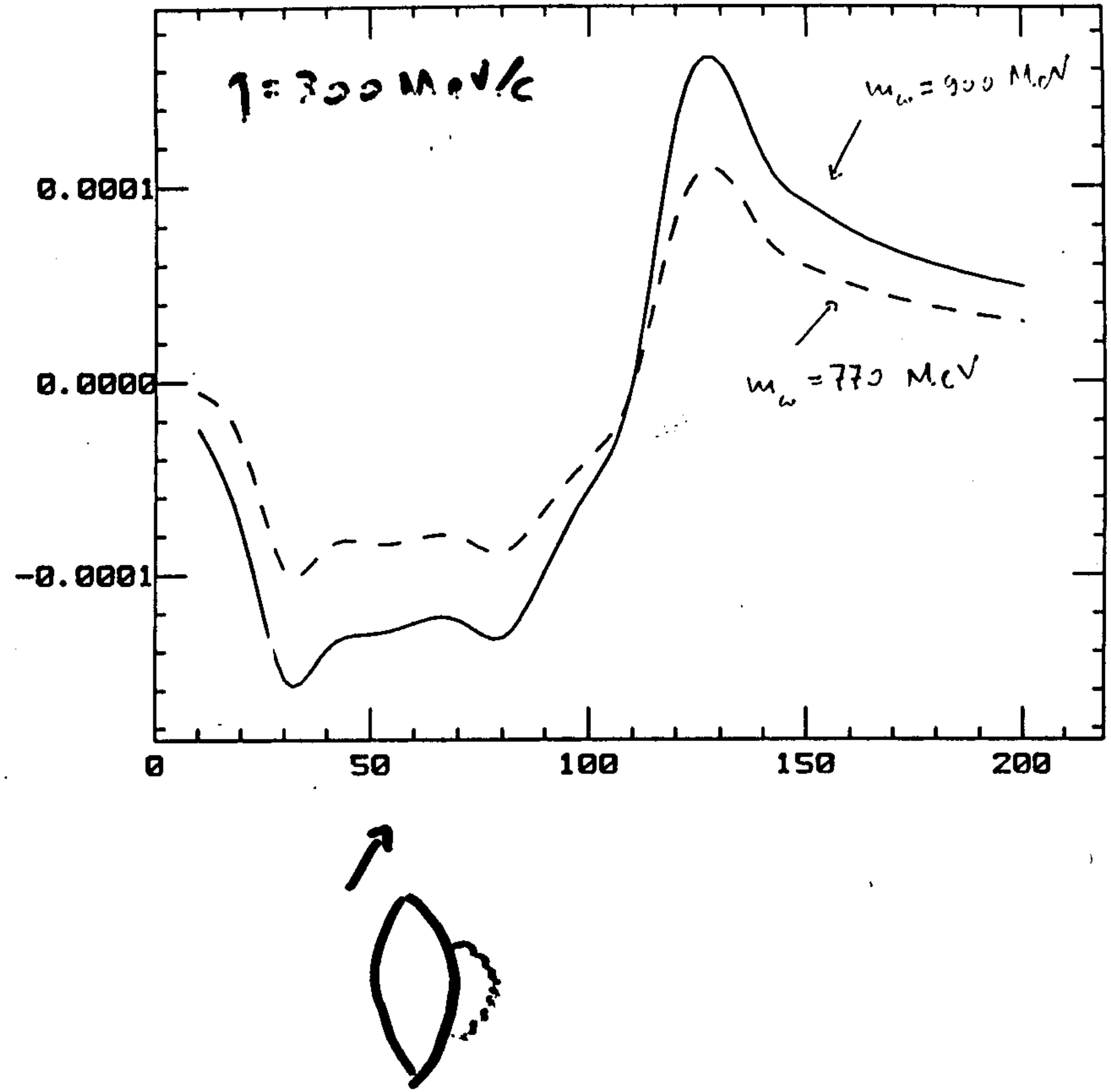
$$\tilde{\Pi}_{\mu\nu}^{\text{mean field}} =$$ 

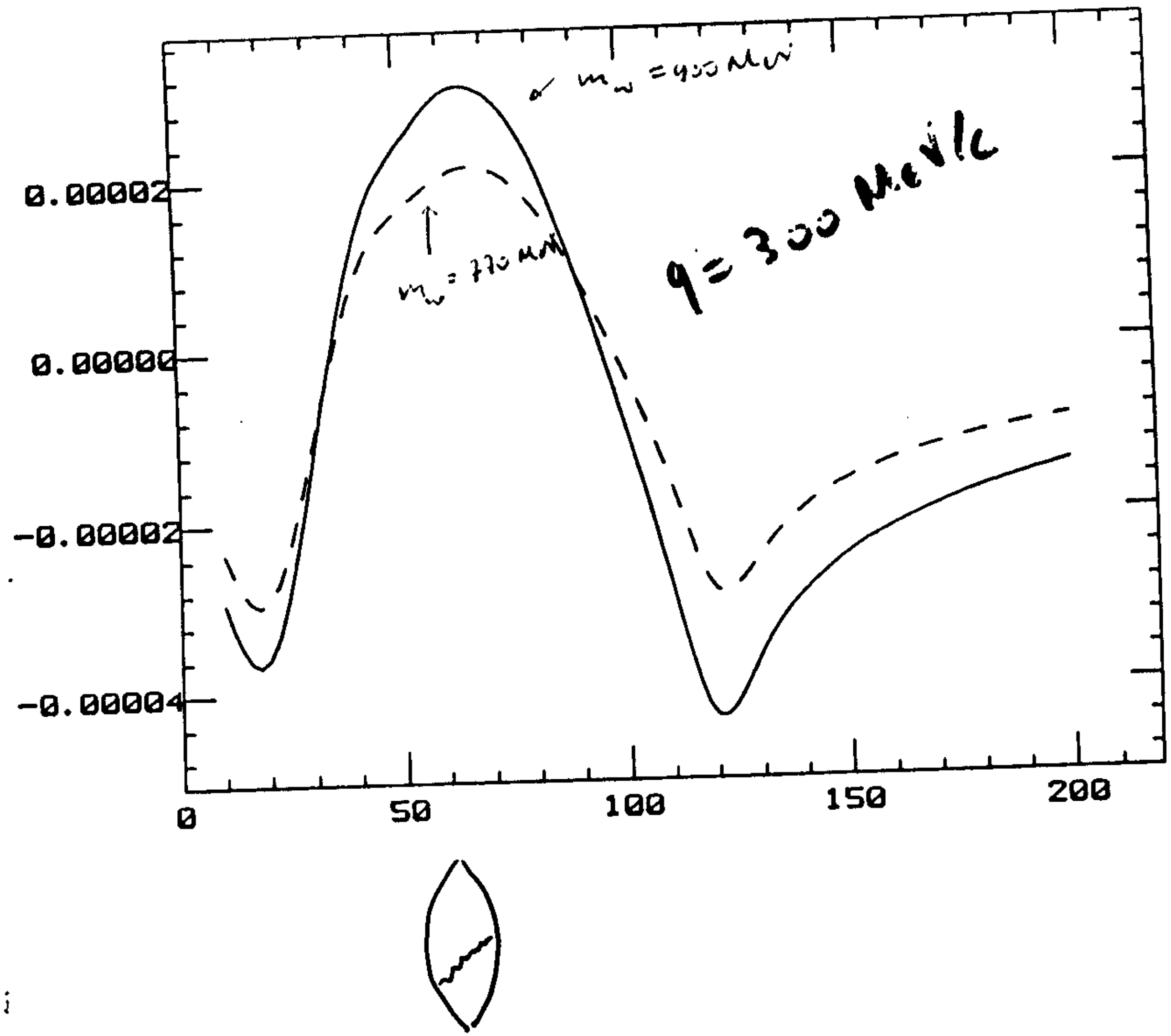
(γ and π do not have the same quantum numbers)

2) 1-loop Corrections:



+ other diagrams scarcely relevant in the longitudinal channel


q = 300 MeV/c
0.0001
0.0000
-0.0001
0
50
100
150
200
m_ω = 900 MeV
m_ω = 770 MeV



0.00002
0.00000
-0.00002
-0.00004
0
50
100
150
200
m_ω = 900 MeV
m_ω = 770 MeV
q = 300 MeV/c

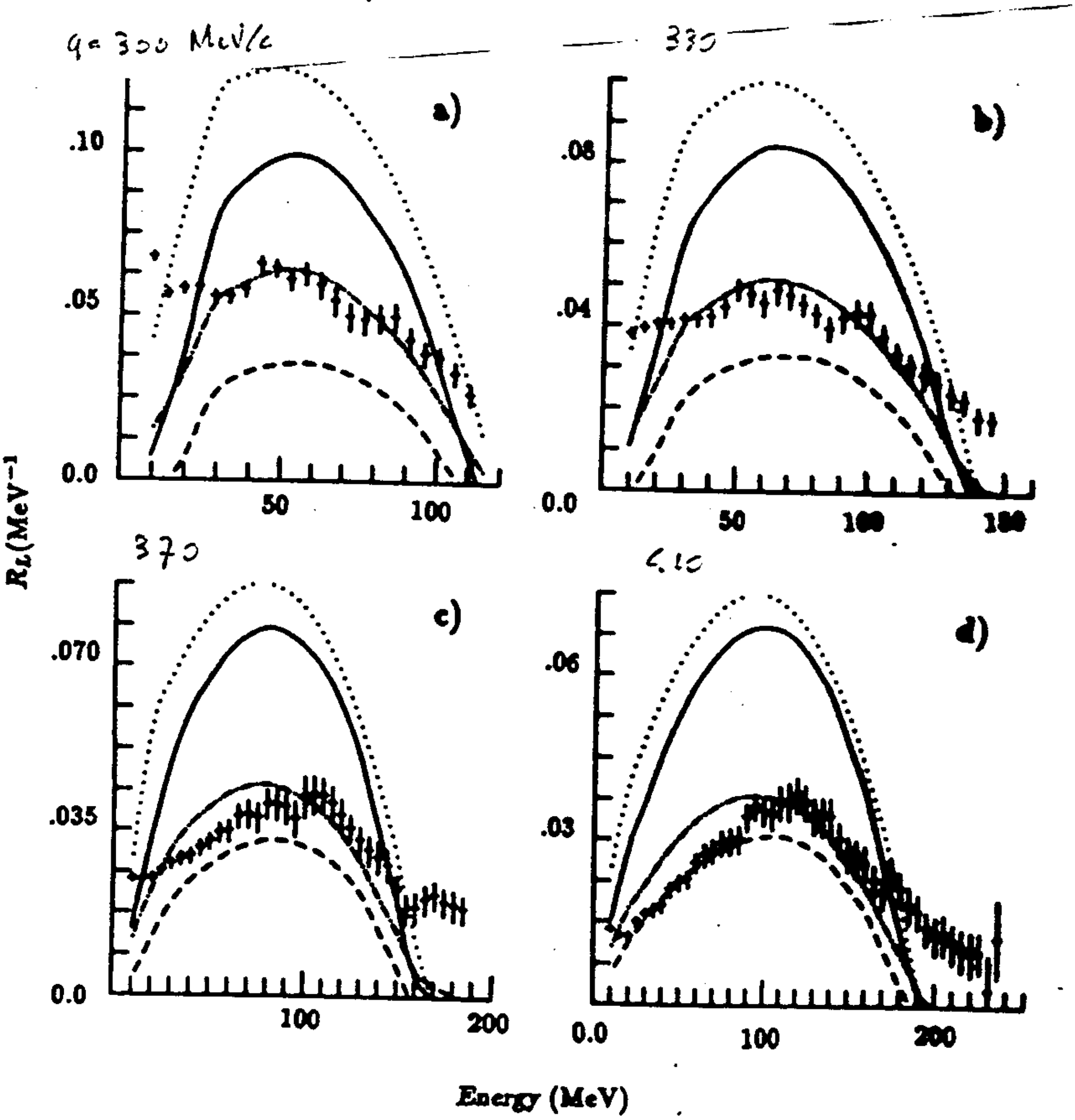
Results for (e,e') on Ca

...... = free (c^m ...)

——— = total

—·—·— = Isovector contribution

- - - - = Isoscalar contribution



Results with RPA

i. e. $\Pi = \Pi^{c} + \Pi^{1(=1)} \longrightarrow$

$\longrightarrow \dfrac{\Pi}{1 - f\Pi}$ or $\dfrac{\Pi}{1 - f'\Pi}$ f, f' Landau parameters

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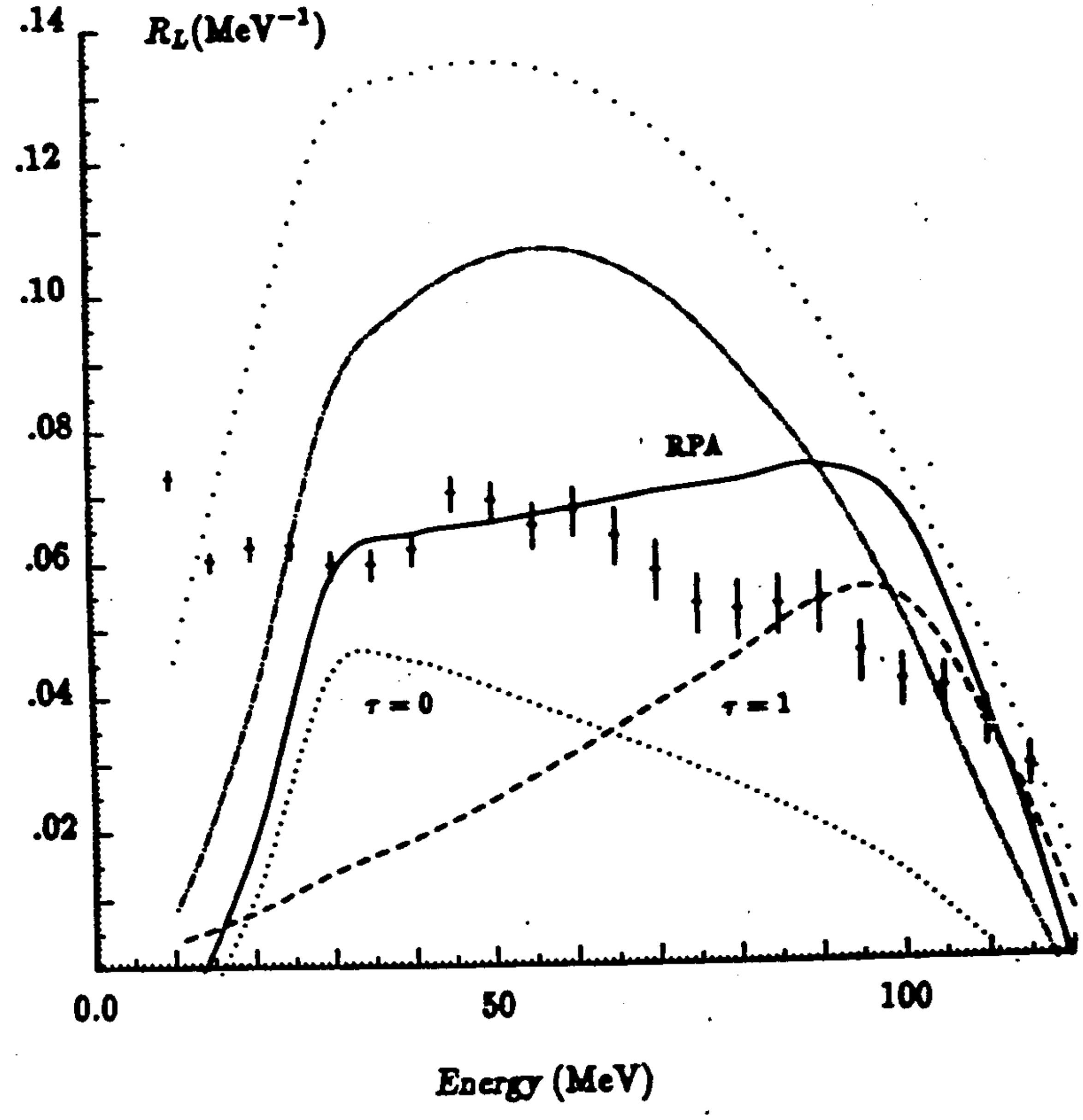
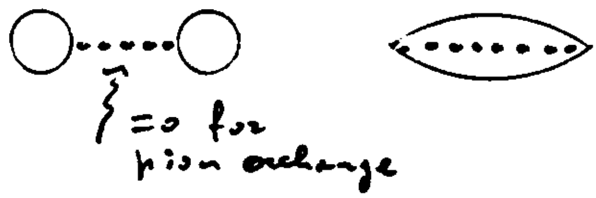
Here $f = -0.7$

$f' = 1.2$

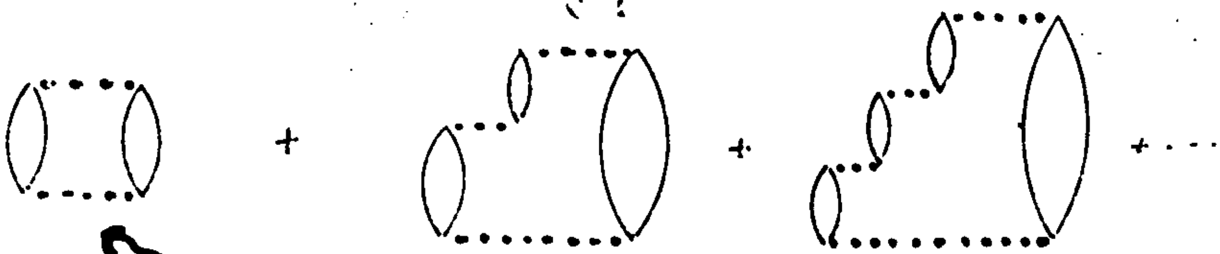


3. Correlations vs Binding Energy

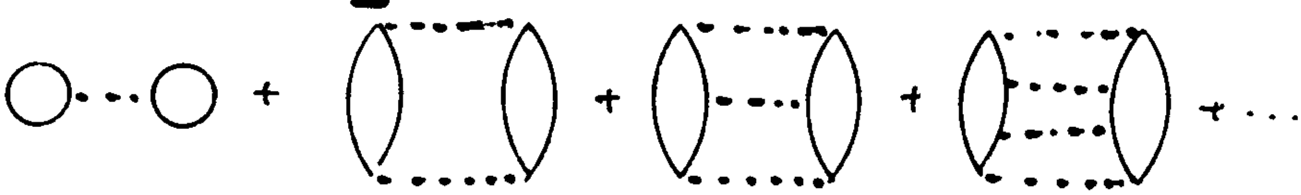
· B.E. : Hartree Fock :



Correlation energy



COMPARE WITH BRUECKNER THEORY



Both series contain the same pathological diagram.

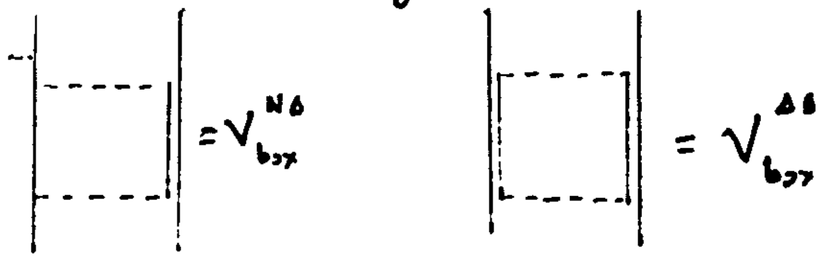
But in Brückner theory with realistic potentials correlations are authomatically Accounted for.

So, no problems in Binding Energy calculations ?

see later

4. Number of Δ's in nuclear matter

Consider the diagrams



(box diagrams)

Their iteration in BHF provides a large contribution to BE.

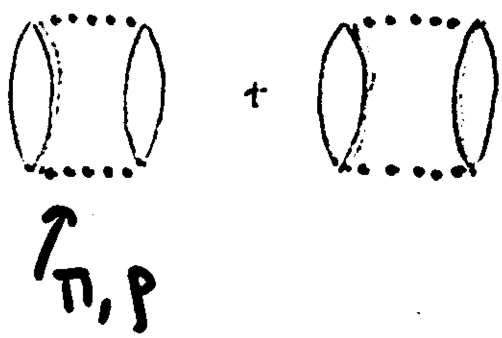
Bonn Potential: Mesons exchange + box diagrams:

Number of Δ's in Nuclear matter:

$$N_\Delta = \frac{\partial}{\partial (M_\Delta - M)} \, B.E.^{box}$$

A crude simplification:

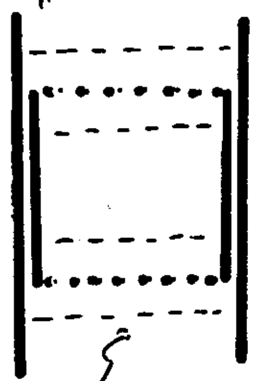
B.E. =



Again problems with the convergence

To ensure the convergence (apart from πNN form factors) it is not sufficient to impose correlations on the box diagram as a whole, but each pion line must be correlated.

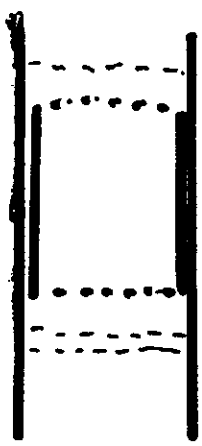
i.e. Diagrams like



ω-meson

are relevant.

Remind that Born potential includes diagrams like



only.

How to cure the disease?

Of course using the correlated version of π (and ρ) exchange

Results for B.E. not yet obtained.

Results for N_0 follow.

Obvious generalization:

 → correlation energy in RPA

$$N_0 = -\frac{\partial}{\partial(\delta\mu)} BE^{corr} \qquad (\delta\mu = \mu_0 - \mu)$$

RESULTS:

g'	$P^{(2)}$	P^{RPA}
0.5	9.26 %	15.89 %
0.6	8.41 %	9.42 %
0.7	8.95 %	6.66 %
0.8	10.97 %	5.66 %

with $m_\omega = 800$.

Remark: without correlation on the π and ρ exchange results are absolutely crazy.

(from R.C., F Conte and U. Lorentini, Phys. Rev 39C (1989) 1588)

5. Δ-Δ correlations v.s. R.E.

How to improve BHF calculations?

Answer: enlarge the phase space

(Coupled channel calculations:

e.g. Faepler-Ohtsuka-Z. Phys A329 (1980) 29

Model interaction:

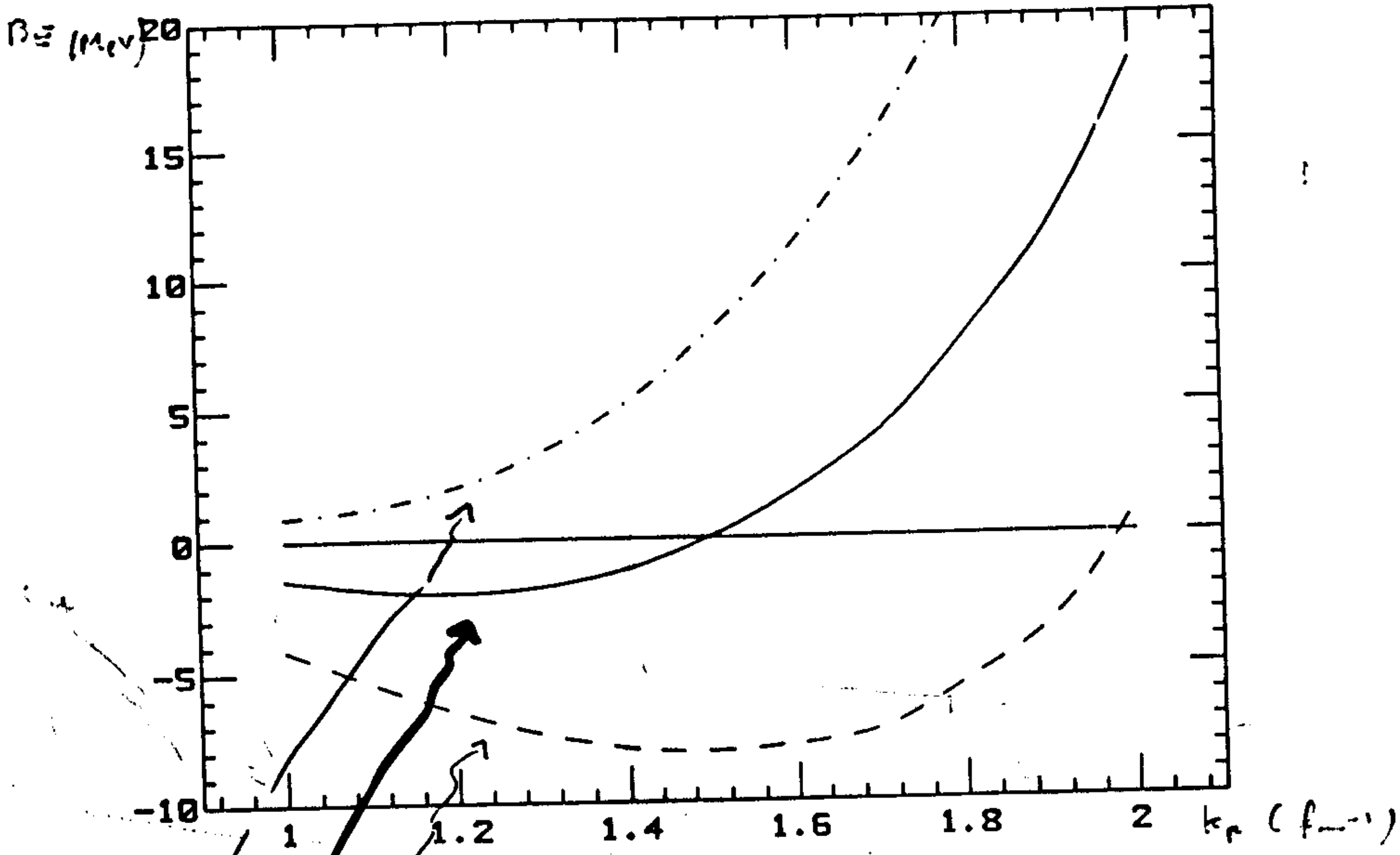
π, ρ, ω exchange between N and Δ

Skipping the complicated details of the solution of the G-matrix equation ...

Results follow:

(with reasonable parameters)

[R.C., Md. A. Matin and P. Saracco, preliminary results]



N-N interaction + Box diagrams

As before, + Δ-N and Δ-Δ ω-exchange

· · + a small Δ-N and Δ-Δ π and ρ exchange

6 CONCLUSIONS

Correlations are important in 1-loop corrections to the mean-field even in evaluating static properties or low momentum transfer reactions =>

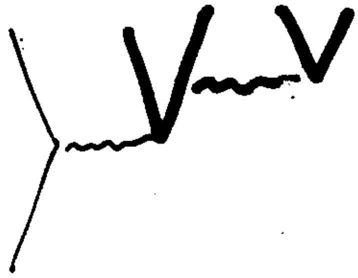
=> Need for → Good microscopical models
→ Experimental informations

How to obtain Experimental informations?

$(p, 2p)$ reactions:
(p, pn)

$(\gamma, 2p)$
(γ, pn)

$(e, e' 2p)$
$(e, e' pn)$



at high momentum transfer

$\sim 1\ GeV/c$ at least