

FUNCTIONAL METHODS AND CORRELATIONS

IN NUCLEAR MATTER

1. Functional techniques
 - 1.1 Effective Action
 - 1.2 Mass Shift
 - 1.3 One loop Corrections
- 2 Longitudinal Response Function
- 3 Correlations vs. Binding Energy
- 4 Number of Δ 's in Nuclear Matter
- 5 Δ - Δ correlations vs. Binding Energy
- 6 Conclusions

1 Functional Techniques

Useful for obtaining new effective actions
and new approximation schemes

Example (used throughout this talk):

Pion Nuclear Physics

Model Lagrangian (simplified scheme)

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\pi + \mathcal{L}_{\pi-N}$$

$$\mathcal{L}_N = \bar{\psi} (i\not{\partial} - M) \psi \quad (\text{or its nonrelativistic limit})$$

$$\mathcal{L}_\pi = \frac{1}{2} (\partial_\mu \bar{\phi})^2 - \frac{f^2}{2} \bar{\phi}^2$$

$$\mathcal{L}_{\pi-N} = i g \bar{\psi} \gamma_5 \vec{\tau} \cdot \vec{\phi} \psi \quad (\text{for instance})$$

When needed $\mathcal{L} \rightarrow \mathcal{L} + \mathcal{L}_{e.m.}$

via minimal coupling

1.1. The Effective Action

Generating Functional:

$$Z = \int D[\bar{\psi}, \psi, \bar{\varphi}] e^{i \int dx \mathcal{L}[\bar{\psi}, \psi, \bar{\varphi}] + J\bar{\psi} + \dots}$$

plus other eventual external sources

Z summarizes the whole hierarchy of the Green's Functions

Note that \mathcal{L} is bilinear in the

Fermionic fields $\bar{\psi}, \psi \Rightarrow$

\Rightarrow Idea: Integrate over $\bar{\psi}, \psi$

[reminds the Hubbard - Stratonvitch transformation and the bosonization of Blaizot]

Identity:

$$Z = \int D[\phi] e^{i S_{eff} + J\phi}$$

with...

$$S_{eff} = \int dx \left\{ \frac{1}{2} (\partial_\mu \phi)^2 - \frac{g}{6} \phi^3 \right\} = \sum_{n=2}^{\infty} \frac{1}{n} \Pi_n \phi^n$$

$$\sum_{n=2}^{\infty} \frac{1}{n} \Pi_n \phi^n = \frac{1}{2} \text{loop} + \frac{1}{3} \text{triangle} + \frac{1}{4} \text{square} + \dots$$

(\bullet = ϕ ; --- = Free Fermion - Green's Function)

[For future reference: $\bullet \circlearrowleft$ = Lindhard Function;

\circlearrowleft and \circlearrowright analytically given by

R.C., P. Surace, N.P. A487
(1988) 279.

Properties of Self:

- Contains pion field only
- ■ The interaction is nonlocal, multilinear and displays one closed fermion loop with any number of vertices
- ■ ■ Renormalization: complicated but possible even inside the nuclear medium: W.M. Alberico, R.C., A.M. Linares and P. Surace, Phys Rev. C 38 (1988) 2388

1.2

Mean Field

Nucleonic level \rightarrow Hartree-Fock
Approximation
(not discussed here)

Pionic level \rightarrow R.P.A.

In fact:

Mean Field = Stationary Phase Approximation

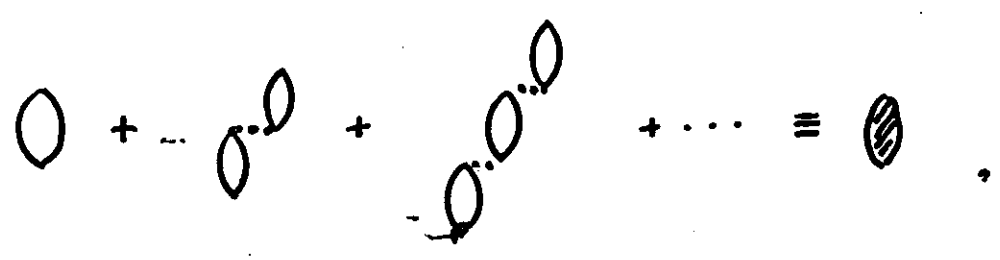
S.P.A equation:

$$\frac{\delta \Sigma_{\text{eff}}}{\delta \phi(x)} = -J(x)$$

Solution for ϕ in presence of
an external source J :

$$\phi = \frac{i}{\square - \mu^2 - \Pi(x)} J \equiv \text{R.P.A. solution}$$

Mass Field Response Function:

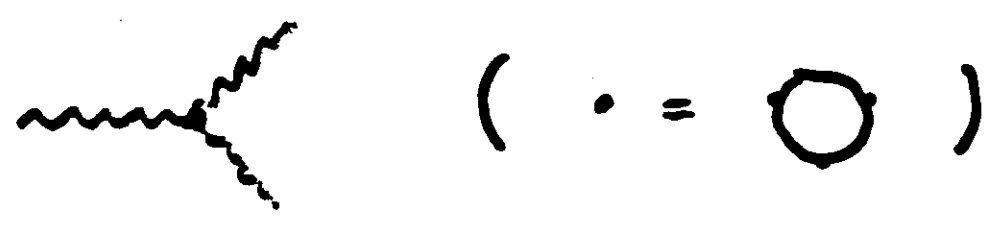


Dressed pion line: $\text{wavy line} = \dots + \dots$

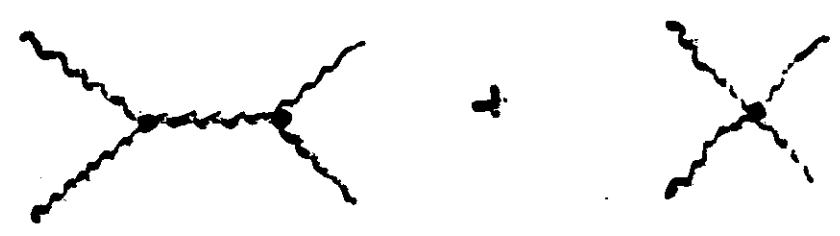
General Mass Field Diagrams:

Tree Graphs in wavy line

Examples $\rightarrow (\pi, 2\pi)$ reaction:



$(2\pi, 2\pi)$ (whatever it will mean)



A relevant Point: 4-momenta involved!

= those of the external probe $\hat{z} k_F$

\Rightarrow Low momentum Probes Explore Low momentum dynamics

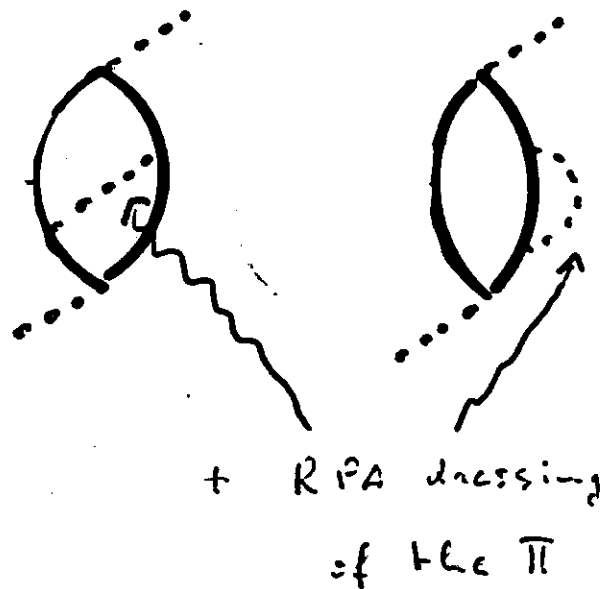
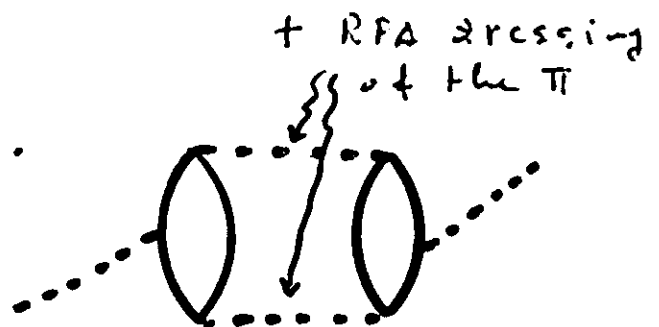
1.3 One Loop Corrections

Example on the Response Function

$$\Pi^{1\text{ loop}} = \text{[diagram 1]} + \text{[diagram 2]}$$

Relevant point (even if seemingly trivial): the loop integrated momentum is not cut by k_F

Microscopically

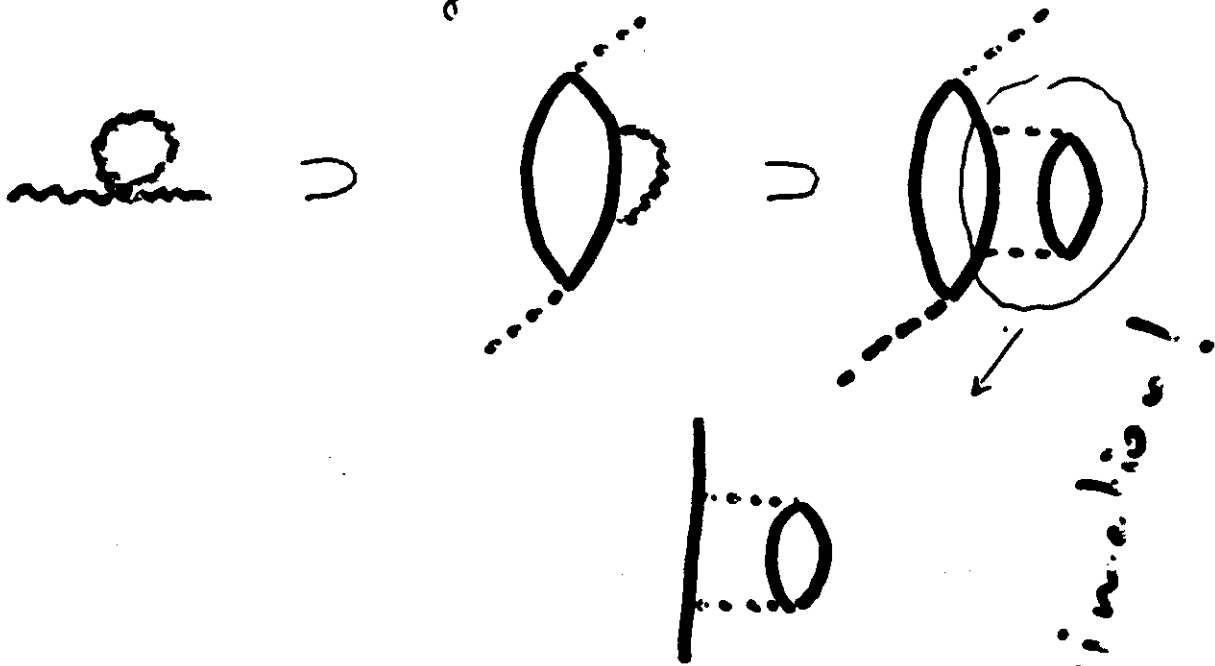


Relevant point:

How does the loop integral converge?

(Renormalization is understood;
we are looking to nuclear medium)

A critical diagram



Put

$$V_{\text{opt}} = -\frac{f^2}{\mu^2} \frac{q^2}{q^2 + \mu^2}$$

$$\text{Loop} = \Pi^{(2)}(q, \omega) \sim \frac{1}{q^2}$$

Naive Approximation!

⇒ The loop integral diverge

How can the divergence be cured?

1) Relativistic effects

$$\cancel{\psi^\dagger (\vec{\sigma} \cdot \vec{\nabla}) (\vec{\sigma} \cdot \vec{\nabla}) \psi} \longrightarrow \bar{\psi} \gamma_5 \not{\partial} \psi$$

now $\int d^3x$ converges

p and s waves mixed together

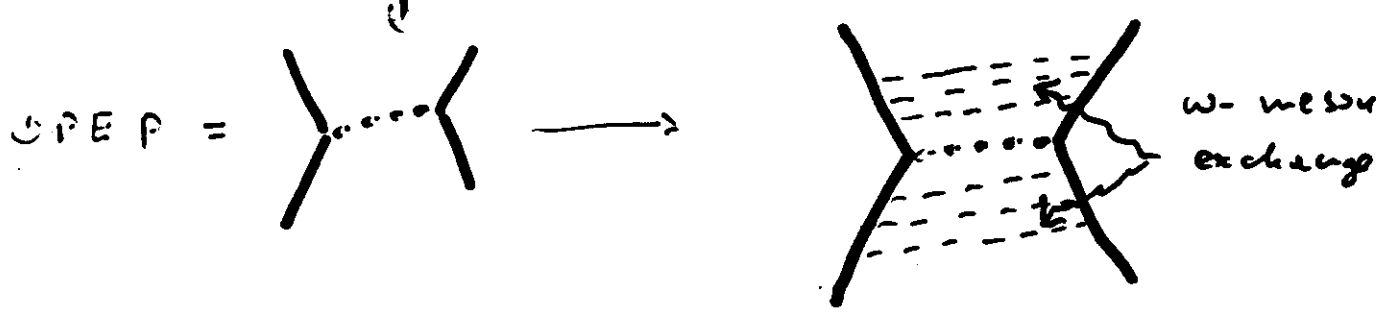
Natural cut-off: The relevant scale is $2M$

2) πNN form factor

Typically $\sigma(q^2) = \frac{\Lambda^2 - \mu^2}{q^2 - \mu^2}$

Typical scale: $1.2 \text{ GeV}/c$

3) Short range correlations



V_{oper} \rightarrow correlated pion exchange

$V_{\text{oper}} \xrightarrow{q \rightarrow \infty} \sim \frac{1}{q^2}$ (at least)

Typical scale: $m_\omega = 770 \text{ MeV}$

Pion dynamics inside the medium. 10

1) Introduction of the Δ

$$\bigcirc \rightarrow \bigcirc + \bigcirc$$

$$\pi^{(1)} \rightarrow \pi_{ph} + \pi_{ch}$$

2) Introduction of g'

$$\bigcirc \cdots \bigcirc \rightarrow \bigcirc \cdots \bigcirc + \bigcirc \overset{\uparrow g'}{=} \bigcirc$$

• Universal choice (for sake of simplicity)

May g' depend upon q and ω ?

(Some indications that g' decreases with increasing ω - R.C., P. Saracco Phys Lett B to be published)

3) How to embody short range correlations?

low energy: $V_{\text{short}} \rightarrow \frac{k^2}{\mu^2} (g' - \frac{q^2}{q^2 + \mu^2})$
 high " $V_{\text{short}}^{\text{cons}} \sim \frac{1}{q^2}$

Idea: $g' - g'(q) = 1 + (g'_0 - 1) \frac{m_\omega^2}{m_\omega^2 + q^2}$

$\begin{matrix} \nearrow & 1 \\ & q \rightarrow \infty \\ \searrow & g'_0 \\ & q = 0 \end{matrix}$

Two parameters:

g'_0 fixes the low energy dynamics

m_ω fixes the high energy dynamics

2. Longitudinal e.m.

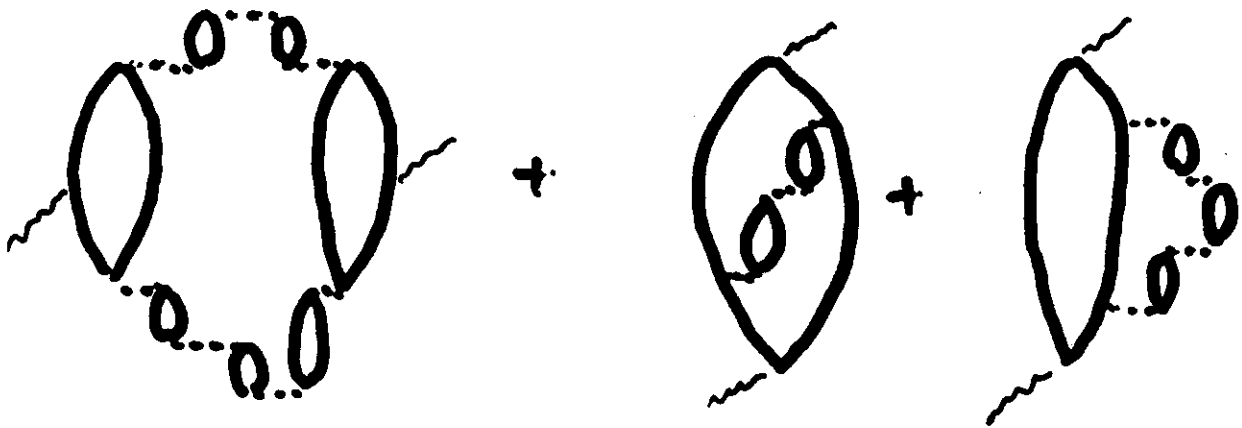
Response Function

1) Mean Field Level

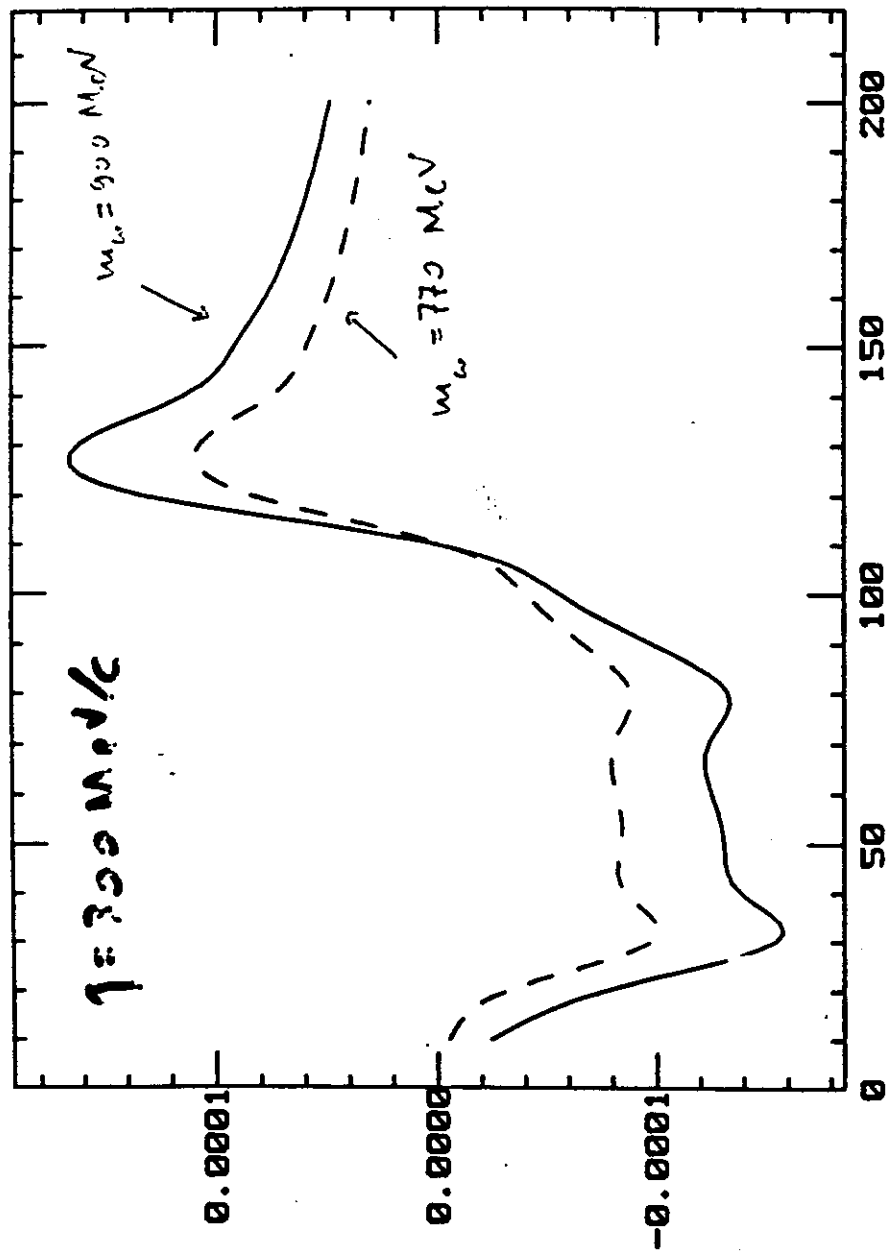
$$\Pi_{\mu\nu}^{\text{mean-field}} = \text{Diagram}$$

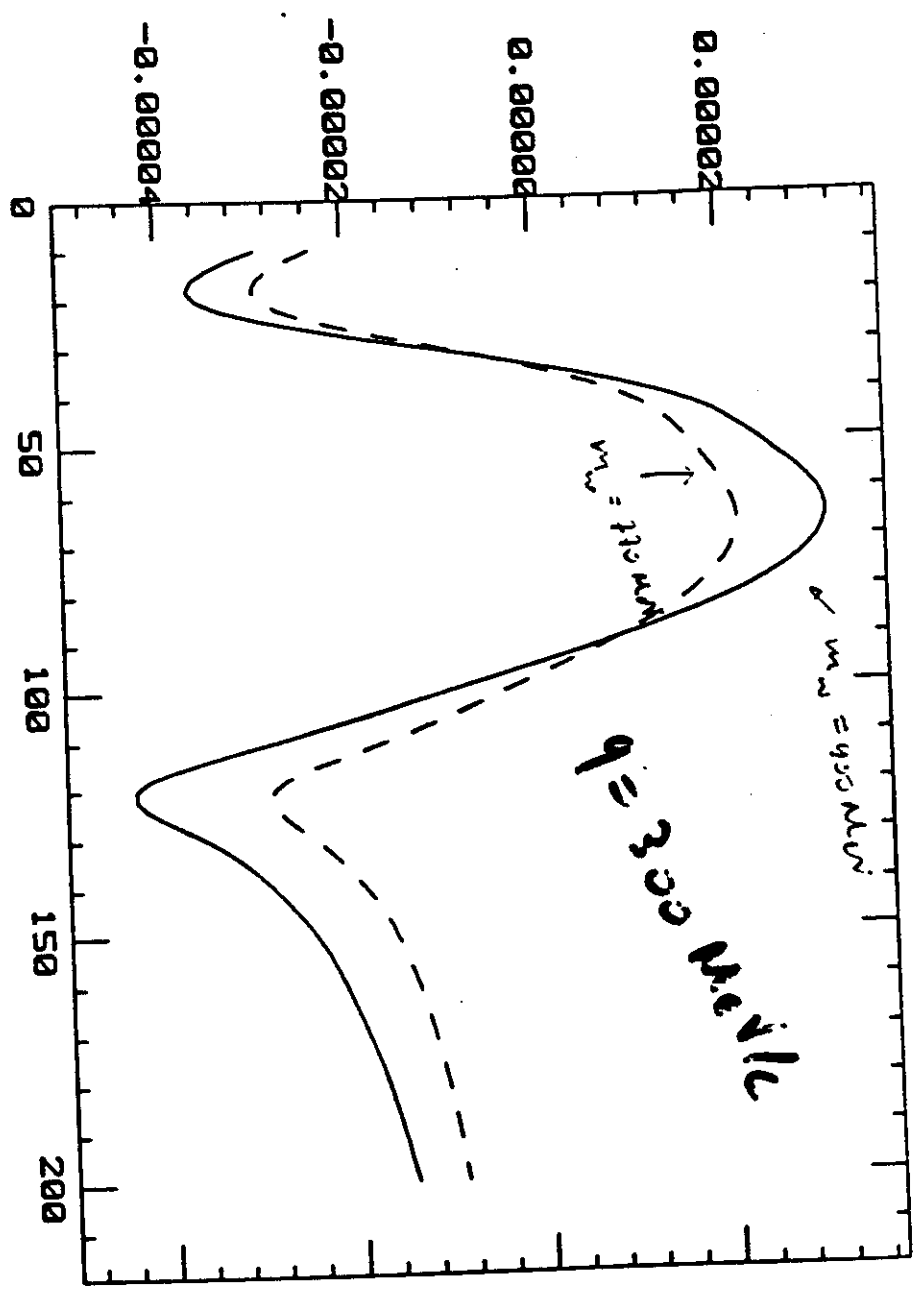
(γ and Π do not have the same quantum numbers)

2) 1-Loop Corrections:



+ other diagrams scarcely relevant in the longitudinal channel





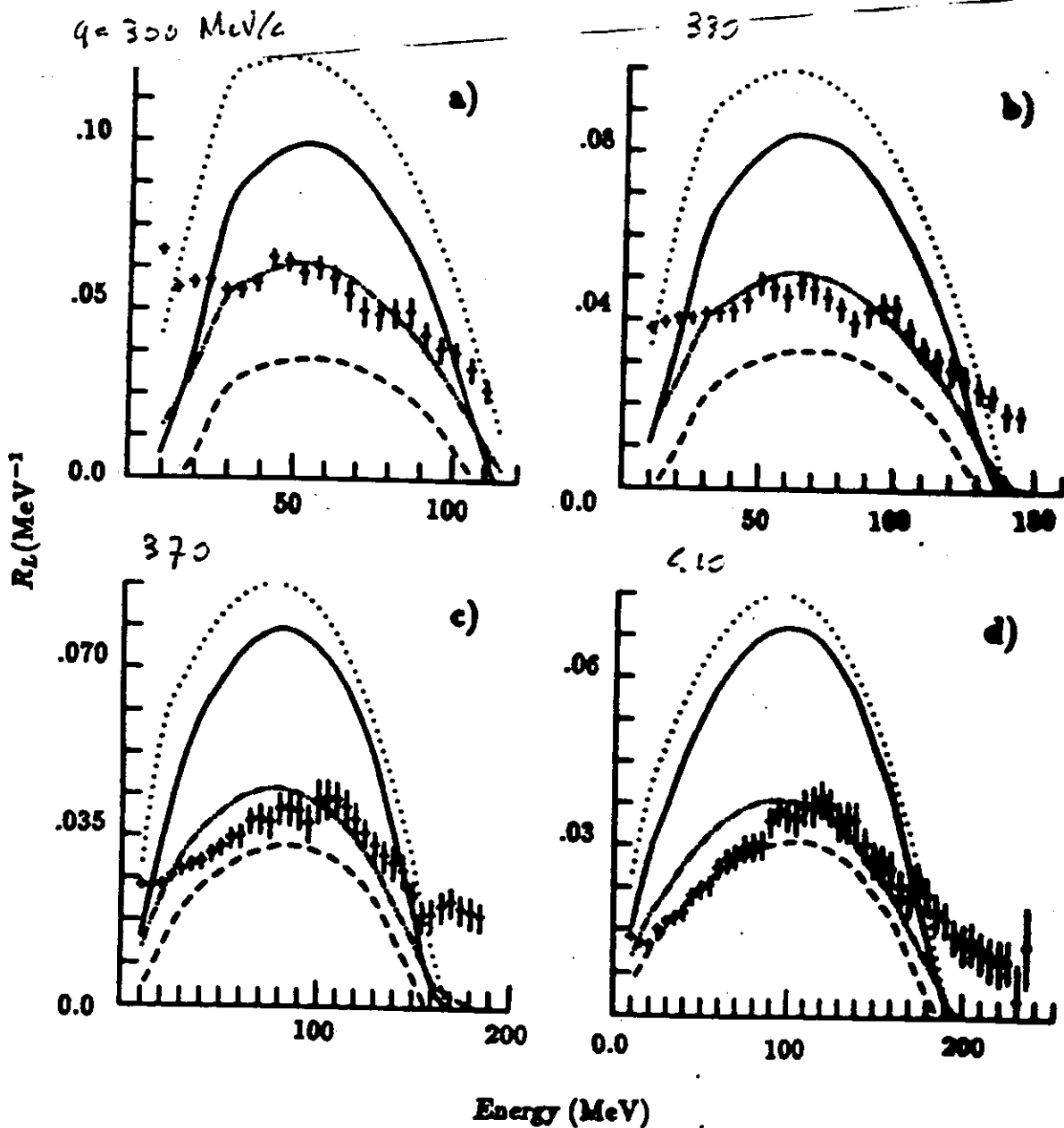
Results for (e, e') on Ca

..... = free (e^+e^- order)

— = total

- · - · - = isovector contribution

- - - = isoscalar contribution



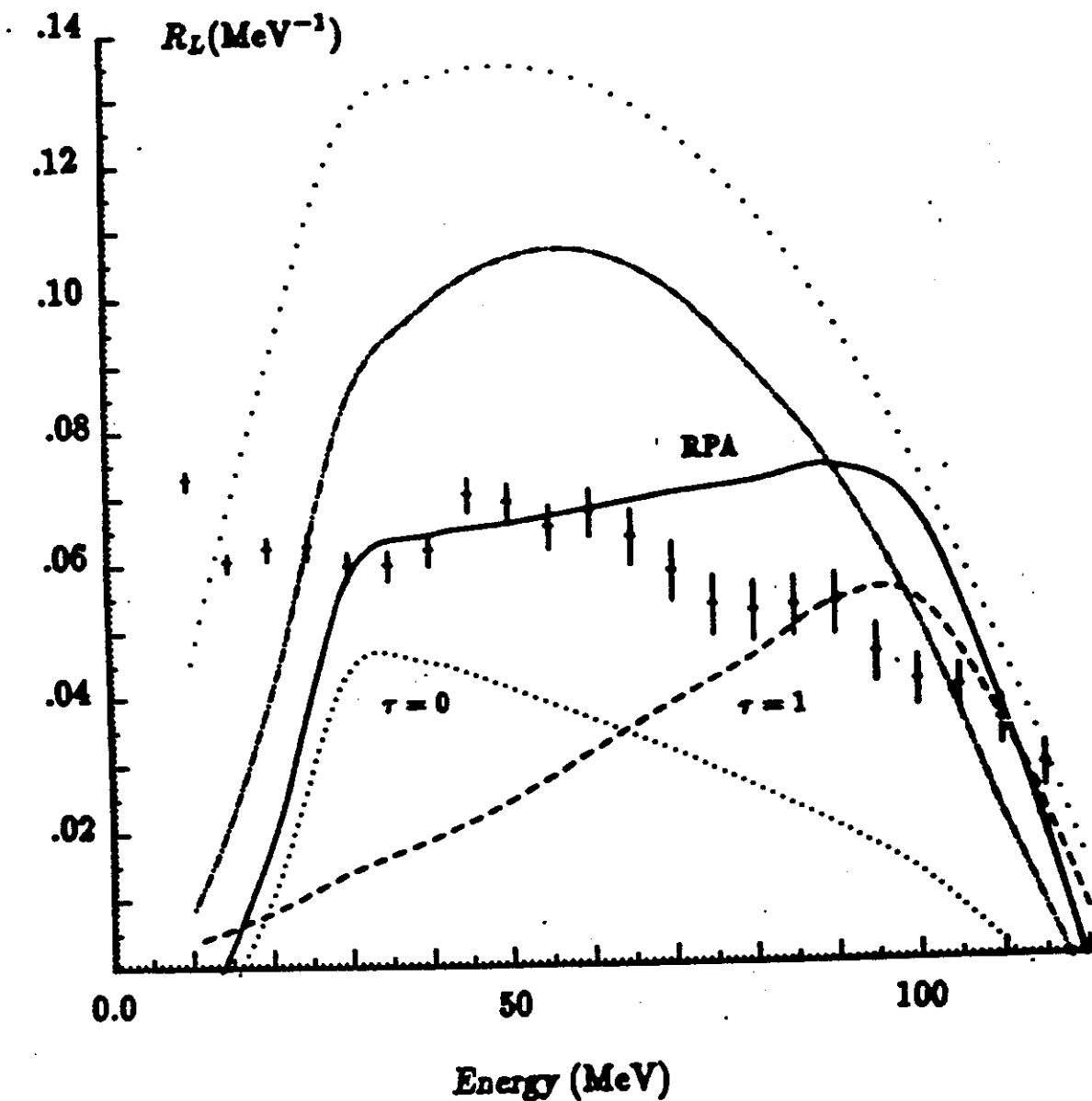
Results with RPA

L. e. $\Pi = \Pi^e + \Pi^{ic:sp} \rightarrow$

$\rightarrow \frac{\Pi}{1 - f\Pi} \approx \frac{\Pi}{1 - f'\Pi}$ f, f' Landau parameters

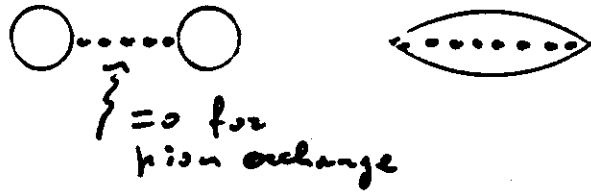
Here $f = -0.7$

$f' = 1.2$

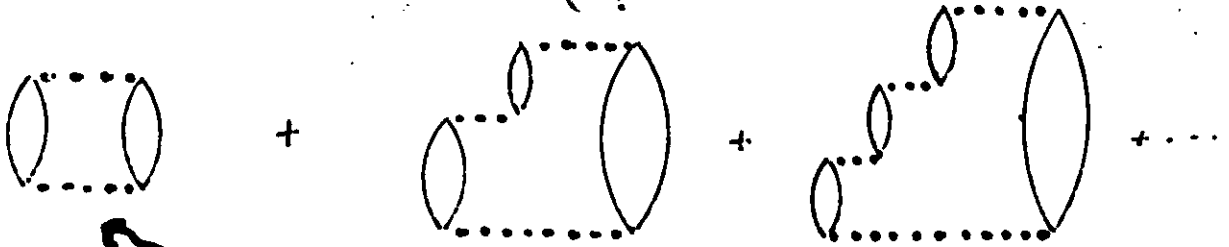


3. Correlations vs Binding Energy¹⁶

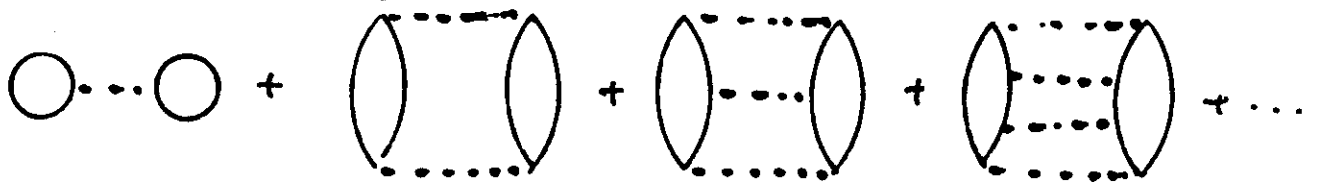
B.E. : Hartree Fock :



Correlation energy



COMPARE WITH BRÜCKNER THEORY



Both series contain the same pathological diagram.

But in Brückner theory with realistic potentials correlations are automatically accounted for.

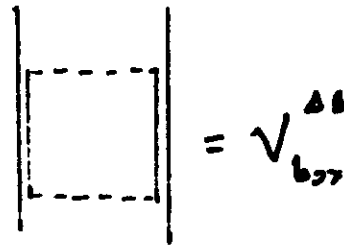
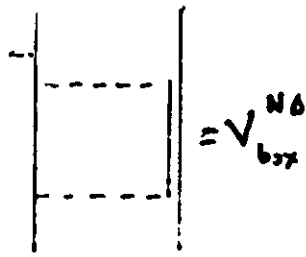
So, no problems in Binding Energy calculations?

see later

4. Number of Δ 's in nuclear matter

17

Consider the diagrams



(box diagrams)

Their iteration in BHF provides a large contribution to BE.

Down Potential: Meson exchange + box diagram:

Number of Δ 's in Nuclear matter:

$$N_{\Delta} = \frac{\partial}{\partial (M_{\Delta} - M)} \text{B.E.}_{\text{box}}$$

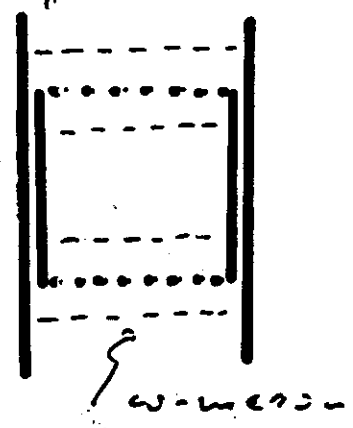
A crude simplification:

$$\text{B.E.} = \text{[diagram 1]} + \text{[diagram 2]}$$

Again problems with the convergence

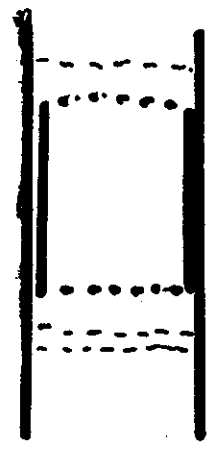
To ensure the convergence (apart from $\pi N N$ form factors) it is not sufficient to impose correlations on the box diagram as a whole, but each piece has to be correlated.

i.e. Diagrams like



are relevant.

Remind that Bonn potential includes diagrams like



only

How to cure the disease?

Of course using the correlated version
of π (and f) exchange

Results for B.E. not yet obtained.

Results for N_0 follow.

Obvious generalization:

$(\vec{0}) \rightarrow$ correlation energy in RPA

$$N_0 = - \frac{\partial}{\partial (\delta M)} RPA^{cor} \quad (\delta M = u_0 - \mu)$$

RESULTS:

g'	$P^{(2)}$	P^{RPA}
0.5	9.26 %	15.89 %
0.6	8.41 %	9.42 %
0.7	8.98 %	6.66 %
0.8	10.97 %	5.66 %

with $u_w = 800$.

Remark: without correlation on the
 π and f exchange results are
absolutely crazy.

(from R.C., F. Conte and U. Lorenzini,
Phys. Rev 39C (1989) 1588)

5. Δ - Δ correlations vs. R.E.

How to improve RHF calculations?

Answer: enlarge the phase space

(Coupled channel calculations:

e.g. Faessler-Ohtsuka-Z. Phys A329
(1980) 29

Model interactions:

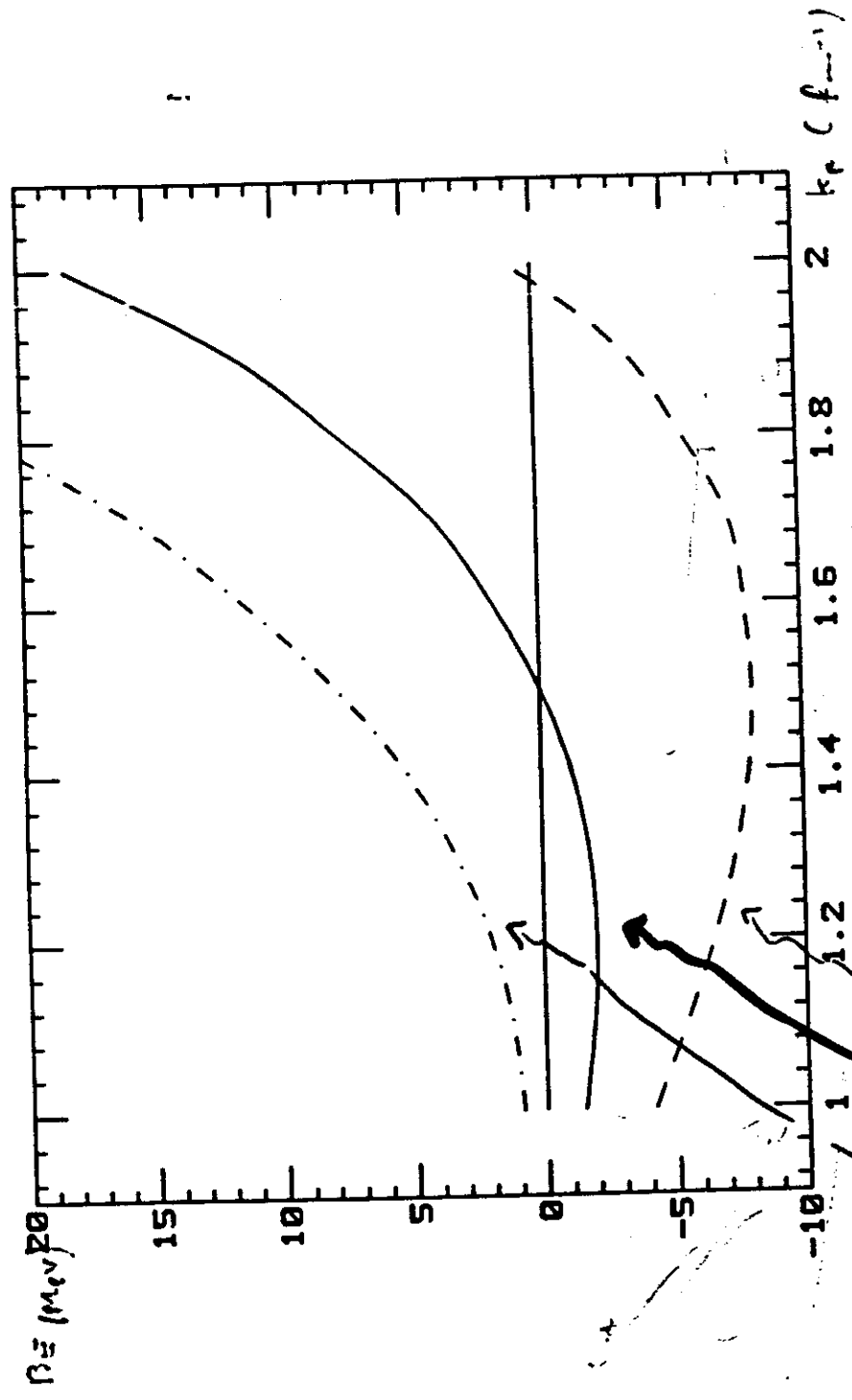
π, ρ, ω exchange between N and Δ

Skipping the complicated details of the solution of the G-matrix equation ...

Results follow:

(with reasonable parameters)

[R.E., M.A. Martin and P. Saracco,
preliminary results]



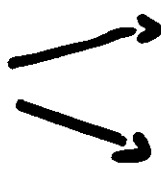
N-N interaction + Box diagrams

As before, + Δ -N and Δ - Δ w-exchange

" " + a small Δ -N and Δ - Δ
 π and ρ exchange

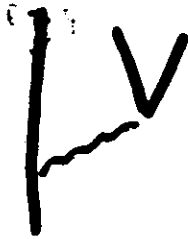
6 CONCLUSIONS

Correlations are important in 1-loop corrections to the mean-field even in evaluating static properties or low momentum transfer reactions \Rightarrow

\Rightarrow Need for  Good microscopical models
Experimental informations

How to obtain Experimental informations?

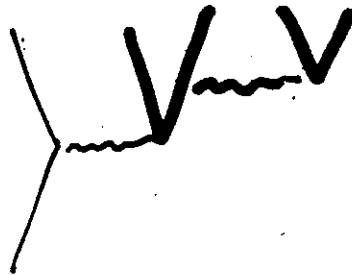
(p, 2p) reactions:
(γ , p α)



(γ , 2p)
(γ , p α)



(e, e' 2p)
(e, e' p α)



at high momentum transfer
 $\sim 1 \text{ GeV}/c$ at least