GROUP THEORETICAL CLASSIFICATION OF MULTI QUARK STATES IN LIGHT NUCLEI. TWO- AND THREE-BODY CORRELATIONS

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1) Group Theoretical classification of $(j)^M$ - configurations in the Quark Shell Model

2) Generalized effective pairing interaction and two-body correlations.

3) The G.T. definition of the nucleon (barion) and three-body correlations

Work inspired by some papers by
BLEULER & al

K. BLEULER, H. HOFESTÄDT, S. MERK, H. R. PETRY
Z. Naturforsh 38A, 705 (1983)

Same + H. BOHR, K. S. NARAIN
P.L. 159 B, 363 (1985)

They propose a nuclear model with 3A quarks, belonging to the up + down sector are enclosed in a unique large spherical Bag, whose radius R is determined by the usual equilibrium condition between the internal and external pressure.

For n-quarks they obtain

$$R \propto r_0 n^{1/3} \; ; \quad E(n.R) = 4\pi B R^3 .$$

$r_0 = 1.3\,\mathrm{fm}$ $\qquad B^{1/4} = 127\,\mathrm{MeV}$

By a suitable choice of the quark mass m and the vacuum pressure B it is possible to reproduce the Mayer-Jensen level scheme of conventional shell model structure, provided of course that $n = 3A$ colourless states are selected.

Spin-orbit comes directly from the Dirac equation.

Once $R(A)$ is given a many-quark state is given by

$$\prod_{r=1}^{3A} q^+_{\alpha_r i_r} |0\rangle \qquad \begin{array}{l} \alpha = 1,2,3 \quad \text{(colour)} \\ i = \{nljm, \tau\} \end{array}$$

but a colourless state is defined as

$$\prod_{\{ijk\}=1}^{A} A^+_{(ijk)} |0\rangle$$

where

$$A^+_{ijk} = \sum_{\alpha\beta\gamma} \epsilon_{\alpha\beta\gamma}\, q^+_{\alpha i}\, q^+_{\beta j}\, q^+_{\gamma k}$$

is a barion creation operator.
If ijk are suitably coupled, a nucleon, a delta, or some exotic barionic state can be obtained.

Nuclear states are labelled by
$L.S.J.M, T, T_z$ etc.

Group Theoretical classification can be obtained easily for pure $(nlj)^N$ configuration; for this purpose it is convenient to give-up the condition for colour-neutrality, which will be imposed later.

Moreover this coloured approach allows for con[illegible] in which a $(nlj)^N$ coloured configuration is coupled to another [illegible] configuration, the two being coupled to a colour singlet.

Pure $(nlj)^N$ configurations, without colour restriction, belong to Irreducible Representations (totally antisymmetric) of the group $SU_{3\cdot 2\cdot(2j+1)}$ and can be analysed and labelled by I.R. of its subgroups

$$SU_3^{\text{Colour}}, \quad SU_2^{T(\text{isospin})}$$

$$SU_{2j+1}^{\text{space}} \supset Sp_{2j+1} \supset R_3$$

$SU_{3\cdot 2\cdot(2j+1)}$ generators have the form

[illegible]

The extension of SU type group to the SO or Sp type dynamical group is convenient both for classification purposes and for a direct connection with residual ([illegible] [illegible]) interactions among quarks.

The orthogonal extension of [illegible] is obtained by the addition of pairs creation (annihilation) operators

[illegible] and [illegible].

The group is $SO_{2 \cdot 3 \cdot 2 \cdot (2j+1)}$

All [illegible] [illegible] are [illegible] [illegible] of dimension

$$[d] = 2^{3 \cdot 2 \cdot (2j+1) - 1}$$

corresponding [illegible]

The dynamical content is trivial: the residual interaction does not depend on the different (colour - spin - isospin) coupling between the two quarks of the pair.

More interesting situation is obtained if $\rho^+\rho^+$, $\rho\rho$, and $\rho^+\rho$ type generators are selected among linear combinations scalar in some space.
There are three possibilities:

1) [illegible]

$$[\text{[illegible]}]^{J=1} \longrightarrow \begin{cases} J = \text{even}, & \text{colour} = [\bar{3}] \\ J = \text{odd}, & \text{colour} = [6] \end{cases}$$

Interaction $\Rightarrow$ Isospin pairing.

[illegible]

The group is $Sp_{2\cdot3\cdot(2j+1)} \supset U_1 \otimes SU_3^C \otimes SU_{2j+1}$

$SU_{2j+1} \rightarrow R_3$

The maximal commuting [illegible] $\in$ [illegible]

SU_2^T

2) [illegible]

$$[\text{[illegible]}]^{J=1} \longrightarrow \begin{cases} T = 0, & \text{colour} = [\bar{3}] \\ T = 1, & \text{colour} = [6] \end{cases}$$

Interaction $\Rightarrow$ angular momentum pairing

[illegible]

The group is $Sp_{2\cdot3\cdot2} \supset U_1 \otimes SU_3^C \otimes SU_2^T$

The maximal [illegible] $\in$ [illegible]

SU_{2j+1}

3) Due to the presence of Colour a third possibility arises:

interaction = super pairing

The group is $SO_6 \supset U_1 \otimes SU_3^c$
(SU_4)

Its I.R.'s are labelled by

v supersenority = number of quarks not coupled to J=0 and T=0

reduced colour $[d_v]$ = I.R. of SU_3^c for superseniority quarks.

external q.n's as $T T_z$ $J M$ etc
and states by quark number N and $[d_N]$ actual colour.

$| v [d_v], N [d_N], T T_z, J M \ldots \rangle$

$\boxed{\text{Each I.R. contains at most 1 colour singlet}}$

For $n l_j = 1 s_{1/2}$ alle I.R^s are shown.

[illegible]

3	[illegible]	
3	N nucleon Δ [illegible]	
6	2 [illegible] deuteron	
9	[illegible] N ? Δ	H^3, He^3
12	4 nucleons $\rightarrow He^4$	

The dynamical situation is also particularly simple in this case ($j = \frac{1}{2}$) since J-pairing or T-pairing interaction can be diagonalized in this scheme and their eigenvalues depends only upon the number of quarks N and the total spin (isospin)

$$E_{S=0}(N,S) = \frac{G_{S=0}}{8}\left[N(N+2) - 4S(S+1)\right]$$

$$E_{T=0}(N,T) = \frac{G_{T=0}}{8}\left[N(N+2) - 4T(T+1)\right]$$

Also the colour coupling of pairs of quarks can be considered and we obtain

$$E_{[\bar{3}]}(N,C) = G_{\bar{3}}\left[\frac{1}{6}N(N+3) - \frac{1}{4}C^2_{SU_3}\right]$$

$$E_{[6]}(N,C) = G_{6}\left[\frac{1}{3}N(N-3) + \frac{1}{4}C^2_{SU_3}\right]$$

where $C^2_{SU_3}$ is the quadratic Casimir invariant

SO_6 (SU_4) generators:

$$N = \sum_{\alpha m t} q^+_{m t \alpha} q_{m t \alpha} \qquad (U_1)$$

$$\Lambda^c_{\alpha\beta} = \sum_{mt} q^+_{m t \alpha} q_{m t \beta} - \delta_{\alpha\beta} \frac{N}{3} \qquad (SU_3^c)$$

$$B^+_{\alpha\beta} = \sum_{mt} \frac{q^+_{m t \alpha} q^+_{-m-t\,\beta} (-)^{j+\frac{1}{2}-m-t}}{\sqrt{2(2j+1)}}$$

$B_{\beta\alpha} =$ h.c.

Superpairing interaction

$$H_{s.pairing} = \frac{G_{s.pairing}}{2} \sum_{\alpha\beta} B^+_{\alpha\beta} B_{\beta\alpha} =$$

$$= \frac{G}{8} \left[C^2_{SU_4} - C^2_{SU_3} - \frac{1}{3} N^2 + (4\Omega + 2) N - 12 \Omega (\Omega + 1) \right]$$

$$\Omega = j + \frac{1}{2}$$

$$C^2_{SU_4} |N, v, \ldots\rangle =$$

$$= \left[C^2_{SU_3} + \frac{1}{3} N^2 - (4\Omega + 2) N + 12 \Omega (\Omega + 1) \right] |N, v, \ldots\rangle$$

$$E_{S\text{-pairing}}(v\,[d_v],\, n\,(d_n)) =$$
$$= \frac{G}{8}\left\{ C^2_{SU_3}(v) - C^2_{SU_3}(n) + (v-n)\left[\frac{1}{3}(v+n) - 2(2\Omega+1)\right]\right\}$$

Isospin pairing and pairing energies can be easily calculated knowing T, J and the seniority + red. isospin.
(if $j > \frac{1}{2}$)

For $j = \frac{1}{2}$ also the effect of Δ coupling can be calculated.

Superpairing coupling gives its maximum contribution, for fixed v and n, in color singlet, where $C^2_{SU_3}(n) = 0$. Some of these are standard nuclear states, others are Δ excited states of how many nucleons?

What is a nucleon (or a Δ) in this model? Up to now the model is algebraic, so must be the questions and the answers provided they are unambiguously formulated.

<u>First</u> definition of a <u>Nucleon</u> in the space of $(j)^n$ configurations :

The j Nucleon is a 3-quarks correlation with quantum numbers $j m {}^1\!/_2 t$, colourless, created by $A^+_{j m 1/2 t} = B^+_{\alpha\beta}\, q^+_{\gamma m t}\, \epsilon_{\alpha\beta\gamma}$

The Number of Nucleons in a $(j)^n$ conf. is the expectation value of

$$N_N^{(3)} = \sum_{mt} A^+_{mt} A_{mt} =$$

$$= \sum_{mt} B^+_{\alpha\beta}\, \epsilon_{\alpha\beta\gamma}\, q^+_{\gamma mt}\, q_{\delta mt}\, \epsilon_{\delta\lambda\mu}\, B_{\lambda\mu} =$$

$$= \Lambda_{4\gamma}\, \Lambda^L_{\gamma\delta}\, \Lambda_{\delta 4}$$

$\alpha = 4$
$\gamma, \delta = 1, 2, 3$

$$C^3_{SU_4} = \sum_1^4 \Lambda_{\alpha\gamma}\Lambda_{\gamma\delta}\Lambda_{\delta\alpha} = \quad + \text{ other Terms}$$

related to $C^3_{SU_3}$, $C^2_{SU_3}$, N^3, N^2, N, $NC^2_{SU_3}$,

all diagonal in SO_6 scheme.

$$N_{NP}\left[j, v(\lambda_v), m(\lambda_m)\right] = \frac{1}{3(12\Omega+6)}\Big\{ C^{1}_{SU_3}(v) - C^{1}_{SU_3}(m) +$$

$$+\left(6\Omega+3-\frac{v}{2}\right) C^{2}_{SU_3}(v) - \left(6\Omega+3-\frac{m}{2}\right) C^{2}_{SU_3}(m) +$$

$$+(v-m)\left[\frac{1}{9}(v^2+vm+m^2) - \Omega(v+m) - (6\Omega+4)\right]\Big\}$$

For $nlj = 1s_{1/2}$ we obtain

Name		$N^{(3)}_{nucleons}$ +	$N^{(3)}_{\Delta}$ =	$N^{(3)}_{colourless}$
Nucleon	N	1	0	1
Delta	Δ	0	1	1
Deuteron	NN	$20/9 = 2\cdot\frac{10}{9}$	$16/9$	4
	$N\Delta$	$8/9$	$28/9$	4
	$\Delta\Delta$	0	4	4
Tritium	NNN	$10/3 = 3\cdot\frac{10}{9}$	$20/3$	10
	$N\Delta\Delta$ or $NN\Delta$	$5/3$	$25/3$	10
He^4		4	16	20

Number of "Δ" is obtained subtracting the number of "N" from the number of colourless 3-q correlations.

In the general case $j > \Sigma$ one can say that for $(j)^{3n}$ conf.

$$N_{\nu}^{(3)} = n\, F(j, \nu, n, \dots)$$

n	$F(j, n)$	$F(\infty, n)$	$j = 7/2$	$j = 3/2$
1	1	1	1	1
2	$\frac{2}{3}\frac{(4\Omega+1)}{(2\Omega+1)}$	$\frac{4}{3}$	$\frac{34}{27}$	$6/5$
2Ω	$\frac{(\Omega+1)(2\Omega+3)}{3(2\Omega+1)}$	$\frac{\Omega}{3}$	$\frac{55}{27}$	$7/5$
$4\Omega-2$	$\frac{10\Omega}{3(2\Omega+1)}$	$\frac{5}{3}$	$\frac{45}{27}$	$4/3$
$4\Omega-1$	$\frac{2}{3}\frac{(4\Omega+1)}{(2\Omega+1)}$	$\frac{4}{3}$	$\frac{34}{27}$	$6/5$
4Ω	1	1	1	1

Are there ambiguities? $\left[(j)^{2\, J=0}, j\right]^{J=j}$ does not fix L which should be equal to ℓ. This can be obtained by imposing L pair $=0$, or without imposition on L pair.

1) Second definition ...
L pair = 0 so that the valence q brings
all q. n.
The preceding result $N_{\nu}^{(3)}$ must be
multiplied by

$$(2j+1)^2 \begin{Bmatrix} \ell & 1/2 & j \\ \ell & 1/2 & j \\ 0 & 0 & 0 \end{Bmatrix} = \frac{2j+1}{2(2\ell+1)}$$

$\ell =$	0	1	2		∞
$j = \ell + \frac{1}{2}$	1	2/3	3/5	- - - - .	1/2
$j = \ell - \frac{1}{2}$	—	1/3	2/5	- - - - .	1/2

2) Third definition of a nucleon
L pair free but the overall $L = \ell$
The correction needs 2 g-j coefficients:
the overall factor is

$$\frac{1}{9}\,\frac{5(2j+1)^2 + 4(\ell-j)(2\ell+1)}{(2\ell+1)(2j+1)}$$

$\ell =$	0	1	2		∞
$j = \ell + \frac{1}{2}$	1	.685	.630	- - - - - .	.555
$j = \ell - \frac{1}{2}$	—	.484	.500	- - - - - .	.555

What else can be done in the light nucl. framework?

The light nuclei radial functions for $j = \ell \pm \frac{1}{2}$ are almost identical so that :-

Fourth definition of a nucleon: the valence quark in one subshell $j\pm$ can be coupled to the quark pair also when this is in the other subshell $j\mp$. One has to add to the preceding figures this new contribution to the number of nucleons, namely $\frac{1}{9} n_\pm N^{(2)}_\mp (\text{9-j coeff})^2$

Colour singlet condition

quarks $j\pm$ $[3]$

diquark $(\bar{3})$ c.c

$[3]\otimes(\bar{3}) = [8]\oplus[1]$

Which gives the standard result

$N^{(3)}_N$ = baryonic number

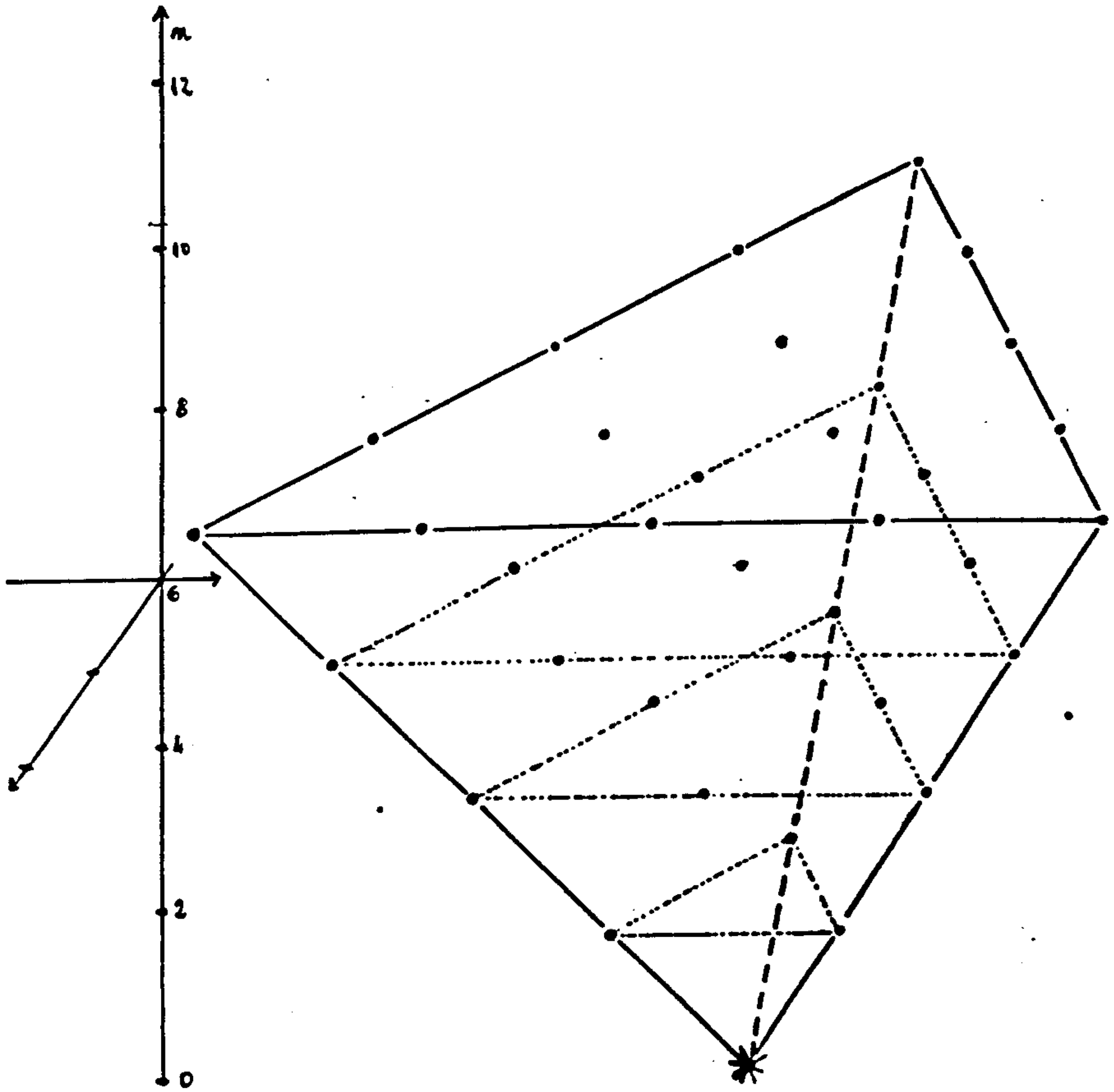
in at least two cases: 1) both subshells are filled 2.) one is filled, the other contains 3 quarks. (with $L_{pair} = 0$!!)

Conclusions + Open problems.

1) Complete basis with coloured states allowing for configuration mixing with 2 body generalized pairing interactions.

2) Critical analysis of the definition of a nucleon (composite particle) in term of 3-body correlation.

- Extension of a nucleons, whatever is its definition; how the $N^{(3)}$ depends on the location and the radius of a sphere inside the nucleon bag BAG?
- Hypernuclei (including S quarks)
- Quark matter.
- $q\bar{q}$ pairs added in the BAG.
- Configuration mixing of many $(j)_i^{n_i}$ configurations with hidden colour and generalized pairing like interactions $\sum_{ij} G_{ij} B_i^+ B_j$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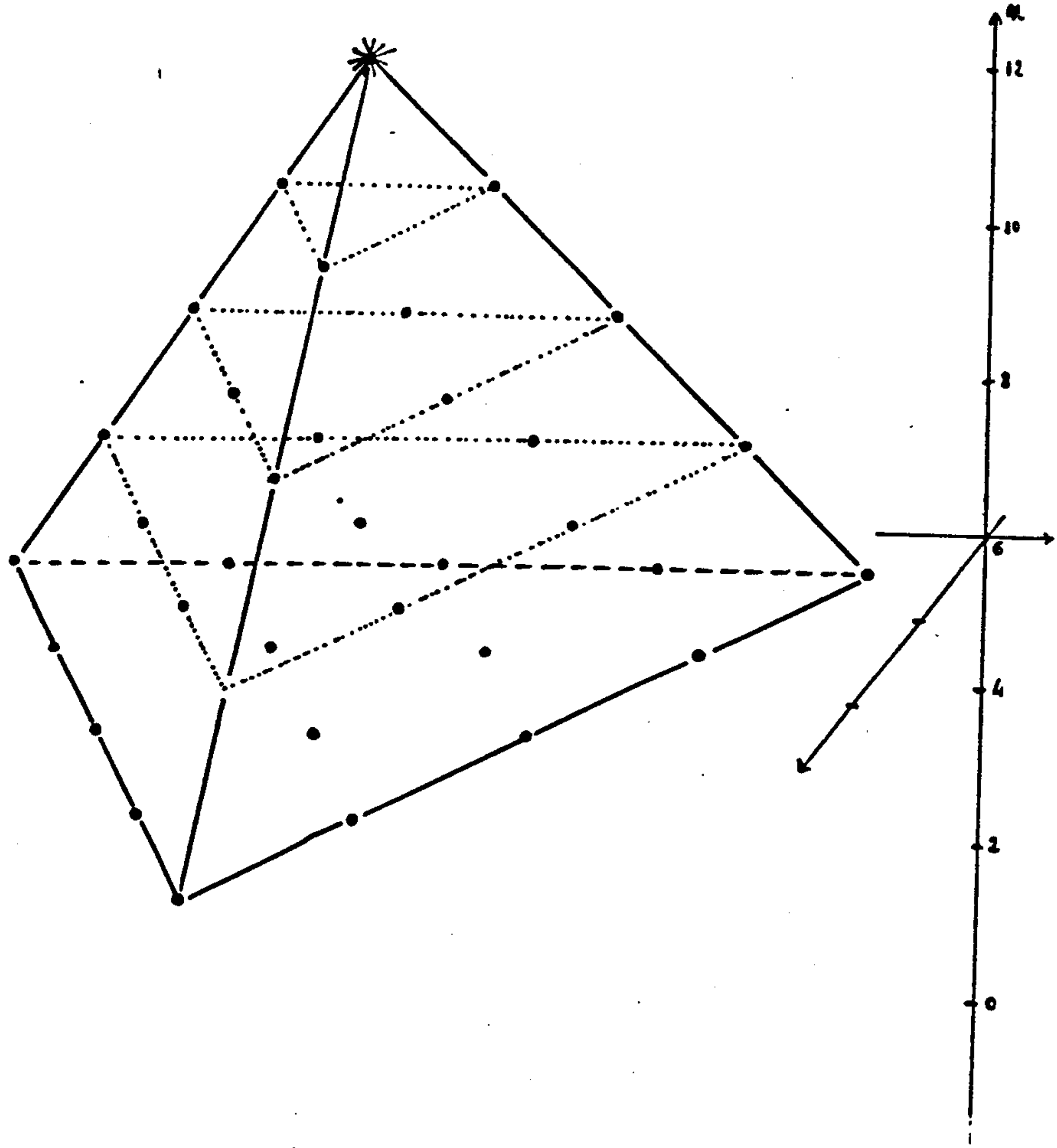

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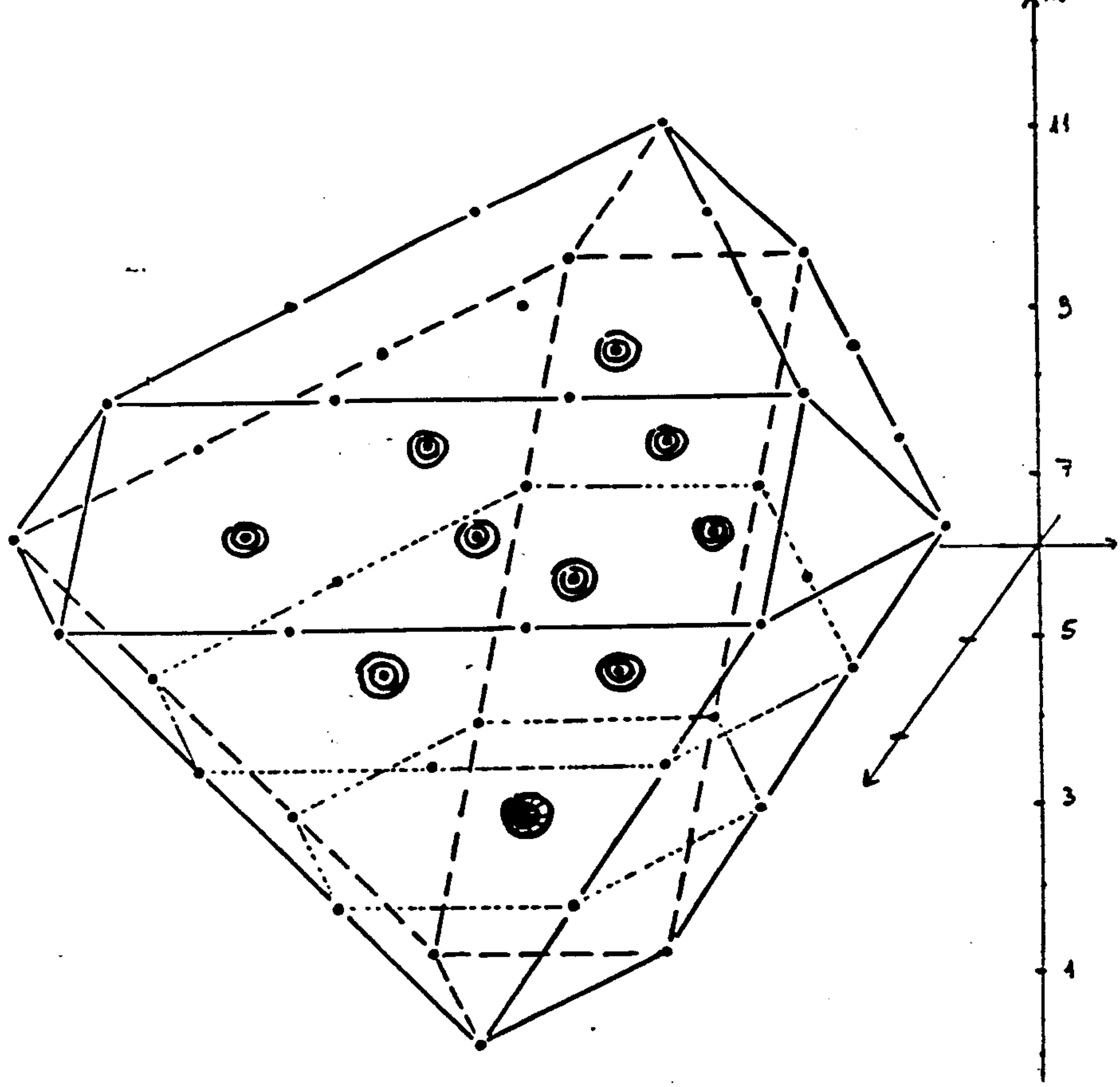

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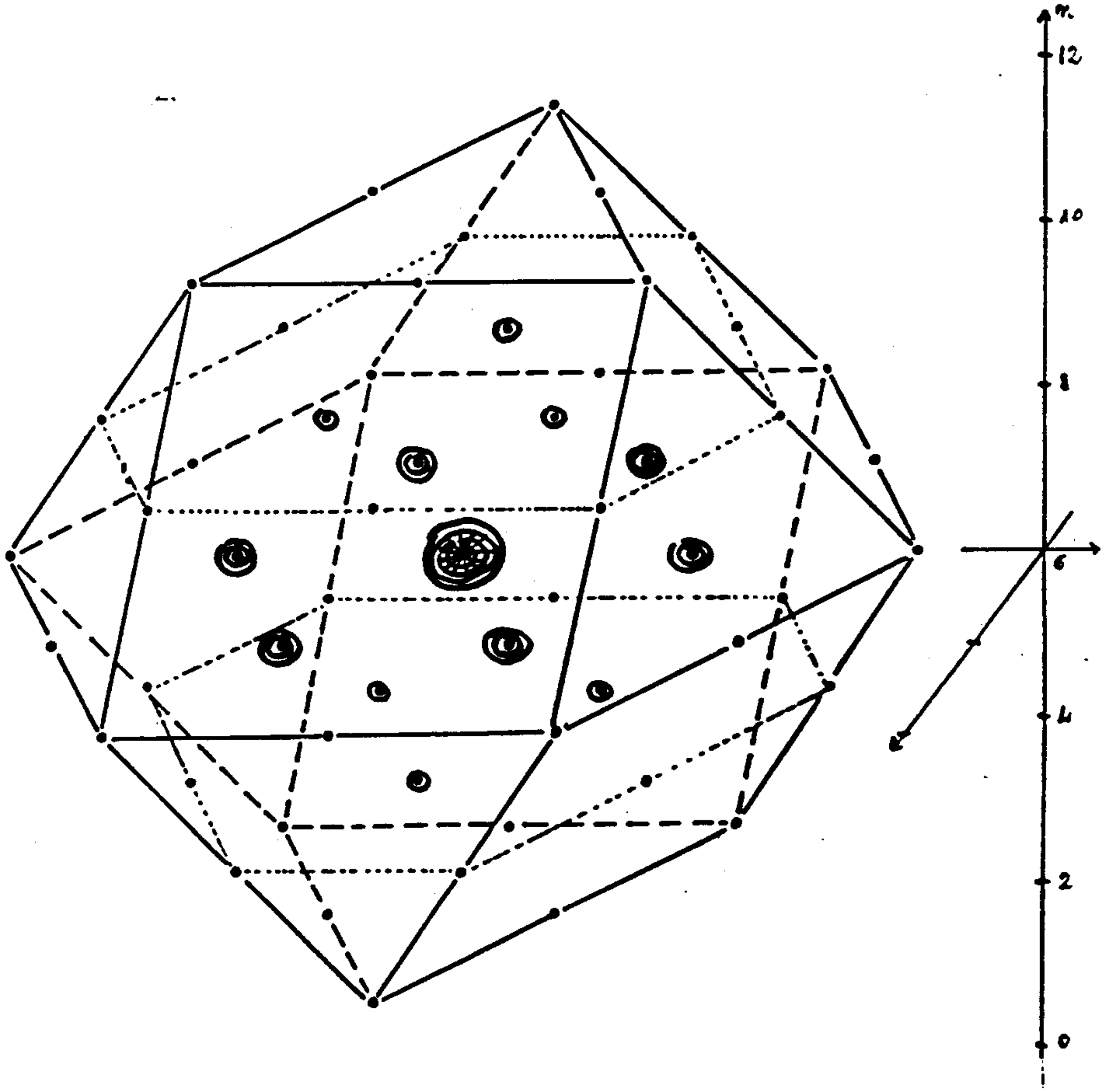

n
12
10
8
6
4
2
0

$V = 2$; $T, S = 1, 0$; $(0, 1)$	$[D]_{SO_C(6)} = [84]$	$\lambda, \mu, \nu = 2,0,2$

n	$[D]_{SU_C(3)}$	λ, μ	$N^{(2)}_{[\bar{3}],1/2,1/2}$	$N^{(3)}_{[1],1/2,1/2}$	$N^{(3)}_{[1],3/2,3/2}$
0	–	–	–	–	–
2	[6]	2, 0	0	0	0
4	[15′]	2, 1	$\frac{1}{2}$	0	$\frac{4}{9}$
4	[3]	1, 0	$\frac{3}{2}$	$\frac{4}{3}$	0
6	[27]	2, 2	$\frac{1}{2}$	0	0
6	[8]	1, 1	$\frac{7}{4}$	$\frac{25}{18}$	$\frac{10}{9}$
6	[1][c]	0, 0	$\frac{5}{2}$	$\frac{20}{9}$	$\frac{16}{9}$
8	$[\bar{15}']$	1, 2	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{8}{3}$
8	$[\bar{3}]$	0, 1	$\frac{5}{2}$	$\frac{20}{9}$	$\frac{40}{9}$
10	$[\bar{6}]$	0, 2	2	2	8
12	–	–	–	–	–

Tabella A.3. : Stati con numero pari di quarks: I.R. [84] di $SO_C(6)$.

Superseniorità, Isospin, Spin, Numero di quarks, Colore, Parametri caratteristici della I.R., Coppie in Superpairing, Correlazioni Nucleoniche e Correlazioni Δ.

(c) Deutone d.