INCLUSIVE SCATTERING of MULTI-GeV ELECTRONS by NUCLEAR MATTER

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#

- Inclusive x-section within PWIA
- Nuclear matter spectral function
- eN vertex
- PWIA results compared to data
- effect of FSI (+ possible relevance of <u>nucleon</u> structure)

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FSI

• INCLUSIVE ELECTRON NUCLEUS X-SECTION

$$e + A \to e' + \text{anything}$$

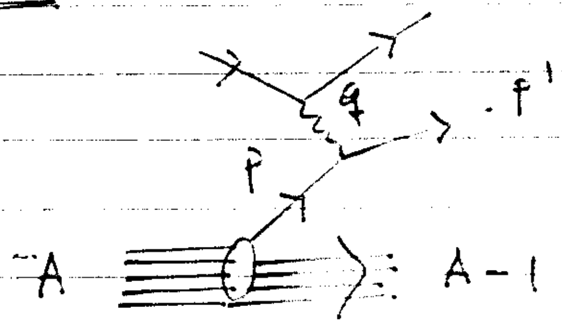
$$\frac{d^2\sigma}{d\Omega\, d\nu} \propto L^{\mu\nu} W^A_{\mu\nu}$$

nuclear tensor

$$W^A_{\mu\nu} = \sum_f \int d^3p_f \, \langle 0 | J^A_\mu | f \rangle \, \delta^{(4)}(P_0 + q - P_f) \, \langle f | J^A_\nu | 0 \rangle$$

The explicit calculation of $W^A_{\mu\nu}$ at large momentum transfer requires a consistent relativistic description of the initial and final nuclear states, as well as of the nuclear current operator

• PWIA



Approximations

$J_\mu^A \sim \sum j_\mu^N$

$|f\rangle \sim |A-1\rangle |p'\rangle$

$$W_{\mu\nu}^A \sim \int d^3p \, dE \, P(p,E) \, \tilde{W}_{\mu\nu}^N$$

• The dynamics of the nuclear target is decoupled from the electromagnetic vertex

• The description of the relativistic motion of the struck proton reduces to a purely kinematical problem.

- SPECTRAL FUNCTION

$$P(p,E) = \frac{1}{\pi} \mathrm{Im} \langle 0 | a_p^{+} (H - E_0 - E - i\eta)^{-1} a_p | 0 \rangle$$

$$H|0\rangle = E_0 |0\rangle$$

Nuclear matter calculation within CBF theory

\# $H = \sum_i p_i^2/2m + \sum_{j>i} v_{ij} + \sum_{k>j>i} v_{ijk}$

Urbana V14 + TNI

\# one-hole and two-hole one particle (correlated) intermediate states

one-hole → two body breakup channel.

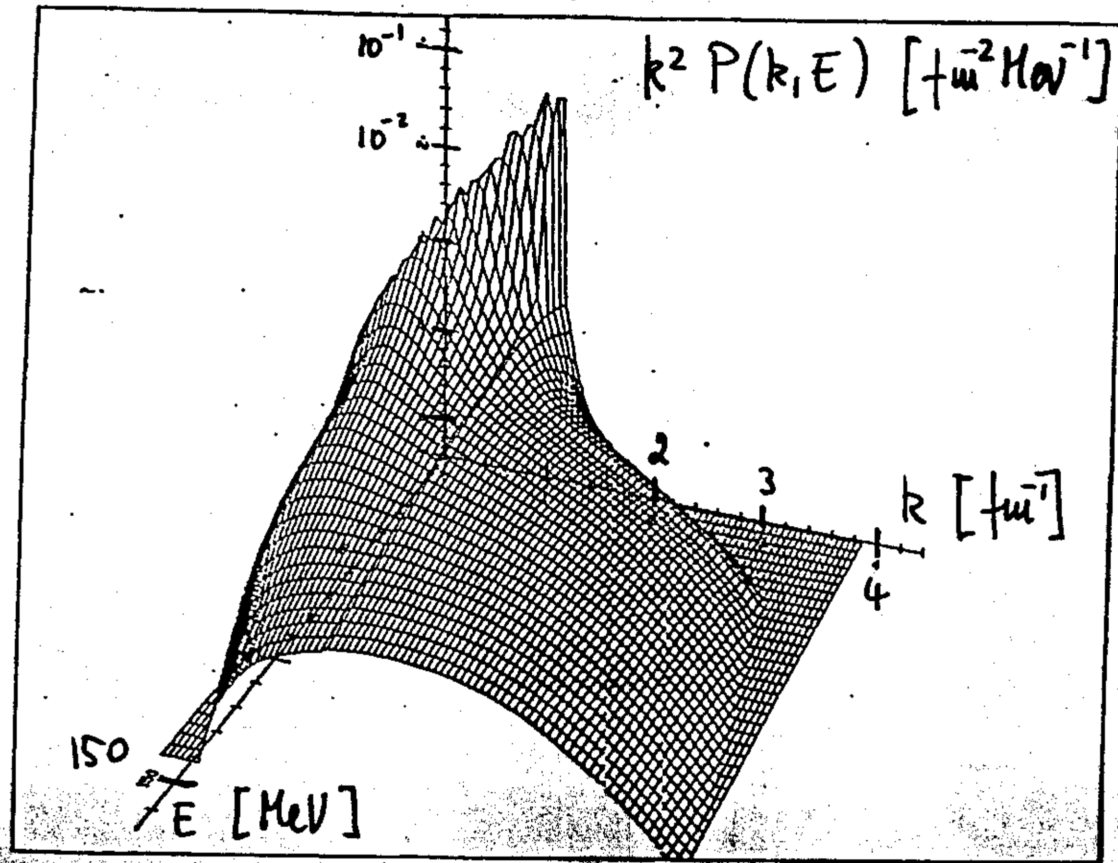
$$P(p,E) \sim \frac{Z(p)\, \mathrm{Im}\, \Sigma(p,E)}{[E + e(p)]^2 + [Z(p)\, \mathrm{Im}\, \Sigma(p,E)]^2}$$

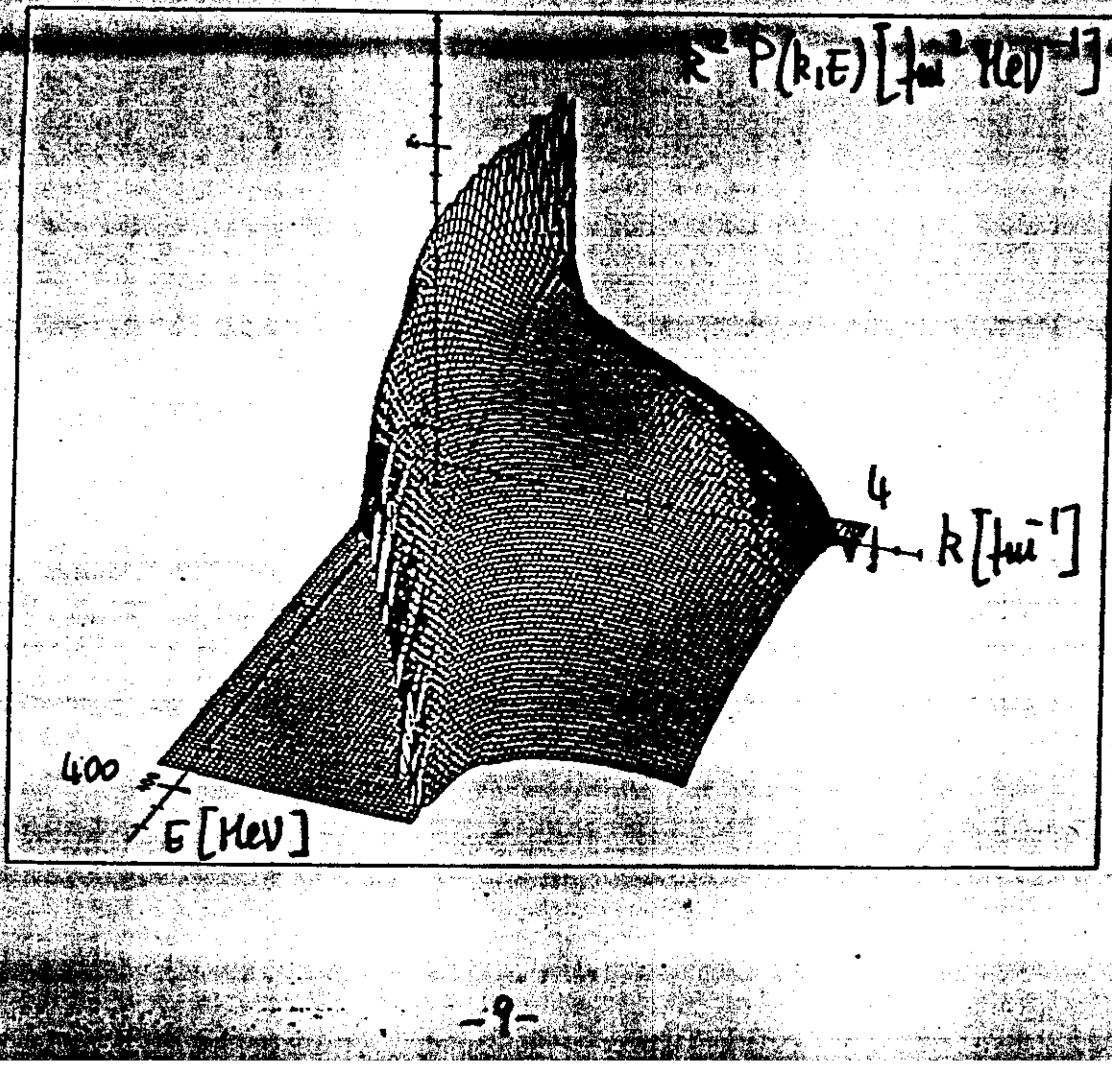
$$e(p) = p^2/2m + \mathrm{Re}\, \Sigma(p,E)$$

two-hole one-particle → three body breakup channel

widespread background, extending up to very large values of E.

NUCLEAR MATTER SPECTRAL FUNCTION
O.B., A. Fabrocini, S. Fantoni Nucl. Phys. A505(89)267 B12





• ELECTRON NUCLEON X-SECTION

Problem the scattering process involves a <u>bound</u> nucleon

⇒ part of the energy transferred by the probe goes into excitation energy of the residual (A-1)-particle system

$$\nu - \tilde{\nu} = \bar{E} + (|\vec{p}|^2 + m^2) - m$$

The larger $|\vec{p}|$, $\bar{E}$ the more the nucleon is off-shell

De Forest's prescription

- use free nucleon spinors and current operators, but replacing

$$q \to \tilde{q} \equiv (\vec{q}, \tilde{\nu})$$

- impose current conservation

$$q_\mu \widetilde{W}^{\mu\nu} = \widetilde{W}_{\mu\nu}\, q^\nu = 0$$

To remove the dependence of the nucleon tensor upon the longitudinal current

- **eN x-section**

$$\sigma_{eN} = \sigma_H \frac{m}{E_p} \left[T_1 \, W_1^N(\tilde{q}, \tilde{q}p) + \frac{1}{m^2} T_2 \, W_2^N(\tilde{q}, \tilde{q}p) \right]$$

\# σ_H Mott x-section

\# $T_1 = 2 \tan^2 \theta/2 + \frac{q^2}{|\vec{q}|^2}\left(\frac{q^2}{\tilde{q}^2} - 1\right)$

$$T_2 = \left(|\vec{p}|^2 - p_{\parallel}^2\right) \tan^2 \theta/2 - \frac{1}{2}\left(\frac{q^2}{|\vec{q}|^2}\right)^2 \frac{(E_p + E_{p'})^2}{4}$$

\# W_1^N W_2^N free nucleon structure functions

- **elastic channel**

$$W_1^N = -\frac{\tilde{q}^2}{2m} G_M^N(\tilde{q}^2)\, \delta(s - m^2) \qquad s = (p + \tilde{q})^2$$

$$W_2^N = \frac{2m}{(1 - \tilde{q}^2/4m^2)} \left\{ \left[G_E^N(\tilde{q}^2)\right]^2 - \frac{\tilde{q}^2}{4m^2}\left[G_M^N(\tilde{q}^2)\right]^2 \right\} \delta(s - m^2)$$

G_E^N, G_M^N : nucleon form factors (Höhler parametrization)

- **inelastic channels**

Bodek - Ritchie parametrization of W_1^N and W_2^N.

- COMPARISON with DATA

Extrapolation of the SLAC NE3 data to infinite A (D. Day et al PRC 40(89)1011)

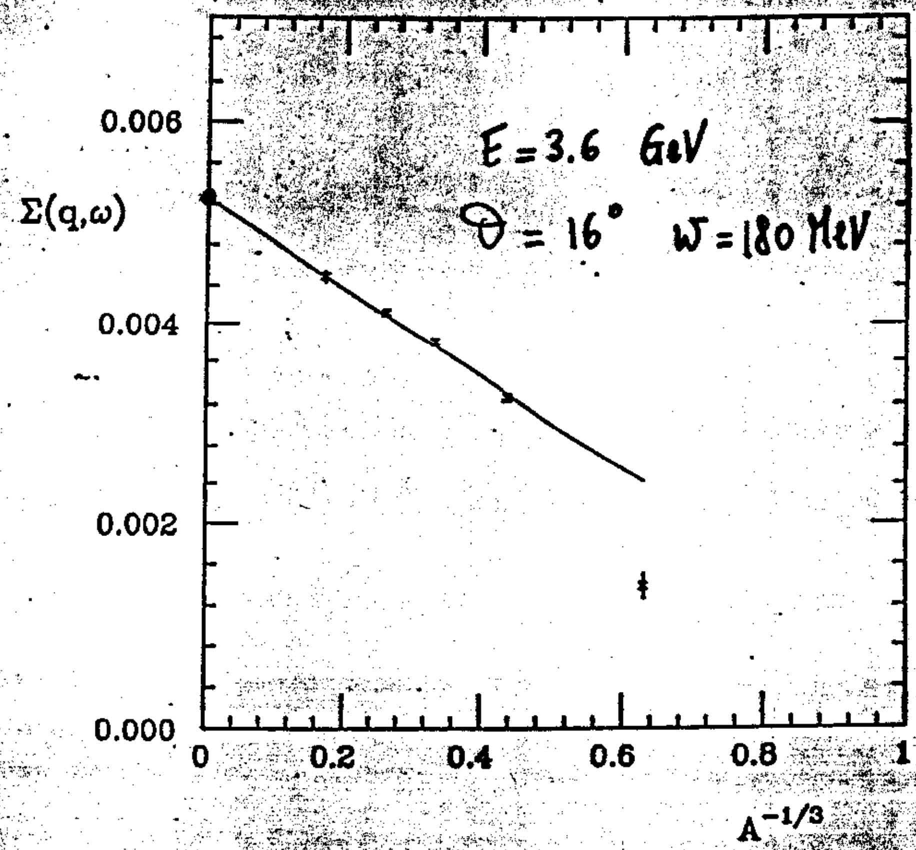
Targets He, C, Al, Fe, Au

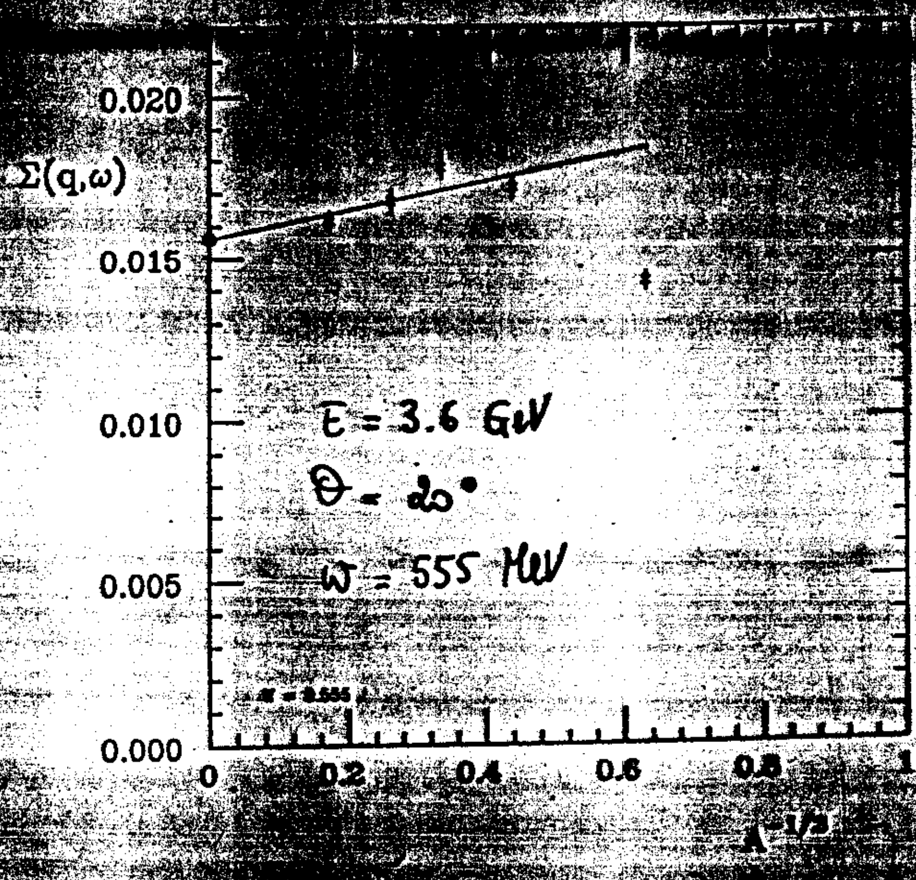
$\varepsilon = 2 \div 4$ GeV $\qquad \theta = 16^\circ \div 20^\circ$

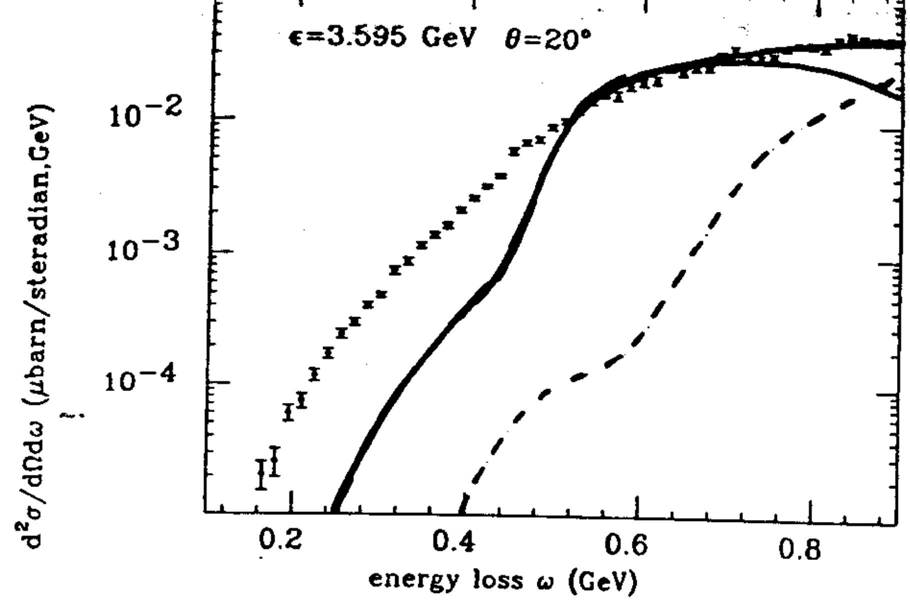
Moss formula for the inclusive x-section

$$\Sigma(|\vec{q}|,\nu) = \frac{1}{A} \frac{d^2\sigma}{d\Omega d\nu}$$

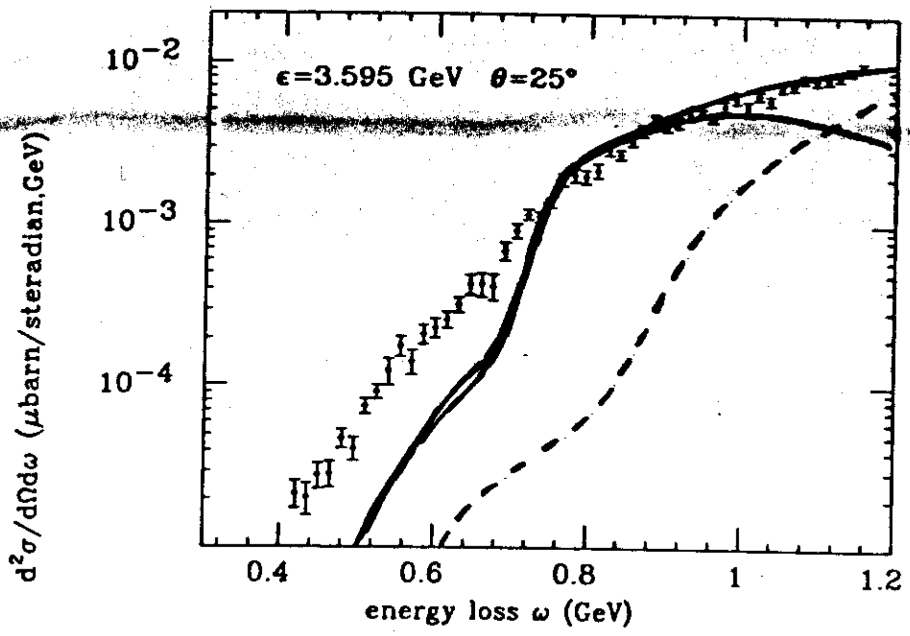
$$\Sigma(|\vec{q}|,\nu) = \Sigma_{NM}(|\vec{q}|,\nu) + \Sigma_S(|\vec{q}|,\nu) A^{-1/3}$$


Σ(q,ω)
0.006
0.004
0.002
0.000
0
0.2
0.4
0.6
0.8
1
A^{-1/3}
E = 3.6 GeV
θ = 16°
ω = 180 MeV

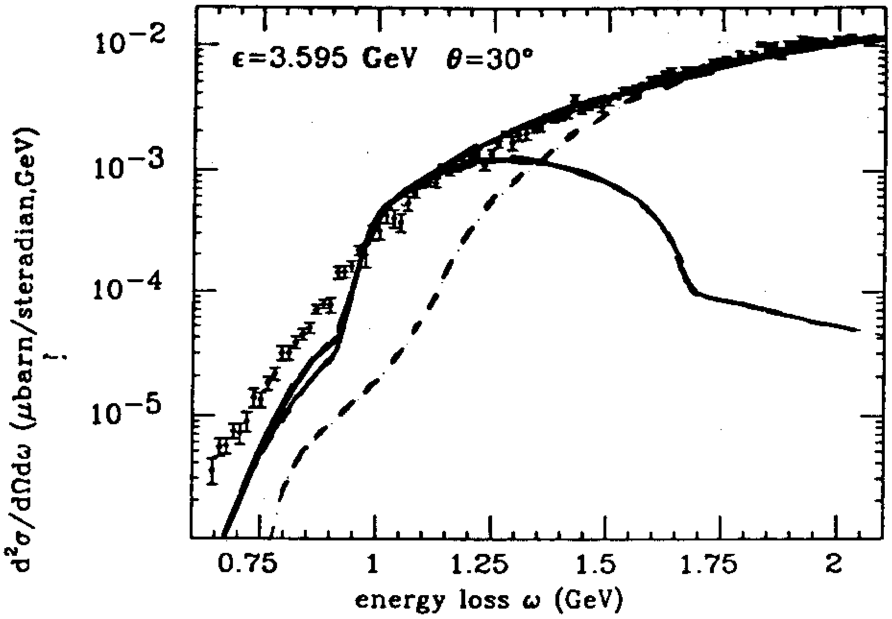

Σ(q,ω)
0.020
0.015
0.010
0.005
0.000
0
0.2
0.4
0.6
0.8
1
A^{-1/3}
E = 3.6 GeV
θ = 20°
ω = 555 MeV



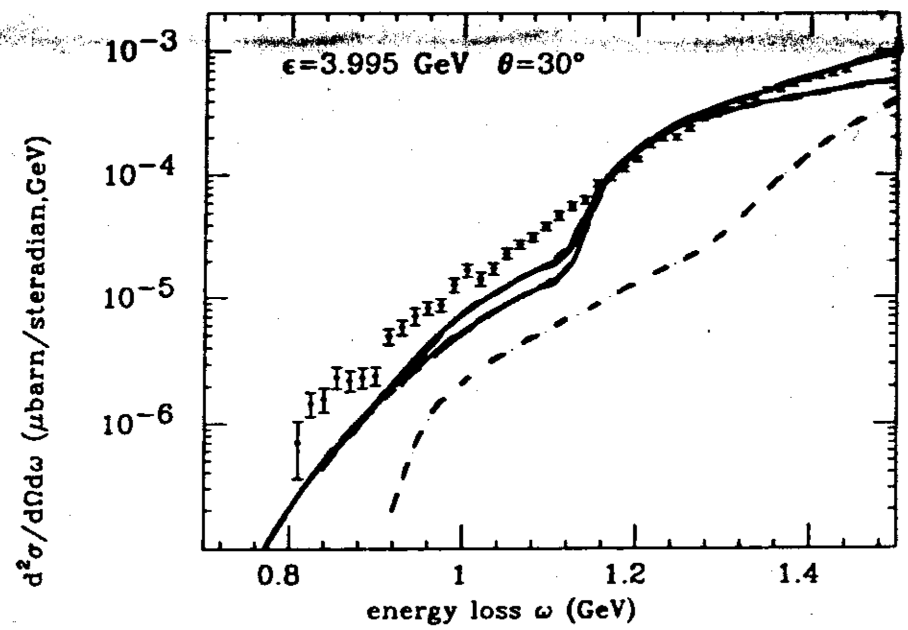
ELASTIC
INELASTIC
TOTAL



PWIA results



— ELASTIC
- - - INELASTIC
— TOTAL



PWIA results

Summary of PWIA results

1. Theory works reasonably well in the region of the quasi free peak and beyond

2. Predicted low energy loss Tail sizeably underestimated

Possible problems

Spectral function incorrect

~~final state interactions of the struck~~

⇒ proton important ⇐

INCLUSION of FINAL STATE INTERACTIONS

$$W^A_{\mu\nu}(\vec{q},\nu) \propto \mathrm{Re}\int_0^t dt \, \langle 0 | J^A_\mu \, e^{-i(H-E_0-\nu)t} \, J^A_\nu | 0 \rangle$$

$$H = H_{IA} + H'$$

$$H_{IA} = H_{A-1} + t_1$$

$$H' = V + iW$$

$$W^A_{\mu\nu}(\vec{q},\nu) = \int_0^\infty W^A_{\mu\nu,IA}(\vec{q},\nu'-V)\, F(\nu-\nu')\, d\nu'$$

$$F(\nu) = \frac{1}{\pi}\mathrm{Re}\int_0^\infty dt \, e^{i\nu t} \, e^{-Wt}$$

- V produces a shift.
  Dirac phenomenology fits of proton-nucleus scattering data indicate $V \sim 25$ MeV
- W is related to the lifetime of the state describing the propagation of the struck particle.

- Simplest approximation (wrong!)

$$\tau = \frac{1}{\rho_0 v \sigma_{eff}}$$

σ_{eff} NN x-section corrected to take into account Pauli blocking

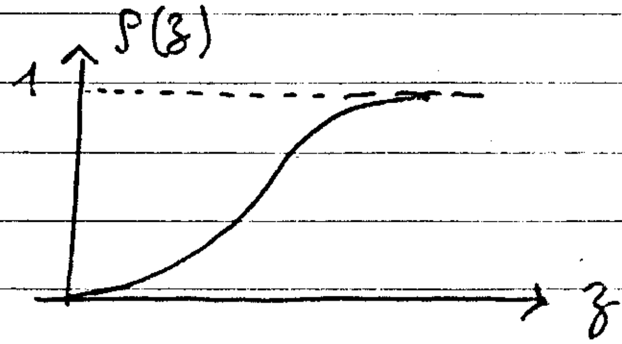
⇒ Lorentzian folding function ⇒ the corresponding response function violates the energy weighted sum rule

- Include correlation effects

The struck particle is surrounded by a hole, created by the strong short range NN repulsion

$$\rho_0 \to \rho(z = vt) = \rho_0 \, g(vt)$$

↑ NN pair correlation function (From CEF calc.)



$$\tau(t) = \frac{1}{\rho_0\, g(vt)\, v\, \sigma_{eff}}$$

$$W = \frac{1}{2}\, \rho\, v\, \sigma_{eff}\, \frac{1}{t} \int_0^t dt'\, g(vt)$$

This treatment of FSI has been extensively used in the past to analize deep inelastic neutron scattering on liquid He.

- **CONNECTION with GLAUBER THEORY of SCATTERING**

Wave function of the struck particle

$$\psi_{p'}(\vec{r}) = e^{ip'z}\,\varphi(\vec{r})$$

$$\frac{d\varphi}{dz} = -\frac{i}{\hbar v}\, v(\vec{b} + \hat{p}'z)\,\varphi(\vec{b} + \hat{p}'z)$$

The equivalent optical potential $v(z)$ felt by the struck particle after travelling a distance $z = vt$ is generated by the (A-1) nucleons in the spectator system (distributed with constant density ρ_0) and by the hole at $z=0$

$$v(z) = \int dx\,dy\; \rho^{(2)}(x,y;x+z,y)\, e^{iQ(x+z)}\, A_{p'}(Q)$$

\# $A_{p'}(Q)$ NN scattering amplitude

\# $\rho^{(2)}(x,y;x+z,y)$ half off-diagonal two body density matrix

Assume now

\# $\rho^{(2)}(x,y;x+z,y) \sim \rho_c\,[\,g(z)\,g(x-y)\,]^{1/2}$

\# $A_{p!}(Q) \sim \delta(Q)$

⇓

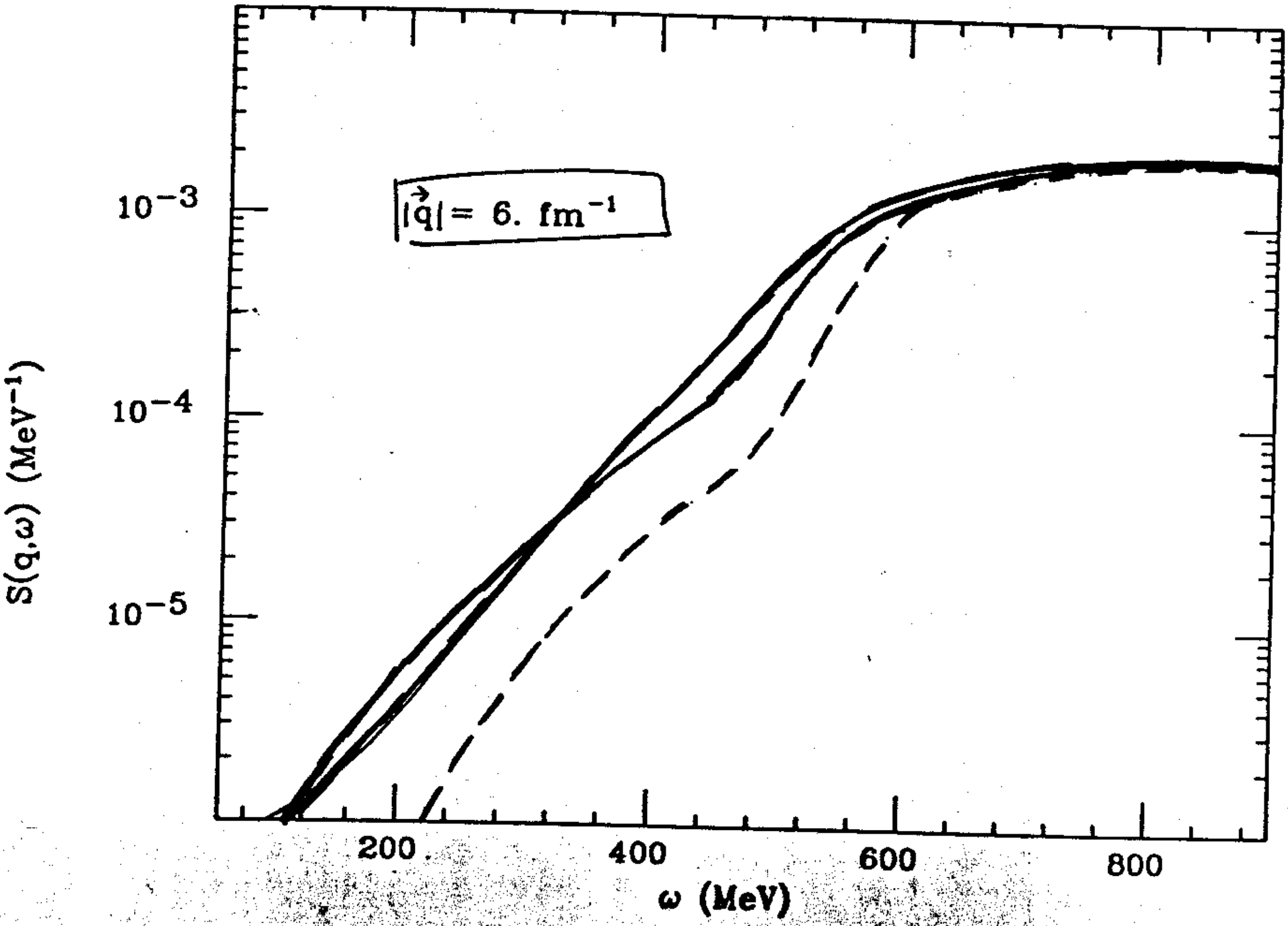
$$v(z) = \rho_c\, v\, \sigma\, g(z)$$

$$\varphi(z) = \exp\left\{-\frac{1}{2}\,\rho_c\, v\, \sigma\, \frac{1}{z}\int_0^z dz'\, g(z')\right\}$$

Since, by definition

$$\varphi(z = vt) = e^{-Wt}$$

the previous expression for W is recovered!



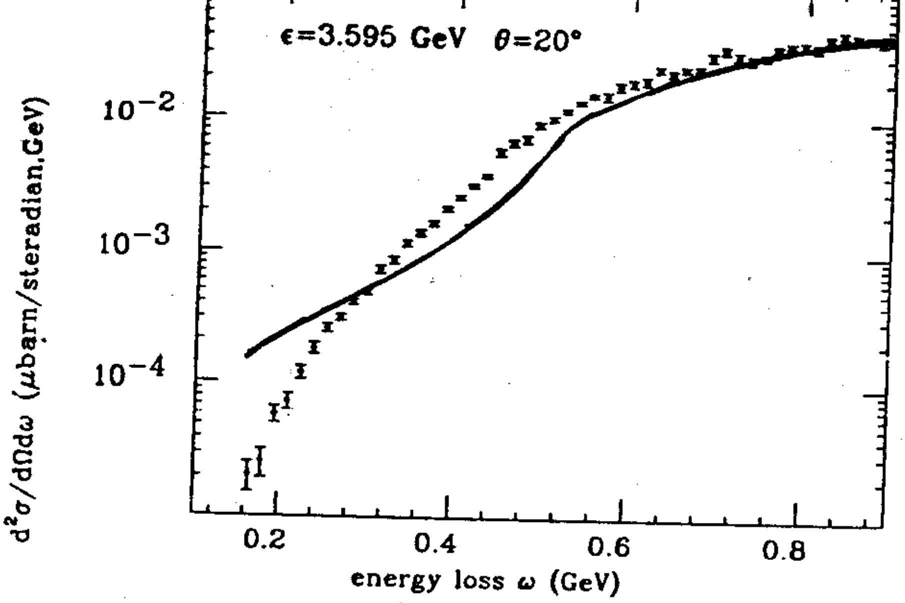
— Full calculation (nonrelativistic)

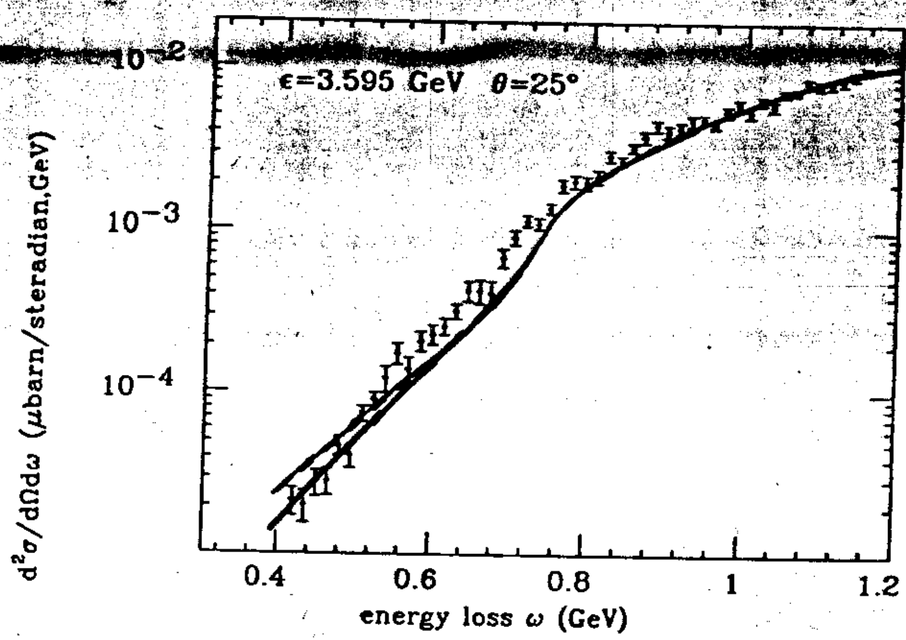
$$S(q,\omega) = \int dt\, e^{i\omega t} \langle \bar{0} | \rho^{\dagger}_{\vec{q}}\, e^{-iHt}\, \rho_{\vec{q}} | \bar{0} \rangle$$

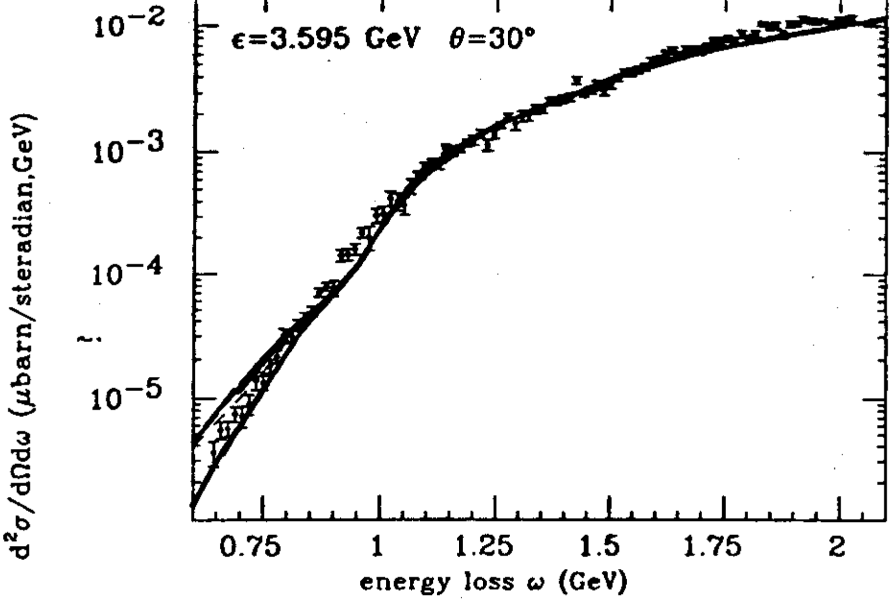
--- Impulse approximation $\qquad \rho_{\vec{q}} = \sum_k a^{\dagger}_{k+q} a_k$

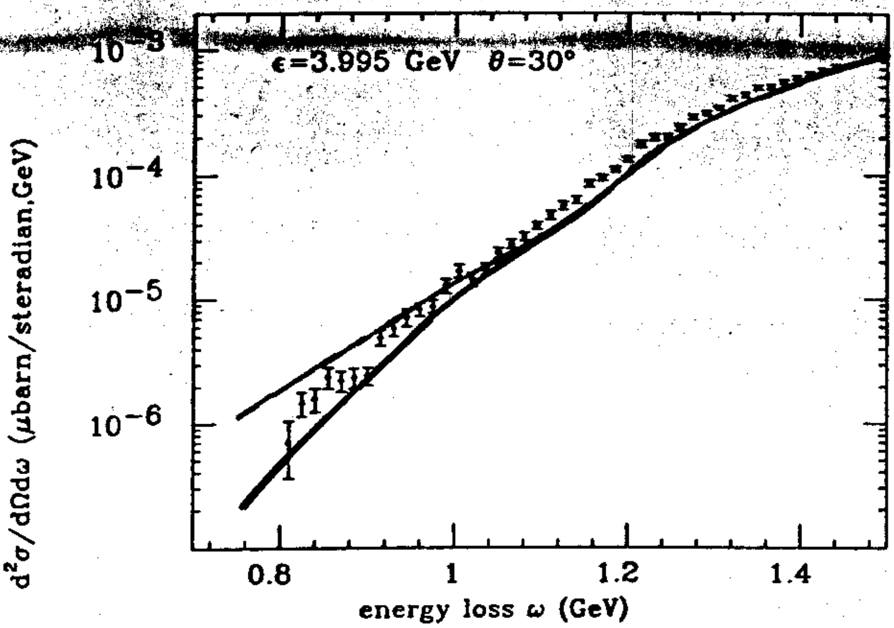
— Impulse approximation + final state interaction with

$$\tau = \frac{1}{\rho \sigma_{NN} g}$$


ϵ=3.595 GeV θ=20°
10−2
10−3
10−4
d²σ/dΩdω (μbarn/steradian,GeV)
0.2
0.4
0.6
0.8
energy loss ω (GeV)


ϵ=3.595 GeV θ=25°
10−2
10−3
10−4
d²σ/dΩdω (μbarn/steradian,GeV)
0.4
0.6
0.8
1
1.2
energy loss ω (GeV)

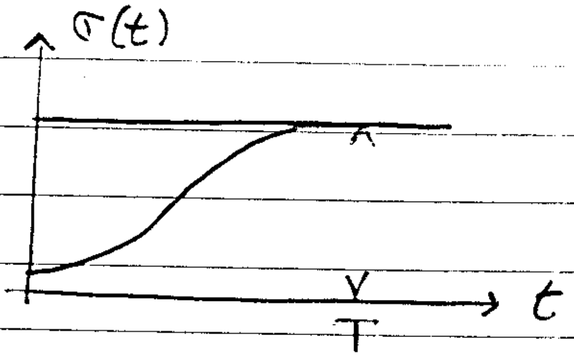

ϵ=3.595 GeV θ=30°
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.75
1
1.25
1.5
1.75
2
energy loss ω (GeV)
d²σ/dΩdω (μbarn/steradian,GeV)


ϵ=3.995 GeV θ=30°
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0.8
1
1.2
1.4
energy loss ω (GeV)
d²σ/dΩdω (μbarn/steradian,GeV)

• Inclusion of an "unconventional" effect

Time evolution of the σ_{NN}

A proton absorbing a virtual photon of very high momentum is in a point like configuration ⇒ σ_{NN} is "small" and evolves in time to reach its normal value.



Farrar, Liu, Frankfurt, Strikman

$$\sigma(z < L = vT) = \sigma_{free}\left[\frac{z}{L}\left(1 - \frac{9\langle k_T^2\rangle}{q^2}\right) + \frac{9\langle k_T^2\rangle}{q^2}\right]$$

$L = 2|\vec{q}|/(\Delta M)^2$ [illegible]

[illegible]

SUMMARY

- Realistic microscopic calculations yield a fairly accurate description of inclusive NM data within "standard" nuclear physics

- FSI effects, which are dominant in the low energy loss tail of the quasi free peak, can be taken into account using a semiclassical approach including short range correlations

- The present approach provides a consistent frame to investigate the possible modification of the NN x-section due to nucleon structure effects.