Hadron wave-function
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DISTORSION OF HADRON WAVE FUNCTIONS IN NUCLEAR MATTER

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Abstract: We report on an efficient way of calculating for high energies the final state interaction of a hadron with a target nucleus. In particular we study the electromagnetic nucleon knock out form factor. In essence, the method rests upon the fact that the plane wave approximation should be a reasonable approximation for high energy projectile. In each partial wave we calculate the difference between the exact result and the plane wave approximation. This is shown to converge at least twice as fast than the separate partial wave expansions of the exact form factor or of the plane wave approximation.

Electromagnetic knock out or production of hadrons from nuclei invariably leads to the study of final state interactions and hence to the evaluation of distorted waves [1,2]. This is not to be viewed only as a nuisance but also as a valuable source of information on the propagation of hadrons in nuclear matter [3,4]. However to discover possible non trivial aspects one must first proceed to a

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precise evaluation of the standard distorted wave which describes the interaction of the outgoing hadron with the nucleus via a complex energy dependent optical potential. This is known to be a difficult technical problem at high energies since in general a very large number of partial waves is needed in a straightforward partial wave expansion [5]. On the other hand one expects on physical grounds that at high energies, around the quasi free kinematics, the final state interaction becomes comparatively less important and the quasi-classical or eikonal approximations can be used as a guidance. This strongly suggests that one should calculate first the plane wave approximation, which physically implies no final state interaction with the residual nucleus, and treat as a correction the difference between the exact result and the plane wave approximation.

We now present a schematic model calculation where we only consider the distorted wave associated to the observed outgoing nucleon.

The transition form factor for a nucleon going from an initial bound state $\varphi_{lm}(\vec{r})$ to a final scattering state $\psi_k^{(-)}(\vec{r})$ with the momentum transfer $\vec{q}$ reads

$$S_{lm}(\vec{k}) = \int d^3r \; \psi_k^{(-)*}(\vec{r}) \; e^{i\vec{q}\vec{r}} \; \varphi_{lm}(\vec{r}) \qquad (1)$$

where $\varphi_{lm}(\vec{r})$ is a solution of the bound state Schroedinger equation ($\varepsilon_{lm} < 0$)

$$H \, \varphi_{lm}(\vec{r}) = \varepsilon_{lm} R_l(r) \; Y_l^m(\hat{r}), \qquad (2)$$

$R_l(r)$ being the radial part of $\varphi_{lm}(\vec{r})$. The single particle Hamiltonian H with the radial finite range potential V(r) is given by

$$H = -\frac{\nabla^2}{2m_N} + V(r) \tag{3}$$

and the scattering state fulfils ($E_k = k^2/2m_N$)

$$H\,\psi_k^{(-)}(\vec{r}) = E_k\,\psi_k^{(-)}(\vec{r}). \tag{4}$$

In the absence of final state interaction the plane wave approximation to the transition form factor is the Fourier transform of the bound state wave function evaluated at momentum $\vec{k}-\vec{q}$

$$S_{lm}^{PW}(\vec{k}-\vec{q}) = \int d^3r\; e^{-i(\vec{k}-\vec{q})\vec{r}}\,\varphi_{lm}(\vec{r}). \tag{5}$$

The integration in (5) can be made more efficient by relying on the range of the potential itself. One uses the Schroedinger equation in momentum space

$$S_{lm}^{PW}(\vec{p}) = \langle\vec{p}\,|\varphi_{lm}\rangle = \langle\vec{p}\,|V|\varphi_{lm}\rangle\,\frac{1}{\varepsilon_{lm}-p^2/2m_N}\,. \tag{6}$$

and

$$\langle\vec{p}\,|V|\varphi_{lm}\rangle = 4\pi\,Y_l^m(\hat{p})(-i)^l\int_0^\infty dr\;r^2\,j_l(qr)\,R_l(r)\,V(r). \tag{7}$$

One thus gets an accurate value of (5) with fewer integration points since the integration range is now cut off by the range of the potential. The exact form factor can now be evaluated as

$$S^{ex}_{lm}(\vec{k},\vec{q}) = S^{PW}_{lm}(\vec{k}-\vec{q}) + A_{lm}(\vec{k},\vec{q}), \qquad (8)$$

where

$$A_{lm}(\vec{k},\vec{q}) = \int d^3r \, [\psi_k^{(-)*}(\vec{r}) - e^{-i\vec{k}\vec{r}}] \, e^{i\vec{q}\vec{r}} \varphi_{lm}(\vec{r}). \qquad (9)$$

We then use a standard partial wave expansion for the difference in the expression above. The scattering wave function can be expanded as

$$\psi_k^{(-)}(r) = 4\pi \sum_{LM} i^L e^{-i\delta_L} u_L(k,r) \, Y_L^M(\hat{r}) \, Y_L^{M*}(\hat{k}), \qquad (10)$$

where $u_L(k,r)$ is the positive energy solution of the radial Schroedinger equation. Equation (9) can then be expressed in terms of the one dimensional integrals:

$$I_{lL\lambda}(k,q) = \int_0^\infty dr \, r^2 \, [e^{i\delta_L} u_L(k,r) - j_L(k\,r)] \, R_l(r) j_\lambda(qr) \qquad (11)$$

where the Bessel functions $j_L(kr)$ arise from expanding the partial wave expansion of the outgoing plane wave in (9). The radial integrals in eq.(11) provide a direct measure of the distortion effects in each partial wave. These effects become clearly more and more important as one goes away from the quasifree region or to large $|\vec{k} - \vec{q}|$.

This technique has been used in the course of some work concerning a high energy approximation for one nucleon knock out form factors [2]. We have reanalyzed and extended the numerical calculations presented in [2] bearing special attention to the convergence of the partial wave expressions. The transition form factor is evaluated for a single particle model, specifically a square well and a Woods-Saxon potentials. We present here results for a square well, whose depth is 46.06 MeV and whose radius is 3 fm. Note that a square well calculation provide, as can be seen in [2], a significant test of the method. The s-wave binding energy is 31.2 MeV and the p-wave one 16.5 MeV. Calculations have been performed for outgoing momenta of 3, 4 and 5 fm^{-1} and momentum transfers q = 3, 4 and 5 fm^{-1}. Typical example of the convergence ($k = q = 4\ fm^{-1}$, $\vec{k} = \vec{q}$) study is presented in the table. The correction $A \equiv A_{00}$, for the s-wave can be very acurately calculated with 16 partial waves whereas the plane wave contribution converges for much larger values (L_{max} > 35). We have obtained similar results for the Woods-Saxon potential and also for the p-wave knock-out form factor. Results of the same quality were obtained for an asymmetric configuration $k \neq q$ and different angles between $\vec{k}$ and $\vec{q}$ in the forward hemisphere (with respect to $\vec{q}$ which we take as the angular momentum quantization axis); typically for $k = 3\ fm^{-1}$, $q = 4\ fm^{-1}$ or $k = 4\ fm^{-1}$, $q = 3\ fm^{-1}$ one needs at most 15 partial waves while at $k = 5\ fm^{-1}$, $q = 5\ fm^{-1}$, 20 partial waves are needed. The number of partial waves required to evaluate the form factor is thus reduced by more than a factor of two in all the cases considered. The main differences between the exact calculation (8) and the plane wave approximation (5) arise essentially from the central partial waves. As a final remark we stress that this technique is straightforwardly applicable in cases where the knock out is included by a non strongly interacting probe such that only one wave function is distorted (see eq.(1)).

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References

1. See for instance: J.V. Noble, Phys. Rev. C17 (1978) 2151
2. F. Cannata, J.-P. Dedonder, L. Lesniak, Phys. Rev. C33 (1986) 1888; see also:
   R.D. Amado, F. Cannata, J.-P. Dedonder, Phys. Rev. C31 (1985) 162

3. See for instance S. Boffi, C. Giusti, F.D. Pacati, Nucl. Phys. A386 (1982) 599;
   F. Cannata, J.-P. Dedonder, J.R. Gillespie, Nucl. Phys. A467 (1987) 636;
   S. Boffi et al, Nuovo Cim. 98A (1987) 291
4. D.F. Geesaman et al, Phys. Rev. Lett. 63 (1989) 734, and also CEBAF proposal PR-89-022 (1989).

5. N.S. Chant, P.G. Roos, Phys. Rev. C27 (1983) 1060;
   S. Frullani and J. Mougey, Advances in Nuclear Physics, ed. by I.W. Negele and E. Vogt (Plenum, N.Y., 1985), Vol. 15.

Table

S-wave knock-out form factor for a square well potential (see text): $q = k = 4\ fm^{-1}$, $\vec{k} - \vec{q} = 0$ (ReA - real part of the final state correction, PW - plane wave approximation, ReS = PW + ReA, ImS - imaginary part)

L	δ_L rad	ReA $fm^{3/2}$	PW $fm^{3/2}$	ReS $fm^{3/2}$	ImS=ImA $fm^{3/2}$
0	0.79	$-0.61 \cdot 10^{-2}$	0.15	$9.27 \cdot 10^{-2}$	$9.39 \cdot 10^{-2}$
2	0.76	-0.27	0.73	0.46	0.44
4	0.72	-0.36	1.15	0.79	0.69
6	0.72	-0.38	1.33	0.96	0.83
8	0.59	-0.31	1.26	0.95	0.64
10	0.43	$1.5 \cdot 10^{-2}$	0.96	0.97	0.45
12	$6.3 \cdot 10^{-2}$	$3.93 \cdot 10^{-2}$	0.61	0.65	$4.08 \cdot 10^{-2}$
14	$3.5 \cdot 10^{-3}$	$2.36 \cdot 10^{-3}$	0.36	0.36	$1.27 \cdot 10^{-3}$
16	$1.0 \cdot 10^{-4}$	$6.50 \cdot 10^{-5}$	0.21	0.21	$2.15 \cdot 10^{-5}$
18	$1.7 \cdot 10^{-6}$	$1.00 \cdot 10^{-6}$	0.12	0.12	$2.08 \cdot 10^{-7}$
20	$1.6 \cdot 10^{-8}$	$9.45 \cdot 10^{-9}$	$6.9 \cdot 10^{-2}$	$6.9 \cdot 10^{-2}$	$1.23 \cdot 10^{-9}$