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SPRING COLLEGE ON PLASMA PHYSICS

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FILAMENTATION AND MODULATIONAL INSTABILITIES OF OF HIGH-POWER LASER BEAMS IN PLASMAS

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PART A

Filamentation instability of high power laser beams in plasmas

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Accounting for three types of plasma slow responses to an intense electromagnetic radiation, the spatial filamentation instability in a uniform unmagnetized plasma has been investigated. The nonlinearities arising from the relativistic electron mass variation and the low-frequency ponderomotive force are included. Both of these nonlinearties are valid for arbitrarily large values of radiation intensity. A new kind of rapidly growing filamentation instability is shown to exist at high pump power.

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I. INTRODUCTION

Recently, high power laser beams are being employed for inertial fusion[1] as well as for the plasma-based beat-wave accelerators[2,3] The inertial fusion requires the anomalous absorption of laser light by the plasma, whereas the plasma-based beat-wave accelerator concept relies on the radiation induced high phase velocity plasma waves that can accelerate an electron to very high energies. Thus, a clear understanding of linear as well as nonlinear propagation characteristics of the electromagnetic radiation through the plasma is of great importance.

In the linear regime, studies of mode conversion, collisional absorption, classical transport, etc. are essential. At moderate laser intensity, there arises a number of nonlinear effects including various kinds of parametric instabilities,[4,5] wave filamentation,[6-8] soliton formation,[9-14] and profile modification.[15] Generally, laser-plasma interaction involves two distinct nonlinearities. The first one comes from the radiation pressure effects[16], whereas the second arises from the relativistic electron mass variation.[17] The combined effects of those two nonlinearities give rise to some new phenomena. For example, several authors[9-14] have accounted for an arbitrary large amplitude nonlinear electron current density and have demonstrated the existence of numerous types of finite amplitude envelope solitons for a one-dimensional problem.

The filamentation instability and self-focusing occur in a multi-dimensional situation. Previous studies[6,7] of the filamentation instability have not included the relativistic electron mass variation nonlinearity and have been restricted to a special class of quasi-static spatial modulation of the radiation. The purpose of this paper is the following. We investigate the filamentation instability of an arbitrary large amplitude electromagnetic radiation including the relativistic electron mass variation as well as the relativistic ponderomotive force nonlinearties. Furthermore, the plasma slow response to an intense electromagnetic wave is of general nature. It is found that at large laser intensity a rapidly growing filamentation instability arises which is absent in the previous investigations[6,7]. Thus, the present work generalizes the analyses of Kaw et al.[6] and Liu and Tripathi[7].

The manuscript is organized in the following fashion. In the next section, we present the basic equations and derive the wave equation retaining a large amplitude nonlinear electron current density. The latter arises from the beating of the relativistic electron quiver velocity with the large scale slow electron number density variation created by the radiation pressure of the intense laser beam.

In Sec. III, we present three types of expressions for the slow electron density variation. The filamentation instability is analyzed in Sec. IV. A numerical analysis of the spatial growth is carried out in Sec. V. Our results are summarized in the last section.

II. BASIC EQUATIONS

We consider a uniform unmagnetized electron-ion plasma in the presence of a circularly polarized electromagnetic wave of the form

$$\vec{E} = E(r)\ (\hat{x} + i\hat{y})\ \exp(ik_o z - i\omega_o t) + c.c., \tag{1}$$

where $\vec{E}$ is the electric field, k_o is the wavevector, ω_o is the wave frequency, and c.c. stands for the complex conjugate.

The nonlinear interaction of a strong electromagnetic wave with the background plasma is governed by the relativistic electron momentum equation

$$(\partial_t + \vec{u}_e\cdot\nabla)\vec{p}_e = -e(\vec{E} + \vec{u}_e\times\vec{B}/c) - T_e\nabla \ln n_e, \tag{2}$$

the Maxwell equation

$$\nabla\times\vec{B} = (4\pi/c)\ \vec{J} + c^{-1}\partial_t\vec{E}, \tag{3}$$

and the Poisson equation

$$\nabla \cdot \vec{E} = 4\pi e (n_i - n_e), \tag{4}$$

where $\vec{v}_e$ is the electron fluid velocity, T_e is the electron temperature, e is the electronic charge, c is the speed of light, $\vec{p}_e = m_o \vec{v}_e / \Gamma$, $\Gamma = (1 - v_e^2/c^2)^{1/2} = (1 + p_e^2/m_o c^2)^{-1/2}$, m_o is the rest mass of the electron, and $\vec{J} = n_i e \vec{v}_i - n_e e \vec{v}_e \equiv \vec{J}_i + \vec{J}_e$ is the current density.

On using the electromagnetic fields

$$\vec{E} = - \nabla \phi - c^{-1} \partial_t \vec{A} \tag{5}$$

and

$$\vec{B} = \nabla \times \vec{A}, \tag{6}$$

where ϕ is the scalar potential associated with the plasma slow motion and $\vec{A}$ is the perpendicular component of the high-frequency vector potential, one[10] can show that Eq.(2) is satisfied by a high-frequency part

$$\vec{p} = m_o c \vec{\Psi} , \tag{7}$$

where $\vec{\Psi} = e\vec{A}/m_o c^2$, and an equation

$$\beta (1 + |\Psi|^2)^{1/2} - \beta = \phi - \ln N_e , \tag{8}$$

where the linear inertia of the plasma slow motion has been neglected for spatial modulation. Equation (8) dictates the

excitation of the large-scale ambipolar field as well as the electron density variation by the relativistic ponderomotive force.[10,16] Furthermore, we have defined $\beta = c^2/v_{te}^2$, $N_e = n_e/n_o$, where $v_{te} = (T_e/m_o)^{1/2}$ is the electron thermal velocity, and n_o is the unperturbed plasma density. We have normalized ϕ by T_e/e, and all spatial lengths by the electron Debye length $\lambda_D = (T_e/4\pi n_o e^2)^{1/2}$. In addition, the plasma is assumed to be unperturbed at $|\vec{r}| \rightarrow \infty$ and accordingly, the boundary conditions $\phi = \Psi = 0$, $N_e = 1$ have been imposed.

On the other hand, from Eqs. (3), (5), and (6), one finds the electromagnetic wave equation

$$\partial_t^2 \vec{A} - c^2 \nabla^2 \vec{A} = 4\pi c \vec{J}_e , \tag{9}$$

where the ion current is noted to be small, and is,therefore, neglected. The right-hand side of Eq.(9) represents the nonlinear electron current density arising from the beating of the relativistic quiver velocity and the large-scale slow electron density variation.

We assume that the nonlinear interaction of an intense laser radiation with the background plasma gives rise to a spatially slowly-varying envelope along the z axis. Thus, within the WKB approximation[18], Eq. (9) can be written as

$$i\alpha \partial_z \psi + \beta \nabla_\perp^2 \psi + \Delta \psi = N_e \psi (1 + |\vec{\psi}|^2)^{-1/2}, \tag{10}$$

where $\alpha = 2k_0 c^2/\omega_{po}^2 \lambda_D$, $\Delta = (\omega_0^2 - k_0^2 c^2)/\omega_{po}^2$, $\omega_{po} = (4\pi n_0 e^2/m_0)^{1/2}$ is the electron plasma frequency, and we have assumed $\partial_z^2 \ll \nabla_\perp^2 (= \partial_x^2 + \partial_y^2)$.

III. MODEL EQUATIONS FOR PLASMA SLOW RESPONSE

In this section, we present three types of model equations which can describe the plasma slow response to the electromagnetic waves. First, we consider the forced-Raman(FR) interaction[12] in which the ions form a neutralizing background and the ambipolar potential is directly created by the radiation pressure[4]. Thus, from Eq. (8), we have

$$\phi = \beta[(1+|\psi|^2)^{1/2} - 1] \quad . \qquad (11)$$

On substituting Eq. (11) into the Poisson equation, one can determine the slow electron density variation[12]

$$N_e^R = 1 + \beta \nabla^2 (1+|\psi|^2)^{1/2} . \qquad (12)$$

Secondly, we investigate the forced-Brillouin(FB) interaction in which the ion density variation obeys the Boltzmann distribution, i.e.,

$$n_i = n_0 \exp(-\sigma\phi) \quad , \qquad (13)$$

where $\sigma = T_e/T_i$, and T_i is the ion temperature. Inserting Eqs. (11) and (13) into the Poisson equation, the electron number density variation for this case is found to be[13]

$$N_e^B = \beta \nabla^2 (1+|\psi|^2)^{1/2} + \exp\{-\beta_b[(1+|\psi|^2)^{1/2} - 1]\} , \qquad (14)$$

where $\beta_b = \beta\sigma$.

Finally, we present the quasi-static (QS) adiabatic response[11] to the electromagnetic radiation. Here, the electron number density variation determined from Eq. (8) is given by

$$N_e = \exp[\phi + \beta - \beta(1+|\psi|^2)^{1/2}] \quad . \qquad (15)$$

Using the quasi-neutrality condition $n_e = n_i$ (valid for long wavelength modulations), one then finds

$$\phi = \beta[(1+|\psi|^2)^{1/2} - 1]/(1+\sigma) \quad . \qquad (16)$$

Eliminating ϕ from Eqs. (15) and (16), we have[11]

$$N_e^S = \exp\{-\beta_s[(1+|\psi|^2)^{1/2} - 1]\} , \qquad (17)$$

where $\beta_s = \sigma/(1+\sigma)$. Note that Eq. (17) is a generalization of Kaw et al.[6] by including the relativistic ponderomotive force.[10,16] Thus, Eqs. (12), (14), and (17) together with Eq. (10) constitute a set of nonlinear equations for the study of the filamentation instability of strong laser beams in a uniform unmagnetized plasma.

IV. FILAMENTATION INSTABILITY

The filamentation instability of a constant amplitude intense laser beam can be investigated following the general method of Karpman[18]. Accordingly, we decompose the potential ψ into two parts:

$$\psi = (\psi_0 + \delta\psi)\, e^{i\delta z} , \tag{18}$$

where ψ_0 (real) is the pump amplitude, $\delta\psi$ ($\ll \psi_0$) is the perturbation, and δ is a nonlinear shift in the wavenumber. From Eq. (18), we have

$$|\psi|^2 = \psi_0^2 + \psi_0\, \delta\psi_0 , \tag{19}$$

where $\delta\psi_0 = \delta\psi + \delta\psi^* \ll \psi_0$.

A. Forced-Raman Interaction

Inserting Eq. (12) into Eq. (10) and using Eqs. (18) and (19), one obtains the nonlinear frequency shift

$$\delta = \alpha^{-1}[\Delta - (1+\psi_0^2)^{-1/2}] , \tag{20}$$

and an evolution equation for the perturbation

$$i\alpha\partial_z\delta\psi + \beta\nabla_\perp^2\delta\psi + \frac{\psi_0^2}{2(1+\psi_0^2)^{3/2}}\delta\psi_0 = \frac{\beta\psi_0^2}{2(1+\psi_0^2)}\nabla^2\delta\psi_0 . \tag{21}$$

Note that in obtaining Eq. (21), we have made use of Eq. (20).

Letting $\delta\psi = X + iY$ in Eq. (21), and separating into real and imaginary parts, we get

$$-\alpha\partial_z Y + \beta\nabla_\perp^2 X + \psi_0^2(1+\psi_0^2)^{-3/2} X = \beta\psi_0^2(1+\psi_0^2)^{-1}\nabla^2 X , \tag{22}$$

$$\alpha\partial_z X + \beta\nabla_\perp^2 Y = 0 . \tag{23}$$

For X and Y varying spatially as

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \tilde{X} \\ \tilde{Y} \end{pmatrix} \exp(i\vec{k}_\perp\cdot\vec{r} + ik_z z) ,$$

Eqs. (22) and (23) may be Fourier transformed and combined to yield

$$k_z^2 = \frac{-\beta k_\perp^2\{\psi_0^2(1+\psi_0^2)^{-1/2} - \beta k_\perp^2\}}{\{\alpha^2(1+\psi_0^2) + \beta^2 k_\perp^2\psi_0^2\}} . \tag{24}$$

For spatial growth along the direction of wave propagation, we set $k_z = -ik_m$ ($k_m > 0$). Then, from Eq. (24), we obtain the amplification rate

$$k_m^R = \frac{\beta^{1/2}k_\perp\{\psi_0^2(1+\psi_0^2)^{-1/2} - \beta k_\perp^2\}^{1/2}}{\{\alpha^2(1+\psi_0^2) + \beta^2k_\perp^2\psi_0^2\}^{1/2}} , \tag{25}$$

with the threshold condition

$$\beta k_\perp^2 = \psi_0^2(1+\psi_0^2)^{-1/2} . \tag{26}$$

B. Forced-Brillouin Interaction

By substituting for the electron number density N_e from Eq. (14) into Eq. (10), using Eqs. (18) and (19), and following the method for analyzing the filamentation instability in Sec. IVA, one finds for the nonlinear wavenumber shift and the spatial growth rate, respectively,

$$\delta = \alpha^{-1}\left[\Delta - (1+\psi_o^2)^{-1/2} \exp\{\beta_b[1-(1+\psi_o^2)^{1/2}]\}\right], \quad (27)$$

and

$$k_m^B = \frac{\beta^{1/2} k_\perp \left(\psi_o^2[\beta_b + (1+\psi_o^2)^{-1/2}] \exp\{\beta_b[1-(1+\psi_o^2)^{1/2}]\} - \beta k_\perp^2\right)^{1/2}}{\{\alpha^2(1+\psi_o^2) + \beta^2 k_\perp^2 \psi_o^2\}^{1/2}} . \quad (28)$$

The instability threshold condition is given by

$$\beta k_\perp^2 = \psi_o^2[\beta_b + (1+\psi_o^2)^{-1/2}] \exp\{\beta_b[1-(1+\psi_o^2)^{1/2}]\} . \quad (29)$$

C. Quasi-static Interaction

Once again, we follow the procedure outlined in Sec. IVA to investigate the filamentation instability caused by the static spatial modulation. Thus, substituting for the electron number density N_e^s from Eq. (17) into Eq. (10), we find

$$\delta = \alpha^{-1}\left[\Delta - (1+\psi_o^2)^{-1/2} \exp\{\beta_s[1-(1+\psi_o^2)^{1/2}]\}\right], \quad (30)$$

and the spatial growth rate

$$k_m^s = \beta^{1/2} k_\perp \alpha^{-1}\left(\psi_o^2(1+\psi_o^2)^{-1}\{\beta_s + (1+\psi_o^2)^{-1/2}\} \exp\{\beta_s[1-(1+\psi_o^2)^{1/2}]\} - \beta k_\perp^2\right)^{1/2}, \quad (31)$$

with

$$\beta k_\perp^2 = \psi_o^2(1+\psi_o^2)^{-1}\{\beta_s + (1+\psi_o^2)^{-1/2}\} \exp\{\beta_s[1-(1+\psi_o^2)^{1/2}]\}, \quad (32)$$

at threshold. In the next section, we present the numerical analyses of the growth rates. In particular, we study the effects of the laser intensity, spectra of the unstable modes, as well as the plasma temperature on the spatial amplification rate.

V. NUMERICAL ANALYSIS

We examine the variation of the spatial amplification rate associated with the three types of plasma slow motions as a function of the wavenumber of the perturbation perpendicular to the direction of propagation($k_\perp$), the incident laser amplitude ψ_o , and β ($=c^2/v_{te}^2$).

Figure 1 exhibits the spatial growth k_m against $k_\perp$, for β =50 (v_{te} =0.14c), k_o =0.1, and four values of ψ_o . We see that as ψ_o increases, the maximum value of k_m and the range of $k_\perp$ over which an instability occurs decrease rapidly for both the forced-Brillouin(FB) and the quasi-static(QS) interactions,with the decline of the former being much more rapid. For ψ_o =0.75, the FB is totally damped, while for ψ_o =1.0, both FB and QS experience no growth. On the other hand, the forced-Raman(FR) interaction displays an opposite behavior, with both k_m-maximum and the range of $k_\perp$ values for wave growth increasing with ψ_e.

An overall dependence of k_m on ψ_e is shown in Fig. 2., where β and k_o assume the values given above, and we fix $k_\perp$ =0.04. We observe that FB and QS interactions cause growth for $\psi_e < 0.65$. As the laser amplitude (and, therefore, the intensity) increases to larger values, only FR causes wave growth. The restricted range of ψ_e values for wave growth associated with FB and QS could be attributed to the behavior of the nonlinear wavenumber shift δ, which in Fig.3 is plotted against ψ_e for ω_e^2/ω_{pe}^2=1.5 and for all other parameters fixed as in Fig. 2. We find that for FB and QS, δ reaches the value of k_o, wavenumber of the incident laser beam,for ψ_e in the range $0.5 \leq \psi_e \leq 0.7$. It appears that at large pump intensity the nonlinear wavenumber shift is so large that the wave propagation properties are destroyed and the wave suffers damping. On the other hand, for FR interaction, the nonlinear wavenumber shift is relatively small for $0 \leq \psi_e \leq 1.0$, and does not hinder wave growth.

Figure 4 displays the dependence of the amplification rate on β. Here, we fix ψ_o=0.5 and $k_\perp$=0.04. We see that for $\beta > 70 (v_{te} \leq 0.12c)$ only FR produces growth.

However, as the plasma temperature increases, β decreases and all three interactions contribute to wave growth, with that due to QS dominating in the range $20 < \beta < 55$ $(0.13 < v_{te}/c < 0.22)$.

The above graphs were drawn for a CO_2 laser (k_o= 6 x 10^3 cm^{-1}) and a dense plasma with an electron number density n_o= 2.0 x 10^{19} cm^{-3}. Thus, the results are applicable to the laser produced plasmas.On the other hand, for plasma-based beat-wave accelerators, one considers a tenuous plasma with an electron number density n_o=4.0 x 10^{15} cm^{-3} and $\omega_o = 50\, \omega_{pe}$. For these values, the variations of k_m with $k_\perp$, ψ_e, and β are found to be the same as shown in Figs. 1, 2, and 4. However, the magnitude of k_m decreases by a factor of seventy. This is due to the significant increase in the parameter α.

VI. DISCUSSION

In this paper, we have investigated the filamentation instability of an intense electromagnetic wave by including an arbitrary large amplitude relativistic electron quiver velocity as well as large-scale electron density variations created by the relativistic radiation pressure. Both of these effects become significant for laser intensity beyond 10^{16} W/cm^2 because of the large quiver velocity v_o($=eE_o/m_o\omega_o$ $= eA_o/m_oc$) which obeys the wellknown scaling $v_o/c = 6\times 10^{-10}$ $\lambda\ I^{1/2}$, where λ is the laser wavelength in microns and I is the laser intensity in W/cm^2. For example, for a CO_2 laser with $\lambda = 10.6\ \mu$m and $I = 10^{16}$ W/cm^2, one finds $v_o \simeq 0.6c$. Clearly, in such cases the effects of relativistic electron mass variations have to be included in the study of laser-plasma interactions.

At present, high power laser beams are under construction for the inertial fusion as well as the plasma-based beat-wave accelerators. In both of these schemes, one must consider various kinds of nonlinear effects that might arise at moderate and also at large pump intensities. Here, we have accounted

for three types of model plasma slow responses to strong laser beams and have documented the existence of a filamentation instability. Since we have allowed for an arbitrary large amplitude laser beam, our findings are valid far beyond the regimes of the previous investigations[6,7]. Furthermore, in contrast to earlier studies[6,7], the present work has pointed out the importance of numerous kinds of plasma slow responses to the radiation including the relativistic ponderomotive force.

According to our investigation, the filamentation instability involving the quasi-static adiabatic spatial modulations is important at low laser power in hot plasmas. At high laser intensity, we found a rapidly growing filamentation instability associated with the forced-Raman interaction. Accordingly, this interaction can have adverse effects on the stability of powerful laser beams because the filamentation instability may lead to the formation of light pipes which can cause local heating. In conclusion, we mention that the filamentation instability should play a crucial role on the nonlinear propagation of strong laser beams in plasmas. Unless a way to cure this instability is found, the ideas of plasma-based beat-wave accelerators and the concept of inertial fusion may remain obscure.

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FIGURE CAPTIONS

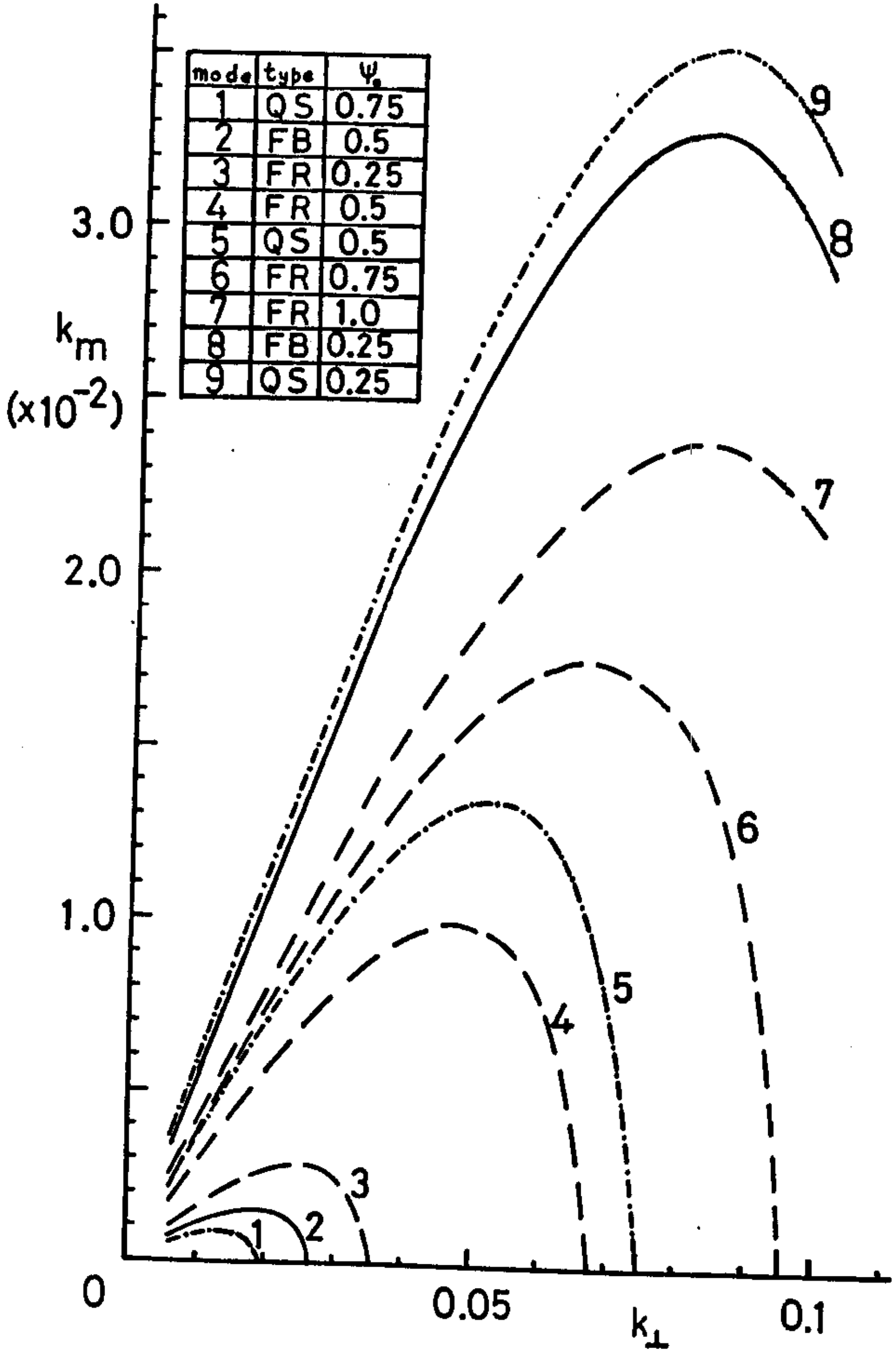
Fig. 1 k_m vs $k_\perp$ for different types of plasma slow responses with β =50, k_o =0.1, σ =1, and Ψ_o =0.25, 0.5, 0.75, and 1.0. FR indicates forced-Raman, FB forced-Brillouin, and QS quasi-static. In the table, each mode is labelled according to the type of interaction and the corresponding Ψ_o .

Fig. 2 k_m vs laser amplitude Ψ_o for the three plasma slow responses, with $k_\perp$ =0.04, β =50, and σ =1. The parameter labelling the curves indicates the type of interaction, as in Fig. 1.

Fig. 3 Plot of nonlinear wavenumber shift δ against Ψ_o , with all other parameters fixed as in Fig. 1. The curves are labelled as in Fig. 2.

Fig. 4 k_m vs β for three types of plasma slow responses, with $k_\perp$ =0.04, σ =1, and Ψ_o =0.5. The curves are labelled as in Fig. 2.

FIGURE 1



mode	type	ψ_o
1	QS	0.75
2	FB	0.5
3	FR	0.25
4	FR	0.5
5	QS	0.5
6	FR	0.75
7	FR	1.0
8	FB	0.25
9	QS	0.25

FIGURE 2

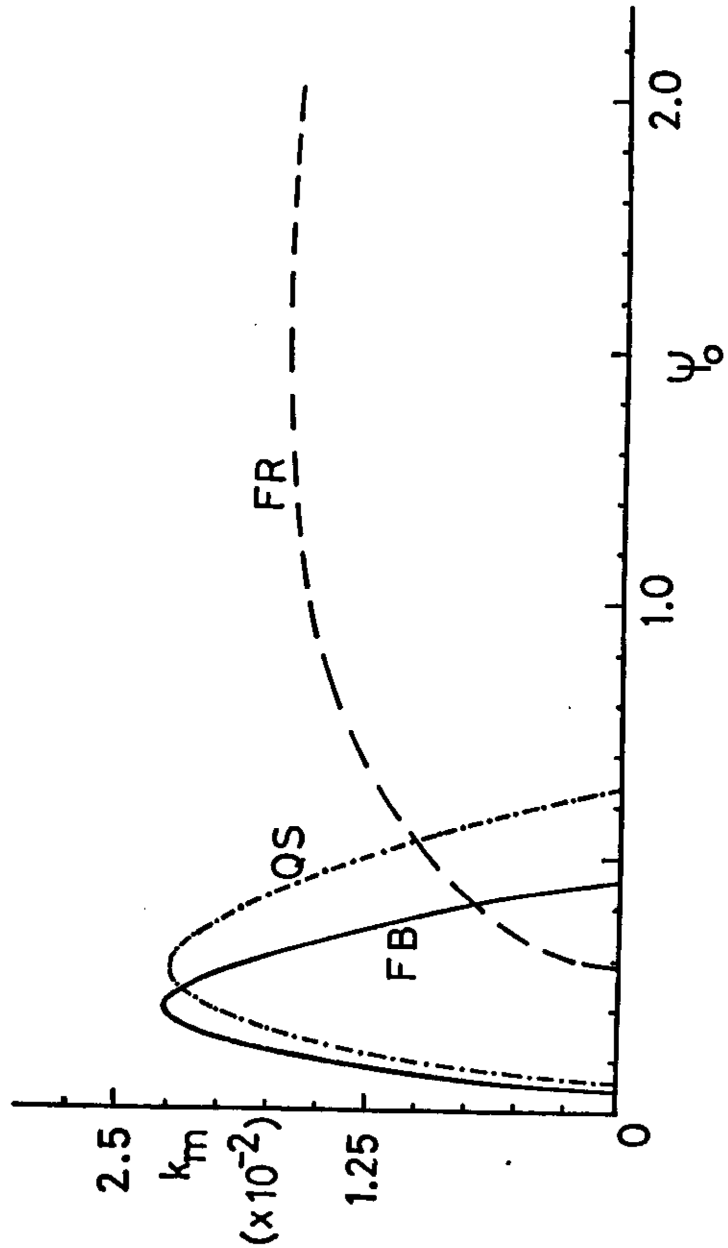


FIGURE 3

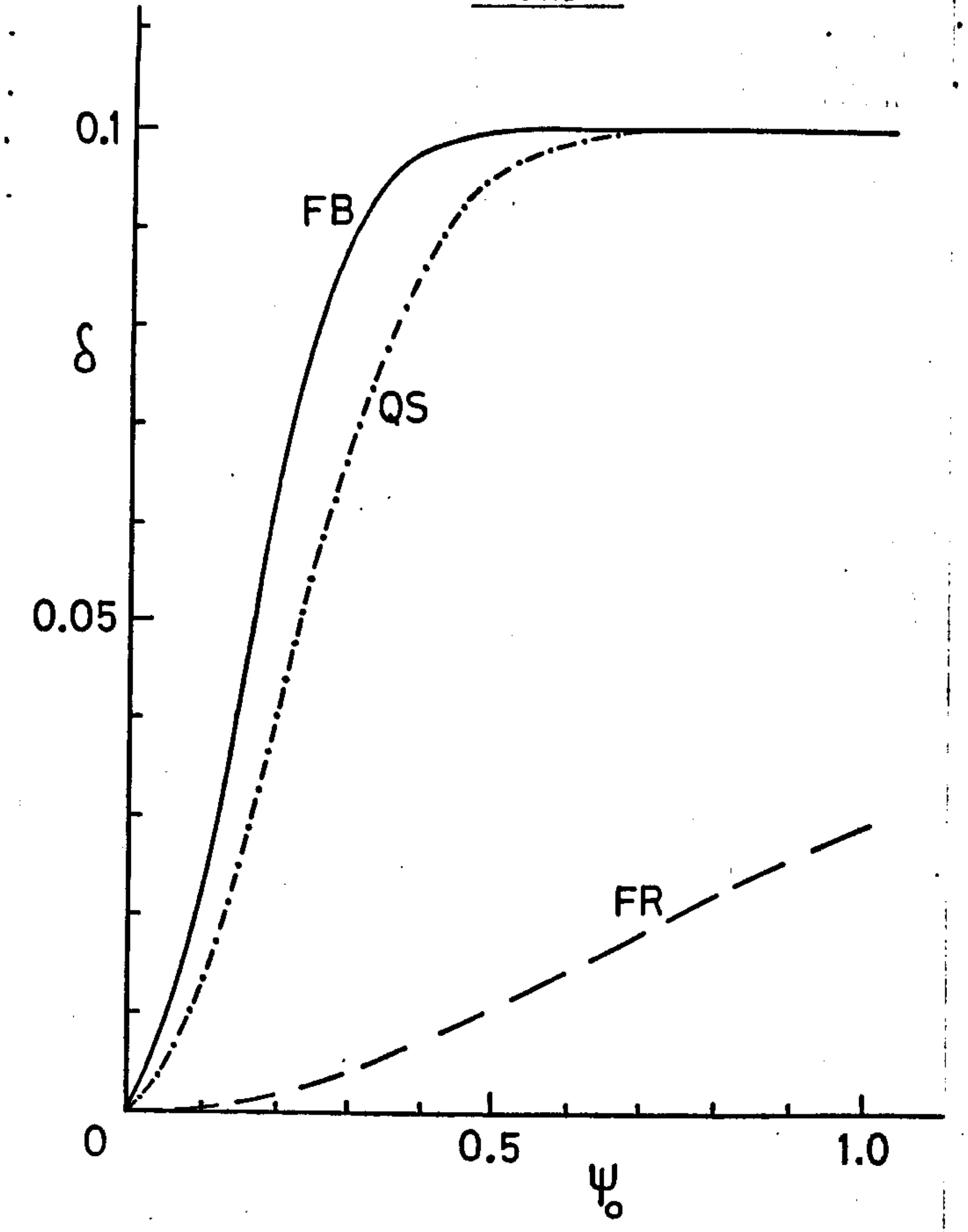
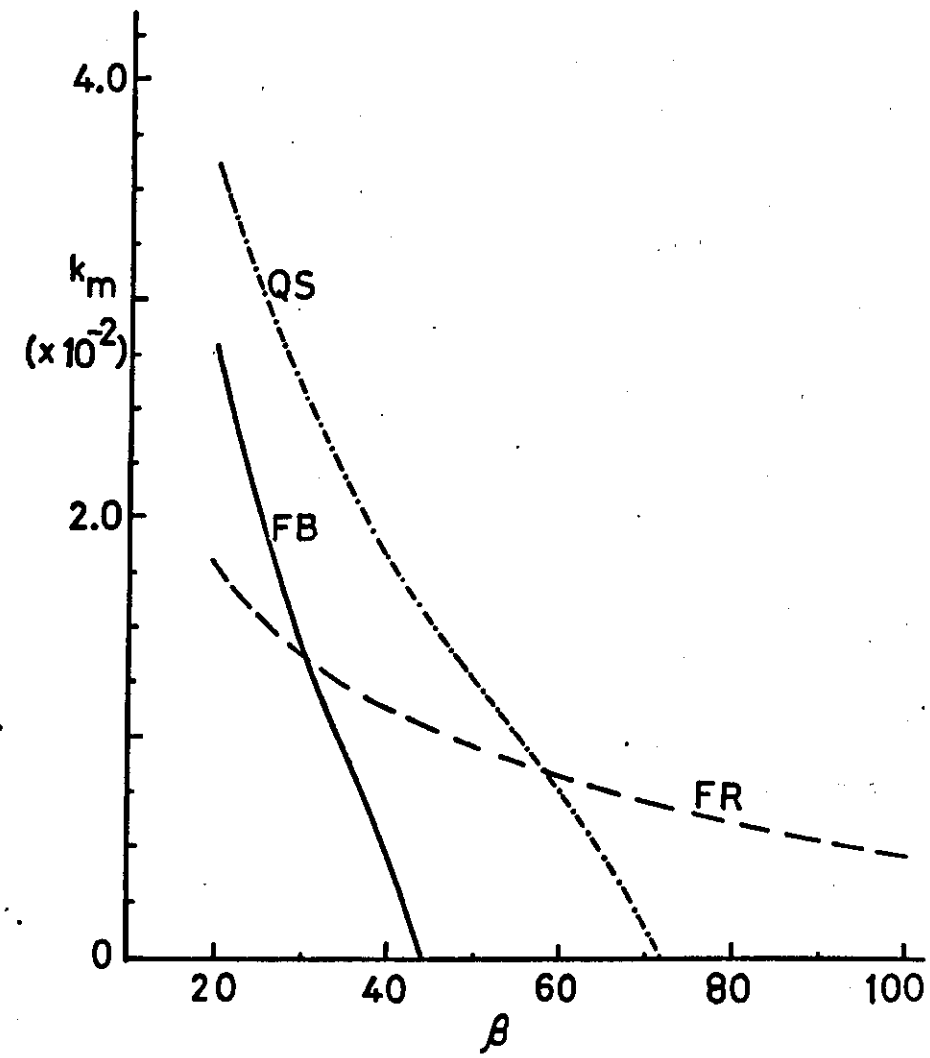


FIGURE 4



PART B

Modulational instability of strong electromagnetic waves in plasmas

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Modulational instability of an arbitrarily large amplituce electromagnetic wave is investigated taking into account the relativistic electron quiver velocity and the relativistic ponderomotive force. The interplay between three kinds of plasma slow responses to the electromagnetic wave is examined. The associated temporal growth rates are graphically exhibited. Relevance of our results to laser driven inertial fusion as well as the plasma-based beat-wave accelerator is pointed out.

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The inertial fusion[1] and the plasma-based beat-wave accelerator[2] concepts rely on the usage of powerful laser beams in plasmas. It is wellknown[3] that when the intensity of CO_2 (Nd glass)lasers exceeds 10^{16} W/cm^2 (10^{17} W/cm^2), the quiver velocity of the electrons approach the speed of light. In such a situation, consideration of the relativistic electron mass variation[4] as well as the relativistic ponderomotive force[5,6] is very essential in the study of nonlinear laser-plasma interactions.

In the past, a number of authors[6-10] have investigated the combined effects of the relativistic electron mass variation and the relativistic ponderomotive force on the formation of one-dimensional envelope solitons. It turns out that the inclusion of the former can give rise to super-sonic envelope solitons consisting of large scale electron density variations and envelopes of localized radiation pulses. Since it is widely thought that the envelope solitons could represent a quasi-stationary state of the modulational instability[11], it is of practical interest to investigate the modulational instability of large amplitude electromagnetic waves.In this paper, we have achieved this by taking into account various kinds of plasma slow responses[8-10] along with the nonlinear current density that arises from the beating of the relativistic electron quiver velocity with the large scale slow electron number density variations caused by the relativistic ponderomotive force.

Consider the nonlinear interaction of the background plasma with a large amplitude circularly polarized electromagnetic wave of the form

$$\vec{\Psi} = \frac{1}{2}\Psi(t,z)\,(\hat{x} + i\hat{y})\,\exp(ik_0 z - i\,\omega_0 t) + c.c., \quad (1)$$

where $\Psi = eA/m_0c^2$, e is the electron charge, m_0 is the rest mass of the electron, c is the speed of light, $\vec{A}$ is the perpendicular component of the vector potential, ω_0 and k_0 are the frequency and the wavevector of the radiation, respectively, and c.c. stands for the complex conjugate.

It is wellknown[6,8-10] that the interaction of the plasma slow motion with the radiation can lead to a slowly varying envelope of waves which is governed by

$$i\epsilon\,\partial_t\psi + i\alpha\,\partial_z\psi + \Delta\psi + \beta\partial_z^2\psi = N_e\psi(1+|\psi|^2)^{-1/2}, \quad (2)$$

where $\partial_t \ll \omega_0$, $N = n_e/n_0$, n_0 is the unperturbed plasma number density, n_e is the electron number density variation created by the radiation pressure, $\epsilon = 2\,\omega_0/\omega_p$, $\alpha = 2k_0c^2/\omega_p^2\lambda_D$, $\Delta = (\omega_0^2 - k_0^2c^2)/\omega_p^2$, and $\beta = c^2/v_t^2$. The electron plasma frequency and the thermal velocity are denoted by $\omega_p = (4\pi n_0 e^2/m_0)^{1/2}$ and $v_t = (T_e/m_0)^{1/2}$, respectively. The space variable is normalized by the electron Debye length $\lambda_D = (T_e/4\pi n_0 e^2)^{1/2}$ and the time by ω_p^{-1}.

We consider three kinds of plasma slow responses and present corresponding expressions for the electron number density variations. We characterize these three responses as the forced-Raman(FR), forced-Brillouin(FB), and quasi-static(QS) interactions. Physically, FR and FB involves[8,10] the balance between the ambipolar and the relativistic ponderomotive potentials, whereas QS assumes an equilibrium between the relativistic radiation pressure, slow electric force, and the electron pressure gradient force[9]. For FR interaction, the immobile ions form a neutralizing background and the corresponding electron number density variation is given by[8]

$$N_e^R = 1 + \beta\partial_z^2(1+|\psi|^2)^{1/2}. \quad (3)$$

On the other hand, for FB interaction, ion dynamics enter. Assuming electrostatically confined ions, we can approximate the ion number density variation by a Boltzmann distribution. The electron number density variation for this case is given by[10]

$$N_e^B = \exp\{\beta_b - \beta_b(1+|\psi|^2)^{1/2}\} + \beta\partial_z^2(1+|\psi|^2)^{1/2}, \quad (4)$$

where $\beta_b = \sigma\beta$ and $\sigma = T_e/T_i$ is the ratio of the electron to the ion tempertaure. Finally, we note that for the QS interaction, the slow ions are electrostatically confined and the electron number density variation follows a modified (by the relativistic ponderomotive potentia) Boltzmann distribution[9]

$$N_e^s = \exp\{\beta_0 - \beta_0(1+|\psi|^2)^{1/2}\} \quad , \qquad (5)$$

where quasi-neutrality has been assumed for the QS interaction. We have denoted $\beta_0 = \sigma/(1+\sigma)$.

Equations (2) to (5) constitute a set of coupled nonlinear equations for the study of the modulational instability of a large amplitude electromagnetic pump wave. Accordingly, we let

$$\psi = (\psi_0 + \delta\psi)\exp(-i\delta t) \quad , \qquad (6)$$

where ψ_0 is real and denotes the amplitude of the pump, $\delta\psi$ ($\ll \psi_0$) is the amplitude of the perturbation, and δ is a nonlinear frequency shift caused by the nonlinear interaction. For the FR interaction, we present a detailed instability analysis. Inserting Eqs. (3) and (6) into Eq. (2), we find the nonlinear frequency shift

$$\delta_R = \epsilon^{-1}\{(1+\psi_0^2)^{-1/2} - \Delta\} \quad , \qquad (7)$$

and an evolution equation for the perturbation

$$i(\epsilon\partial_t + \alpha\partial_z)\delta\psi + \beta\partial_z^2\delta\psi + \frac{\psi_0^2\,\delta\psi_0}{2(1+\psi_0^2)^{3/2}} = \frac{\beta\psi_0^2}{2(1+\psi_0^2)}\partial_z^2\delta\psi_0 \, , \qquad (8)$$

where $\delta\psi_0 = \delta\psi + \delta\psi^*$, and the asterix denotes the complex

conjugate. Substituting $\delta\psi = X + iY$ into Eq. (8), separating real and imaginary parts, we have

$$-(\epsilon\partial_t + \alpha\partial_z)Y + \beta\partial_z^2 X + \frac{\psi_0^2 X}{(1+\psi_0^2)^{3/2}} = \frac{\beta\psi_0^2}{(1+\psi_0^2)}\partial_z^2 X \quad , \qquad (9)$$

and

$$(\epsilon\partial_t + \alpha\partial_z)X + \beta\partial_z^2 Y = 0 \, . \qquad (10)$$

Letting

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \tilde{X} \\ \tilde{Y} \end{pmatrix}\exp(iKz - i\Omega t) \quad , \qquad (11)$$

we can Fourier analyze Eqs. (9) and (10), and combine them to obtain the nonlinear dispersion relation valid for an arbitrary large amplitude pump. The result is

$$(\Omega - \alpha K/\epsilon)^2 = (\beta K^2/\epsilon^2[1+\psi_0^2])\{\beta K^2 - \psi_0^2(1+\psi_0^2)^{-1/2}\} \, , \qquad (12)$$

where Ω and K are the frequency and the wavenumber associated with the plasma slow motion. For the modulational instability, we set $\Omega = \alpha K/\epsilon + i\gamma$ in Eq. (12) and obtain the growth rate

$$\gamma_R = \frac{\beta^{1/2}K}{\epsilon(1+\psi_0^2)^{1/2}}\{\psi_0^2(1+\psi_0^2)^{-1/2} - \beta K^2\}^{1/2} \, . \qquad (13)$$

Threshold is given by

$$\psi_0^2(1+\psi_0^2)^{-1/2} = \beta K^2 . \tag{14}$$

The above analysis can be repeated to investigate the modulational instability involving the forced-Brillouin and the quasi-static interactions. For the FB interaction, the nonlinear frequency shift, the growth rate, and threshold are, respectively, given by

$$\delta_B = \epsilon^{-1}\left[(1+\psi_0^2)^{-1/2}\exp\{\beta_b-\beta_b(1+\psi_0^2)^{1/2}\}-\Delta\right] , \tag{15}$$

$$\gamma_B = \frac{\beta^{1/2}K}{\epsilon(1+\psi_0^2)^{1/2}}\left\{\psi_0^2[\beta_b+(1+\psi_0^2)^{-1/2}]\exp\{\beta_b-\beta_b(1+\psi_0^2)^{1/2}\}-\beta K^2\right\}^{1/2} , \tag{16}$$

and

$$\psi_0^2[\beta_b+(1+\psi_0^2)^{-1/2}]\exp\{\beta_b-\beta_b(1+\psi_0^2)^{1/2}\} = \beta K^2 . \tag{17}$$

On the other hand, for the QS interaction, we have

$$\delta_s = \epsilon^{-1}\left[(1+\psi_0^2)^{-1/2}\exp\{\beta_s-\beta_s(1+\psi_0^2)^{1/2}\}-\Delta\right] , \tag{18}$$

$$\gamma_s = \frac{\beta^{1/2}K}{\epsilon}\left\{\frac{\psi_0^2}{(1+\psi_0^2)}[\beta_s+(1+\psi_0^2)^{-1/2}]\exp\{\beta_s-\beta_s(1+\psi_0^2)^{1/2}\}-\beta K^2\right\} , \tag{19}$$

and at threshold

$$\psi_0^2(1+\psi_0^2)^{-1}[\beta_s+(1+\psi_0^2)^{-1/2}]\exp\{\beta_s-\beta_s(1+\psi_0^2)^{1/2}\} = \beta K^2 . \tag{20}$$

We have carried out the numerical analyses of the growth rates. Figure 1 shows the variation of the growth rate γ for the three types of plasma slow responses with K, the wavenumber of the modulation, for different values of the amplitude ψ_0 of the pump wave. We see that for FB and QS both the maximum growth rate and the range of K-values over which an instability occurs decrease with increasing ψ_0. For $\psi_0 = 0.75$, FB is totally damped and for $\psi_0 = 1$ both FB and QS interactions do not cause the modulational instability. On the other hand, FR displays an opposite behavior. The plot of the nonlinear frequency shift δ against ψ_0 is displayed in Fig. 2 for the three types of interactions. We note that for FB and QS, the nonlinear frequency shift attains a value $-\Delta/\epsilon$ for ψ_0 in the interval $0.5 < \psi_0 < 0.7$. The variation of γ with β is shown in Fig. 3. We observe that for $\beta > 70$ ($v_t \leq 0.12$ c) only FR interaction participates in the modulational instability. For increasing plasma temperature, β decreases, and all the three interactions contribute to the modulational instability, with QS dominating in the range $20 < \beta < 50$.

In summary, we have investigated the modulational instability of a large amplitude electromagnetic wave taking into account the finite amplitude nonlinear electron current density arising from the beating of the relativistic electron quiver velocity with the large scale slow electron density variations created by the relativistic ponderomotive force of the electromagnetic waves. The present results are thus valid far beyond the regime of the small amplitude calculations[12] which accounted for a special class of non-resonant ion-sound modulations. Our results can have relevance to inertial fusion as well as the plasma-based beat-wave accelerator because in both of these programs usage of strong electromagnetic waves are under consideration. The modulational instabilit, as described here, may lend support to the formation of finite amplitude envelope solitons.[6-10] The latter may interact with the background plasma leading to bulk heating or a high-energy electron population. Both of these effects can play an important role in nonlinear laser-plasma interactions.

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Figure Captions

Fig. 1: γ vs K for different types of plasma slow responses with β =50, k_0=0.1, σ =1, ω_0^2/ω_p^2 =1.5 and Ψ_0 =0.25, 0.5, 0.75, and 1.0. FR indicates forced-Raman, FB forced-Brillouin, and QS quasi-static. In the table, each mode is labelled according to the type of interaction and the corresponding Ψ_0 .

Fig. 2: Plot of the nonlinear frequency shift δ against Ψ_0 , with all other parameters fixed as in Fig. 1. The parameter labelling the curves indicates the type of interaction, as in Fig. 1.

Fig. 3: γ vs β for three types of plasma slow responses, with K=0.04, σ =1, Ψ_0 =0.5. The curves are labelled as in Fig. 2.

FIGURE 1

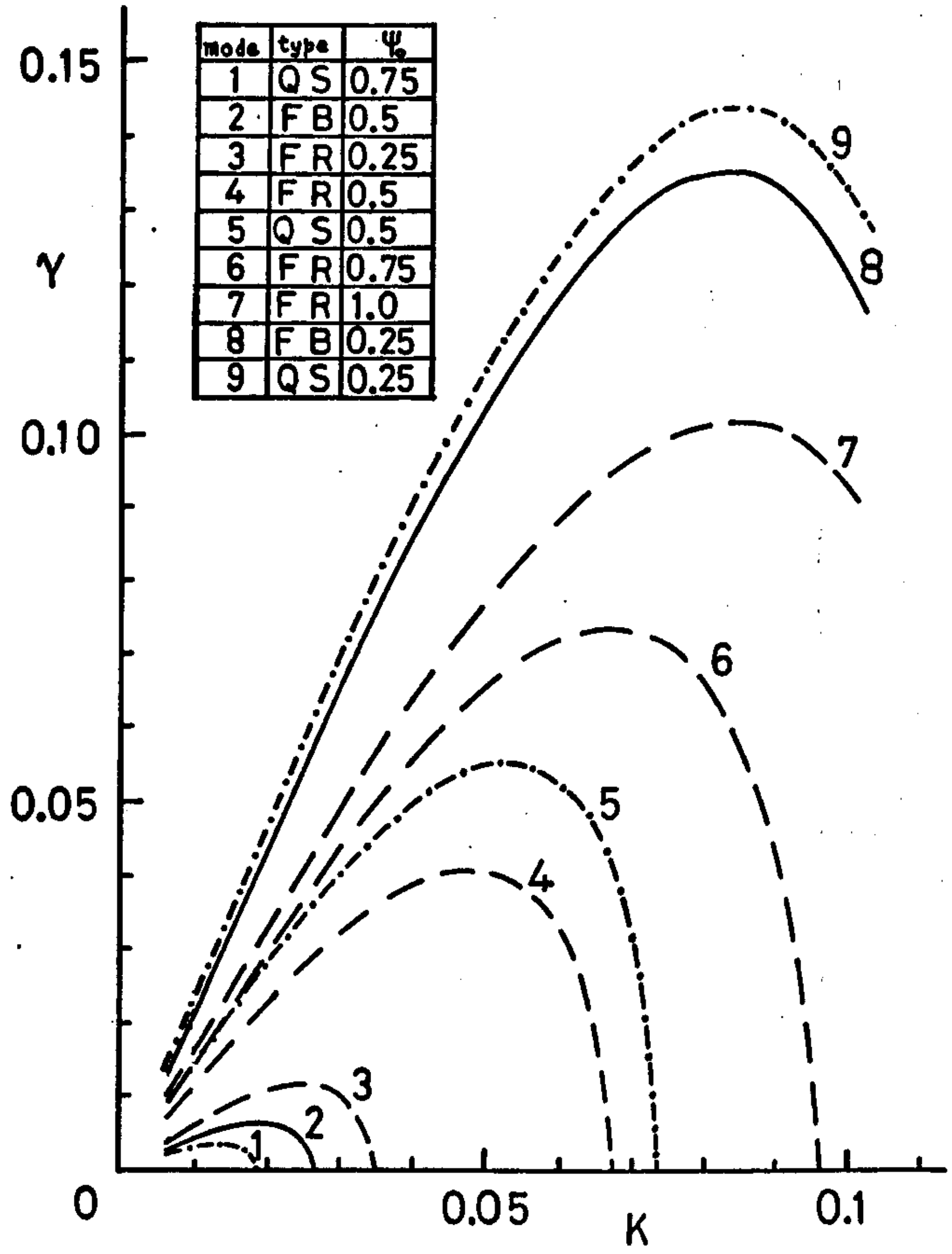

FIGURE 2

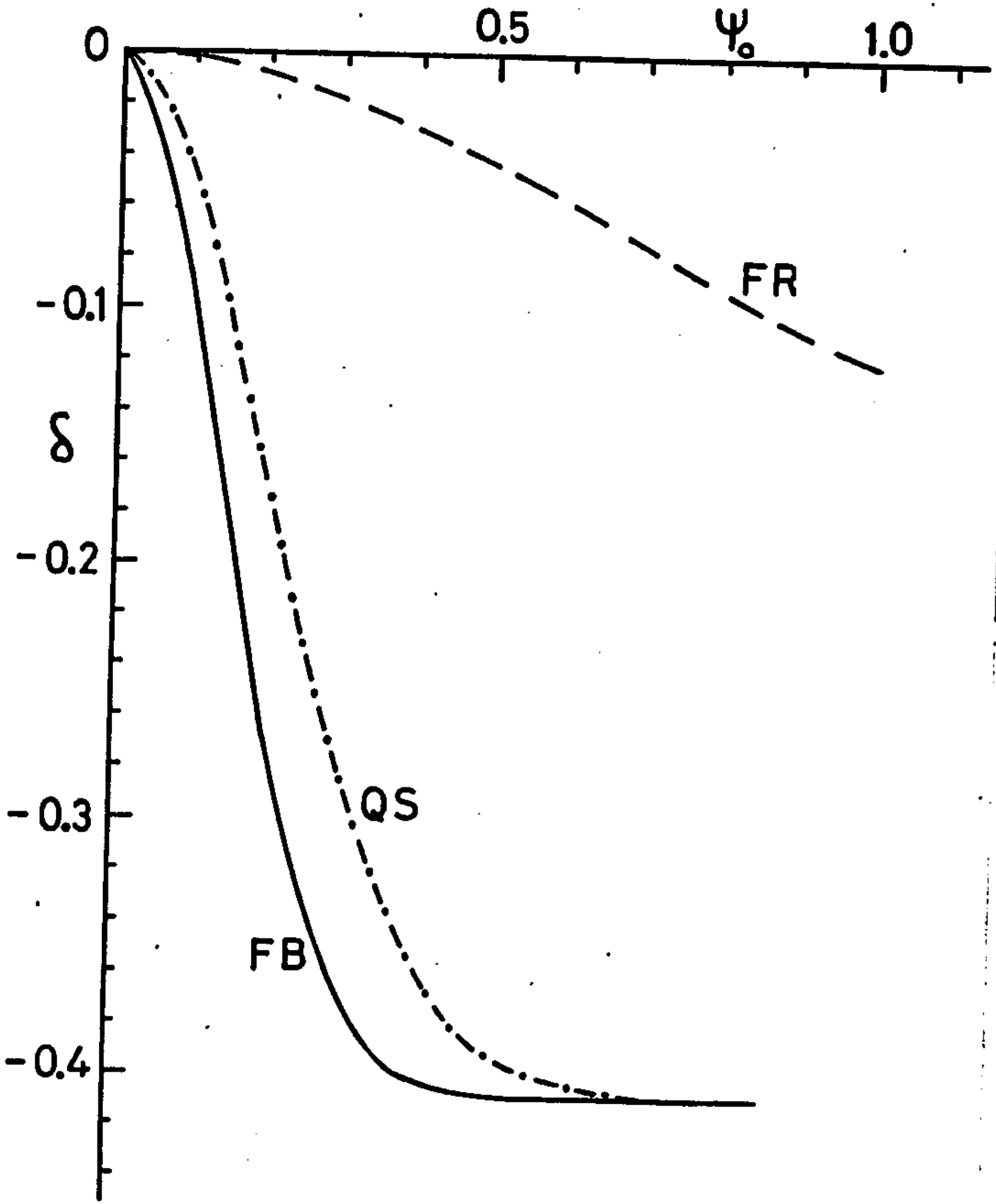

FIGURE 3