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SPRING COLLEGE ON PLASMA PHYSICS

(25 May = 19 June 1987)

POSSIBLE INSTABILITIES IN THE BEAT WAVE ACCELERATOR

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Abstract

In this article the concept of the beat wave accelerator is studied with emphasis put on the plasma physics. An important effect is the relativistic nature of the electrons oscillating in the electric field of the beat wave. Various instabilities are presented which could limit the overall efficiency of the accelerating process.

* This work was carried out as a contribution to the RAL study group on the Beat-Wave Accelerator.

Introduction

The study of generating beat waves in plasmas has been going on for more than ten years, with the first experiments being done in 1971 (Stansfield, Nodwell and Meyer (1971)). Kaufmann et. al., (1972) and Rosenbluth et. al., (1973) considered theoretically the generation of Langmuir waves in Tokomaks by beating two laser beams whose frequency difference matched the plasma frequency. The aim was to heat the plasma with the resultant plasma wave, which decayed by collisional damping or Landau damping. The problem naturally arose in the study of Laser fusion with Stimulated Raman and Brillouin scattering. The production of very high energy electrons was a result. In an article by Lin and Dawson (1974) describing the generation of fast particles in laser plasma experiments the concept of beat wave generation was discussed. Modification experiments in the ionosphere (Wong et. al., 1978) have also us ! the beat wave concept to create large amplitude plasma waves with a full theory being developed for production of ion sound waves by Fried et. al., (1979). Recently the idea for using such a plasma process as an alternative method of high energy acceleration has been proposed by Tajima and Dawson (1979), Joshi et. al., (1981) and Ruth and Chao (1982) and is now commonly called the laser beat wave accelerator (Lawson (1983)),

The beat wave accelerator depends on the generation of a large amplitude plasma mode with a phase velocity close to the velocity of light. The generation of such a plasma mode is possible by beating together two laser beams of frequencies and wavenumbers $(\omega_1, \underline{k}_1)$ and $(\omega_2, \underline{k}_2)$ such that the beat wave has $w = \omega_1 - \omega_2$ and $\underline{k} = \underline{k}_1 - \underline{k}_2$. The process is related to stimulated forward Raman scattering which can be considered as the single pump treatment. The general equations describing the beat wave mechanism and stimulated Raman scattering are therefore the same.

The plasma mode generated by the beat process grows linearly at first whereas it grows exponentially from noise in the Raman process, these two processes will therefore compete with each other if the Raman process is sufficiently fast. The saturation level of the plasma mode will, however, be determined by nonlinear processes which have still to be fully investigated.

Similar nonlinear saturation processes will operate in both the beat wave process and the Raman process. The Raman process, however, can be detrimental to the operation of the beat wave accelerator since in the Raman process different modes can be excited resulting in laser light being scattered out of the interaction region. The beat wave process is intrinsically non-linear because of the large amplitude waves involved. There are nonlinear problems associated with the laser beams as well as the large amplitude plasma wave. A study of the beat wave accelerator will therefore involve the development of nonlinear methods in both analytic and computational treatments.

Model and derivation of the nonlinear Equations

The beat wave accelerator concept using laser beams to generate large amplitude Langmuir waves in a plasma is very similar to the four wave Raman forward scattering in an under-dense plasma. Stimulated Raman scattering is normally considered as a three wave process where the incident transverse (laser beam) decays into another transverse wave and a Langmuir wave. The beat wave accelerator relies on two laser beams beating together in a plasma, producing a beat disturbance at the local plasma frequency. In an under-dense plasma where $\omega_0 >> \omega_{\rm pe}$, ω_0 is the laser frequency and $\omega_{\rm pe}$ is the plasma frequency, the forward Raman scattering becomes important. For phase matching the wavenumber k_{ℓ} of the Langmuir mode is much smaller than the laser wavenumber $k_{\beta} \ll k_{o},$ under these conditions we must consider an up-shifted or anti-Stokes transverse component as well as the down shifted Stokes component, since both can be considered to be resonant with the initial laser wave and the Langmuir wave. The instability then becomes a "four wave" process with the incident laser beam ($_{\omega_0}, \ \underline{k_0}$) decaying into a Stokes wave $(\omega_1, \underline{k}_1)$ and an anti-Stokes wave $(\omega_2, \underline{k}_2)$ together with a density disturbance at $(\omega_{\rho}, \underline{k}_{\rho})$. To describe this effect we will consider the coupling process to conserve momentum exactly and energy only approximately with the relations

$$\frac{\mathbf{k}}{\mathbf{c}} = \frac{\mathbf{k}}{\mathbf{c}}_{1,2} \stackrel{\text{def}}{=} \frac{\mathbf{l}}{\mathbf{l}}, \qquad \frac{\mathbf{\omega}_{0}}{\mathbf{c}} \stackrel{\text{def}}{=} \frac{\mathbf{\omega}_{1,2}}{\mathbf{c}}$$

In writing these relations we assume there is a frequency mismatch in the system, this allows coupling to both the upper and lower sidebands.

The plasma model we use to analyse the problem is the relativistic two fluid equations together with Maxwell's equations and Poisson's equations. The use of a relativistic treatment is necessary when we come to examine the longitudinal beat wave which is driven to very large amplitudes such that the quiver velocity in the longitudinal electric field approaches the velocity of light. We will show later that the relativistic corrections ultimately saturate the growth of the best wave.

Starting from the equations:

$$\frac{\partial \mathbf{a}_j}{\partial \mathbf{t}} + \underline{\nabla} \cdot \mathbf{n}_j \mathbf{v}_j = 0$$
 (1)

$$\left(\frac{\partial}{\partial z} + \underline{\mathbf{v}}_{j}, \underline{\hat{z}}\right) \gamma_{j} \mathbf{v}_{j} + \frac{\mathbf{K} \mathbf{T}_{j}}{\mathbf{n}_{j} \mathbf{m}_{j}} \underline{\nabla} \mathbf{n}_{j} + \mathbf{v}_{j} \underline{\mathbf{v}}_{j} = \frac{\mathbf{q}_{j}}{\mathbf{m}_{j}} (\underline{\mathbf{E}} + \underline{\mathbf{v}}_{j} \mathbf{x} \underline{\mathbf{B}})$$
(2)

$$\underline{\nabla} \cdot \underline{\mathbf{E}} = \frac{1}{\varepsilon_0} \sum_{j} n_j q_j \qquad -(3)$$

$$\underline{\nabla} \times \underline{E} = -\frac{1}{2} \circ \frac{\partial \underline{H}}{\partial t}$$
 -(4)

$$\underline{\nabla} \times \underline{\mathbf{H}} \approx \underline{\mathbf{J}} + \varepsilon_0 \frac{\partial \underline{\mathbf{L}}}{\partial \mathbf{t}}$$
 (5)

where
$$J = \sum_{j} n_{j} c_{j} v_{j}$$
, $\gamma_{j} = (1 - v_{j}^{2}/c^{2})^{-\frac{1}{2}}$

and j = i,e, we obtain the following equations for a plane polarized electromagnetic wave $E_{\rm T}$ and an electrostatic density perturbation δn .

$$\left(\frac{\partial^2}{\partial t^2} + v_2 \right) = \frac{\omega_1 \lambda_e}{\omega_T^2} - \frac{1}{2t} + \omega_{pe}^2 - c^2 \underline{v}^2 \right) \underline{E}_T = \frac{-en}{\varepsilon_0} \underline{e} \underline{v} \cdot (\underline{v}_e, \underline{v}_e) + \frac{e}{\varepsilon_0} - \frac{\partial}{\partial t} (n_e v_e)$$
(6)

$$\left(\frac{\partial^2}{\partial t^2} + v_{\underline{a}_{\underline{a}_{\underline{a}_{\underline{a}}}}}\right) = \frac{\partial}{\partial t} - \frac{\mu \mathbf{T}_{\underline{e}}}{\underline{\mathbf{w}}_{\underline{e}}} \nabla^2 + \omega_{p\underline{e}}^2\right) \delta n = \frac{3}{2} \omega_{p\underline{e}}^2 - \frac{\mathbf{v}_{\underline{e}}^2}{c^2} \delta n - n_{\underline{o}_{\underline{a}_{\underline{a}}}} \left[(\underline{\mathbf{v}}_{\underline{e}}, \nabla) \underline{\mathbf{v}}_{\underline{e}} + \frac{e}{m_{\underline{e}}} (\underline{\mathbf{v}}_{\underline{e}}, \underline{\mathbf{w}}) \right] - (7)$$

where 4 is the electron ion collision frequency. The left hand side of equation (7) contains the relativistic mass correction term $\frac{3}{2}\omega_{pe}^2 \frac{ve}{c^2} \delta n$ which results in a frequency shift and the pondermotive force due to the high frequency transverse field and also the nonlinear coupling to the low frequency ion sound modes which can give rise to the Langmuir modulational instability (Bingham and Lashmore-Davies (1979)). The relativistic correction term is important whenever the quiver velocity in the longitudinal field approaches c, the quiver velocity in the transverse fields is always much less than c for the case considered. Beat Wave Generation

The equation describing the generation of a longitudinal beat wave produced by two high frequency transverse waves is obtained from equation (7). Assuming that the wave fields (electromagnetic and electrostatic) are given by products of a slowly varying amplitude, (on the time scale of the pump frequency) times the plane wave determined by the linear dispersion relation. The transverse waves E are represented by:

$$\underline{E}_{T} = \operatorname{Re} \{ \boldsymbol{\mathcal{E}}_{j} (\underline{x}, \iota) \exp i(\underline{k}_{j}, \underline{x} - \boldsymbol{\omega}_{j} \iota) \} ; \quad j = 0, 1, 2,$$

and the density perturbation δn by: $\delta n = \text{Re} \left\{ N(\underline{x}, t) \text{ exp } i(\underline{k}_{\ell}, \underline{x} - \omega_{\ell} t) \right\}$

Using the small amplitude approximation and neglecting damping, relativistic effects, pump depletion and other non-linear processes the equation for the longitudinal plasma wave becomes

$$\frac{\partial \mathbf{N}(\mathbf{t})}{\partial \mathbf{t}} = -\mathbf{i} \frac{\mathbf{n}_{o} \mathbf{e}^{2}}{4\mathbf{m}_{e}^{2} \mathbf{e}_{o} \mathbf{m}_{1}} - \frac{\mathbf{k}_{o}}{\mathbf{a}_{e}^{2}} \mathbf{\mathcal{E}}_{o} \mathbf{\mathcal{E}}_{1}$$
(8)

Letting $N/n_{\rm e} \simeq \Lambda(t) e^{1\,\varphi t}$ we find the longitudinal plasma wave grows linearly in time with

$$\Lambda(t) = \Lambda(o) + \frac{e^2 \boldsymbol{\ell}_0 \boldsymbol{\ell}_1}{4m_e^2 \omega_0 \omega_1 c^2} \omega_p e^t$$
 (9)

In this approximation the wave amplitude would grow until A(t) = 1 i.e. when the electron quiver velocity in the longitudinal wave equals c. The amplitude however will saturate well before reaching this level by pump depletion and non-linear effects such as the relativistic correction to the plasma frequency. If we include the relativistic effects the equation for the density perturbation can be written as:

$$\mathbf{i} \frac{\partial \mathbf{N}}{\partial t} + \frac{3}{16} \frac{\omega_{\mathbf{pe}}}{\alpha_{\alpha}^{2}} \|\mathbf{N}\|^{2} \mathbf{N} = \frac{n_{o} e^{2} \mathbf{k}}{4m_{e}^{2} \omega_{o} \omega_{1} \omega_{\mathbf{pe}}} \boldsymbol{\mathcal{E}}_{o} \boldsymbol{\mathcal{E}}_{1}^{*} \mathbf{\boldsymbol{k}}$$
 -(10)

If we include spatial variation as well as temporal variation the equation becomes a driven non-linear Schrödinger equation. From equation (10) the wave amplitude paturates when $\frac{\partial N}{\partial L} = 0$, thus wave growth stops when:

$$\mathbf{A} = \left(\frac{4}{3} \frac{\mathbf{e}^2 \boldsymbol{\ell}_0 \boldsymbol{\ell}_1}{\mathbf{m}_0^2 \mathbf{u}_{-1} \mathbf{e}^2}\right)^{\frac{1}{3}} \sim (11)$$

For the parameters given in the Ruth and Chao design model and also in the first RAL study report Lawson (1983) (see Table 1) the wave is found to saturate well before the wave breaking limit, at a value given by:

$$\frac{eE}{m_e \omega_{re} c} = \frac{\delta n}{n} \simeq 0.15$$
 -(12)

therefore wave breaking as a thermalization process is not a problem in this model. This saturation level for the Langmuir feld sets the upper limit for the effective peak accelerating field to be 1.8GV/m. Although other processes such as the modulational instability and filamentation of the Langmuir wave due to the relativistic effect must be taken into account. These will produce a broadening in frequency and the creation of spikes in space and the formation of solitons.

To describe the modulational instability we write the density disturbance ∂n as the sum of a pump wave and two other components the Stokes and anti-Stokes waves $\partial n = \operatorname{Re} \{ N_o(\mathbf{x},t) \text{ ext } i(\underline{k}_o,\underline{\mathbf{x}}-\omega_o t) + N_{1,2}(\underline{\mathbf{x}},t) \text{ exp } i(\underline{k}_{1,2},\underline{\mathbf{x}}-\omega_{1,2}t) \}$ where N_j($\underline{\mathbf{x}},t$) is determined by the non-linear interaction. Using a perturbation procedure on equation (7) and neglecting pondermotive force effects we obtain the following equations for these waves:

$$\begin{array}{l} \left(\frac{\partial}{\partial t} + \gamma_{\ell}\right) \, N_{0} = i\Gamma \left[\left| N_{0} \right|^{2} N_{0} + \left| N_{1} \right|^{2} N_{0} + \left| N_{2} \right|^{2} N_{0} + 2N_{0} \star N_{1} N_{2} \, e^{i\left(\delta_{1} + \delta_{2}\right)^{T}} \right] \\ \left(\frac{\partial}{\partial t} + \gamma_{\ell}\right) \, N_{1} = i\Gamma \left[\left| N_{0} \right|^{2} N_{1} + \left| N_{1} \right|^{2} N_{1} + \left| N_{2} \right|^{2} N_{1} + N_{0}^{2} N_{2} \star e^{-i\left(\delta_{1} + \delta_{2}\right)^{T}} \right] \\ \left(\frac{\partial}{\partial t} + \gamma_{\ell}\right) \, N_{2} = i\Gamma \left[\left| N_{0} \right|^{2} N_{2} + \left| N_{1} \right|^{2} N_{2} + \left| N_{2} \right|^{2} N_{2} + N_{0}^{2} N_{1} \star e^{-i\left(\delta_{1} + \delta_{2}\right)^{T}} \right] \\ \gamma_{\ell} = v_{e\,i}/2 \, , \\ \text{where } \Gamma = \frac{3}{16} \, \frac{\omega_{pe}}{n^{2}} \, \text{is the coupling coefficient and } \delta_{1,2} = \omega_{0} - \omega_{1,2} \, \text{is the frequency mismatch. A fuller derivation of these equations will be presented in a future publication. Solving these equations for N = a constant and assuming the amplitudes N_{1} e^{i\left(\delta_{1} t\right)} \, \text{and } N_{2}^{\star} \, e^{-i\delta_{2}t} \, \text{vary as exp} \, (-i\omega t) \, \text{results in the following dispersion velation for } N_{0} >> N_{1} N_{2} \end{array}$$

$$(\omega - \delta_1 + i\gamma_{\ell})(\omega + \delta_2 + i\gamma_{\ell}) - (\delta_1 + \delta_2) K = 0$$
 -(14)
here $K = \Gamma [N_{\alpha}]^2$

Solving this equation we obtain the following threshold for instability:

-5-

-4-

$$\kappa = -[\gamma \boldsymbol{\ell}^{2} + \Delta^{2}]/\Delta$$

where $\Delta = \delta_1 + \delta_2$ i.e. we have instability only when $\Delta < 0$. However, from the definitions of δ_1 and δ_2 we find that:

$$\Lambda \simeq \frac{-\left[\mathbf{k}_{MOD}\right]^2 \left[\mathbf{v}_{TV}\right]^2}{\omega_0}$$

and so A is negative definite, $k_{MOD} = k_0^{-\frac{1}{2}} + k_{1,2}^{-1}$ is the modulation wavenumber. The growth rate resulting from (14) can be expressed as:

$$\frac{\gamma_g}{\omega_o} = -\frac{\gamma_e}{\omega_o} + \kappa_s \left(\frac{2K}{\omega_o} - \kappa_s^2\right)^{\frac{1}{2}} - (15)$$

where $\kappa_{s} = \frac{1}{\sqrt{2}} \frac{\kappa_{MOD} v_{\tau c}}{\omega_{c}}$, and the threshold can be expressed as $\frac{3}{16} W \rho / nm_{c} c^{2} + \frac{1}{4} \frac{\gamma_{c}}{\omega}$.

where $W_{l} = \frac{1}{2} e_{0} |\mathbf{E}_{l}|^{2}$. This threshold is much less than amplitude levels reached in most laboratory experiments. The Langmuir modulational instability (Bingham and Lashbore-Davies (1979)) due to pondermotive effects becomes important for $\frac{W_{l}}{W_{l}} = \frac{v_{l}}{v_{l}} e_{l}^{\frac{1}{2}}$ which is a much smaller threshold than the relativistic value. Nowever, the pondermotive effects involve the ions motion and so time scales for this process to occur will be the ion time scales $\frac{v_{l}}{\omega_{pi}}$, where $\frac{w_{p1}}{w_{p1}}$ is ion plasma frequency, which is much longer than the time for the relativistic effects to occur. For short time scales the relativistic term can be the dominant one. Modulational type instabilities indicate the onset of strong Langmuir turbulence with the generation of a broad frequency spectrum and cavity formation. Parametric three wave decay processes are forbidden when the condition $\frac{\lambda}{e^{\lambda}DE^{2}}$ $(m_{e}/m_{1})^{\frac{1}{2}}$ is satisfied (Bingham and Lashmore-Davies (1979)). For the parameters considered in the Ruth and Chao design this condition is satisfied.

Saturation by particle trapping can also contribute to the final level of the Langmuir wave field. Coffey (1971) has shown that the amplitude of the Langmuir wave saturates at a level given by:

$$\frac{\ell}{\sqrt{4 \cos \frac{\pi}{9} \frac{v}{v_{\rm ph}}}} = \left(1 - \frac{1}{3} \left(1 - \frac{8}{3} \left(1 - \frac{8}{3}\right)^{\frac{1}{4}} + 2 \left(1 - \frac{1}{2}\right)^{\frac{1}{4}}\right)$$

where $\beta = \frac{2|\mathbf{v}|^2}{10} \frac{|\mathbf{v}|^2}{|\mathbf{b}|}$. For the Ruth and Chao model the wave saturates at the level

 $\delta n/n \simeq 0.8$, for a plasma temperature of 10 eV, which is much higher than that determined by relativistic effects. In determining the saturation due to particle trapping the plasma temperature must be determined. The plasma temperature can change due to non-linear heating by the large amplitude Langmuir wave, this will be discussed in a later report.

Stimulated Raman Scattering

One of the drawbacks in the beat wave accelerator scheme is the fact that the high powered laser beams can under-go non-linear scattering processes which result in some of the laser energy being scattered out of the interaction zone. Stimulated Raman scattering is one such non-linear process, as mentioned before the beat wave process is a special case of stimulated Raman forward scattering. To describe stimulated Raman scattering, we can use the same set of equations (6) and (7), however, instead of dealing with two pump waves we will consider only one pump and two high frequency scattered waves. When the frequency of the plarma is much less than the laser frequency we need to consider coupling to both upper and lower sidebands, this inherently produces a frequency mismatch in the system with the result that the low frequency density perturbation is a driven response with its frequency being determined by the laser parameters. As before we assume that the waves can be described by a linear phase times a slowly varging amplitude in space and time,

$$E_{\text{TO},1,2} = \Re_{\mathbf{e}} \{ \boldsymbol{\mathscr{E}}_{0,1,2}(\underline{\mathbf{x}},t) \text{ exp } \mathbf{i}(\underline{\mathbf{k}}_{0,1,2},\underline{\mathbf{x}}-\boldsymbol{\omega}_{0}t) \}$$

$$\delta \mathbf{n} = \Re \{ \Re(\underline{\mathbf{x}},t) \text{ exp } \mathbf{i}(\underline{\mathbf{k}}_{\boldsymbol{\ell}},\underline{\mathbf{x}}) \}$$

where E_{T0} represents the pump wave and E_{T1} , E_{T2} are the Stokes and anti-Stokes waves respectively. The density perturbation is assumed to be a driven response, its frequency of oscillation will be determined from the dispersion relation.

Expanding the distribution function for the transverse waves about their linear values we obtain the following reduced equations for the Stokes and ant:-Stokes waves:

$$\left(\frac{\partial}{\partial t} + \underline{v}_{1}, \frac{\partial}{\partial \underline{x}} + Y_{T}\right) \hat{\boldsymbol{\mathcal{E}}}_{1} \quad (\underline{x}, t) = -ic_{T} \hat{\boldsymbol{\mathcal{E}}}_{0} N \star e^{-i\delta_{1}t}$$
(16)

$$\left(\frac{\partial}{\partial \mathbf{t}} + \underline{v}_2, \frac{\partial}{\partial \underline{x}} + \gamma_T\right) \boldsymbol{\ell}_1 \quad (\mathbf{x}, \mathbf{t}) = -\mathbf{i} \mathbf{c}_T \boldsymbol{\ell}_0 \mathbf{N} + \mathbf{e}^{-\mathbf{i} \delta_2 \mathbf{t}} \qquad -(17)$$

where v_1 , v_2 are the group velocities, $c_T = \frac{e^2}{4m_e \epsilon_0 \omega_0}$ is

is the coupling

coefficient and $\delta_{1,2} = \omega_0 - \omega_{1,2}$ is the frequency mismatch with $\hat{c}_1 = k \rho \cdot v_0 - \frac{c^2 k^2}{2\omega_0} \ell$ $\delta_2 = -k \rho \cdot v_0 - c^2 k \rho^2 / 2\omega_0$ and, $v_0 = \frac{c^2 k}{\omega}$ is the group velocity of the rump wave. The equation for the density perturbation is given by:

$$\left(\frac{\partial^2}{\partial t^2} - v_{Te}^2 \nabla^2 + \omega_{pe}^2 + \gamma_{\ell} \frac{\partial}{\partial t}\right) N = -C \left[\boldsymbol{\ell}_0 \boldsymbol{\ell}_1^* e^{-i\delta_1 t} + \boldsymbol{\ell}_0^* \boldsymbol{\ell}_2 e^{i\delta_2 t} \right]$$
(18)

where $C = \frac{n_0 e^2 k_1^2}{2\pi^2 \omega^2}$ is the coupling coefficient and γ_1 represents damping collisional of Candau damping. Introducing the new amplitudes $\alpha_1 = \mathcal{E}_1^* e^{-i\delta_1 t}$ and $\alpha_2 = \mathcal{E}_2 e^{i\delta_2 t}$ and assuming α_1 , α_2 and N vary as exp(iwt), we obtain the following dispersion relation for $\underline{k}_{\ell}^{<<} \underline{k}_0$ from equations (16), (17) and (18).

$$[(\omega - \frac{k}{\ell} \cdot \frac{v}{o} + i\gamma_{T})^{2} - \delta^{2}][\omega^{2} - \omega_{pe}^{2} - v_{Te}^{2} k_{\ell}^{2} + i\omega\gamma_{e}] - \frac{1}{2}\delta^{2}\omega_{pe}^{2} \frac{v_{o}s_{e}}{c^{2}} = 0 \qquad -(19)$$

where v_{osc} is the quiver velocity in the pump field, $\delta = \frac{-c^2 k_{p}^2}{2\omega}$.

This dispersion relation is well known and has been solved for a number of different cases (Nishikawa (1968), Bingham and Lashmore-Davies (1976)). For $\underline{k} \rho^{[1]} \underline{v}_0$ we have forward scatter, with the short wavelength pump wave being modulated by the long wavelength electrostatic wave resulting in the generation of high frequency transverse sidebands and bunching of the plasma particles. This is the single pump analogue of the beat wave process the growth rate and threshold obtained from equation (19) are given by:

$$\gamma = \frac{1}{2\sqrt{2}} \frac{\omega_{p}^{2}}{\omega_{0}} \frac{v_{p}^{2}}{c} -(20)$$

$$\frac{v_{osc}^{2}}{c^{2}} = 8\gamma_{T} \frac{\omega_{p}}{\omega_{p}^{2}} -(21)$$

The plasma wave frequency is given by the real part of ω i.e. Re $_{\star} = \mathbf{k}_{\ell} \left(1 - \frac{\omega^2}{\omega^2}\right)^{\frac{1}{2}} c$

This "four-wave" modulational instability results in the broadening of the laser frequency and generation of broad-band plasma oscillations.

For the parameters used in the Ruth and Chao model $\gamma = 5 \times 10^8 \, \text{sec}^{-1}$, $I_{LASER} \approx 10 \, I_{THRESHOLD}$ where I_{LASER} is the laser intensity.

For $\underline{k}_{\ell} \pm \underline{k}_0$ we have a purely growing instability with a standing longitudinal wave set up whose wavenumber is perpendicular to the laser propagation direction resulting in the break-up of the beam into filaments with a

growth rate $\gamma = \frac{\omega_{pe}}{\sqrt{2}} \frac{v_{osc}}{c}$ and threshold $\frac{v_{osc}^2}{c^2} = \frac{\gamma_T}{2\omega}$. For the parameters given in the Ruth and Chao design $\gamma \approx 2.5 \times 10^{11} \text{sec}^{-1}$, $I_{LASER} \approx 9I_{THRESHOLD}$.

The above processes are "four wave" modulational type instabilities resulting when \underline{k}_{ℓ} is either parallel or perpendicular to \underline{k}_{o} , for intermediate cases where $\underline{k}_{\ell} \cdot \underline{k}_{o} = k_{e} k_{o} \cos \theta$, $\theta \neq 0$, $\pi/2$, the process becomes a three wave resonant side-scatter instability with the following resonance condition for forward side scattering:

$$k_{\ell} = k_{o} \cos\theta - (k_{o}^{2} \cos^{2}\theta - \frac{2\omega_{p}e^{\omega}o}{c^{2}})^{\frac{1}{2}}$$



Thus results in a scattering angle $\theta_s^z \sin^{-1} \left(\frac{2\omega_{\text{pe}}}{\omega_o}\right)^{\frac{1}{2}}$ with a growth rate $\gamma \approx \frac{\omega_{\text{pe}}}{\sqrt{2}} \frac{v_{\text{osc}}}{c}$ and threshold $\frac{v_{\text{osc}}}{c^2} \approx \frac{\gamma_e \gamma_T}{2\omega_{\text{pe}}}$. This corresponds to a scattering angle θ_s^z 5°, growth rate $\gamma \approx 2.5 \times 10^{-9} \text{ sec}^{-1}$ and a threshold level I_{THRESHOLD} \approx ${\rm I}_{\rm LASER}/1000~{\rm fcr}$ the Ruth and Chao model. This instability results in the laser light being scattered out of the interaction column and poses a serious threat to the laser beat-wave accelerator scheme. Other instabilities associated with the laser beams are the filamentation and self-focusing instabilities (Bingham and Lasymore-Davies (1976), Max et al (1974)) these cause the incident plane wave to break up into a number of filaments of higher laser intensity. The laser intensity used in the Ruth and Chao design is 16 times the threshold intensity and the growth length is of the order of 5cm. A summary of the different types of .nstabilities can be found in Table 2. The parameters used to prepare table 2 correspond to those used in the design study (Lawson (1983)). The three wave forward Raman process is the most serious instability since it has the fastest growth rate. For laser pulse lengths greater than about 50 psec the loss of ene-gy from the laser through this process becomes the dominant loss process, with the light being scattered at an angle of 5° out of the main beam. The four wave processes are not quite so serious, however, over a long propagation path frequency broadening due to these processes could become a serious problem, also the possibility of large radial electric fields being set up by the

perpendicularly standing Langmuir wave could disrupt the electron beam. The filamentation process is also seen to be important for propagation lengths greater than 5cm. This process will amplify any spatial non-uniformities o_{\perp} the laser beam.

Conclusion

In this report we have discussed some of the plasma physics problems associated with the beat wave accelerator. We have shown that an important effect is the frequency shift, due to the relativistic mass correction of the Langmuir wave. This relativistic term determines the saturation level of Langmuir wave ($\delta n/n = 0.15$), this limits the effective peak accelerating field E_z to 1.86V/m, it also contributes to frequency broadening and cavity formation (breaking the beam up into smaller wavepackets). The different type of instabilities associated with the large amplitude Langmuir wave and the laser beams have been discussed and are shown to lead to important effects which require further investigation.

Effects such as plasma heating due to either the Langmuir wave or the laser beam will be treated in a future report.

Table l

List of laser and plasma parameters used in Ruth and Chao design and also in the Lawson (1983) report.

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LASER - Neodymium

| WAVELENGTH λ_{0} | = 1.06 µm | |
|--------------------------|---------------------------------------|----|
| FFEQUENCY w | = 1.78 x 10 ¹⁵ Rad/s | ec |
| POWER, P | = 8.5×10^{13} WATTS | |
| PULSE DURATION, T | = 100 psec | |
| SPOT SIZE, O | = 0.09 cm | |
| INTENSITY, I | $= 3.3 \times 10^{15} \text{ W/cm}^2$ | |

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PLAFMA - HYDROGEN
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| DENSITY, n _o | = | $1.6 \times 10^{16} \text{cm}$ |
|--|----|--------------------------------|
| FREQUENCY, w | 4 | 7.2 x 10^{12} Rad/sec |
| EECTION LENGTH, L | = | 5 m |
| ² LASMA IEMP., T _e | ~ | 1-10 eV |
| COLLISICN FREQUENCY, vei | ς, | $1.5 \times 10^{10} sec^{-1}$ |
| _ASER DAMPING, Y _T | - | 5.9 x 10^{7}sec^{-1} |
| INVERSE BREMSSTRAHLUNG, $\tau_{B}^{}$ (ti-e scale) | æ | 16 nsec |
| ABSURPTION LENGTH, L | Ŧ | 5 m |
| QUITER VELCCITY, Mosc | | 4.9 x 10 ⁻² c |
| PEAT ACCELERATING FIELD E | Ŧ | 1.8 GV/m |

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Table 2

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Summary of growth rates and thresholds for the various laser-plasma interactions. The plasma and laser parameters used are given in Table 1, they correspond to those used in the design study (Lawson (1983)).

| | Threshold W/m | Growth rates |
|---|-----------------------------|------------------------------|
| Best-Wave process | - | 4.3 x 10 ⁹ |
| 3-Vave Forward Raman Scattering | 3.3 x 10^{12} | 2.5 x 10^{11} |
| 4-have Forward Raman Scattering $(\underline{k}_{\ell} \frac{1}{2} \underline{k}_{0})$ | 3.6×10^{14} | 5×10^8 |
| 4-Wave Forward Raman Scattering $(\underline{k} \underline{\ell} \perp \underline{k}_0)$ | 3.6 x 10^{14} | 2.5 x 10 ¹¹ |
| Filamentation | 2 x 10 ¹⁴ | 5 cm |