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SPRING COLLEGE ON PLASMA PHYSICS

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A REVIEW OF PLASMA ACCELERATORS

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Abstract

Future progress of elementary particle physics depends on new generations of accelerators in the multi-TeV range. It is generally agreed that leptons (electrons) should be the right projectiles in linear colliders. An efficient way of accelerating electrons is in longitudinal plasma waves. The corresponding relativistic dynamics are rewiewed. High power lasers are well suited for creating such waves thanks to nonlinear interaction processes (beat waves). Gradients over 1 GeV/m can be foreseen. Preliminary results from theory, numerical simulation and experiment, are encouraging. An alternative solution is the wake field due to a relativistic charge traveling through a plasma. Both methods may provide focusing of the accelerated particle beam.

Table of contents

1. Large accelerators in the future.

Since the days of Ampere who was the first to accelerate objects, thanks to the electromagnetic interaction [1], many methods, based on that same force, were invented to impart large energies to microscopic bodies. Thus, the fundamental laws of our Universe are investigated in details: the larger the energy, the deeper and the more accurate are the results. The evolution of accelerators is best seen on the Livingston chart [2] recently updated [3]. It shows that optimal performances of accelerating processes follow an exponential growth (figure 1).

Elementary particle physicists are still demanding higher energies. Gigantic machines are being built. In Europe, at C.E.R.N., the L.E.P. (Large Electron Positron Collider) is under construction. This circular ring whose circumference is 27 km will operate in 1989 at the 2x80 G.e.V. level. In the United States, at S.L.A.C. the S.L.C. (Stanford Linear Collider) is in the testing stage with energy 2x50 G.e.V. while the S.S.C. (Superconducting Super Collider) project is to be approved. However, the location of this 96 km circular proton-antiproton collider (2x20 T.e.V.) is not decided yet.

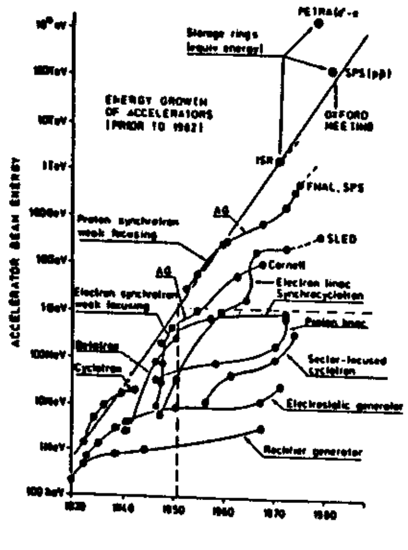

Fig. 1. The Livingston chart after Voss [3]

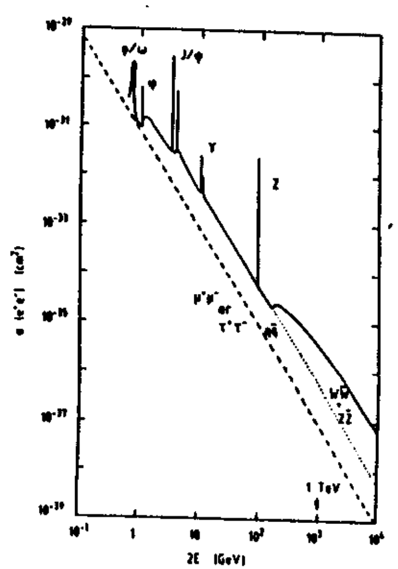

Fig. 2. Cross sections for e^+e^- events after Amaldi [7]

In the next generations of accelerators, energies up to a few tens of T.e.V. are needed. In that range, a number of interesting physical problems are worth being investigated experimentally [4][5]. It is generally agreed that in the future, electron-positron colliders should be built. Indeed, these particles are, so far, elementary: therefore, the whole acceleration energy is available for a given process between elementary objects. On the contrary, protons and antiprotons are built up of three quarks each: consequently about 1/6th of the acceleration energy is left for a fundamental process. Electrons in linear colliders are the best choice in order to avoid losses through synchrotron radiation [6]. Now, no more than about 20 M.e.V./m can be reached using conventional R.F. techniques applied to electron acceleration with the consequence of tremendously large sizes. Much higher accelerating gradients are then mandatory.

Another requirement is of importance. Indeed cross sections between elementary objects such as quarks and leptons vary as the reciprocal energy squared: figure 2 [7]. In order to observe a reasonable number of events per unit time, one has to increase accordingly the luminosity :

$$\mathcal{L} = N^2hf/A \qquad (1\text{-}1)$$

In this definition, N is the number of accelerated particles in a bunch, f the repetition rate of bunches passing through the experimental area, A the beam cross section, and h a reduction factor due to the pinching of encountering bunches of particles with opposite charges. The number of expected events is the product of the cross section times the luminosity. The largest luminosity ever, $10^{32}cm^{-2}s^{-1}$, was achieved at C.E.R.N. with proton-antiproton intersecting storage rings.

The challenge accelerator builders are faced to, can be stated as follows:

accelerate along straight lines

- electrons
- up to energies over 1 T.e.V.
- in a machine the size of which compares with that of present day devices
- with a luminosity $10^{34}cm^{-2}s^{-1}$.

To reach that goal, new concepts were devised in the past few years, some of which make use of plasmas and laser plasma interaction.

The reason for plasmas is the following: in every accelerating device, particles travel " in vacuo ", actually rarefied gas inside pipes whose walls are made of either conducting or dielectric material. Larger gradients imply higher electric fields which may cause electrical breakdown in

the residual gas or along the walls. Thus, the necessary fine tuning of the cavities is destroyed. This is the main limitation to increasing gradients. Inside a plasma, there is no such limitation since matter is already ionised. Furthermore plasmas are able to propagate large electric fields in electron plasma waves.

Now, breakdown thresholds in H.F. electromagnetic systems increase with frequency. Optical frequencies are specially attractive. When focusing a laser beam with a lens or a mirror, high intensities, I, are readily obtained. The corresponding electric fields are given independantly of wavelength by:

$$E = (2/\varepsilon_0)^{1/2}(I/c)^{1/2} \qquad (1\text{-}2)$$

with the result: 1.8 Gigavolt/m for 10^{12} Wcm^{-2} (a 1967 achievement), 1.8 Teravolt/m for 10^{18} Wcm^{-2} (to day's performance with the most advanced high power lasers). Laser plasma interaction provides a way to take advantage of the laser capability for producing high fields.

When a high intensity laser beam is focused onto a solid surface, matter is strongly heated. The temperature is so large that a plasma is formed and set into motion. It was observed long ago in such experiments that energetic charged particles (ions or electrons) are emitted. Their velocities are inconsistent with the measured temperatures in the plasma plume. This phenomenon was never satisfactorily explained. Many mechanisms were proposed for particle acceleration in the laser driven plasma: gasdynamical rarefaction, influence of self generated magnetic fields, electrostatic fields associated with charge separation, wave particle interaction... None of these could be identified as the single origin of the observed results. In 1979, it was shown by Tajima and Dawson [8], with the help of particle simulation that a particular process, the resonant beating of two laser waves whose frequency difference is equal to the plasma frequency, is able to drive a high amplitude longitudinal plasma wave in which electrons can be accelerated to very high energies. This was the starting point for many theoretical, numerical and experimental investigations which show that laser plasma interaction is indeed a promising way to reach the large accelerating gradients required in future devices.

An alternative way of creating high amplitude plasma waves was recently propsed [9]. The passage of ultrarelativistic charged objects (usually bunches of particles) through ionized matter induces electrostatic oscillations which form a wake following the driving charge. Thus, a traveling wakefield is obtained in which secondary particles can be efficiently accelerated up to energies which exceed the individual energies of the particles in the driving bunch.

2. Electron acceleration in a longitudinal plasma wave.

Assume a longitudinal electron wave propagates with a well defined amplitude and phase through a cold plasma in the z direction. Denoting by E_0 the field amplitude, by ω_p the frequency, by k_0 the wavenumber and by ϕ_0 an initial phase, the relativistic equation of motion for an electron is

$$dv/dt = -(e/m_e)(1-v^2/c^2)^{3/2}E_0\sin(k_0 z-\omega_p t+\phi_0), \qquad (2\text{-}1)$$

where m_e is the rest mass. The equation is better rewritten in terms of the Lorentz factor

$$\gamma = 1/(1-v^2/c^2)^{1/2} \qquad (2\text{-}2)$$

and with the similarity variable

$$\xi = k_0 z - \omega_p t \qquad (2\text{-}3)$$

One then gets the differential system

$$d\gamma/dt = -(eE_0/m_e c)(1-1/\gamma^2)^{1/2}\sin(\xi+\phi_0)$$
$$d\xi/dt = ck_0(1-1/\gamma^2)^{1/2} - \omega_p \qquad (2\text{-}4)$$

from which is derived the (ξ,γ) phase space representation displayed in Fig. 3.

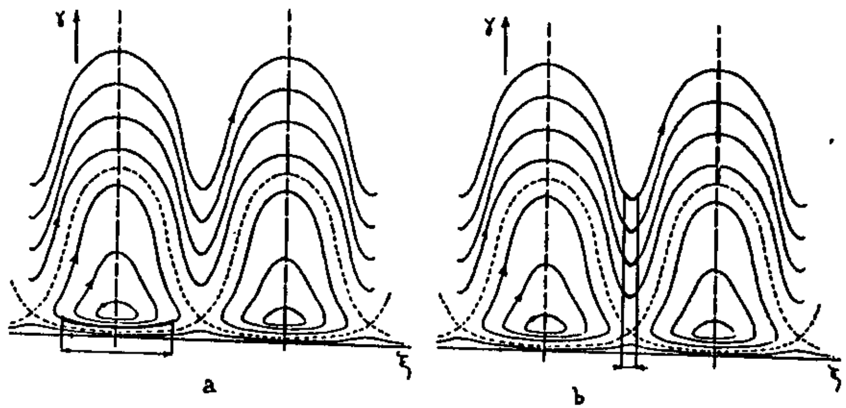

Fig. 3. γ,ξ phase plot. a) acceleration of trapped electrons, b) acceleration of passing electrons.

The figure shows that the following can be accelerated to high energies: i) electrons with trapped trajectories and initial low energy, ii) electrons with sharply defined initial phase on passing trajectories associated with velocities greater than the phase velocity of the wave

$$u_R = \omega_p/k_0. \qquad (2\text{-}5)$$

Then one can imagine an accelerating device with several stages. In the first one, an electron beam with a uniform longitudinal density and velocity u_R is bunched within a quarter period and half wavelength in ξ space (figure 4b). After energy filtering, high energy bunches are injected with proper phasing into successive stages (figure 4c). Note that an initially spatialy uniform monoenergetic electron beam with an energy well above trapping is slightly bunched with a global energy loss: free electron laser regime (figure 4d).

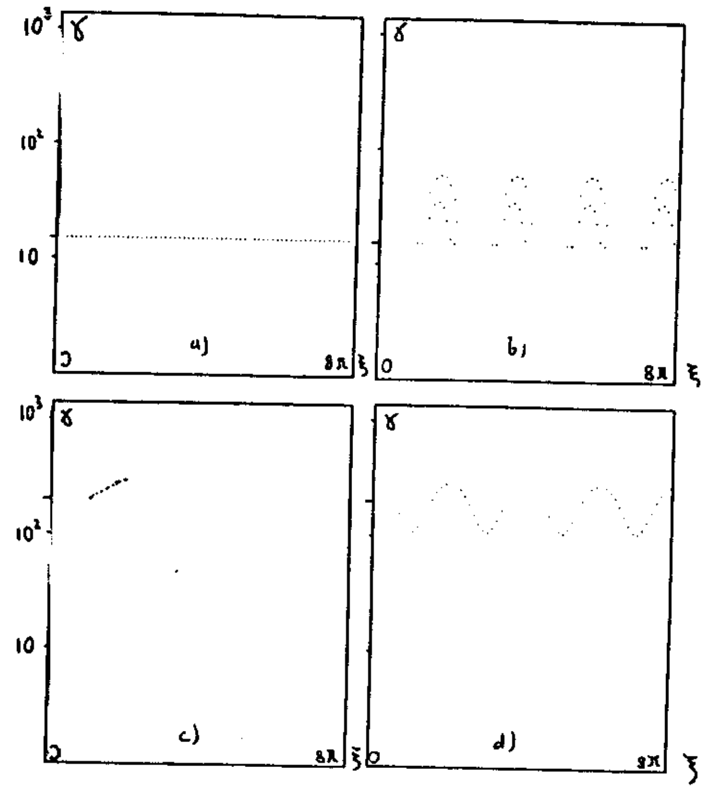

Fig. 4. Electron behaviour in γ,ξ phase space. a) initial state of a completely trapped electron beam; b) bunching of the trapped electrons of fig. a; c) acceleration of a bunch of passing electrons; d) behaviour of a high energy electron beam: slight bunching and global energy loss, free electron laser regime.

3. Acceleration energy and length.

The first order ordinary differential equation in $d\gamma/d\xi$ resulting from (2-1) is readily integrated to give

$$ck_0\gamma - \omega_p(\gamma^2-1)^{1/2} - ck_0\gamma_0 + \omega_p(\gamma_0^2-1)^{1/2} = (eE_0/m_0c)(\cos\xi - \cos\xi_0). \qquad (3\text{-}1)$$

The maximum value of the second factor in the right hand side is 2. It corresponds to the largest $\Delta\gamma$ along a passing trajectory which assuming that both γ_0 and γ are much greater than 1, is

$$\Delta\gamma = \gamma - \gamma_0 \simeq (2eE_0/m_0c\omega_p)\beta_R/(1-\beta_R), \quad \beta_R = (1-1/\gamma_R^2)^{1/2} = \omega_p/ck_0 . \qquad (3\text{-}2)$$

The maximum energy that an electron may acquire in the process is then

$$W_A = m_0c^2\Delta\gamma \simeq (4eE_0/m_0c\omega_p)m_0c^2\gamma_R^2 = 4eE_0\gamma_R^2c/\omega_p. \qquad (3\text{-}3)$$

Now, there exists an absolute maximum for the plasma wave amplitude: density perturbation equal to the background density, the so called wavebreaking condition. An equivalent statement is: all electrons in the plasma are trapped in the wave. In a reference frame moving with the phase velocity (henceforth called " moving frame "), most of the plasma electrons are traveling with a velocity $-\omega_p/ck_0$. They will be trapped provided the potential energy in the wave is at least equal to their energy $\gamma_R m_0 c^2$. Integrating the Poisson equation over half a wavelength and noting that the particle number in a wavelength is a relativistic invariant as well as the longitudinal field, one finds the condition:

$$2eE_0\gamma_R/k_0 = m_0\gamma_Rc^2 \qquad \text{i.e.} \qquad E_0 = m_0c\omega_p/e . \qquad (3\text{-}4)$$

Substituting in (3-3), the upper limit for the electron energy turns out to be:

$$W_{Amax} = 2m_0c^2\gamma_R^2 \quad . \qquad (3\text{-}5)$$

In the moving frame, electrons are accelerated over at most half a wavelength. Since there is no phase locking, electrons in the wave are to be decelerated whenever they propagate beyond that distance. The corresponding acceleration length l_A in the laboratory frame is given by

$$W_A = eE_0l_A \qquad \text{hence} \qquad l_A = 2\gamma_R^2c/\omega_p. \qquad (3\text{-}6)$$

The length of the accelerating plasma column should not exceed l_A and the extension of the bunches along the z axis has to be less than a small fraction (e.g. 1/10th) of the wavelength $2\pi/k_0$. After (3-3) and (3-6) the accelerating gradient is in general

$$W_A/l_A = 2eE_0 = cm_0\omega_p(n/n_0) \qquad (3\text{-}7)$$

where n is the amplitude of the density perturbation. The upper limit is

$$(W_A/l_A)_{max} = m_0c\omega_p. \qquad (3\text{-}8)$$

In all cases, (W_A/l_A) is proportional to the square root of the background plasma density.

The main limitation in the process comes from the acceleration length which results from wave particle dephasing. For given plasma and laser conditions, it puts an upper boundary on the energy a particle may acquire in a single pass. Such an effect can be overcome by superimposing on the plasma wave a uniform magnetic field whose direction is perpendicular to the wave-vector. Thus the particle velocity has a transverse component (Fig. 5). This slightly reduces the longitudinal component which can be kept equal to the phase velocity u_R. An obvious analogy with surfing, inspired the word " surfatron " coined for this process [10]. Let x be the coordinate parallel to the transverse component of the velocity. When phase locking occurs, the accelerated particle velocity tends to

$$v_x \to (c^2-u_R^2)^{1/2}, \quad v_z \to u_R, \qquad (3\text{-}9)$$

while its energy increases indefinitely according to

$$d\gamma/dz \sim (eB/m_ec)\gamma_R \qquad (3\text{-}10)$$

where B is the applied magnetic field. The particle travels at an angle θ with respect to the wave vector.

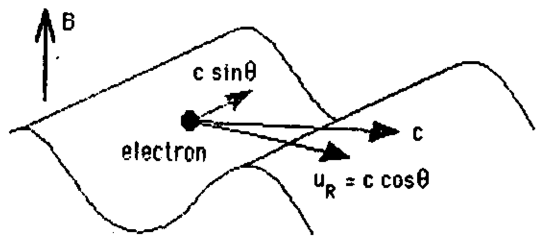

Fig. 5. Particle and wave propagation in the surfatron

4. Growth and saturation of a resonant beat wave.

When two electromagnetic waves act on an electron, the v×B force has longitudinal components with the frequency $\omega_1-\omega_2$. When this difference is equal to the plasma frequency ω_p, an electron plasma wave is resonantly driven. It is then shown [11] that the longitudinal electric field obeys the forced oscillator equation

$$d^2E/d\xi^2 + E = -2(e/m_e)k_pA_1A_2\cos\xi \qquad (4\text{-}1)$$

where A1 and A2 are the vector potentials of the laser waves. The solution of this equation has an amplitude unphysically growing to infinity, with a phase locking at $\pi/2$. The actual amplitude is expected to saturate, thanks to various possible processes: pump depletion, wave-breaking, relativistic oscillatory motion of the electrons in the wave, cascading towards lower frequencies by Raman scattering, collisional or Landau damping... Let Γ be some phenomenological damping coefficient. Accounting for relativistic electron oscillations results in a negative nonlinear cubic term added to the left hand side of (4-1) whereas cascading results in a positive cubic term. Both processes may be included in a single term with a phenomenological coefficient α. Finally one gets a Duffing equation

$$d^2E/d\xi^2 + \Gamma dE/d\xi + (1-\alpha E^2)E = -2(e/m_e)k_pA_1A_2\cos\xi \qquad (4\text{-}2)$$

whose solutions are investigated analytically or numerically [12]. An example is given in Fig. 6: it shows, in the absence of damping, that the amplitude varies periodically on a slow time scale: nonlinear period. There is no phase locking and numerical simulations have evidenced some turbulence creeping in after the first maximum [13]. This leads to the use of short laser pulses whose duration does not exceed the nonlinear period.

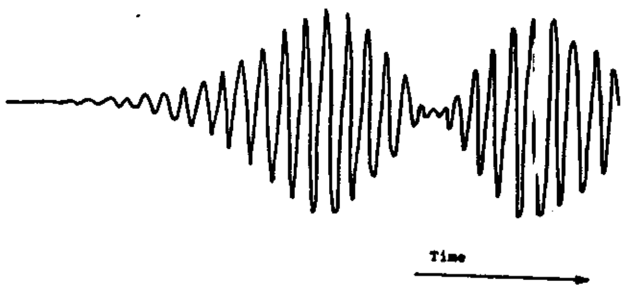

Fig. 6. Longitudinal plasma wave amplitude: growth and saturation by relativistic detuning (cubic term in (4-2)).

However, this conclusion might be modified after remarking that the saturation is due to detuning induced by the growth of the cubic term. The detuning can be compensated by a spatially or temporally variable plasma density, or by a variable pump frequency (" chirped " laser pulses). Provided the compensation is not too large, the maximum amplitude is substantially increased (up to a factor of 3), and the short time constraint is somewhat relaxed: see Fig. 7.

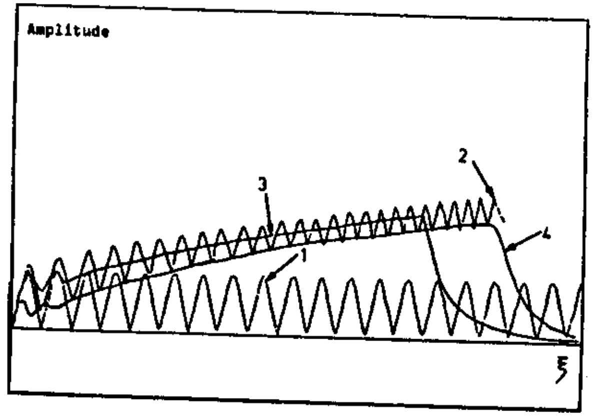

Fig. 7. 1) No damping, fixed plasma density; 2) no damping, linearly increasing plasma density; 3) damping, linearly increasing plasma density; 4) damping, linearly increasing plasma density with initial detuning.

In any case, the amplitude cannot grow beyond the value for which all electrons in the plasma are trapped, i.e. the density perturbation in the wave is equal to the plasma density itself (wave-breaking condition). This sets an absolute limit to the expected energy of the accelerated particles.

5. Acceleration prospects.

The accelerating gradient i.e. the relativistically invariant longitudinal electric field in the wave is simply related to the relative electron density perturbation $\delta n/n$

$$E = c(m_e n_e/\varepsilon_o)^{1/2}\,\delta n/n \qquad (5\text{-}1)$$

where n_e is the unperturbed plasma density. Introducing the interaction parameter

$$\Lambda = (e/m_o c)^2 (E_1/\omega_1)(E_2/\omega_2) \qquad (5\text{-}2)$$

and the Lorentz factor γ_R associated with u_R, one gets for the energy in the laboratory frame that an electron may acquire

$$W_A = 2m_o c^2 \gamma_R^2 (16\Lambda/3)^{1/3}. \qquad (5\text{-}3)$$

Note that the phase velocity u_R is equal to the group velocity of the laser waves with mean frequency ω_a ($=(\omega_1-\omega_2)/2$), and

$$\gamma_R = \omega_a/\omega_p. \qquad (5\text{-}4)$$

From (5-3) and (3-6) an effective accelerating gradient is derived

$$W_A/l_A = m_e c\omega_p (16\Lambda/3)^{1/3}. \qquad (5\text{-}6)$$

The above results are valid provided one has

$$(16\Lambda/3)^{1/3} \leq 1 \qquad (5\text{-}7)$$

the equality sign corresponding to wavebreaking. A quantitative summary is presented in Table 1: the data, laser intensities, acceleration energies and lengths refer to wave-breaking conditions. They represent absolute maxima.

Table 1

MAXIMUM ENERGIES AND ACCELERATION LENGTHS

Laser wavelength (µm)	10	1	0.25	
Intensity (Wcm^{-2})	1.3×10^{15}	1.3×10^{17}	2×10^{18}	
Electron density (cm^{-3})				Gradient (GeV/m)
10^{15}	10 GeV 4 m	1 TeV 400 m	16 TeV 6.4 km	2.5
10^{16}	1 GeV 13 cm	100 GeV 1.3 m	1.6 TeV 210 m	6
10^{17}	100 MeV 4 mm	10 GeV 40 cm	160 GeV 6.4 m	25
10^{18}	10 MeV 0.13 mm	1 GeV 1.3 cm	16 GeV 21 cm	60
10^{19}		100 MeV 0.4 mm	1.6 GeV 6.4 mm	250

High laser frequencies associated with low-density plasmas yield data of interest to high-energy physicists. However, maintaining the required beam intensity over long distances might seem unrealistic, even in the case of self focusing.

On the other hand, low frequency lasers irradiating high density plasmas provide comparatively modest accelerations but over very short lengths. Such conditions correspond to actual experiments on laser plasma interaction being done in many laboratories. Since all numbers shown in Table 1 obey scaling laws, this means that results relevant to particle acceleration can be easily obtained in present day experiments . Foreseeable accelerators are likely to be made of a number of stages built according to the intermediate part of Table 1.

Now, the main limitation in the electron energy comes from the existence of an acceleration length due to phase slippage. The " Surfatron " may be thought about as a way to increase the acceleration length, hence the energy. Table 2 shows a comparison between the surfatron and the ordinary beat-wave. Since the interaction parameter need not be as high, another advantage of the surfatron is the reduction of the required laser intensity.

Table 2

COMPARISON BEAT WAVE/SURFATRON

λ_0 (µm)	10		0.25	
n_0 (cm^{-3})	10^{17}		10^{19}	
$(16\Lambda/3)^{1/3}$	0.2		0.2	
γ_R	10		40	
	Beatwave	Surfatron	Beatwave	Surfatron
γ	40	2×10^3	640	2×10^6
W_A (GeV)	0.02	1	0.32	10^3
l_A (cm)	0.4	20	0.64	1.9×10^3
B (T)		2		5
θ (degrees)		5.7		1.5

This last point deals with another severe limitation of the ordinary beat-wave i.e. the necessity of tremendously high laser intensities over large distances. An alternative scheme was proposed to set up beat waves with a moderate laser intensity [14]. As it is well known in the free electron laser, beating can be obtained between an electromagnetic wave (ω_1, k_1) and a spatially alternating magnetic field (" wiggler " with maximum field B_2, frequency $\omega_2=0$ and wave-number

k_2), in the presence of a relativistic electron beam (driving beam). Doing so inside a plasma may induce the growth of a longitudinal wave with frequency ω_1 and phase velocity

$$u_R = \omega_1/(k_1+k_2). \qquad (5\text{-}8)$$

There exists a divergence for γ_R at the cutoff

$$\omega_p^2 = 2c\omega_1 k_1 - c^2 k_2^2. \qquad (5\text{-}9)$$

The problem of the saturation of the longitudinal wave is the same as before and is dealt with by the same methods. It turns out that when ω_1 is much greater than the cutoff value, the acceleration length is a fraction of the wiggler wavelength. Matching the acceleration length to the wiggler size requires conditions close to cutoff which will be holding in the following. Taking into account the relativistic correction, yields an energy and an acceleration length:

$$W_A = 2m_0c^2[(\omega_p^2/\omega_1^2)\gamma_R^2+\gamma_R](16\Lambda/3)^{1/3}, \qquad l_A = 2\gamma_R^2 c/\omega_1. \qquad (5\text{-}10)$$

The interaction parameter is

$$\Lambda = (E_1/\omega_1)(B_2/k_2) \qquad (5\text{-}11)$$

where B_2/k_2 can easily be made a large number. The comparison with the ordinary beatwave is given in Table 3. The effective gradient is usually smaller. However, as it is clear from the data presented in Table 4, moderate laser intensities are indeed sufficient if one wants to build up a proof-of-principle experiment using state-of-the-art wigglers.

The laser wiggler beat wave in presence of an electron beam is also able to accelerate charged particles directly thanks to the ponderomotive potential. Indeed, in the right hand side of the second order equation of motion, the forcing term can also be considered as an equivalent longitudinal electric field whose amplitude is

$$E_{eq} = 2(e/m_0\gamma_R)k_0A_1A_2 = 2(\omega_p m_0 c/e\gamma_R)\Lambda \qquad (5\text{-}12)$$

and whose periodicity is the same as that of the driven plasma wave if any. If no plasma wave is to be generated, the corresponding accelerating mechanism is the so called inverse free electron laser [15](I.F.E.L., a rather misleading denomination...). The field E_{eq} is proportionnal to Λ whereas, E_0 goes as $\Lambda^{1/3}$. The two are equal for a critical value

$$\Lambda_c = (2/3)^{1/2}(\omega_p\gamma_R/\omega_1+\omega_1/\omega_p)^{3/2}. \qquad (5\text{-}13)$$

It turns out that Λ_c is exceedingly large. For $\Lambda \leq \Lambda_c$, E_0 is larger than E_{eq}. The presence of a plasma

thus greatly enhances the accelerating power of the I.F.E.L.. Since the $\Lambda^{1/3}$ variation results from the same saturation mechanism via relativistic detuning, the 2 laser beat wave scheme also rates better than the inverse free electron laser for electron acceleration at least for the available and predictable laser intensities.

Table 3

2 LASER BEAT WAVE / LASER WIGGLER BEAT WAVE

	2 LASERS	LASER/WIGGLER
Phase velocity u_R	$(\omega_1-\omega_2)/(k_1-k_2)$	$\omega_1/(k_1+k_2)$
Lorentz factor γ_R^2	$(\omega_0/\omega_p)^2$	$(k_1+k_2)^2/[(2k_1+k_2)k_2-\omega_p^2/c^2]$
Interaction parameter Λ	$(e/m_0c)^2(E_1/\omega_1)(E_2/\omega_2)$	$(e/m_0c)^2(E_1/\omega_1)(B_2/k_2)$
Wave-breaking condition	$Z=(16\Lambda/3)^{1/3}=1$	$Z=\gamma_R\omega_1^2/(\gamma_R\omega_p^2+\omega_1^2)$
Energy W_A	$2m_0c^2\gamma_R^2Z$	$2m_0c^2[(\omega_p/\omega_1)^2\gamma_R^2+\gamma_R]$
Acceleration length l_A	$2\gamma_R^2c/\omega_p$	$2\gamma_R^2c/\omega_1$
Gradient W_A/l_A	$m_0c\omega_pZ$	$m_0c\omega_1[(\omega_p/\omega_1)^2+\gamma_R^{-1}]Z$

Table 4

ACCELERATION BY LASER-WIGGLER BEAT WAVE IN A PLASMA

(Wiggler: $\lambda_2 = 10$cm, $l_A = 1$m, $B_2 = .6$T)

LASER TYPE	CO_2	Nd	KrF
Wavelength µm	10	1	0.25
Intensity Wcm^{-2}	5×10^{10}	5×10^{12}	8×10^{13}
Plasma density cm^{-3}	2×10^{15}	2×10^{16}	8×10^{16}
γ_R	540	1.7×10^3	3.4×10^3
W_A GeV	0.6	2.3	7

6. Preliminary experiments.

High (a few M.e.V. L..) energy electrons were observed long ago as a by-product of laser plasma interaction experiments. Most of the time a visible or infrared almost single line high power laser is aimed at a solid surface. It is difficult to identify a given process in such a brute force action: many mechanisms take place at the same time and compete in the energy transfer from the field to the dense, hot and usually inhomogeneous resulting plasma. However it was possible to interpret, thanks to the comparison with numerical simulations, some data on electron energy spectra in terms of laser induced acceleration [16].

Specific beat wave experiments have been devised, starting with CO_2 lasers which can easily emit 2 lines (wavelengths 9.6 and 10.6µm) whose frequency difference matches the plasma frequency at density 10^{17} cm^{-3}. The objectives are: i) evidence the driven plasma wave and evaluate its amplitude; ii) accelerate electrons coming from a source outside the plasma. Early attempts at U.C.L.A. on the first point were made using 2 counterpropagating laser beams along the axis of a θ-pinch [17]. More recently, 2 laser beams propagating in the same direction were used to irradiate transversally a high pressure He arc. Thomson scattering of a ruby laser line is the canonical diagnostic. From the scattered spectrum, a longitudinal electric field

$$0.3 < E_0 < 1 \text{ G.e.V./m}$$

was infered [18]. A different approach to verify electron acceleration by a resonant beat wave was implemented in Canada by F. Martin and coworkers [19]. In this experiment, a two frequency high power CO_2 laser is used in three different ways: i) it produces the plasma by breaking down a supersonic gas jet; ii) it drives the beat wave; iii) it creates an electron emitting auxilliary plasma by impact onto a nearby Aluminum target. A small group of these electrons in a narrow energy band around .64 M.e.V. is selected by a focusing dipole magnet, then sent through the plasma in the same direction as the beating laser beams. Finally the electrons enter an analysing spectrometer (figure 8). Preliminary measurements have evidenced electron energies up to 2 M.e.V indicating acceleration by about 1 G.e.V./m over 1.5 mm. This experiment also shows that a laser created plasma might be a very convenient medium for electron acceleration. Gas breakdown through multiphoton ionization has created fairly uniform plasmas [20]. Other possibilities are foils exploding after laser impact or long (~ 1cm) plasma plumes obtained by focusing a laser beam onto a metal surface with a cylindrical lens [21].

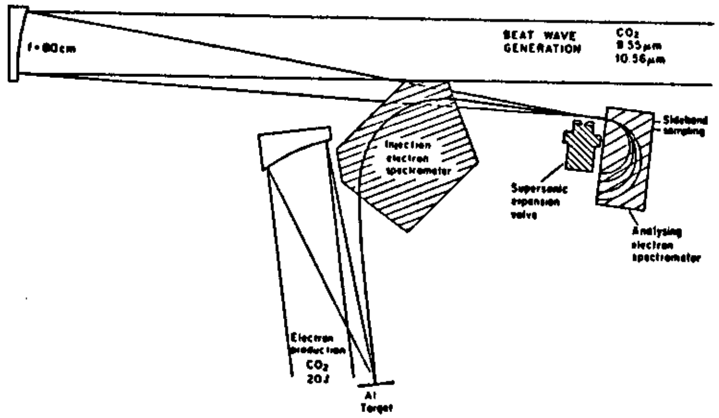

Fig. 8. Experimental set up for electron acceleration after [19].

7. Wakefields.

An electric charge traveling with a relativistic velocity through a plasma induces a wake of electrostatic oscillations. Consider first the case of a point charge q with velocity u_0 close to c. The corresponding charge density is expressed in cylindrical coordinates around the direction of u_0 as:

$$\varrho_q = q(\delta(r)/2\pi r)\delta(z-u_0t). \qquad (7\text{-}1)$$

The equation for electron density oscillations is then

$$\partial^2 n/\partial t^2 + \omega_p^2 n = (en_0/m\varepsilon_0)\varrho_q = (q\omega_p^2/eu_0)(\delta(r)/2\pi r)\delta(z-u_0t) \qquad (7\text{-}2)$$

whose solution is a Green's function

$$n = (q\omega_p/eu_0)\sin\omega_p(t-t')Y(t-t') = n_w\sin\omega_p(t-t')Y(t-t') \qquad (7\text{-}3)$$

where $Y(t-t')$ is Heaviside's step function. Using Poisson's equation, the corresponding electric field is found. Its longitudinal component is ($u_0 \sim c$) [22]

$$E_z(r,z) = -(qk^2/2\pi\varepsilon_0)K_0(kr)Y(t-z/c)\cos\omega_p(t-z/c) - E_{wz}Y(t-z/c)\cos\omega_p(t-z/c) \qquad (7\text{-}4)$$

where $k = \omega_p/c$ and $K_0(kr)$ is a modified Bessel function. The corresponding longitudinal profile is shown on figure 9.

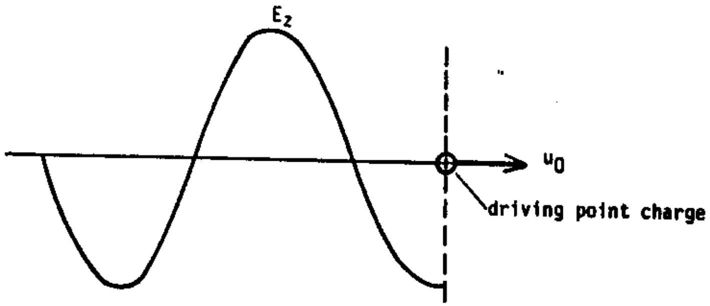

Fig. 9. Longitudinal wakefield due to a point charge.

In the case of an extended driving bunch, $\rho(r,z)$, an integration of E_z has to be performed over the spatial domain occupied by the charges. The driving bunch with length l and maximum particle density n_{bmax} undergoes a retarding electric field whose mean value $\langle E_{rd} \rangle$ is approximated by some coefficient μ times the maximum E_{rd}. An important parameter is the transformer ratio R. In a one dimensional situation there is no on axis divergence of E_{wz} and

$$R = E_{wz}/E_{rd} = \mu k l (n_{bmax}/n_w). \qquad (7\text{-}5)$$

The value of R is a measure of the efficiency of the wakefield as an accelerator.

Now, a particle or a bunch accelerated in a longitudinal plasma wave also induces a wakefield. For an infinitely thin charge in a one dimensional situation

$$E_z = (\sigma/\varepsilon_0) Y(t-z/c) \cos\omega_p (t-z/c) \qquad (7\text{-}6)$$

which to be compared to the field in the wave E_0. It turns out that the two are equal when the number of elementary charges (e.g. electrons) per unit surface to be accelerated is [23]

$$N_{max} = c(\varepsilon_0 m_0/n_0 e^2)^{1/2} n_b \qquad (7\text{-}7)$$

where n_0 is the background density in the plasma and n_b is the amplitude of the density oscillation (both per unit area). When these charges are properly phased with respect to the plasma wave there is no more oscillation behind them. (7-7) indicates the maximum permitted beam loading in plasma wave acceleration.

In general the electric field in the wake has a transverse component which is readily calculated after (7-4), using the Panofsky-Wenzel theorem [24]. Electric field lines form a 2 dimensional pattern which is schematically shown on figure 10.

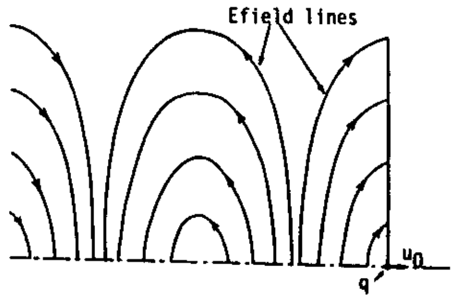

Fig. 10. Electric field line in the 2 dimensional wake of a point charge.

The transverse component of the electric field is thus able to alternatively focus and defocus a bunch of particles to be accelerated. Regions in the wake extending over a quarter wavelength are both accelerating and focusing. This is a very attractive property. This situation is also encountered in the resonant beat wave whenever the laser beams have a suited transverse structure: ideally a cylindrical symetry with an intensity decreasing monotonically from the axis.

So far no experimental results have been obtained on wakefields in a plasma. In an experiment under completion at Argonne National Laboratory in the U.S.A., two electron bunches whose longitudinal spacing can be adjusted, are sent either into a plasma or into a specially designed R.F. cavity which is also able to produce a wakefield [25].

8. Conclusion.

Provided their amplitude is high enough, electron plasma waves are promising for elementary particle acceleration. Theoretical, numerical and experimental work has begun on resonant beat waves and wakefields. This way towards very high energies is taken seriously by particle physicists although significant applications are not expected before long. About 20 years are ahead of us to:

i) on the one hand, investigate in details laser interaction and wake generation, check the scaling laws, reach the required amplitudes, evaluate new concepts such as the " Surfatron " or laser-wiggler beating in a plasma;

ii) on the other hand, develop laser sources suited to the job: short wavelength (≤ 1μm), high power picosecond pulses, efficiency larger than 10%. For state of the art lasers, such requirements are highly contradictory: for instance, CO_2 lasers have a high efficiency, but the wavelength (10μm) is too high; or it is difficult to get high energy short pulses of 0.25 μm radiation from a KrF laser...

However, the use of the focusing properties of transversally inhomogeneous plasma waves is to be applied to actual accelerators in the near future. Plasma lenses migth be the first application of plasmas in the field of high energy physics.

References

1. A.M. Ampère, *Théorie mathématique des phénomènes électrodynamiques uniquement déduite de l'expérience*, Paris (1827).
2. M. Stanley Livingston, *High Energy Accelerators* , Interscience (1954).
3. G.A. Voss, in *The Challenge of Ultra-high Energies* , ECFA-RAL Oxford (1982).
4. E. Eichen, I. Hinchliffe, K. Lane, C. Quigg, *Rev. Mod. Phys*. 56 (1979) 579.
5. U. Amaldi, Workshop on Advanced Accelerator Concepts, Madison August 1986, to appear as A.I.P. Proceedings.
6. B. Richter, in *Laser Acceleration of Particles*, A.I.P. Proceedings n° 130 (1985).
7. U. Amaldi, *Nucl. Inst. Met.* A243 (1986)312.
8. T. Tajima & J.M. Dawson, *Phys. Rev. Lett* . 43 (1979) 267.
9. P. Chen, J.M. Dawson, R.W. Huff, T. Katsouleas, *Phys. Rev. Lett.* 54 (1985) 693.
10. T. Katsouleas & J.M. Dawson, *Phys. Rev. Lett* . 51 (1983) 392.
11. M.N. Rosenbluth & C.S. Liu, *Phys. Rev. Lett* . 29 (1972) 701.
12. J.P. Matte, F. Martin and P. Brodeur, INRS preprint (1986); J.L. Bobin, workshop on Advanced Accelerator Concepts, Madison August 1986, to appear as A.I.P. Proceedings.
13. C. Joshi, W. Mori, T. Katsouleas, J.M. Dawson, J.M. Kindel, D.W. Forslund, *Nature*, 311 (1984) 525.
14. J.L. Bobin, *Optics Comm.* 55 (1985) 413.
15. C. Pellegrini, in *The Challenge of Ultra-high Energies* , ECFA-RAL Oxford (1982).
16. C. Joshi, T. Tajima, J.M. Dawson, H.A. Baldis, N.A. Ebrahim, *Phys. Rev. Lett.* 45 (1981)267.
17. B. Amini, F.F. Chen, *Phys. Rev. Lett.* 53(1984)1441.
18. C.E. Clayton, C. Joshi, C. Darrow and D. Umstadter, *Phys. Rev. Lett.* 54 (1985) 558.
19. F. Martin, P. Brodeur, J.P. Matte, H. Pépin and N. Ebrahim, *S.P.I.E. Proc.* 664 (1986) 20.
20. A.E. Dangor et al. *C.L.I.C. note 29*, C.E.R.N. (1986).
21. F. Amiranoff, C. Labaune, private communication.
22. T. Katsouleas, Phys. Rev. A33 (1986) 2056.
23. T. Katsouleas, S. Wilks, P. Chen, J.M. Dawson, J.J. Su, preprint P.P.G. 952 (1986) submitted to *Particle Accelerators* .
24. W. Panofsky, W. Wenzel, *Rev. Sci. Instrum.* 27 (1956) 967.
25. J.B. Rosenzweig, Workshop on Advanced Accelerator Concepts, Madison August 1986, to appear as A.I.P. Proceedings.