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THEORY OF WAVE RADIATION BY SOLITONS IN PLASMA

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The current interest in the study of solitons is due to their significance for the understanding of wave dynamics in various fields of physics. Solitons - nonlinear solitary waves - arise in various physical processes having one common characteristic: in these processes, wave dynamics is defined by the nonlinearity, on the one hand, and by the dispersion, on the other, - the factors generating two opposite tendencies in the wave process development. The essential importance of solitons has been especially clearly revealed after the discovery of exact methods of the solution of Cauchy problem for a whole class of nonlinear wave equations. It has been established that the arbitrary initial disturbance decays into a set of solitons /1,2/. Apparently, solitons play a fundamental role in the nonlinear wave dynamics analogous to that of plane waves in linear problems.

The modern conception of "soliton" has been introduced by Zabusky and Kruskal, who investigated by computer methods Korteweg--de Vries (KdV) equation simulating the ionacoustic wave propagation in plasma /3/.

Side by side with KdV equation, nonlinear Shrödinger equation (NSE) has been one of the first nonlinear equations describing the nonlinear wave dynamics in plasma with equal participation of nonlinearity and dispersion. The equilibrium of these two factors creates the conditions for the arisal of solitonic structures. It is typical nonlinear dispersion medium, and it is not more chance that the conception of "soliton" arose just at the investigation of the equations describing wave dynamics in plasma.

Eere we present the theory of the process of wave radiation by solitons described by the equations of Zakharov's equations type. First, we investigate the ionacoustic wave radiation at the Langmuir soliton motion in the inhomogeneous plasma and demonstrat the main points of our approach to the problem of low-frequency (LF) wave radiation by the solitons of the envelopes of high-frequency (HF) wave fields.

1. Ionacoustic wave radiation at the Langmuir soliton motion in the inhomogeneous plasma

We investigate the Langmuir soliton dynamics in the inhomogeneous plasma within the frames of Zakharov's equations system modified for the case of inhomogeneous plasma. In the dimensionless form, it can be written as follows:

$$i\frac{\partial E}{\partial t}+\frac{\partial^2 E}{\partial x^2}-\Delta n(x)E-\delta n E=0 \tag{1}$$

$$\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right)\delta n=\frac{\partial^2 |E|^2}{\partial x^2} \tag{2}$$

Here E is slowly varying (in time) complex amplitude of the Langmuir wave, δn - LF variation of plasma density caused by the action of HF pressure force, $\Delta n(x)$ describes the equilibrium plasma density deviation from the uniform distribution. The system of equations (1),(2) has the following motion integrals: N - the number of quanta, H- energy:

$$N=\int |E|^2 dx \tag{3}$$

$$H=\int\left(\left|\frac{\partial E}{\partial x}\right|^2+(\Delta n(x)+\delta n)|E|^2+\frac{u^2+\delta n^2}{2}\right)dx \qquad (4)$$

where u (mass velocity of plasma) is determined from the equation $\frac{\partial u}{\partial x}=-\frac{\partial \delta n}{\partial t}$. Besides, there exist two more motion integrals

$$\int \delta n dx = const \; ; \; \int u dx = const \qquad (5)$$

They are connected with LF plasma motion described by Eqn(2).

In the absence of inhomogeneity ($\Delta n(x)=0$), one more motion integral would exist corresponding to the momentum of "Langmuir plasmons + inacoustis disturbance field" system:

$$P=\int\left(\frac{i}{2}\left(E\frac{\partial E^*}{\partial x}-E^*\frac{\partial E}{\partial x}\right)+u\delta n\right)dx \qquad (6)$$

Plasma inhomogeneity leads the momentum nonconservation, and the following equation may be written for the latter:

$$\frac{\partial P}{\partial t}=-\int |E|^2\,\frac{\partial \Delta n(x)}{\partial x}\,dx \qquad (7)$$

Thus, the inhomogeneity creates the force directed against the gradient of the equilibrium plasma density.

The partial solution to the system of equations (1),(2) in the homogeneous plasma ($\Delta n(x)=0$) is well-known. It is the soliton-selfconsistent localized distribution of HF field and plasma density disturbance:

$$E=\mathcal{E}_m \operatorname{sech}[k_o(x-\bar{x}(t))]\exp(i\phi)$$

$$\delta n=-\frac{|E|^2}{1-V^2} \qquad (8)$$

where $\quad k_o=[2(1-V^2)]^{-\frac{1}{2}}\mathcal{E}_m \; ; \quad V=\dot{\bar{x}}(t)\equiv\frac{d\bar{x}(t)}{dt} \; ;$

$$\frac{\partial\phi}{\partial x}=\frac{V}{2} \; ; \; \frac{\partial\phi}{\partial t}=k_o^2-\frac{V^2}{4} \; ; \; V,\mathcal{E}_m=const \qquad (9)$$

Apparently, the soliton propagates in the homogeneous plasma at a constant subacoustic velocity (in dimensionless variables $V^2<1$). Note also that soliton (8) represents a stable solution of the system (1),(2) within the uniform region where $\Delta n(x)=0$ /4/. We are aimed at the investigation of the soliton dynamics at the transition from the homogeneous region into the inhomogeneous one, where $\Delta n\neq 0$. Thus, the following initial problem is stated: let the solitary wave propagating in the homogeneous plasma as a soliton (8) collide upon an inhomogeneity at a certain moment of time. The subsequent evolution of soliton at its motion into the inhomogeneous region is of interest. According to the stated problem, we seek for the solution of the system (1),(2) in the general form assuming $|E|=\mathcal{E}(x-\bar{x}(t),t)$, where $\bar{x}(t)$ (the coordinate of the HF field localization centre) is, generally speaking, a nonlinear function of time. It denotes that the soliton as a whole moves in a nonuniform manner, and the explicit $|E|$ dependence on t takes into account the possible distortion of the solitary wave "form". In this case, the solution of Eqn(2) may be represented as

$$\delta n(x,t)=-\frac{|E(x-\bar{x}(t),t)|^2}{1-\dot{\bar{x}}^2(t)}+\delta n_s(x,t) \qquad (10)$$

where $\delta n_s(x,t) = \frac{1}{2}\int_{t_0}^{t} dt' \left\{ \frac{\ddot{x}(t')}{(1-\dot{x}(t'))^2} |E(x^-,t')|^2 - \frac{\ddot{x}(t')}{(1+\dot{x}(t'))^2} |E(x^+,t')|^2 \right\} +$

$$+\frac{1}{2}\int_{t_0}^{t} dt' \left\{ \frac{1}{(1-\dot{x}(t'))} \frac{\partial |E(x^-,t')|^2}{\partial t'} + \frac{1}{1+\dot{x}(t')} \frac{\partial |E(x^+,t')|^2}{\partial t'} \right\}, \quad x^{\pm} \equiv x \mp t \pm t' - \bar{x}(t) \tag{11}$$

Eqn(11) is a general solution of wave equation (2). It is obtained under the conditions corresponding to our initial problem, namely, the existance of soliton moving in the homogeneous plasma at the initial moment of time $t = t_0$.

The solution (10), to a certain extent, is of formal character until we find the solution of integro-differential equation obtained instead of the system (1),(2):

$$i\frac{\partial E}{\partial t} + \frac{\partial^2 E}{\partial x^2} - \Delta n(x)E + \frac{|E|^2 E}{1-\dot{x}^2(t)} - \delta n_s E = 0 \tag{12}$$

where δn_s is determined from (11).

In Eqn(10) for $\delta n(x,t)$, three terms of different character may be singled out. The first term corresponds to the density perturbation moving with HF field. Two other terms are united in $\delta n_s(x,t)$. The connection of this part of perturbation δn with the field E is nonlocal. It is evident from Eqn(11) that the arisal of two latter terms in the expression for δn is connected with the soliton nonstationarity,i.e. with its acceleration $\ddot{x}(t)$ and shape change at the motion in the inhomogeneous plasma. Besides, it is mostly important that the part of density perturbation connected with $\delta n_s(x,t)$ is not localized already in the region of HF field concentration. In may detach from HF field and propagate afterwords in the form of ionacoustic waves, i.e. the

soliton radiates LF waves.

Therefore, the term $\delta n_s E$ in Eqn(12) is considered as a reverse response of the radiation to the source, i.e. as the radiation recoil. We assume that such reverse action action of radiation upon the soliton is weak: $|\delta n_s| \ll |E|^2$. Respectively, we seek for the solution of Eqn(12) in the following form:

$$E = E_0 + E_1 + \dots$$
$$|E_0| \gg |E_1| \gg |E_2| \dots \tag{13}$$

Then we obtain the following hierarchy of equations:

$$i\frac{\partial E_0}{\partial t} + \frac{\partial^2 E_0}{\partial x^2} - \Delta n(x)E_0 + \frac{1}{1-V_0^2}|E_0|^2 E_0 = 0 \tag{14}$$

$$i\frac{\partial E_1}{\partial t} + \frac{\partial^2 E_1}{\partial x^2} - \Delta n E_1 + \frac{1}{1-V_0^2}\left(2|E_0|^2 E_1 + E_0^2 E_1^*\right) = -\frac{V^2(t)-V_0^2}{(1-V_0^2)^2}|E_0|^2 E_0 + \delta n_s(E_0)E_0 \tag{15}$$

Here $\delta n_s(E_0)$ implies the expression (11) with the solution of Eqn(14) (acceleratedly moving soliton) substituted instead of E :

$$E_0 = \mathcal{E}_m \,\mathrm{sech}[k_0(x-\bar{x}(t))]\exp(i\phi) \tag{16}$$

where $k_0 = [2(1-V_0^2)]^{-\frac{1}{2}}\mathcal{E}_m$, $V = \dot{\bar{x}}(t) \equiv \frac{d\bar{x}(t)}{dt}$

$$\frac{\partial \phi}{\partial x} = \frac{V}{2}, \quad \frac{\partial \phi}{\partial t} = k_0^2 - \frac{V^2}{4} - \Delta n(\bar{x}(t)) + \bar{x}(t)\left.\frac{\partial \Delta n(x)}{\partial x}\right|_{x=\bar{x}(t)} \tag{17}$$

V is not constant any more, and we obtain it from the "motion equation" /5/ :

$$\frac{dV}{dt} = -2 \left.\frac{\partial \Delta n(x)}{\partial x}\right|_{x=\bar{x}(t)} \qquad (18)$$

This equation may be obtained using Eqn(7) for the momentum, keeping in mind that $(\Delta\ell/L_H) \ll 1$ (where $\Delta\ell$ is the soliton width, and $L_H \sim \left|\frac{d}{dx} \ln \Delta n(x)\right|^{-1}$)

The equation of the second approximation (15) allows us to define the soliton shape change E_1 due to the accelerated soliton motion and to the "scattering" of the arising ionacoustic perturbation on the soliton proper (the term $\delta n_s E_0$ in the right-hand part of Eqn(15)). The field E_1 is concentrated within the region of soliton localization. In fact, we should firstly note that the "source" of the field E_1 is localized in the region of the soliton concentration. Secondly, outside the source localization region, the equation for E_1 becomes Shrödinger equation for the "particle" in the given "field" $\Delta n(x)$. Its solution under natural zero initial conditions (at $t = t_0$: $\delta n_s = 0$, $V(t) = V_0$, $E_1 = 0$) is rather trivial: $E_1(x,t) = 0$ $(|x-\bar{x}(t)| \gg \Delta\ell)$. The plasmon generated within the soliton localization region cannot leave this region and remains locked in the density well.

It is evident from the structure of Eqn(11) for δn_s that the magnitude $|\dot{V}\Delta\ell/(1 \pm V)^2 \ll 1$ represents a small parameter allowing to neglect, in the first approximation, the reverse influence of radiation upon the soliton. It means that during the passage of the distance of the soliton width order $\Delta\ell \sim 1/k_0$ by the ionic sound, the change of the soliton velocity is negligible in comparison with the ionacoustic velocity. This condition reminds

of a well-known condition of the radiation dipolity in the radiation theory. It is clear from Eqn(11), as well as from Eqns(14) and (15) that, to the first nonzero approximation in this small parameter, the radiation is caused by the accelerated soliton motion, and not by its deformation making the contribution of the second order in the small parameter. Therefore, in the so called dipole approximation, we may write the following expression for δn_s proceeding from Eqn(11) and using Eqn(16)

$$\delta n_s(x,t) = \frac{1}{2} \frac{\ddot{\bar{x}}(t-x+\bar{x}(t^+))\Delta\ell \mathcal{E}_m^2}{(1-\dot{\bar{x}}(t-x+\bar{x}(t^+)))^3} \left(th \frac{x-\bar{x}(t)}{\Delta\ell} \; th \frac{x-t+t_0-x_0}{\Delta\ell} \right) +$$

$$+ \frac{1}{2} \frac{\ddot{\bar{x}}(t+x-\bar{x}(t^-))\Delta\ell \mathcal{E}_m^2}{(1+\dot{\bar{x}}(t+x-\bar{x}(t^-)))^3} \left(th \frac{x-\bar{x}(t)}{\Delta\ell} - th \frac{x+t-t_0-x_0}{\Delta\ell} \right) \qquad (19)$$

where $t^\pm$ is determined from the equation

$$x \pm (t - t^\mp) - \bar{x}(t^\mp) = 0$$

Subsequent calculations depend on the specific shape of the plasma inhomogeneity profile $\Delta n(x)$. However, one may conclude from the analysis of the structure of Eqn(19) that $\delta n_s(x,t)$ represents plasma density perturbation which is not concentrated in the soliton localization region, and whose fronts become more and more distant from the soliton centre $\bar{x}(t)$ moving to the left and to the right with the ion-sound velocity. Here, it is appropriate to give the analogy between the charged particle and the soliton, which explains well the physical sense of the investigated phenomenon. A part of the density perturbation, propagating subsequently in the form of ionacoustic waves, breaks away from the accelerated soliton which undergoes profile deformation side by side with acceleration (i.e. soliton radiates), in the same manner as the electromagnetic field created by the accelerating charged particle breakes away from the latter and propagates in the form of radiation.

To estimate the rate of energy and momentum loss due to the ionacoustic wave radiation by the soliton, we apply the equations for the energy and momentum densities:

$$\frac{\partial \mathcal{P}}{\partial t} = -\frac{\partial}{\partial x}\left(2\left|\frac{\partial E}{\partial x}\right|^2 - \frac{1}{2}\frac{\partial^2 |E|^2}{\partial x^2} + |E|^2 \delta n + \frac{\delta n^2}{2} + \frac{u^2}{2}\right) - |E|^2 \frac{\partial \Delta n(x)}{\partial x}$$

$$\frac{\partial \mathcal{H}}{\partial t} = -\frac{\partial}{\partial x}\left(i\left(\frac{\partial E}{\partial x}\frac{\partial^2 E^*}{\partial x^2} - \frac{\partial E^*}{\partial x}\frac{\partial^2 E}{\partial x^2}\right) + i(\Delta n + \delta n)\left(E\frac{\partial E^*}{\partial x} - E^*\frac{\partial E}{\partial x}\right) + (\delta n + |E|^2)u\right)$$

where

$$\mathcal{P} = \frac{i}{2}\left(E\frac{\partial E^*}{\partial x} - E^*\frac{\partial E}{\partial x}\right) + u\delta n$$

$$\mathcal{H} = \left|\frac{\partial E}{\partial x}\right|^2 + (\Delta n + \delta n)|E|^2 + \frac{u^2 + \delta n^2}{2}$$

and perform the integration of these expressions over the "volume" whose boundaries are located in the "wave zone" far from the radiation source, i.e. $|x^{\pm} - \overline{X}(t)| \gg \Delta \ell$, where $X^{\pm}$ are right-hand and left-hand limits of the integration region. Then we obtain the following expressions for the momentum and energy fluxes passing in a unit of time across the planes located in the points $X^{\pm}$ normally to X-axis:

$$\frac{\partial}{\partial t}\int_{x^-}^{x^+} \mathcal{P}\, dx = -\left(\delta n_s^2(x^+,t) - \delta n_s^2(x^-,t)\right) - \int_{x^-}^{x^+} |E_0|^2 \frac{\partial \Delta n}{\partial x}\, dx \qquad (20)$$

$$\frac{\partial}{\partial t}\int_{x^-}^{x^+} \mathcal{H}\, dx = -\left(\delta n_s^2(x^+,t) + \delta n_s^2(x^-,t)\right) \qquad (21)$$

It follows from (20) that the momentum change of "soliton + acoustic field" system occurs both due to the sound radiation and under the action of the inhomogeneity. Under certain conditions, the momentum may remain unchanged at the sound radiation: the difference between two positively defined values in the right-hand part of Eqn(20) points to this fact.

The intensity of energy loss, as it should be expected, according to (21) depends on the squared amplitude of the radiated waves.

Note that Eqns(20),(21) are general; it means that they define the momentum and energy change not only for an isolated soliton radiating ionacoustic waves, but also for other localized sources, for example, groups of interacting solitons or other nonstationary localized formations representing solutions of the system of equations (1),(2).

2. Radiation of ionacoustic waves by solitons at the passage through the "transition" layer

We consider the soliton overcoming the "transition" plasma layer, i.e. region of finite width in which plasma density undergoes a jump, changing its value. Outside the layer plasma is assumed to be homogeneous. It is clear from the above consideration that the soliton should accelerate at the passage of the transition layer and, hence, it should radiate. After overcoming this inhomogeneous layer, the soliton moves with a constant velocity again, and ceases to radiate.

We characterize the transition layer by the parameter h - the differential of the plasma density values on both sides of the layer, and by the parameter K defining the spatial scale of the "jump" region. The transition layer is given by the following model function

$$\Delta n(x) = \frac{h}{2}\left(1 + th\, \kappa x\right) \qquad (22)$$

In this case the equation of the soliton motion (18) acquires the form

$$\ddot{\bar{x}}(t) = -\frac{h\kappa}{ch^2 \kappa \bar{x}(t)} \qquad (23)$$

We assume that at the initial moment of time $t = t_0$ the soliton is located to the left of the transition layer, in the point with the coordinate x_0 $(x_0 < 0,\ \kappa/x_0/ \gg 1)$, and moves to the right velocity V_0. Proceeding with the above-stated analogy between the soliton and a classical particle, we also assume that its "kinetic energy" is much higher than the "potential" barrier height, i.e. $V_0^2/(4|h|) \gg 1$. Then, solving Eqn(23), we may write the expressions for the explicit time dependences of the velocity and the acceleration of the soliton centre:

$$\dot{\bar{x}}(t) = V_0\left(1 - \frac{h}{V_0^2}\right) - \frac{h}{V_0}\, th\left(\kappa V_0 (t - t_0) + \kappa x_0\right) \qquad (24)$$

$$\ddot{\bar{x}}(t) = -\, h\kappa\, \mathrm{sech}^2\left(\kappa V_0 (t - t_0) + \kappa x_0\right) \qquad (25)$$

Eqns(24,25) show that the difference of the soliton velocities before and after overcoming the density jump $\Delta V = 2h/V_0$ and the soliton undergoes the acceleration during the interval of time $\Delta t \simeq 1/\kappa V_0$ equal to the time necessary for the soliton to travel through the transition layer.

Having obtained the explicit expression for the soliton acceleration, we may calculate the ionacoustic wave radiation

using Eqn(1): $\delta n_s(x,t) = \frac{\mathcal{E}_m^2 \kappa h}{2 V_0^3 \kappa_0}\left(G^+(x,t) + G^-(x,t)\right)$

$$G^{\pm}(x,t) = \pm \frac{V_0^3}{(V_0 \mp 1)^3}\, \frac{th\, \kappa_0 (x - \bar{x}(t)) - th\, \kappa_0 (x \mp t \pm t_0 - x_0)}{ch^2 \frac{\kappa V_0}{V_0 \mp 1}\left(t \mp x - t_0 + \frac{x_0}{V_0}\right)} \qquad (26)$$

The analysis of this expression shows that the density perturbation looks like two pulses of opposite polarities. At the drag of the soliton, the sound is radiated in the form of density "well" along the direction of the soliton motion, and in the form of density "hump" - along the opposite direction. At the soliton acceleration $(h < 0)$, the "hamp" propagates forward along the motion direction, while the well - in the opposite direction. As it has been expected, the spatial width of these pulses is defined by the sound velocity multiplied by the time of the soliton travel through the transition layer, which equals $c_s/\kappa V_0$ in the dimensional form. Estimating the total energy removed by the radiation using Eqn(27), we obtain $W = \frac{4}{3}\frac{h^2 \kappa}{V_0 \kappa_0^2}\mathcal{E}_m^4$

Similar IAW radiation is caused by the passage of "supersonic" soliton of the electromagnetic (e.m.) wave of the transition layer. The character of high-power e.m. wave propagation in plasma is defined both by striction nonlinearity and by the relativistic "weighting" of the electrons oscillating in the HF wave field. The account the latter effect leads to the possibility of the e.m. wave propagation in the form of soliton moving at the velocity exceeding the ionacoustic velocity. Such solitons may be described within the frames of the system of equations (1),(2) in which the first equation is substituted by the following equation /6,7/:

$$i\frac{\partial E}{\partial t} + \frac{\partial^2 E}{\partial x^2} - \Delta n(x)E + \delta n E + \gamma |E|^2 E = 0 \qquad (27)$$

where $\gamma = \frac{3}{2} T_e/(m_e c^2)$ for linearity polarized e.m. wave. Assuming $\gamma \gg |1 - \dot{x}^2(t)|^{-1}$, we restrict ourselves to the consideration of the "supersonic" soliton: $\dot{x}^2(t) \gg 1$, since $\gamma \ll 1$. Note that Eqns(24) and (25) for the velocity and acceleration are valid for the supersonic soliton. Eqn(26) for the radiation is also valid, but one should keep in mind that, in contrast to the "subsonic" Langmuir soliton, we should assume $V_0 \gg 1$ in (26). Besides, one should take into account that the condition $|\ddot{x}| \ll k_c \dot{x}^2$ is satisfied when obtaining Eqn(26), which denotes that during the soliton travel across the distance of the order of its width, its velocity change is small. Taking into account the facts stated above, the analysis of Eqn(26) shows that the IAWs radiated by the "supersonic" soliton represent two pulses propagating to the right and the left at the ion sound velocity. Soon after their formation, both pulses rapidly begin to lag behind the "supersonic" soliton. The energy removed by the radiation in this case equals $W = \frac{4}{3} \frac{k^2 \kappa}{V_0^6 k_0^2} \mathcal{E}_m^4$

Typical patterns of space-time distribution of radiated IAWs and solitons are given in Figs. 1,2 and 3. Fig.1 shows a "subsonic" Langmuir soliton and the IAW pulses radiated by it at various moments of time beginning from the moment of their generation. Fig.2 shows the pattern of the spatial density distribution in the radiated wave (1) at the moment when the "supersonic" soliton (2) is situated in the middle of the "transition" layer (the vertical scale of the soliton (2) being reduced), And Fig.3 gives the spatial distribution of the radiated IAWs (1) and "supersonic" soliton (2) at various moments of time after the soliton (2) has passed through the "transition" layer.

3. Ionacoustic wave radiation at the collision of Langmuir solitons

3.1. Collision of solitons

The system of Zakharov's equations (1),(2) describes not only single solitons of the type (8), but also allows the solution in the form of the ensemble of N solitons with, generally speaking, different velocities and amplitudes located at the distances exceeding their effective widths. In the general case, it is imposiible as yet to describe analytically the subsequent evolution of such a multisoliton structure of the self-consistent field distribution and plasma density within the frames of the system (1),(2), i.e. to follow the processes occuring at the mutual approach of the solitons when they begin to interact. However, if we neglect the inertia of ions and describe the system within the frames of NSE, the pattern of the soliton interaction is the following. Approaching each other, the solitons merge. We obtain a non-stationary formation which decays again after a certain time into solitons coinciding exactly with the initial ones. The only result of the collision is the change in the soliton phases: the rapid soliton becomes shifted forward along the direction of its motion with respect to the place where it should have been situated without the collision with another soliton, while the slower soliton becomes shifted backward. Here the total shift of each soliton is equal to the algebraic sum of its shifts at the pair-wise collisions, which points to the pair-wise character of the interaction. Thus, after a certain time interval, the multisoliton structure is restored again, but it has the form of N solitary pulses located one after another with increasing velocity.

The non-stationary character of the soliton dynamics at their interaction point to the necessity of taking the ion inertia into account. It has been shown by numerical experiments [8,9] aimed at the investigation of the Langmuir soliton formation and interaction dynamics within the frames of the system of equations (1),(2), that the soliton-soliton interaction is accompanied by the IAW radiation. In Ref. [10] an attempt of the analytical approach to the solution of the IAW "drag radiation" problem at the soliton interaction has been made. In this paper, the authors neglected the reverse influence of the radiated waves upon the solitons in the assumption of weak interaction; however, the soliton collision was presented as a simple linear superposition of soliton fields. A similar problem was analytically investigated also in Ref.[11]. However, while neglecting the reverse influence of the radiation upon the soliton dynamics, the authors have proceeded from the fact that the soliton collision is described by the multisoliton solutions of NSE, which is not always reduced to the linear superposition of solitons even in their "weak interaction".

Thus, we consider two solitons of the type (8) in the homogeneous plasma

$$E_j = \sqrt{2(1-V_j^2)}\, k_j \,\mathrm{sech}\, k_j(x-x_{oj}-V_j t)\exp(i\varphi_j)$$

$$\delta n_j = -2k_j^2\,\mathrm{sech}^2 k_j(x-x_{oj}-V_j t) \qquad (28)$$

$$\varphi_j = \frac{V_j}{2}x + \left(k_j^2 - \frac{V_j^2}{4}\right)t + \varphi_j\,,\ \varphi_j = const\,,\ j=1,2\,.$$

Let these solitons be located at a large distance at the initial moment of time $t=t_0$. We also assume that the solitons approach each other in the course of time. Until the inequality $|\bar{x}_{o1}(t)-\bar{x}_{o2}(t)| \gg \max\left(\frac{1}{k_1},\frac{1}{k_2}\right)$ is valid, where $\bar{x}_{oj}(t) \equiv x_{oj}+V_j t$, $j=1,2$, the solitons will propagate towards each other, without any mutual influence.

We seek for δn in the form

$$\delta n = -\varkappa|E|^2 + \delta n_s \qquad (29)$$

where $\varkappa>0$ is constant of the order of a unity.

Then the system of equations (1),(2) acquires the form $(\Delta n \equiv 0)$:

$$i\frac{\partial E}{\partial t} + \frac{\partial^2 E}{\partial x^2} + \varkappa|E|^2 E - \delta n_s E = 0 \qquad (30)$$

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\delta n_s = \varkappa\frac{\partial^2|E|^2}{\partial t^2} + (1-\varkappa)\frac{\partial^2|E|^2}{\partial x^2} \qquad (31)$$

It is clear from Eqn(31) that $\delta n_s \equiv 0$ if we consider the soliton moving at a constant velocity $V=\sqrt{1-\varkappa}$. Evidently, the density perturbation due to the soliton interactin is connected with δn_s. The term $\delta n_s E$ in Eqn(30) may be considered as a reverse response of the perturbation to the source.

We seek for the solution of the system of equations (30,31) in the form of series:

$$E = E_0 + E_1 + \cdots$$

$$\delta n_s = \delta n_{s1} + \delta n_{s2} + \cdots$$

$$|E_0| \gg |E_1| \gg |E_2| \ldots\,,\ |\delta n_{s1}| \gg |\delta n_{s2}| \ldots \qquad (32)$$

Neglecting the response of the pertirbation in the first approximation, i.e. $\delta n_s E$, we obtain from (30),(31)

$$i\frac{\partial E_0}{\partial t}+\frac{\partial^2 E_0}{\partial x^2}+\varkappa|E_0|^2E_0=0 \tag{33}$$

$$\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right)\delta n_{s1}=\left(\varkappa\frac{\partial^2}{\partial t^2}+(1-\varkappa)\frac{\partial^2}{\partial x^2}\right)|E_0|^2 \tag{34}$$

$$i\frac{\partial E_1}{\partial t}+\frac{\partial^2 E_1}{\partial x^2}+2\varkappa|E_0|^2E_1+\varkappa E_0^2E_1^*=\delta n_{s1}E_0 \tag{35}$$

$$\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right)\delta n_{s2}=\left(\varkappa\frac{\partial^2}{\partial t^2}+(1-\varkappa)\frac{\partial^2}{\partial x^2}\right)(E_0E_1^*+E_0^*E_1) \tag{36}$$

Now it is clear that in order to find plasma density perturbation in the first approximation, we should first solve Eqn(33) under the respective initial conditions. Since at the initial moment of time we deal with two solitons moving towards each other, the corresponding solution of Eqn(33) should describe the process of the "collision" of these solitons. It is the so-called two-soliton solution of NSE /2/ describing the "scattering" of the given solitons on each other. We represent the squared modulus of two-soliton solution as

$$\frac{\varkappa}{2}|E_0|^2=\frac{\kappa_1^2}{ch^2x_1}+\frac{\kappa_2^2}{ch^2x_2}+\frac{\partial^2}{\partial x^2}\ln\Big[1-\frac{th\alpha}{2-th\alpha}(th x_1 th x_2+th x_1-th x_2)-$$

$$-\frac{1-e^{-\alpha}}{2ch\alpha-sh\alpha}\cdot\frac{\cos\varphi}{ch x_1\, ch x_2}\Big] \tag{37}$$

where

$$x_j=\kappa_j(x-x_{0j}-V_j t),\quad \varphi=\omega t-\kappa x+\varphi_0$$

$$\omega=\kappa_1^2-\kappa_2^2+\frac{V_2^2-V_1^2}{2},\quad \kappa=\frac{V_2-V_1}{2},\quad \varphi_0=const. \tag{38}$$

$$\alpha=-\ln\left(1-\frac{16\kappa_1\kappa_2}{(V_2-V_1)^2+4(\kappa_1+\kappa_2)^2}\right)$$

This solution corresponds to the initial distribution in case of the soliton with the amplitude κ_1 located to the left of the second soliton.

While the inequality $\overline{X}_{02}(t)-\overline{X}_{01}(t)\gg\max\left(\frac{1}{\kappa_1},\frac{1}{\kappa_2}\right)$ is fulfilled, the third term in Eqn(37) is exponetially small, and the solution represents the sum of two separate solitons moving towards each oth While the solitons approach each other, the contribution of the thi term increases, and by the moment of time when $\overline{X}_{02}(t)\approx\overline{X}_{01}(t)$ the representation (37) acquires, in a certain sense, a formal character. since it is already impossible to single out separate solit In another limit with $\overline{X}_{01}(t)-\overline{X}_{02}(t)\gg\max\left(\frac{1}{\kappa_1},\frac{1}{\kappa_2}\right)$, we deal again with two solitons, but now their centres are shifted:

$$\frac{\varkappa}{2}|E_0|^2=\frac{\kappa_1^2}{ch^2\kappa_1(x-x_{01}-\frac{\alpha}{\kappa_1}-V_1t)}+\frac{\kappa_2^2}{ch^2\kappa_2(x-x_{02}+\frac{\alpha}{\kappa_2}-V_2t)} \tag{39}$$

The first (more rapid) soliton seems to be effectively accelerated at the interaction with the second (slower) soliton which undergoes a drag. It may be qualitatively explained as the consequence of the soliton motion across the density well prepared by the other soliton. However, such an explanation is not always useful, especial taking into account the fact that one cannot speak about separate solitons at their mergence, since it is impossible to single them ou on the background of non-stationary pattern. However, in case of wea interaction, the representation (37) as a sum of "solitons + interaction" is unformal even for the moments of time when the solitons become overlapped, and the above explanation of the shift of soliton centres is quite satisfactory.

It is evident from Eqns (37),(38) that in case of weak interaction of solitons the value

$$\epsilon = \frac{16\kappa_1\kappa_2}{(V_2-V_1)^2+4(\kappa_1+\kappa_2)^2} \ll 1 \tag{40}$$

plays the role of a small parameter.

Taking (40) into account, we may write down the following expression instead of (37)

$$\frac{\varkappa}{2}|E_0(x,t)|^2 = \sum_{s=1}^{2}\frac{\kappa_s^2}{ch^2\kappa_s(x-\bar{x}_s(t))} - \epsilon\frac{\kappa_1\kappa_2}{ch^2x_1 ch^2x_2} - \frac{\epsilon}{2}\frac{\partial^2}{\partial x^2}\frac{\cos\phi}{ch x_1 ch x_2} \tag{41}$$

where

$$\bar{x}_s(t) = x_{0s} + V_s t + \frac{\epsilon}{2\kappa_s}\left(th\, x_{3-s} + (-1)^{s-1}\right), \quad s=1,2 \tag{42}$$

Eqns(41),(42) allow us to vizualize the process of the approach of the more rapid soliton to the slower one "falling" into the density well of the second soliton and accelerating at the same time. Thus, it passes the region occupied by the second soliton at the average velocity exceeding its initial velocity, and, as a result, becomes shifted forward along the motion direction.

The slowing-down of the second soliton may be explained in a similar way. When the more rapid soliton overtakes it, it becomes as if "drawn into" the density well of the rapid soliton, acquiring the acceleration directed opposite the motion. Thus, the second soliton travels across the region occupied by the first soliton at the average velocity below its initial velocity. Therefore, the second soliton becomes shifted backwards with respect to the place where it would be located if the collision with the first soliton did not

occur.

Thus, Eqn(42) for the coordinates of the soliton centres clerly shows that the shift of the soliton centres does nor occur in a jump-like manner. It is accumulated gradually and represents the result of the accelerated soliton motion during their collision. It is evident from (41) that the solitons undergo both the acceleration and the profile deformation at their collision.

3.2. IAW radiation at the soliton collision

Having established the character of soliton interaction at their collision, we can find now the generated IAW radiation using Eqn(34). Without any restriction of generality, we assume that $V_1 = -V_2 = V > 0$ and $x_{02} = -x_{01} = x_0 > 0$. Then, substituting into the right-hand part of Eqn(34) the solution of Eqn(33), its modulus squared in the "weak interaction" approximation being given by Eqn(41), and taking into account the fact that $\varkappa = (1-V^2)^{-1}$, we come to the following equation for δn_{s1} (hereinafter the index "1" is omitted):

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\delta n_s = -\epsilon\frac{\partial^2}{\partial x^2}A(x,t) \tag{43}$$

where

$$A(x,t) = -\frac{4\kappa_1\kappa_2V^2}{ch^2x_1 ch^2x_2} - \frac{\omega^2-\kappa^2V^2+4\kappa_1\kappa_2V^2 th\,x_1 th\,x_2}{ch\,x_1 ch\,x_2}\cos\phi + 2V\frac{(\omega+\kappa V)\kappa_2 th\,x_2 - (\omega-\kappa V)\kappa_1 th\,x_1}{ch\,x_1 ch\,x_2}\sin\phi \tag{44}$$

Analysing the source for δn_s, we can see that it is nonzero only during the soliton collision, until their interaction lasts, - during the time interval $\Delta t_{int} \simeq (\kappa_1+\kappa_2)/(2\kappa_1\kappa_2V)$. After that it tends exponentially to zero again while the solitons move away from each other.

other.

In Eqn(41), the terms oscillating at the frequency ω in the course of time may be singled out. The last term in Eqn(4') which, in its turn, corresponds to the conventionally periodic of the "bound" state formed by the solitons, is responsible for them. Evidentle, the oscillating character of the source for δn_s may be revealed if $\Delta t_{int} > \omega^{-1}$

We investigate the IAW radiation in two cases: first, we consider the case of the collision of solitons with greatly differing amplitudes. Then we'll investigate the space and time distribution of the radiation field at the collision of solitons with nearly equal amplitudes.

Thus, we assume for definiteness that $K_1 \gg K_2$. Then we obtain for δn_s :

$$\delta n_s(x,t) = \frac{\epsilon}{2}\frac{\omega^3}{\kappa_1}\left(G^+(x,t) + G^-(x,t)\right)$$

$$G^{\mp}(x,t) = \frac{\operatorname{arctg}(\operatorname{sh}\kappa_1(x-vt+x_0)) - \operatorname{arctg}(\operatorname{sh}\kappa_1(x\mp t\mp x_0))}{\operatorname{ch} 2 v\kappa_2 (x \mp t \pm \frac{x_0}{v})} \times$$

$$\times \sin\left((\omega-\kappa v)(x\mp t)+\varphi^{\pm}\right), \qquad \varphi^{\pm} = const \tag{45}$$

Note that in our calculations we restrict ourselves to the terms of the order of ϵ .

As Eqn(45) shows, δn_s represents a sum of two terms. The first term corresponds to the radiation propagating at the ionic sound velocity ($C_s=1$ in dimensionless units) along X-axis in the positive direction, and the second term - to that propagating in the negative direction. Thus, we have obtained a physically reasonable pattern: ionacoustic waves are radiated during the soliton collision in the course of time equal to the time of the solitons passage through each other, i.e. to the interaction time $\Delta t_{int} \simeq 1/2K_2V$ $(K_1 \gg K_2)$ Naturally, the spatial width of the radiated field localization will

equal the time interval Δt_{int} multiplied by the IAW velocity. Since in the case under consideration $\omega \gg \kappa_2 V$, the radiation fi[eld] represents packets of rapidly oscillating waves, i.e. trains propa[ga]ting at the ionacoustic velocity to the left and to the right. Eqn[(45)] confirms, as well, the conclusion drawn on the basis of numerical [cal]culations 8 that the greater the amplitudes of the interacting [so]litons differ, the shorter is the radiated sound wavelength. We ob[tain] the total energy removed in the form of radiated IAW: $W = \frac{\pi^2}{4}\epsilon^2 \frac{\omega^6}{\kappa_1^2 \kappa_{\cdot}}$

Now let us consider the case of the collision of solitons with very close amplitude values: $|\kappa_1-\kappa_2|/\kappa_1 \sim \epsilon \ll 1$. It has turned out that in this case the first term in Eqn(44) makes the principal con[t]ribution to the radiation, which points to the fact that in this case the radiation is due to the accelerated motion of solitons during their collision. Other terms in Eqn(44) give spatially localized plasma density perturbations "Breaking out" during the soliton collision and exponentially attenuating with their gradual separat[ion] (while the solitons move away from each other). We obtain a rather awkward complex expression for δn_s . For the times $t > X_0/V$, i.[e.] when the solitons have already separated after their collision, and the action of IAW source has stopped, the expression for δn_s is simplified: $\delta n_s(x,t) = -8\epsilon V^3 \kappa_2^2\left(G^+(x,t) + G^-(x,t)\right)$

$$G^{\pm}(x,t) = \frac{\pm 1}{\operatorname{ch}^2 2\kappa_2 V(x\mp t\pm \frac{x_0}{V})}\left[\frac{6}{\operatorname{th}^3 2\kappa_2 V(x\mp t\pm\frac{x_0}{V})} - \frac{6-2\operatorname{th}^2 2\kappa_2 V(x\mp t\pm\frac{x_0}{V})}{\operatorname{th}^4 2\kappa_2 V(x\mp t\pm \frac{x_0}{V})} 2V\right. \tag{46}$$

The analysis of this expression shows that the radiated IAW pr[o]pagate in both directions from the colliding solitons in the form [of] two pulses, each of them representing the alternation of compressi[on] $(\delta n_s > 0)$ and rarefaction $(\delta n_s < 0)$ regions. The forefront of each pu[lse] is formed by the compression wave immediately followed by the rarefaction region. Such a radiation pattern may be visually explained if we compare it with the process of IAW radiation by the soliton at its accelerated motion in the inhomogeneous plasma and remind that the acceleration of each soliton in the process of collision changes its sign once.

Here we should mention, as well, the IAW radiation by so-called "pulsating" soliton. The thing is that the system of two solitons with different amplitudes being at rest with respect to one another may form a bound state localized in space and oscillating at the velocity defined by the difference of the squared amplitudes of the solitons. Such a bound two-soliton state is called "pulsating soliton" /12/.

The squared modulus $|E_0|^2$ of the pulsating soliton may be written as follows:

$$|E_0|^2 = 4E_m^2\left(\frac{1}{ch^2 k_0 x} + \frac{1}{k_0}\frac{\partial}{\partial x}\,\frac{\frac{3}{4} th k_0 x \cdot \sin^2\omega_0 t}{ch^4 k_0 x - \frac{3}{4}\sin^2\omega_0 t}\right) \tag{47}$$

Comparing (47) and (37), one may see that Eqn(47) describes a "superposition" of two solitons with equal (in the present case - zero) velocities and different amplitudes. It is localized within the spatial region with the width of $1/k_0$ and pulsates with the frequency $2\omega_0$. IAW radiation by such a pulsating soliton is similar to the process of acoustic wave radiation by a solid periodically changing its shape with the volume remaining unchanged, in a liquid or in a gas. In the wave zone $|x| \gg \Delta\ell \sim 1/k_0$ the density perturbation has a form typical for the radiation:

$$\delta n_s(x>0,t) = \frac{3}{2}E_m^2\frac{\omega_0^2}{k_0}\sum_{p=1}^{\infty} a_p \sin 2\omega_0 p(t-x)$$

where a_p are the coefficient rapidly decreasing with the growing number p: a_1=1.894; a_2 = -1.470..., a_5=0.017, $|a_p|>|a_{p+1}|$. The loss of pulsating soliton energy by IAW radiation equals, in the average, approximately $55\omega_0^3 E_m^4$ during the pulsation period.

4. Alfven wave generation by the Langmuir soliton

Nonlinear propagation of HF waves in magnetoactive plasma, in contrast to the isotropic one, is accompanied by the perturbation of both plasma density and magnetic field. In this section we'll show that a packet of Langmuir potential HF waves in the form of a soliton may generate a LF vortex motion in the magnetoactive plasma, which is radiated in case of the packet nonstationary behaviour in the form of the transverse eigenmodes of magnetoactive plasma-Alven waves.

The nonlinear dynamics of Langmuir waves in the magnetoactive plasma has been investigated in many papers /13-15/. The frequency of these waves is close to the plasma frequency, and therefore, they are sensitive to the LF plasma density variation δn caused by the force of HF pressure. In the weak external magnetic field $(\omega_{Be}^2 \ll \omega_{pe}^2)$ the frequency of these waves equals

$$\omega = \omega_{pe}\left(1 + \frac{3}{2}(k r_D)^2 + \frac{1}{2}\frac{\omega_{Be}^2}{\omega_{pe}^2}\frac{K_\perp^2}{K^2}\right) \tag{48}$$

where ω_{Be} and ω_{pe} are electron cyclotron and plasma frequencies, r_D - Debye radius, $K_\perp$ - wave vector component normal to the magnetic field.

At the LF motion caused by the force of HF pressure, electric currents may be induced. Hence, LF perturbations of the magnetic field may arise side by side with LF density perturbation. Due to the dispersion singularity (the last term in (47) is small), the former make a negligible contribution to the nonlinear dynamics of HF waves (for instance, to the solitary wave formation). However, as it will be shown below, the generation of LF magnetic field is of independent interest: firstly, a packet of potential Langmuir waves may generate vortex perturbations, and secondly, at the non-stationary behaviour of HF wave packets, LF perturbations are radiated in the form of Alfven eigenwaves.

Let us consider plasma with the equilibrium density varying along the external magnetic field $\vec{B}_0(0,0,B_0)$. We assume that the characteristic inhomogeneity length greatly exceeds the scale of the localization of HF oscillations. Neglecting plasma drift due to the inhomogeneity ($U_d \ll C_S, C_A$, where U_d is the drift velocity, C_S and C_A - ionacoustic and Alfven velocities), we also assume that all LF values depend on one coordinate z. We represent the electric potential of the Langmuir wave in the form

$$\Phi = \frac{1}{2}\psi(t,z)\exp(i\vec{k}\vec{r} - i\omega t) + c.c. \tag{49}$$

where we can write the following equation for slowly varying function /13,15/:

$$i\frac{\partial\psi}{\partial t} + iv_g\frac{\partial\psi}{\partial z} + q\omega_{pe}r_0^2\frac{\partial^2\psi}{\partial z^2} - \frac{\omega_{pe}}{2}\frac{\Delta n(z)}{n_0}\psi - \frac{\omega_{pe}}{2}\frac{\delta n}{n_0}\psi = 0 \tag{50}$$

$$v_g = \frac{\partial\omega}{\partial k_z} = k_z\left(3\omega_{pe}r_0^2 - \frac{\omega_{Be}^2}{\omega_{pe}}\frac{k_\perp^2}{k^4}\right) \tag{51}$$

$$q = \frac{3}{2} + \left(\frac{3}{2}\frac{k_z^2}{k_\perp^2} - \frac{1}{2}\right)\frac{k_\perp^4}{k^4(kr_0)^2}\frac{\omega_{Be}^2}{\omega_{pe}^2} \tag{52}$$

and $\Delta n(z)$ describes the profile of plasma inhomogeneity. Assumi[ng] that the velocity of HF wave packet displacement is much below the ionacoustic velocity, we obtain for δn /16/:

$$\frac{\delta n}{n_0} = -\frac{k^2|\psi|^2}{16\pi n_0 T_e} \tag{53}$$

Then Eqn(50) has a solution in the form of acceleratedly movi[ng] soliton /5/:

$$|\psi| = \psi_0 \operatorname{sech}\frac{z - \bar{z}(t)}{\Delta\ell} \tag{54}$$

where $\Delta\ell = \left(64q\pi n_0 T_e/(k\psi_0)^2\right)^{1/2} r_0$, and the coordinate of the solit[on] centre $\bar{z}(t)$ is defined by the equation

$$\ddot{\bar{z}}(t) = -qv_{Te}^2\frac{\partial}{\partial z}\frac{\Delta n(z)}{n_0}\Big|_{z=\bar{z}(t)} \tag{55}$$

From the equations of magnetic hydrodynamics for the transver[se] LF pertirbations of the magnetic field $\delta B_\perp$, we obtain the equat[ion]

$$\left(\frac{\partial^2}{\partial t^2} - C_A^2\frac{\partial^2}{\partial z^2}\right)\delta B_\perp = \frac{k^2 k_\perp}{8\pi m_i n_0 \omega_{pe}}\frac{\partial^2|\psi|^2}{\partial t\,\partial z} \tag{56}$$

Hence, the source of $\delta B_\perp$ is the so-called nonstationary ponderom[o]tive force /17/. Here we present a general expression for the pond[e]romotive force of HF quasimonochromatic electromagnetic field

$\vec{\mathcal{E}} = 1/2\,\vec{E}(t,\vec{r})e^{-i\omega t} + c.c.$ acting in the cold collisionless magnetoactive plasma /17/:

$$\vec{F} = \frac{1}{16\pi}\left\{\vec{\nabla}(\varepsilon_{\alpha\beta}-\delta_{\alpha\beta})E_\beta E_\alpha^* + \left(\vec{B}_0 \times rot\,\frac{\partial \varepsilon_{\alpha\beta}}{\partial \vec{B}_0}E_\beta E_\alpha^*\right)\right\} +$$

$$+\frac{i}{16\pi}\left\{\frac{1}{\omega}\frac{\partial}{\partial t}\left((\hat{\varepsilon}-\hat{I})\vec{E}\times rot\,\vec{E}^*\right) + \frac{\partial \varepsilon_{\alpha\beta}}{\partial \omega}\frac{\partial E_\beta}{\partial t}\vec{\nabla}E_\alpha^* - c.c.\right\} \quad (57)$$

Here $(\hat{\varepsilon})_{\alpha\beta} \equiv \varepsilon_{\alpha\beta}$ is the dielectric tensor, $(\hat{I})_{\alpha\beta} \equiv \delta_{\alpha\beta}$ unit 3-tensor. Note that the most frequent mistakes were made by various authors just at the determination of the "nonstationary" part of the ponderomotive force (the last two terms in Eqn(57)).

In our case, when $\vec{\mathcal{E}} = -\nabla\Phi$, it is just the nonpotentiality of the nonstationary ponderomotive force (the last term in (57)) which causes the arisal of Alfven type LF perturbations in the magnetoactive plasma.

The solution of Eqn(56) in the dipole approximation (i.e. neglecting the change in the soliton velocity during the time $\Delta\ell/C_A$, $|\ddot{\bar{z}}|\frac{\Delta\ell}{C_A} \ll C_A$ /18/) if the following:

$$\frac{\delta B_\perp}{B_0} = \frac{\dot{\bar{z}}(t)}{C_A^2-\dot{\bar{z}}^2(t)}\,\frac{k_\perp k^2 \Psi_0^2}{8\pi m_i n_0 \omega_{pe}}\,sech^2\frac{z-\bar{z}(t)}{\Delta\ell} + \frac{9}{4}\,\frac{k_\perp\Delta\ell}{\omega_{pe}C_A^2}\left(\frac{k\Psi_0}{B_0}\right)^2 \times$$

$$\times\left(G^+(z,t) + G^-(z,t)\right) \quad (58)$$

where $G^\pm = \dot{\bar{z}}(t^\pm)\ddot{\bar{z}}(t^\pm)\left[th\frac{z\pm C_A t-\bar{z}(0)}{\Delta\ell} - th\frac{z-\bar{z}(t)}{\Delta\ell}\right]$; $t^\mp \equiv t \mp (z-\bar{z}(t))/\Delta\ell$

The first term of this expression represents the magnetic field perturbation localized within the region of soliton and moving with the latter. It corresponds to the bending of the magnetic induction lines. The subsequent terms describe the momenta of Alfven perturbations propagating to the left and to the right of the soliton at the velocity C_A. Evidently, their existance is due to the accelerated soliton motion.

Let us consider the case of the square-law inhomogeneity $\Delta n(z) = \frac{1}{2}\frac{z^2}{L^2}$. According to (55), the soliton will accomplish the oscillatory motion at the frequency $\omega_0 = \sqrt{q}\,\frac{v_{Te}}{L}$. One may choose

$$\bar{z}(t) = -\frac{v_0}{\omega_0}\cos\omega_0 t \quad (59)$$

where v_0 is the maximum soliton velocity. We obtain from (58) the expression for the space and time distribution of the magnetic field perturbation in the Alfven wave:

$$\frac{\delta B_A}{B_0} = \frac{3}{8}\left(\frac{k\Psi_0}{B_0}\right)^2\frac{k_\perp\Delta\ell}{\omega_{pe}}\,\frac{\omega_0 v_0^2}{C_A^2}\left(G^+(z,t) + G^-(z,t)\right)$$

$$G^\pm = \sin 2\omega_0\left(t \mp \frac{z-\bar{z}(t)}{C_A}\right)\left[th\frac{z\mp C_A t-\bar{z}(0)}{\Delta\ell} - th\frac{z-\bar{z}(t)}{\Delta\ell}\right] \quad (60)$$

As we should have expected, the perturbation distribution is of periodic character with the spatial period $\pi C_A/\omega_0$. Outside the points $z = \pm C_A t$ the field drops to zero at a distance of the order of the soliton width $\Delta\ell$, i.e. the radiation front width is determined by the soliton width.

Now we estimate the energy removed by the radiated Alfven wave. Using (59) we obtain the expression for Poynting's vector averaged over the soliton oscillation period:

$$S = C_A\,\frac{9}{64}\left(\frac{k\Psi_0}{B_0}\right)^4\left(k_\perp\Delta\ell\,\frac{\omega_0 v_0^2}{\omega_{pe}C_A^2}\right)^2\frac{B_0^2}{8\pi} \quad (61)$$

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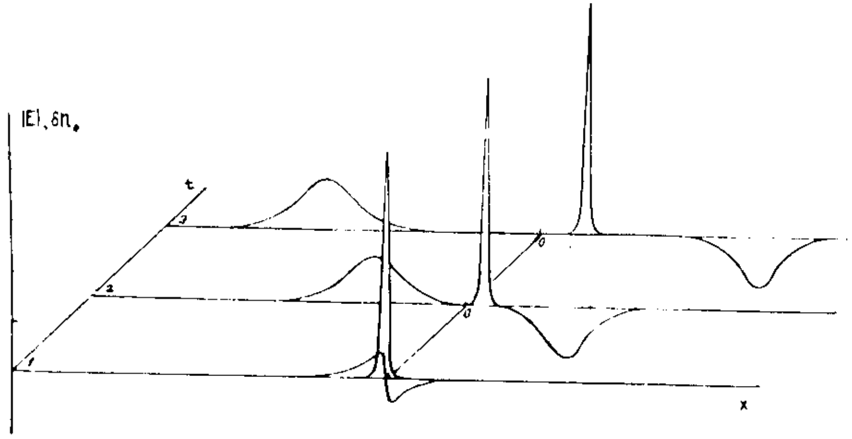

Fig.1

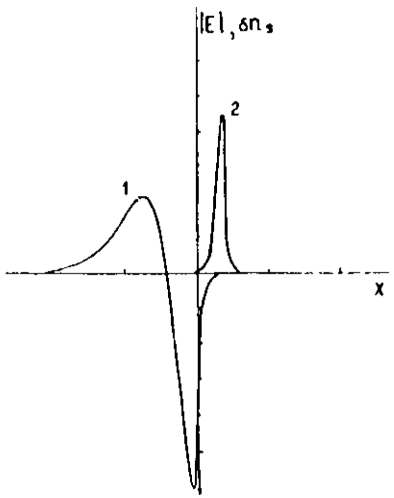

Fig.2

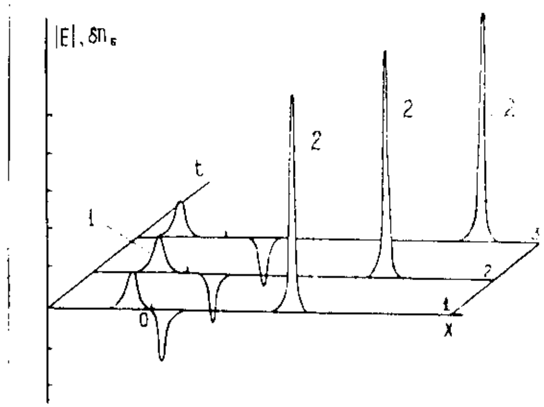

Fig.3