INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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SPRING COLLEGE ON PLASMA PHYSICS

(25 May - 19 June 1987)

ROLE OF WAVE PARTICLE INTERACTION IN SPACE PLASMAS

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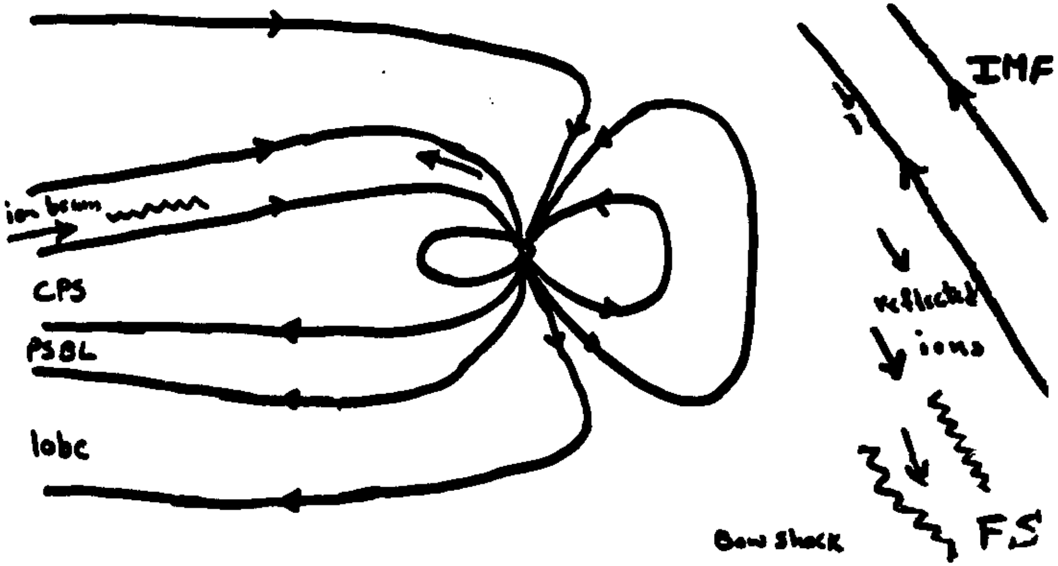
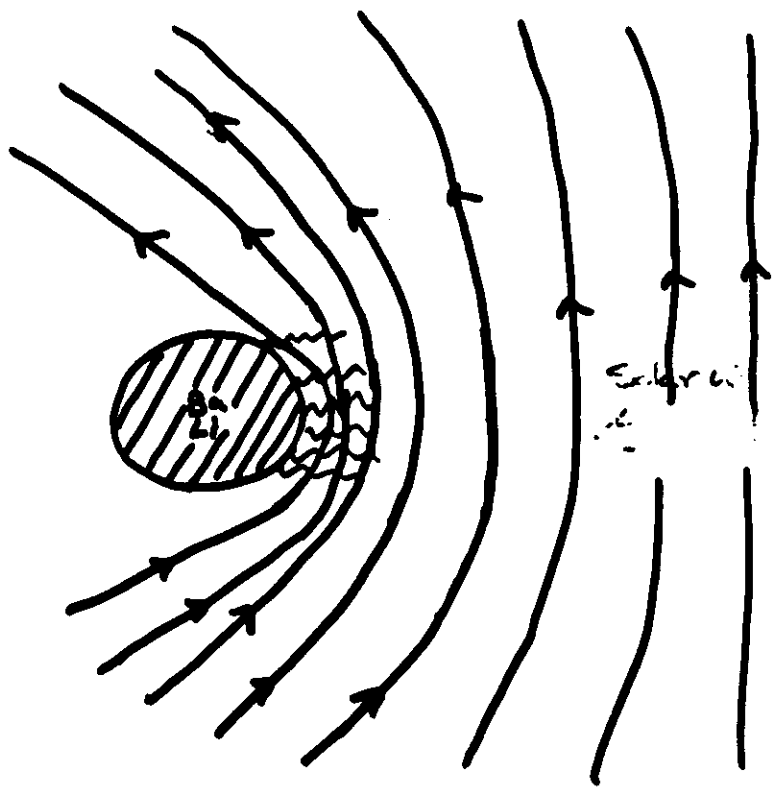


1. Broad Band Electrostatic Noise (PSBL)
2. Magnetic Noise Burst (PSBL)
3. Electrostatic waves in the foreshock and at the foot of quasi-perpendicular shocks

with:

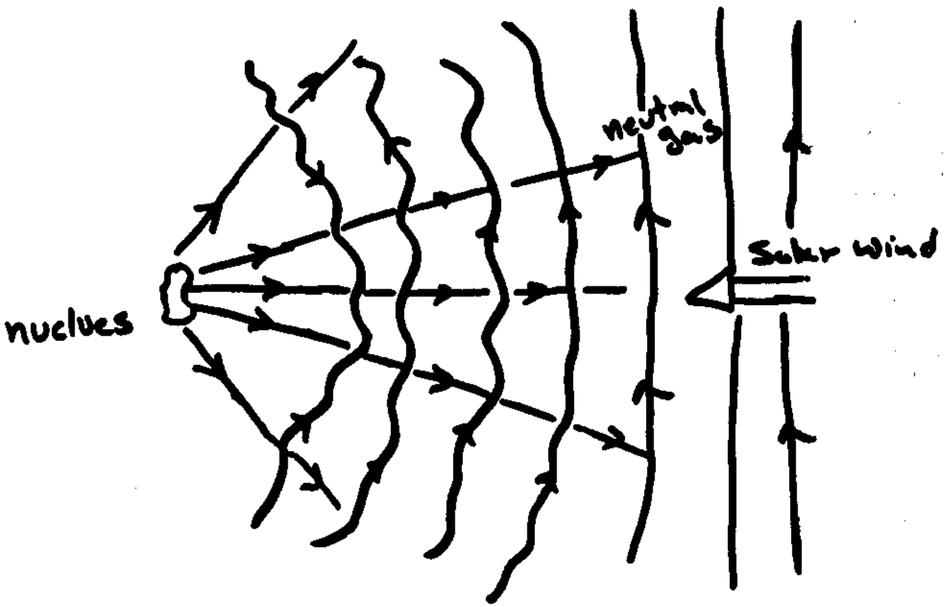
{ K. Akimoto
{ S.P. Gary



1. Nature and the nonlinear evolution of the electrostatic waves observed during AMPTE solar wind releases.

with:

{ K. Akimoto
{ D. A. Gurnett
{ T. Z. Ma
{ R. R. Anderson



1. Kinetic nature of Solar wind mass loading

2. Nature of Cometary shocks

with :

D. Winske

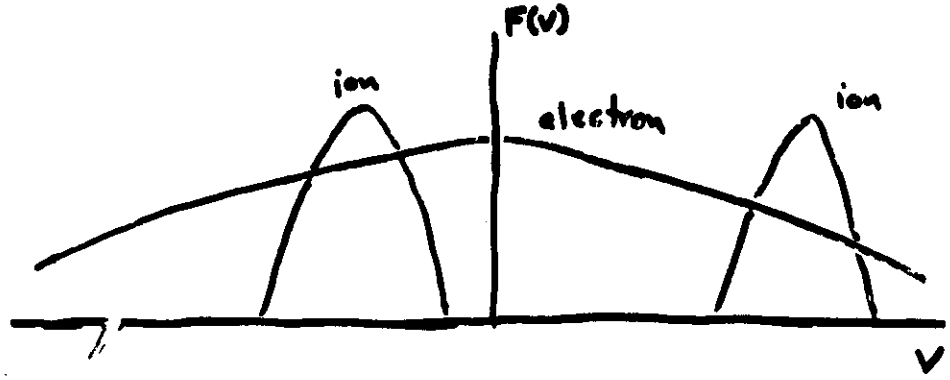
Some Possible ion beam instabilities

Electrostatic :

1. electron/ion-acoustic
2. ion/ion-acoustic
3. electron/ion cross-field
4. ion/ion cross-field

Electromagnetic :

1. ion/ion right-hand resonant
2. ion/ion right-hand nonresonant
3. electron/ion whistler



Lecture 1:

1. Linear analysis of ion-acoustic type instabilities, and the cross-field inst. Use AMPTE waves and BEN as examples.

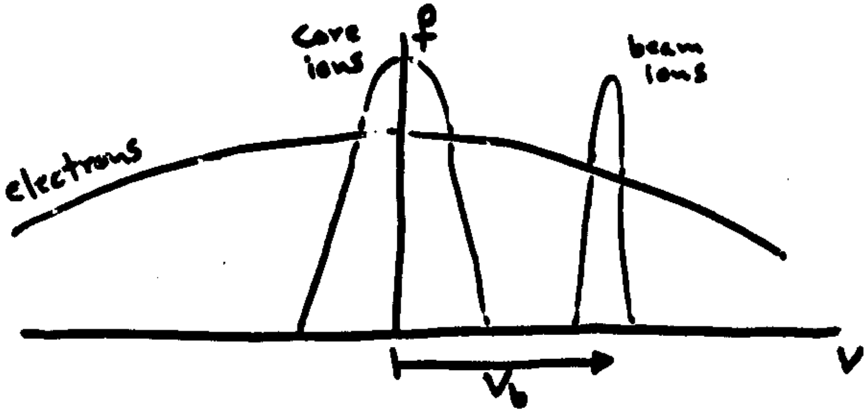
2. Nonlinear analysis of ion-acoustic type instabilities using electrostatic particle simulations. Briefly describe the two kinds of simulation techniques utilized in the study.

Lecture 2:

1. Linear analysis of ion/ion right-hand resonant and nonresonant instabilities and how they relate to MHD modes.

2. Investigate the nonlinear evolution of these electromagnetic instabilities via the hybrid (fluid electrons, particle ions) simulation technique which will be briefly described. The simulations are applied to the problem of solar wind mass loading and cometary bow shocks.

Ion Acoustic type Instabilities



$$1-\sum_{\alpha} \frac{1}{2K^2\lambda_\alpha^2}\, Z'\left(\frac{\omega - K\cos\theta\, \bar{V}_\alpha}{\sqrt{2}\, K\, V_\alpha}\right) = 0 \quad (1)$$

K = wave number , λ_α = Debey length

ω = wave frequncy , $\bar{V}_\alpha$ = drif speed

V_α = thermal speed , θ = angle between $\vec{K}$ & $\vec{V}_\circ$

eq. (1) results in two instabilities :

electron/ion-acoustic

ion/ion-acoustic

These instabilities are insensetive to magnetic field.

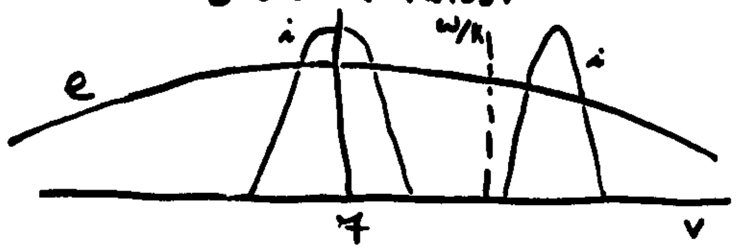
electron/ion-acoustic :

1. A Kinetic instability due to landau resonance with the electrons. Similar to current driven ion acoustic instability.

2. Has acoustic-type dispersion relation i.e. $\omega \sim K C_s$, and its maximum growth occurs at $\theta = 0$

3. Growth rates increase with increasing V_b.

4. The ratio of electron to ion temperature (T_e/T_i) is important in determining the growth rates.

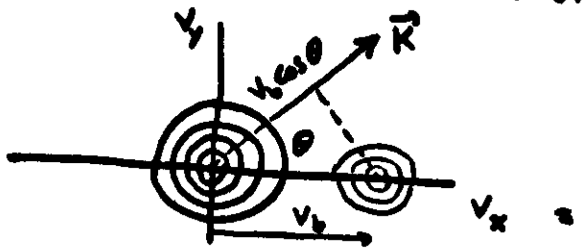


ion/ion-acoustic :

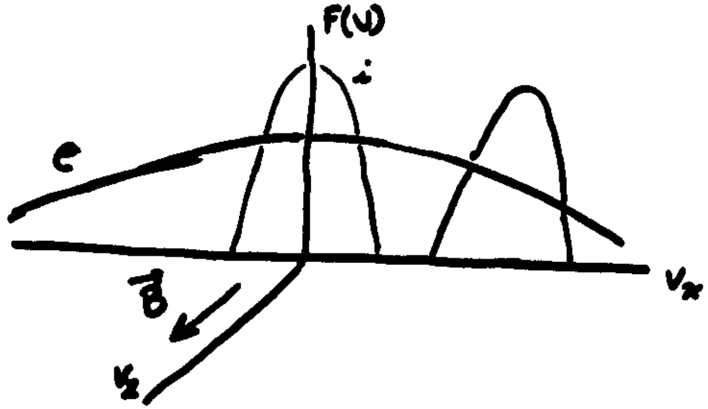
1. This instability is kinetic when the ion beam temperature is high, and is fluid when the beam temp. is small. It is possible for the waves to be in Landau res. with ions and get grow due to fluid instability

2. Has acoustic type dispersion relation. Maximum growth occurs at $\theta = 0$ if $V_b \le 2c_s$ and it occurs at oblique angles if $V_b > 2c_s$

3. The ratio T_e/T_i is important.



electron/ion and ion/ion cross-field inst. :

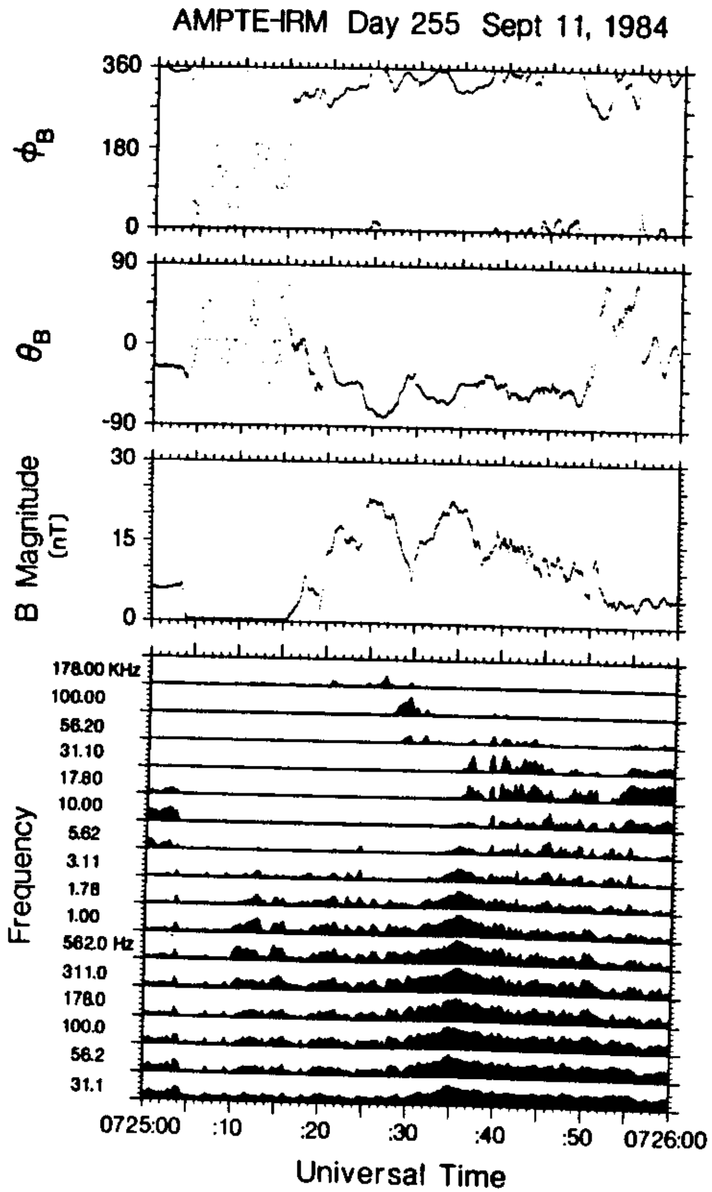


1. electrons are magnetized while the ions are non magnetized.

2. waves have frequencies near the lower hybrid frequency (f_{LH}).

3. The growth rates are maximum when flow is perpendicular to the magnetic field.

4. when β_e is small waves are electrostatic. As β_e increases electromagnetic effects become important and the waves stabalize.



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11

B-G85-65

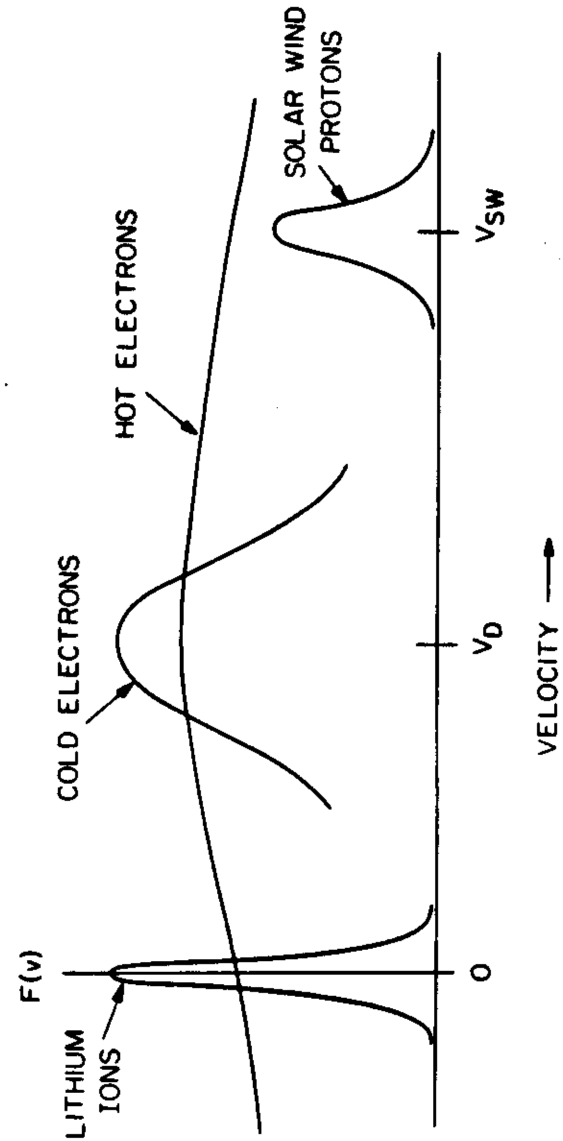
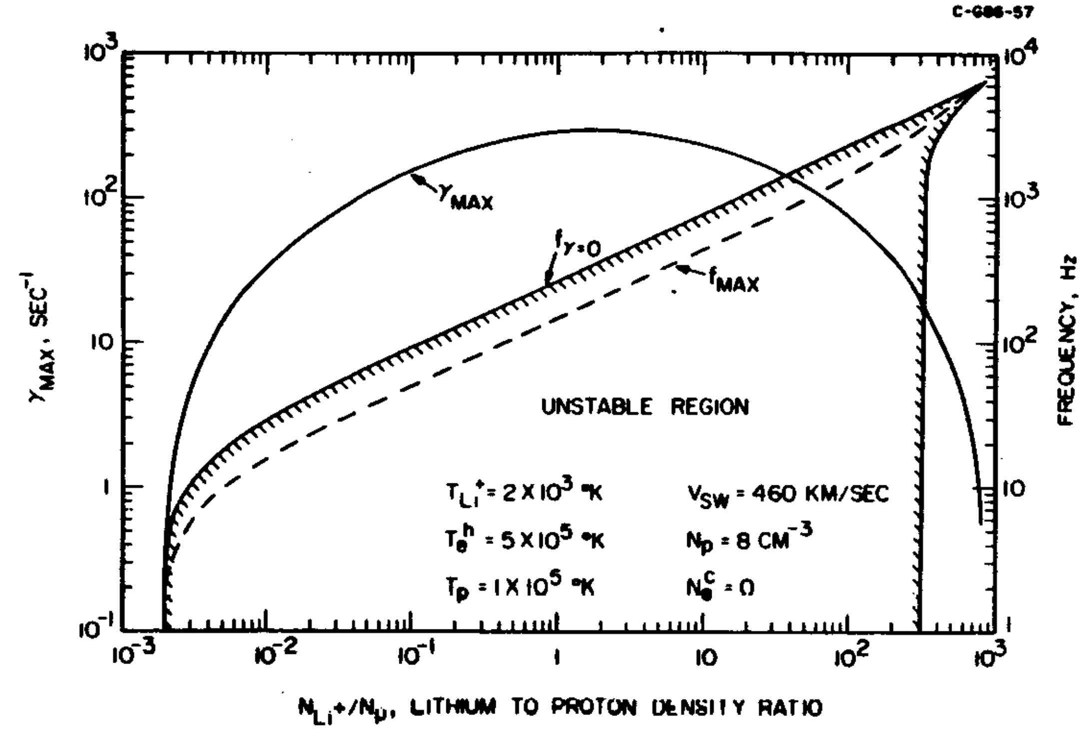
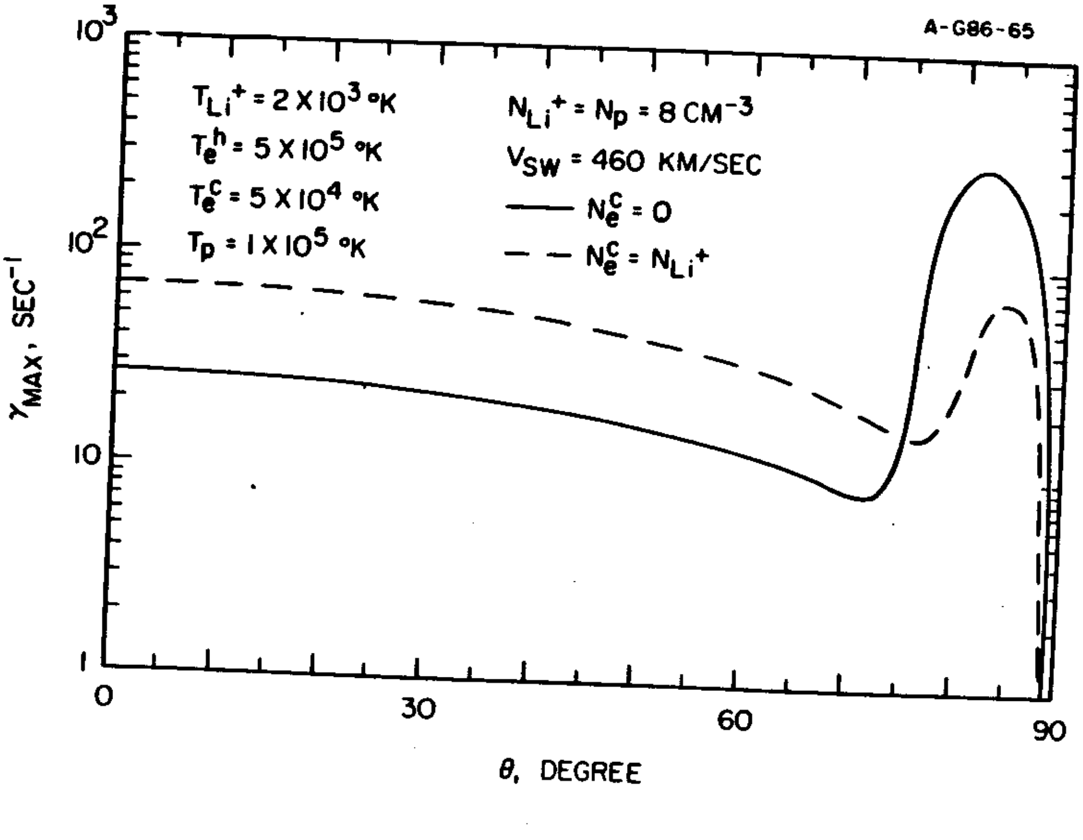


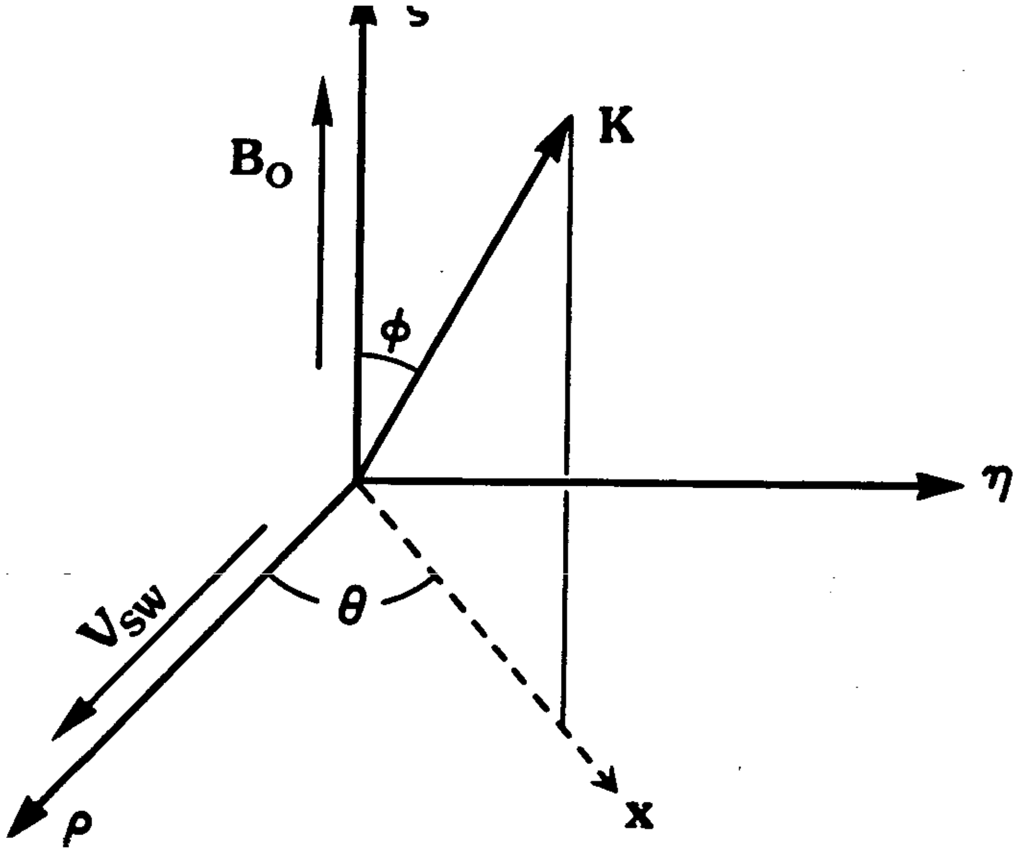
Figure 8


C-G86-57
γ_{MAX}, SEC^{-1}
FREQUENCY, Hz
N_{Li^+}/N_p, LITHIUM TO PROTON DENSITY RATIO
γ_{MAX}
$f_{\gamma=0}$
f_{MAX}
UNSTABLE REGION
$T_{Li^+} = 2 \times 10^3$ °K
$T_e^h = 5 \times 10^5$ °K
$T_p = 1 \times 10^5$ °K
V_{SW} = 460 KM/SEC
$N_p = 8$ CM^{-3}
$N_e^c = 0$

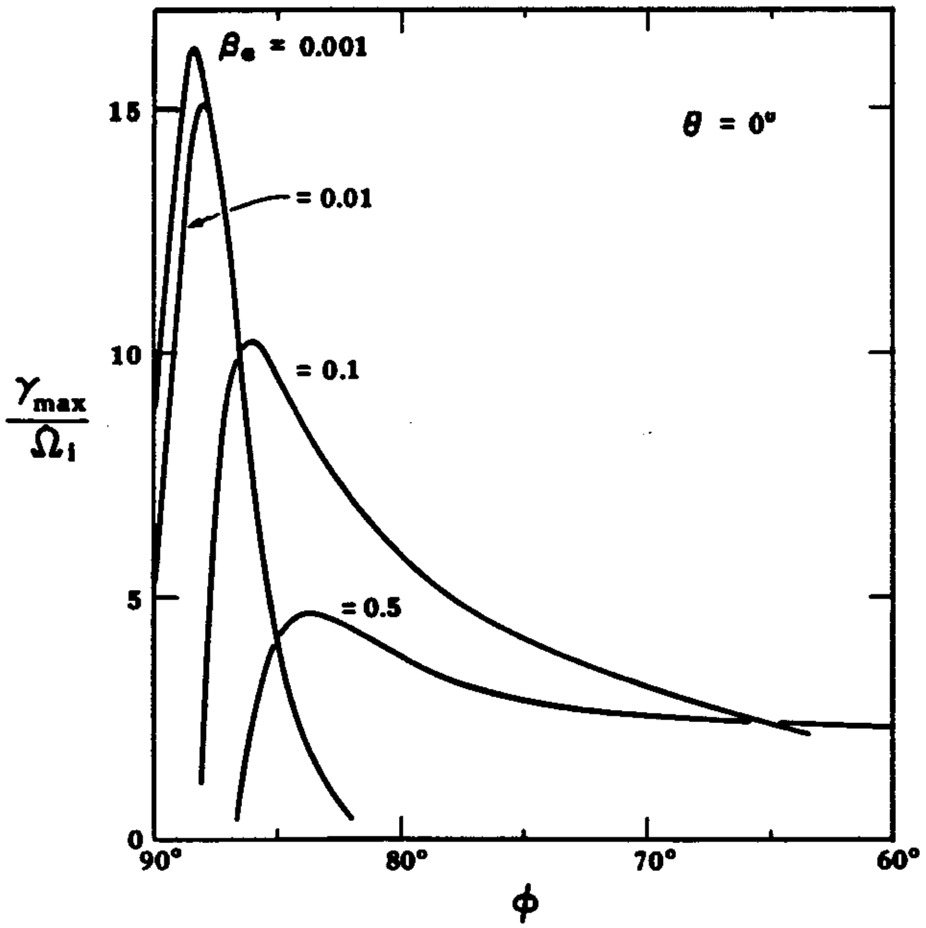
13


A-G86-65
γ_{MAX}, SEC^{-1}
θ, DEGREE
$T_{Li^+} = 2 \times 10^3$ °K
$T_e^h = 5 \times 10^5$ °K
$T_e^c = 5 \times 10^4$ °K
$T_p = 1 \times 10^5$ °K
$N_{Li^+} = N_p = 8$ CM^{-3}
V_{SW} = 460 KM/SEC
$N_e^c = 0$
$N_e^c = N_{Li^+}$

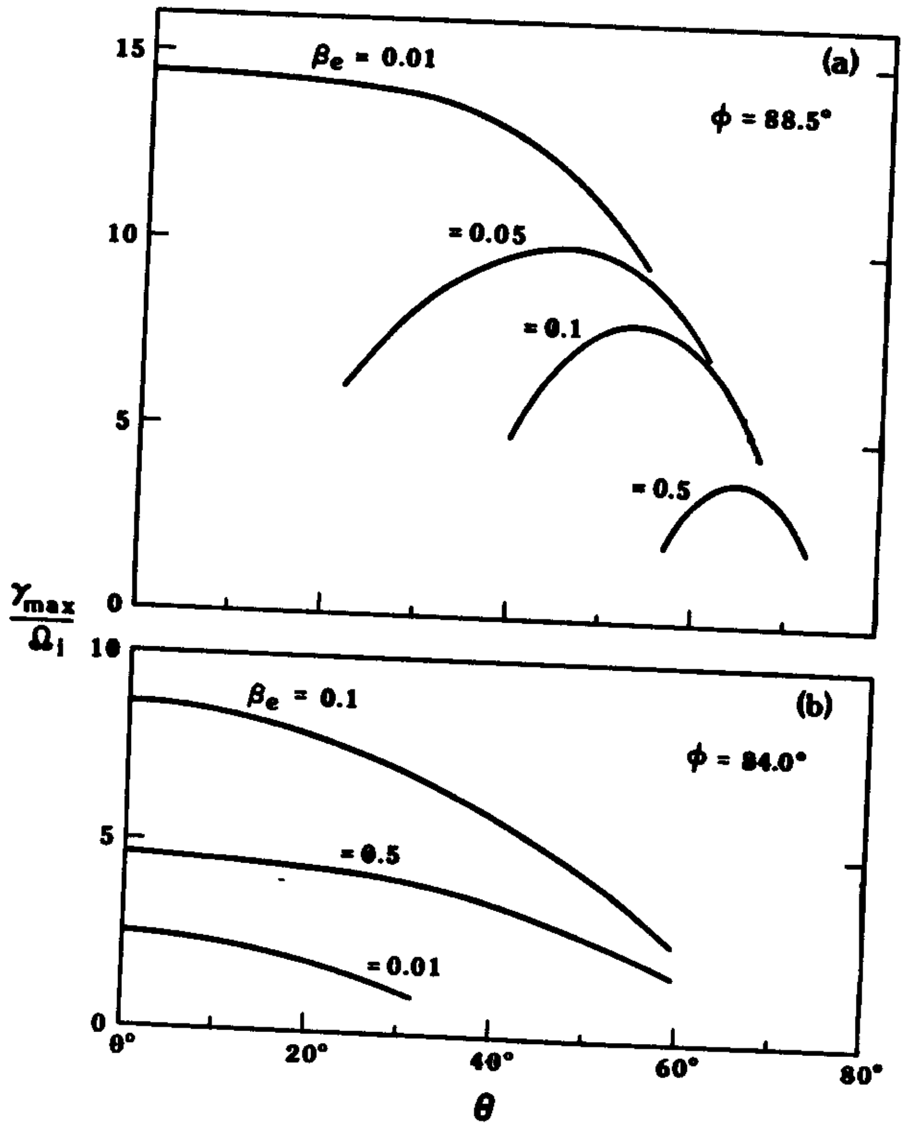
12



ζ
B_O
K
φ
η
V_{SW}
θ
ρ
x

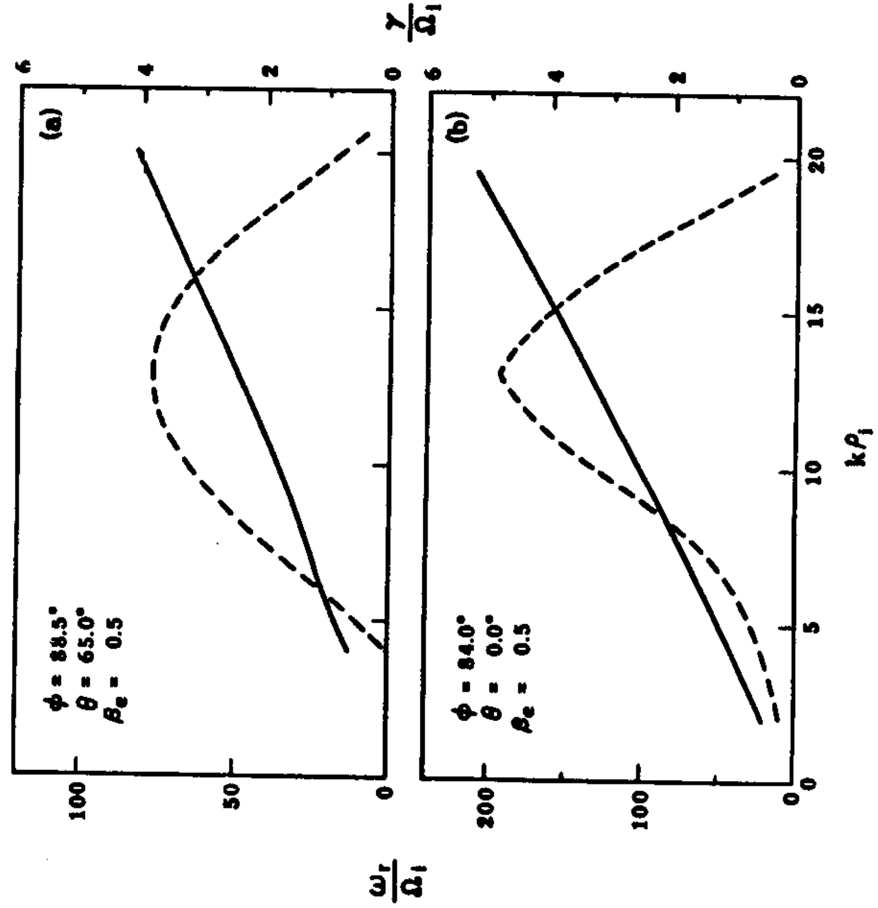


β_e = 0.001
= 0.01
= 0.1
= 0.5
θ = 0°
$\frac{\gamma_{max}}{\Omega_i}$
0
5
10
15
90°
80°
70°
60°
φ



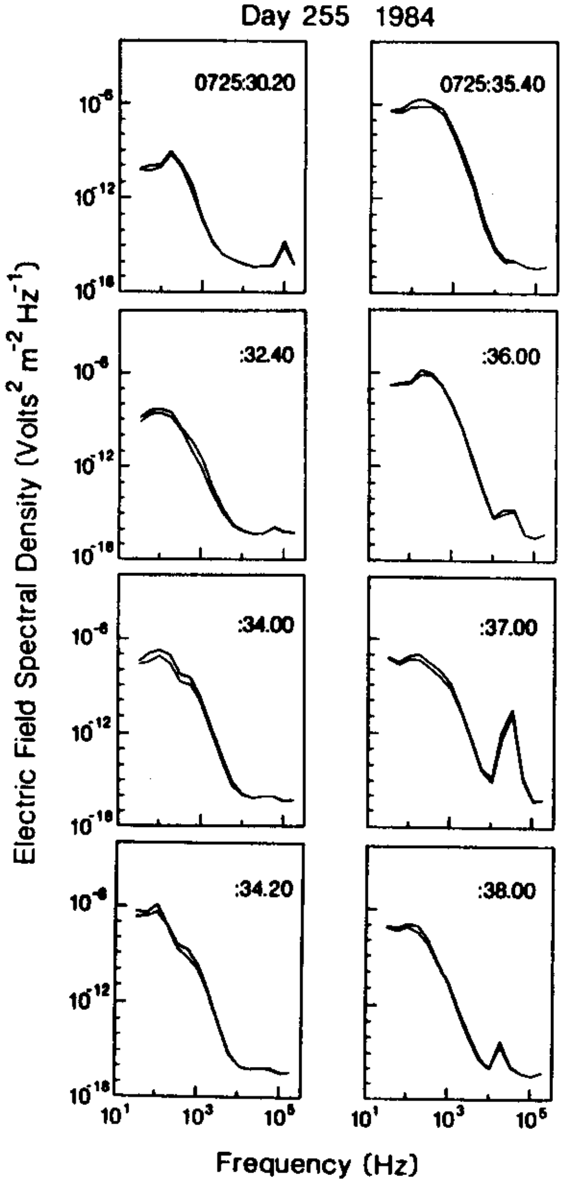
$\frac{\gamma_{max}}{\Omega_i}$
15
10
5
0
β_e = 0.01
= 0.05
= 0.1
= 0.5
(a)
ϕ = 88.5°
10
5
0
β_e = 0.1
= 0.5
= 0.01
(b)
ϕ = 84.0°
0°
20°
40°
60°
80°
θ

16

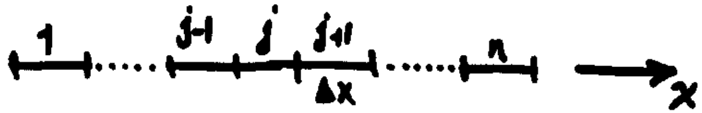


$\frac{\gamma}{\Omega_i}$
(a)
ϕ = 88.5°
θ = 65.0°
β_e = 0.5
(b)
ϕ = 84.0°
θ = 0.0°
β_e = 0.5
$\frac{\omega_r}{\Omega_i}$
$k\rho_i$

17



Full Particle Electrostatic code :



solve $\begin{cases} m\dfrac{dV}{dt} = qE & \text{Lorentz force eq.} \\[2ex] \dfrac{d^2\phi}{dx^2} = -\dfrac{dE}{dx} = -\rho & \text{Poisson's eq.} \end{cases}$

$$\left.\frac{d^2\phi}{dx^2}\right|_j = \frac{\phi_{j+1} - 2\phi_j + \phi_{j-1}}{\Delta x^2} = -\rho_j = -(n_{i,j} - n_{e,j})$$

$$A_j\,\phi_{j+1} + B_j\,\phi_j + C_j\,\phi_{j-1} = D_j$$

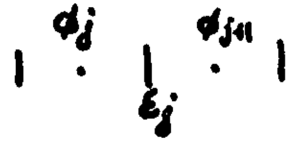
$$\phi_j = E_{j-1}\,\phi_{j-1} + F_{j-1}$$

where $E_{j-1} = \dfrac{-C_j}{A_j E_j + B_j}$ and $F_{j-1} = -\dfrac{C_j}{E_{j-1}}(D_j - A_j F_j)$

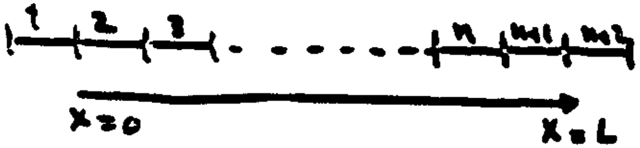
Use B.C.s to solve the recursive eqs. for

E and F

$$E_j = -\frac{\phi_{j+1} - \phi_j}{\Delta x}$$



Periodic boundary conditions :



Boundary conditions :

$$\begin{cases} \text{a. Periodic} \Rightarrow \phi_2 = \phi_{n+2} & (1) \\ \text{b. set zero level of } \phi \Rightarrow \phi_2 = 0 & (2) \end{cases}$$

then

$$(1) \Rightarrow \phi_{n+2} = E_{n+1}\,\phi_{n+1} + F_{n+1} = 0 \Rightarrow \begin{cases} E_{n+1} = 0 \\ F_{n+1} = 0 \end{cases}$$

Calculate E_n, F_n ; E_{n-1}, F_{n-1} ; $\cdots$; E_2, F_2

$$(2) \Rightarrow \phi_3 = E_2\,\phi_2 + F_2 = F_2$$

Calculate $\phi_4, \phi_5, \cdots, \phi_{n+1}$

Plasma Parameters

$m_{Li} = 6 m_p = 600 m_e$

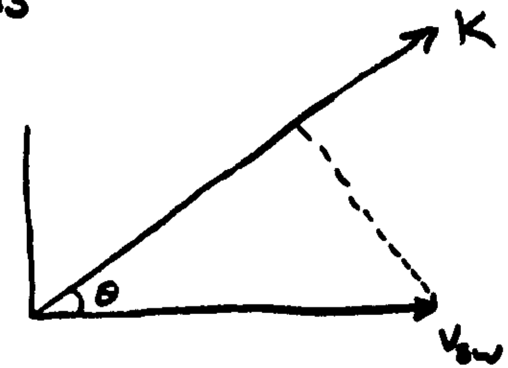
$V_{sw} = 0.71\, V_e$

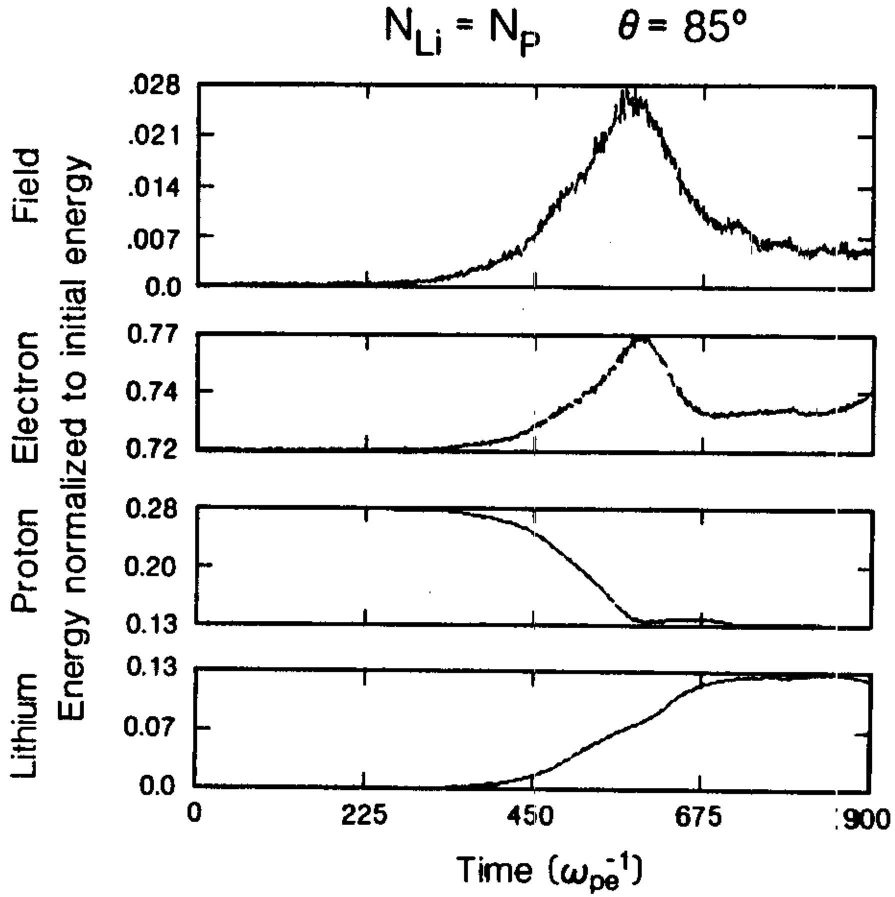
$V_p = 0.01\, V_e$

$V_{Li} = 0.26 \times 10^{-2}\, V_{ep}$

$N_{Li} = N_p$

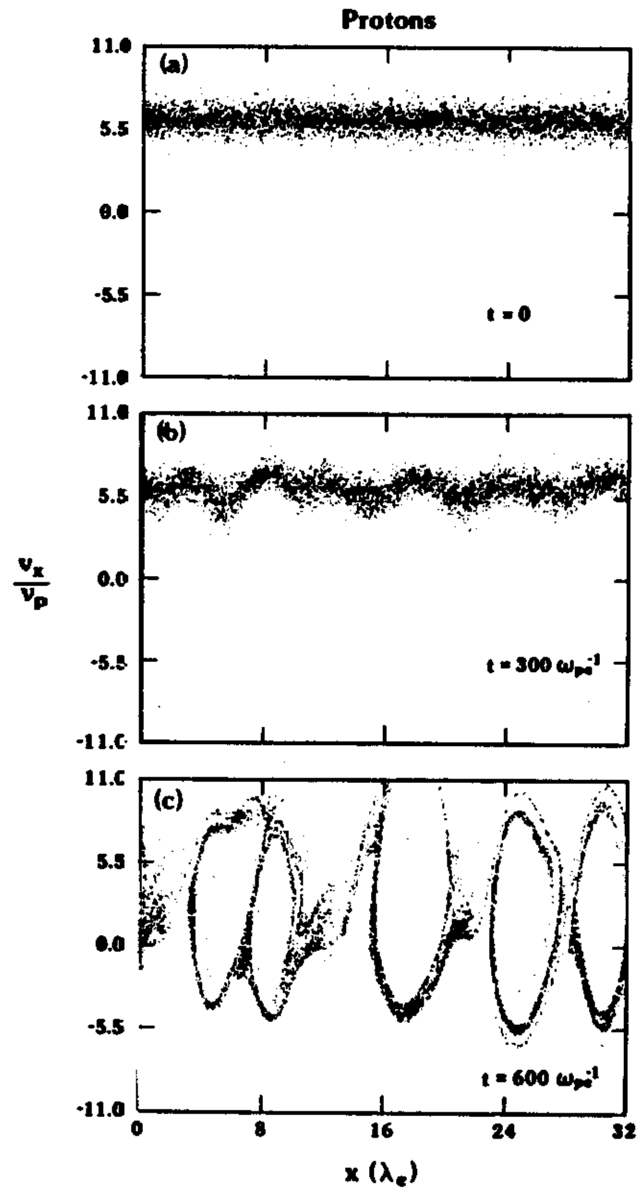
$\theta = 85^{\circ}$





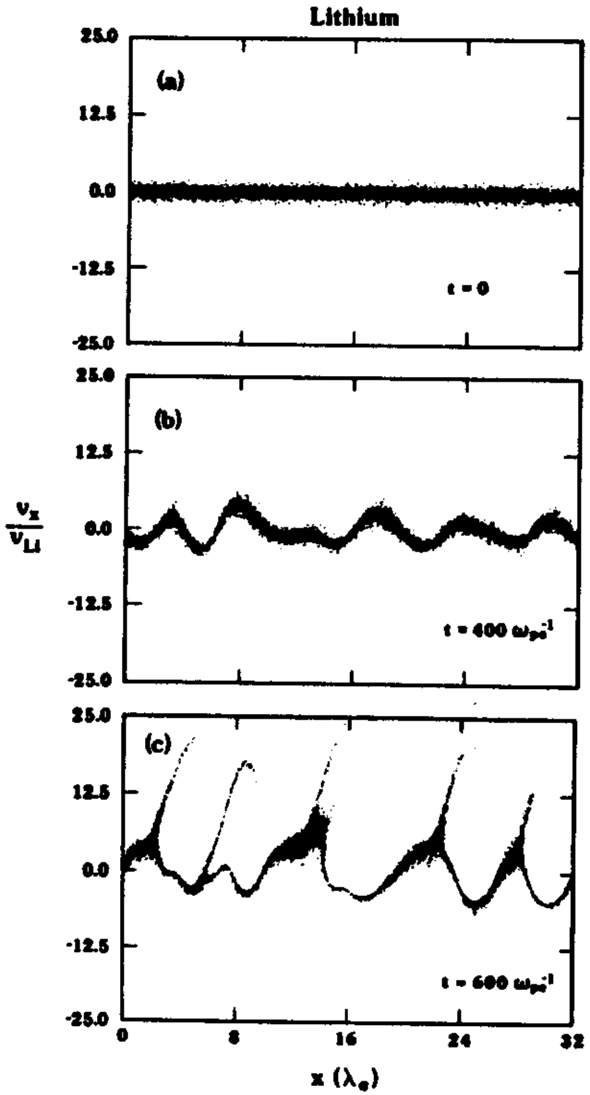
$N_{Li} = N_P$ $\theta = 85°$
Energy normalized to initial energy
Field
Electron
Proton
Lithium
.028
.021
.014
.007
0.0
0.77
0.74
0.72
0.28
0.20
0.13
0.13
0.07
0.0
0
225
450
675
900
Time (ω_{pe}^{-1})

22

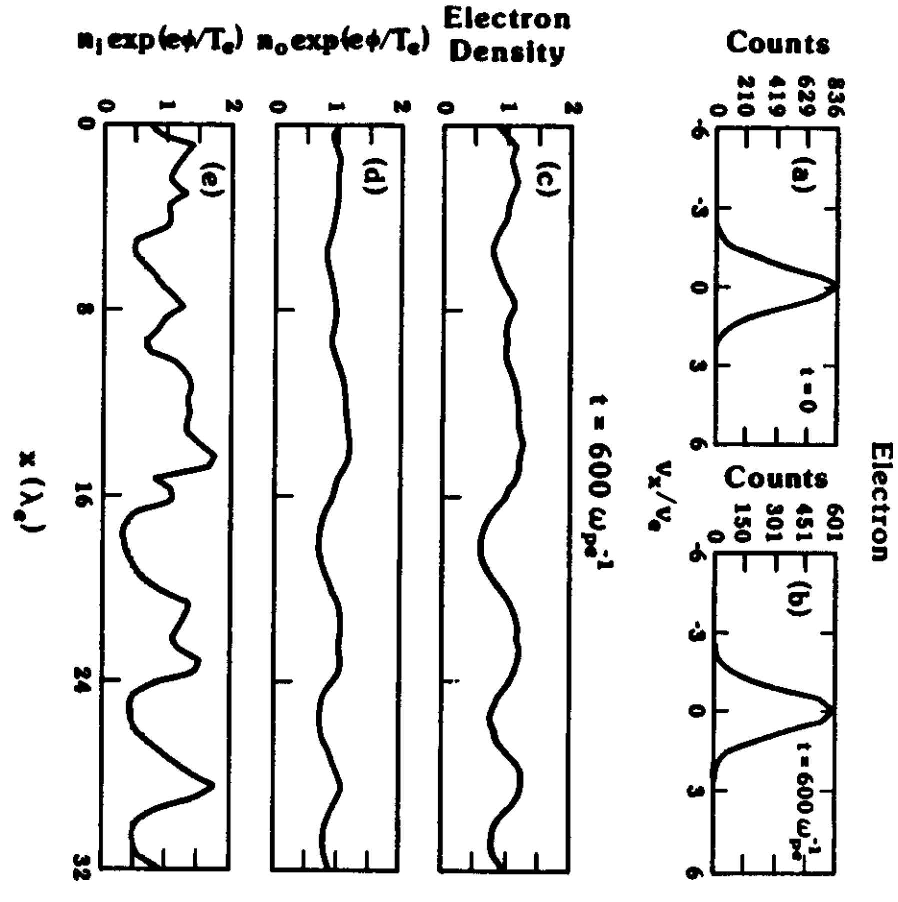


Protons
(a)
t = 0
(b)
t = 300 ω_{pe}^{-1}
(c)
t = 600 ω_{pe}^{-1}
$\frac{v_x}{v_p}$
11.0
5.5
0.0
-5.5
-11.0
0
8
16
24
32
x (λ_e)

23



Lithium
(a)
t = 0
(b)
t = 400 ω_pe^-1
(c)
t = 600 ω_pe^-1
v_x / v_Li
x (λ_e)



Electron
Counts
(a)
t = 0
(b)
t = 600 ω_pe^-1
v_x/v_e
Electron Density
n_o exp(eφ/T_e)
n_i exp(eφ/T_e)
(c)
(d)
(e)
t = 600 ω_pe^-1
x (λ_e)

Electrostatic Simulations with Boltzman electrons

$$\frac{d^2\phi}{dx^2} = -\rho = -\left(n_i - \exp(\phi/T_e)\right)$$

$$D(\phi) \equiv \frac{d^2\phi}{dx^2} + \rho(\phi)$$

$$D(\phi^{t+\Delta t}) = D(\phi^t) + \frac{dD}{d\phi}\Big|_{\phi^t}(\phi^{t+\Delta t} - \phi^t) + \cdots = 0$$

$$\frac{dD}{d\phi} = \frac{d^2}{dx^2} + \frac{d\rho}{d\phi} = \frac{d^2}{dx^2} - \frac{1}{T_e}\exp(\phi^t/T_e)$$

$$\frac{d^2\phi^t}{dx^2} + \left(n_i - \exp(\phi^t/T_e)\right) + \frac{d^2\phi^{t+\Delta t}}{dx^2} - \frac{\phi^{t+\Delta t}}{T_e}\exp(\phi^t/T_e)$$

$$-\frac{d^2\phi^t}{dx^2} + \frac{\phi^t}{T_e}\exp(\phi^t/T_e) = 0$$

$$\boxed{\frac{d^2\phi^{t+\Delta t}}{dx^2} - \frac{\exp(\phi^t/T_e)}{T_e}\phi^{t+\Delta t} = -\left(n_i - \exp(\phi^t/T_e)\right) - \frac{\phi^t}{T_e}\exp(\phi^t/T_e)}$$

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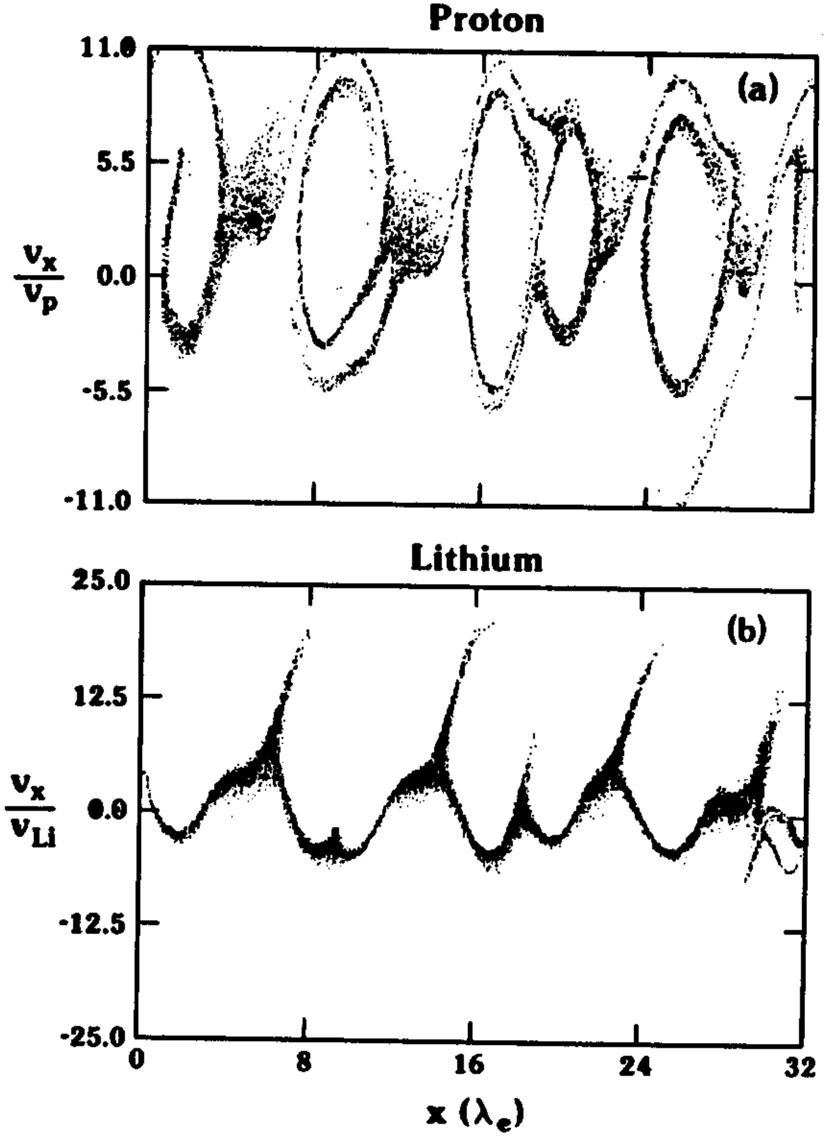
$t = 600\ \omega_{pe}^{-1}$



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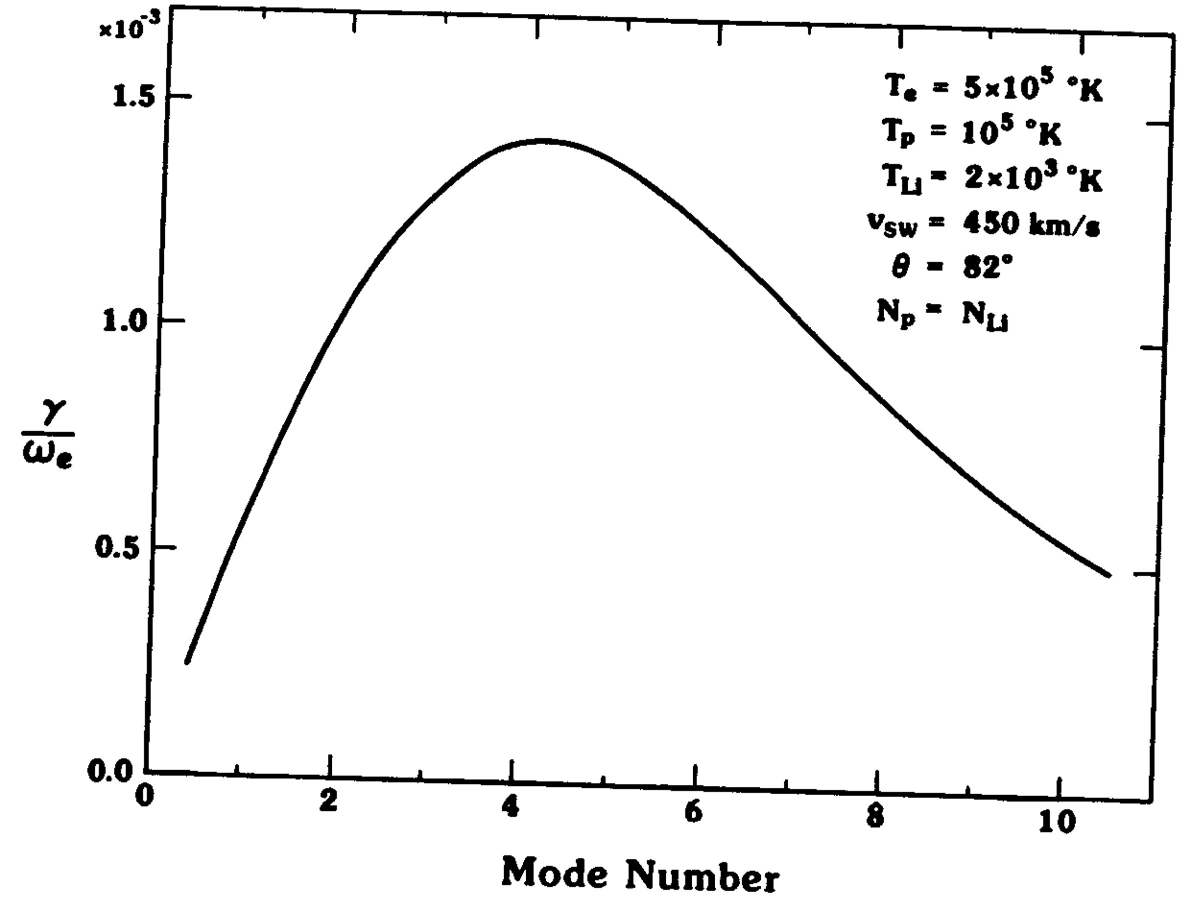
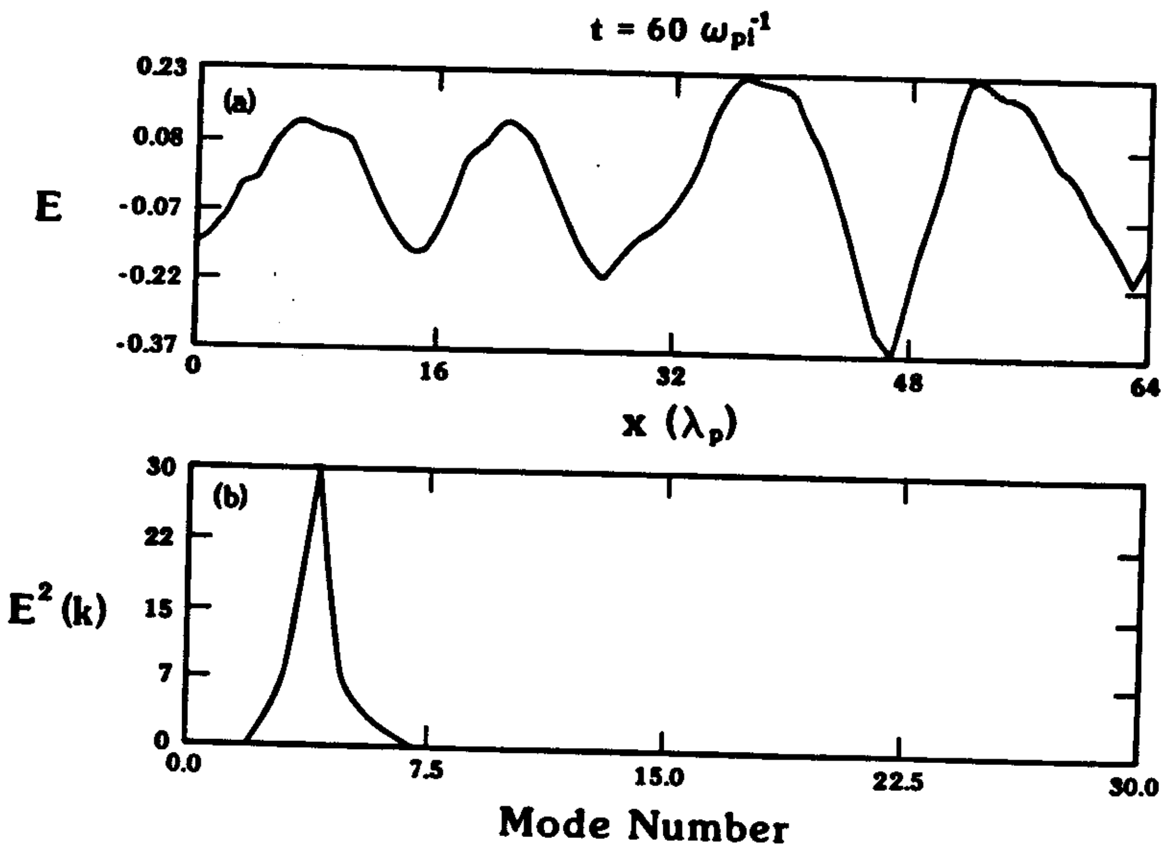
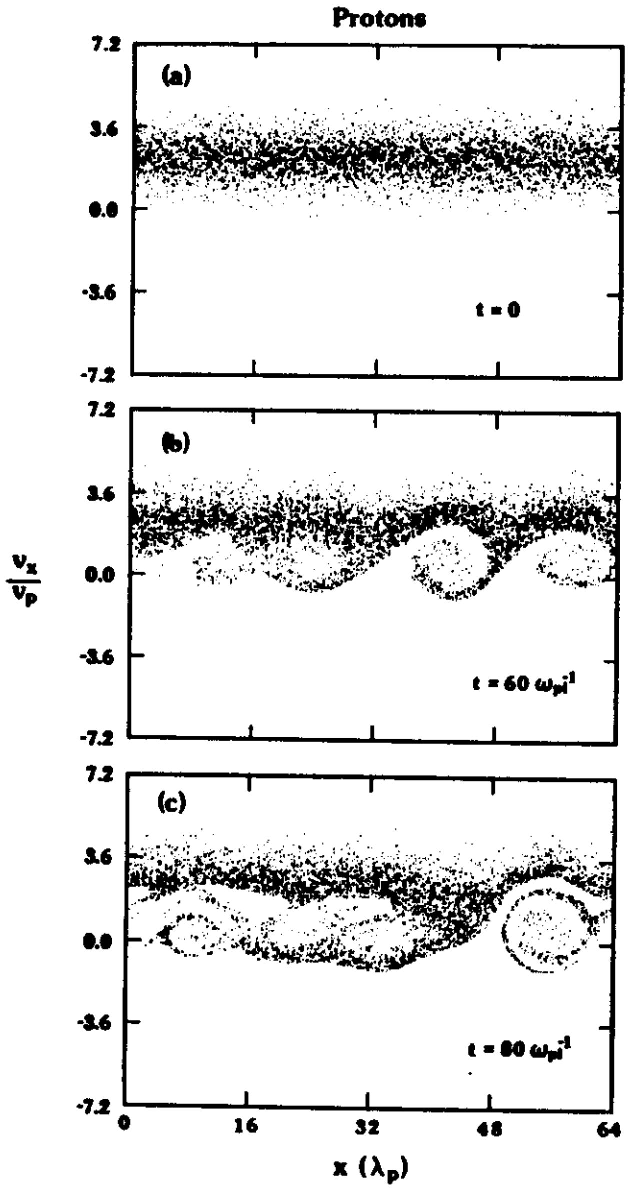
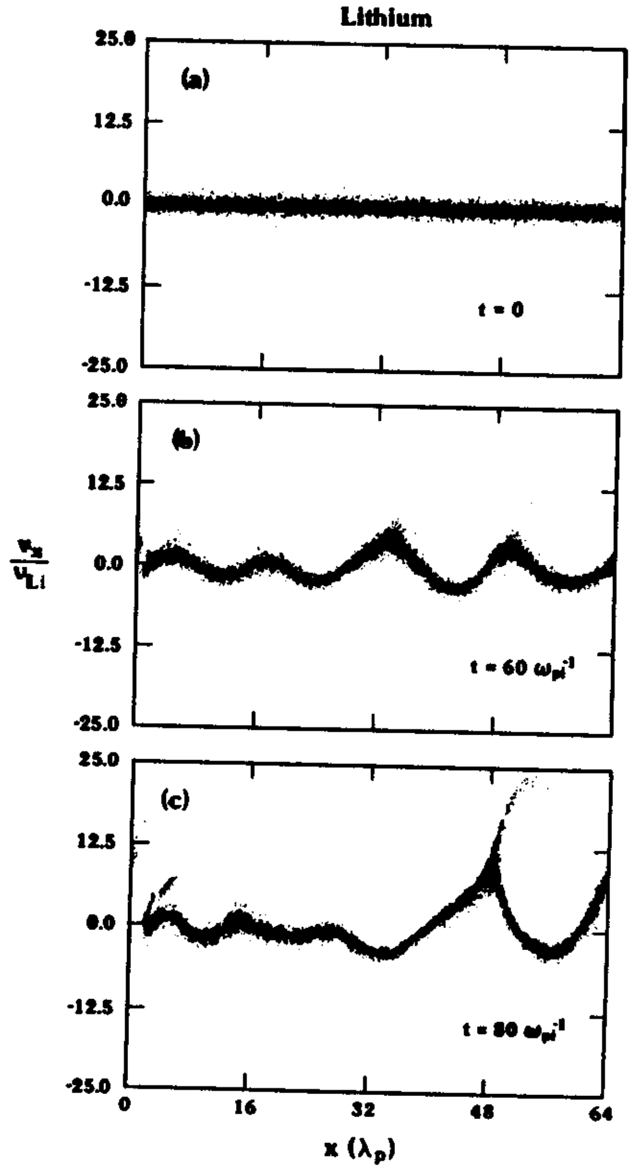


×10^-3
1.5
1.0
0.5
0.0
0
2
4
6
8
10
Mode Number
γ/ω_e
T_e = 5×10^5 °K
T_p = 10^5 °K
T_Li = 2×10^3 °K
v_SW = 450 km/s
θ = 82°
N_p = N_Li

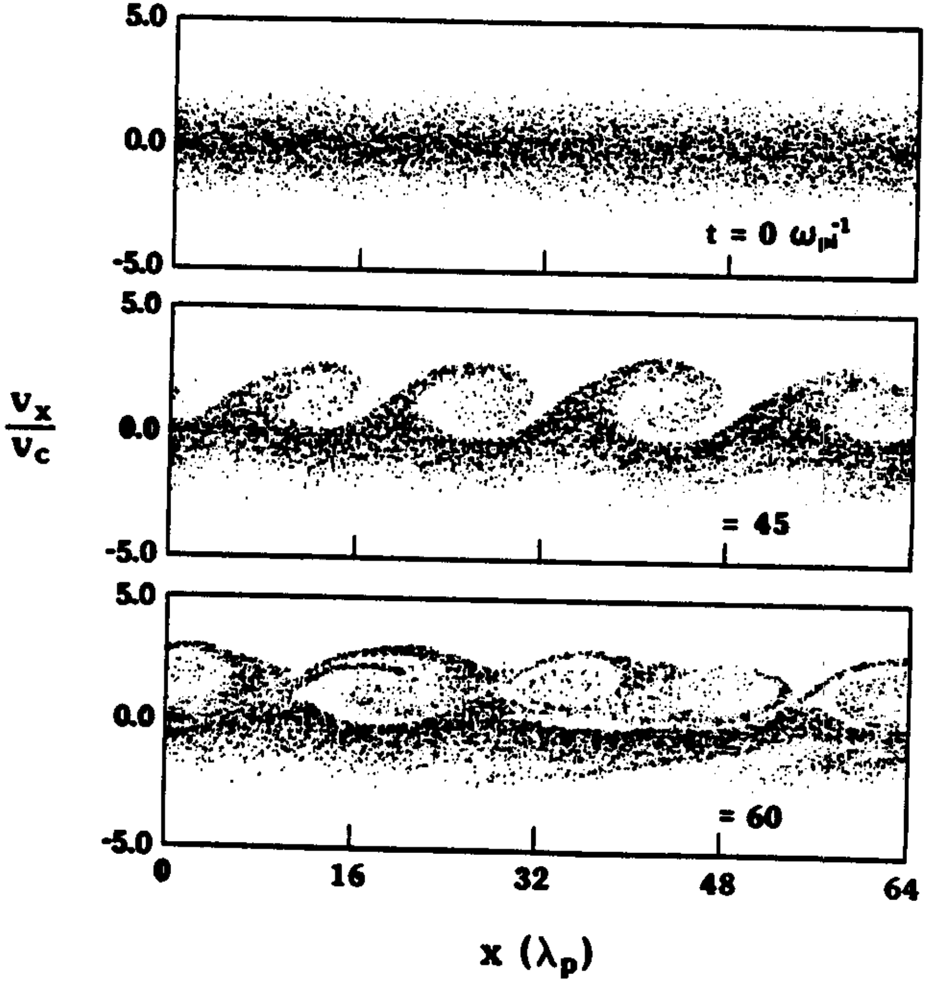


t = 60 ω_pi^-1
(a)
E
0.23
0.08
-0.07
-0.22
-0.37
0
16
32
48
64
x (λ_p)
(b)
E^2 (k)
30
22
15
7
0
0.0
7.5
15.0
22.5
30.0
Mode Number

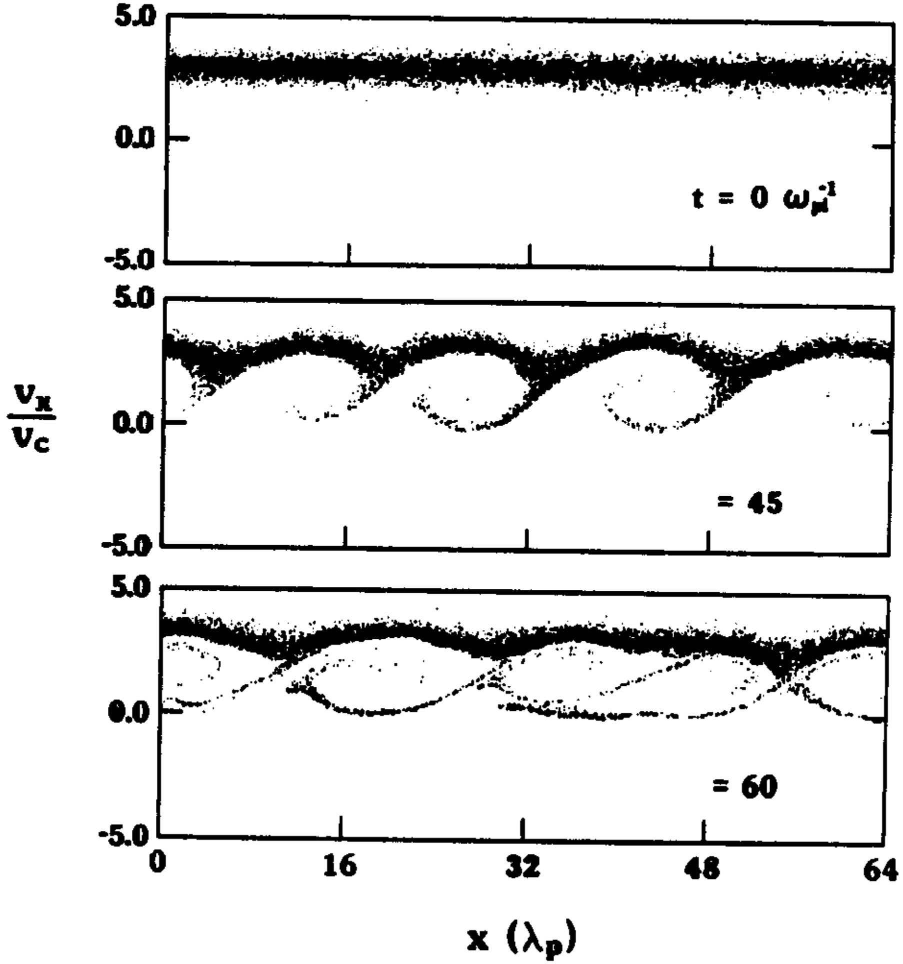

Protons
(a)
t = 0
(b)
t = 60 ω_{pi}^{-1}
(c)
t = 80 ω_{pi}^{-1}
$\frac{v_x}{v_p}$
7.2
3.6
0.0
-3.6
-7.2
0
16
32
48
64
x (λ_p)


Lithium
(a)
t = 0
(b)
t = 60 ω_{pi}^{-1}
(c)
t = 80 ω_{pi}^{-1}
$\frac{v_x}{v_{Li}}$
25.0
12.5
0.0
-12.5
-25.0
0
16
32
48
64
x (λ_p)

Protons (Core)



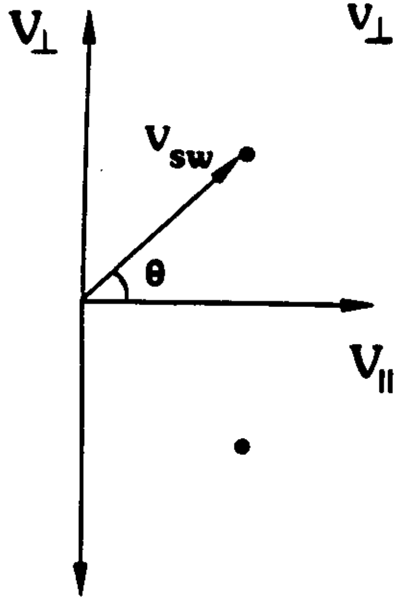
Protons (Beam)

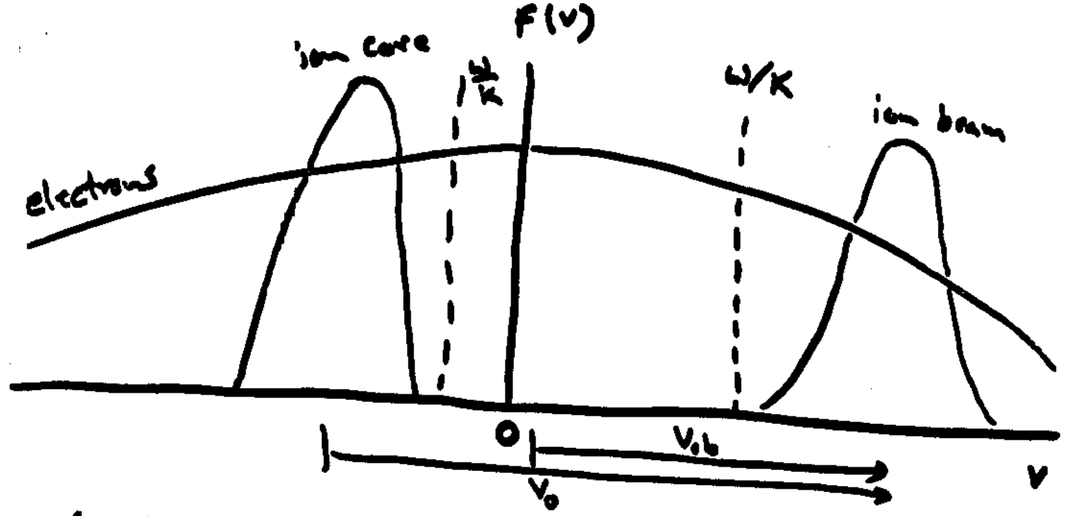


BEAM - RING

$$v_{\parallel} = v_{sw} \cos\theta$$

$$v_{\perp} = v_{sw} \sin\theta$$





1. ion/ion resonant instability:

$$\omega \simeq k_{\parallel} V_{ob} - \Omega_b$$

$$\omega > 0$$

2. ion/ion non resonant instability

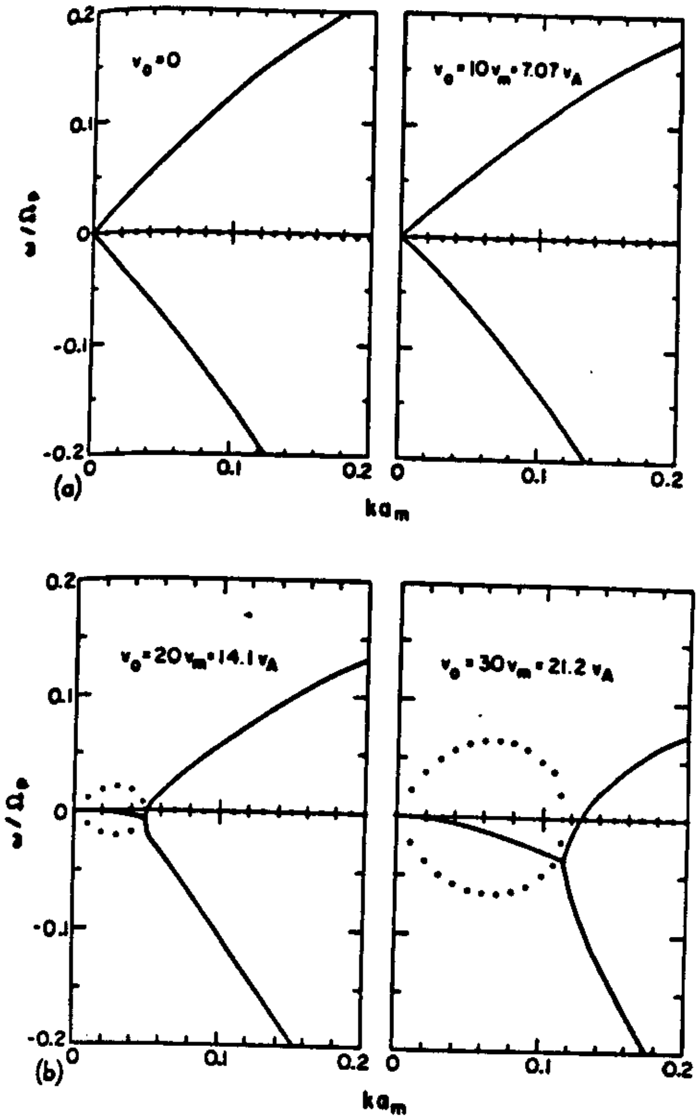
$$\omega < 0$$

$$1 - \frac{k^2 \lambda^2}{\omega^2} + \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} \eta_{\alpha} Z(\eta_{\alpha}^{\pm}) = 0$$

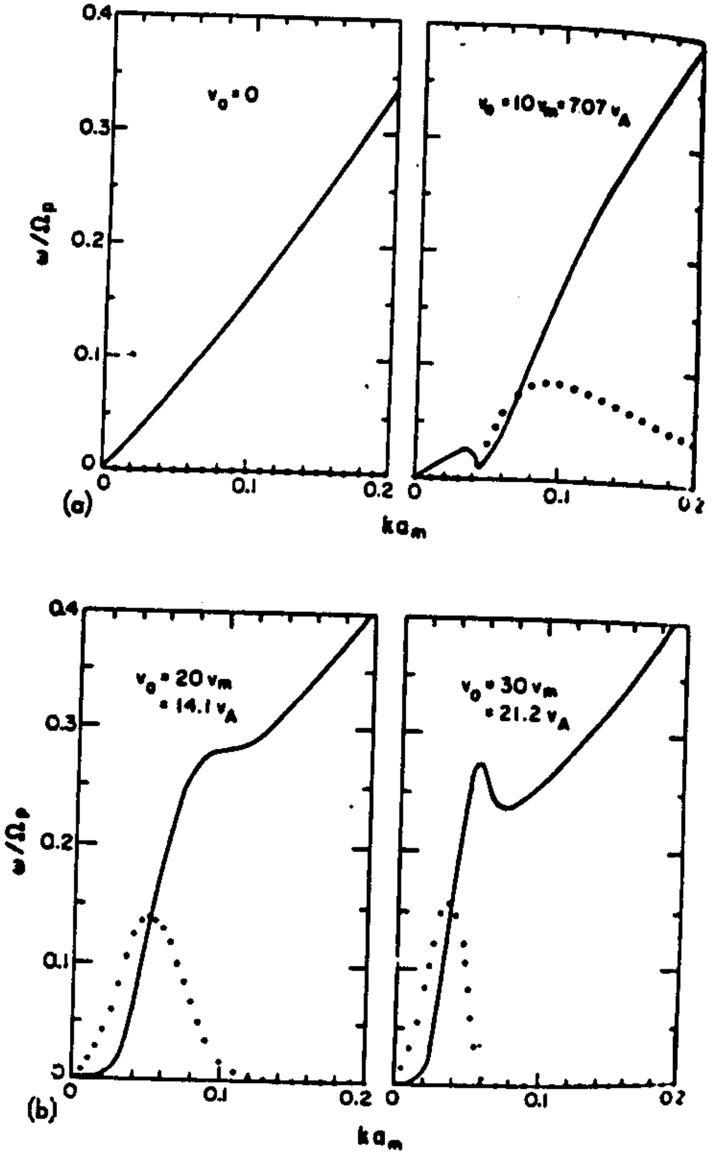
$$\eta_{\alpha} = (\omega - k_{\parallel} V_{o\alpha}) / \sqrt{2}\, k V_{\alpha}$$

$$\eta_{\alpha}^{\pm} = (\omega - k_{\parallel} V_{o\alpha} \pm \Omega_{o\alpha}) / \sqrt{2}\, k V_{\alpha}$$

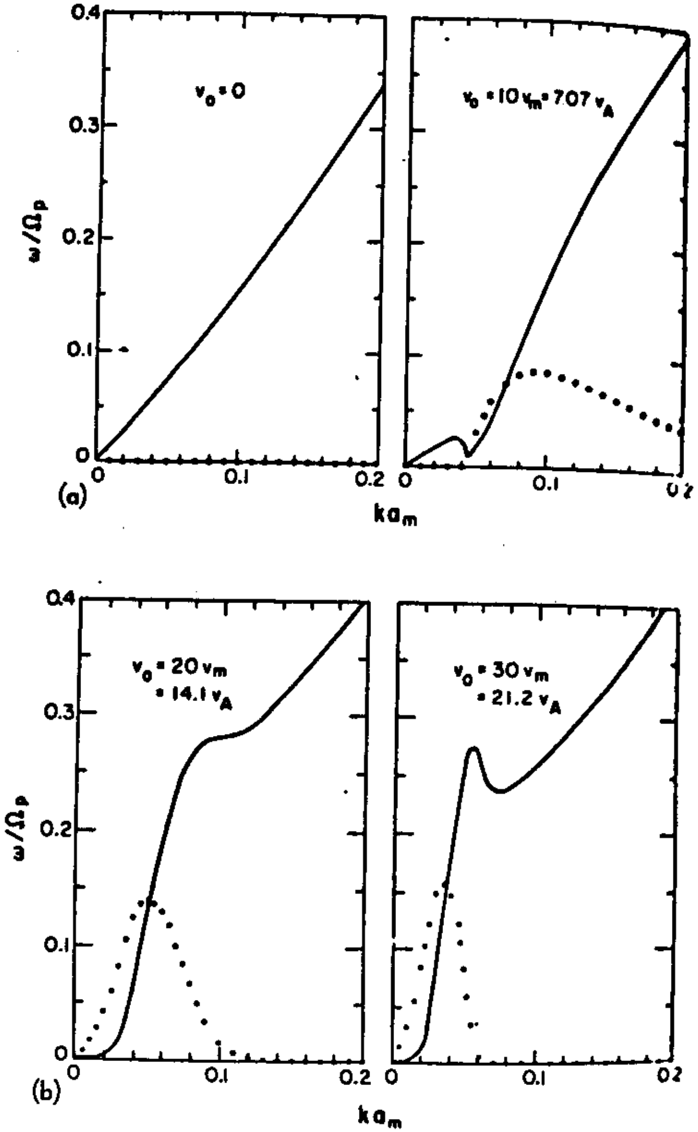
From Gary etal. (1984)



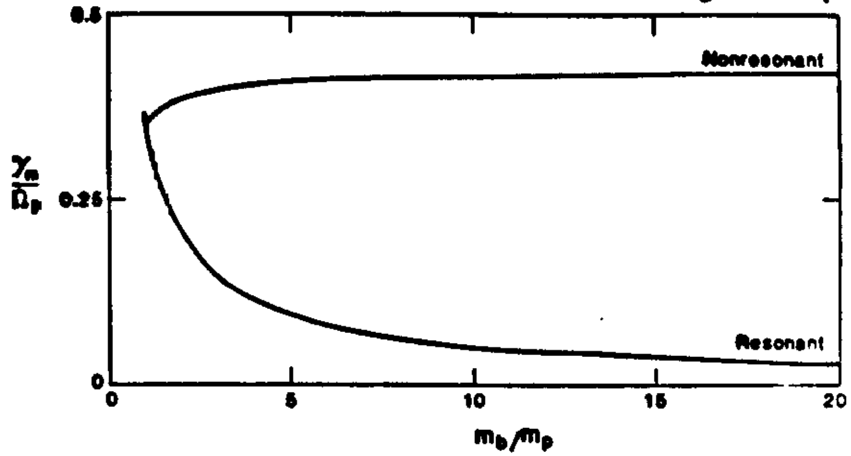
From Gary etal. (1984)



From Winske and Gary 1986



$V_0 = 10\, V_A$



resonant : $\gamma \sim \Omega_p \left(\frac{n_b}{2n_i}\right)^{1/3} \left(\frac{m_p}{m_b}\right)^{2/3}$

nonresonant : $\gamma \sim \frac{\Omega_p}{2} \frac{n_b}{n_i} \frac{V_0}{V_A}$

when $\frac{n_b}{n_i} \ll 1$ and $\frac{V_0}{V_A} \gg 1$

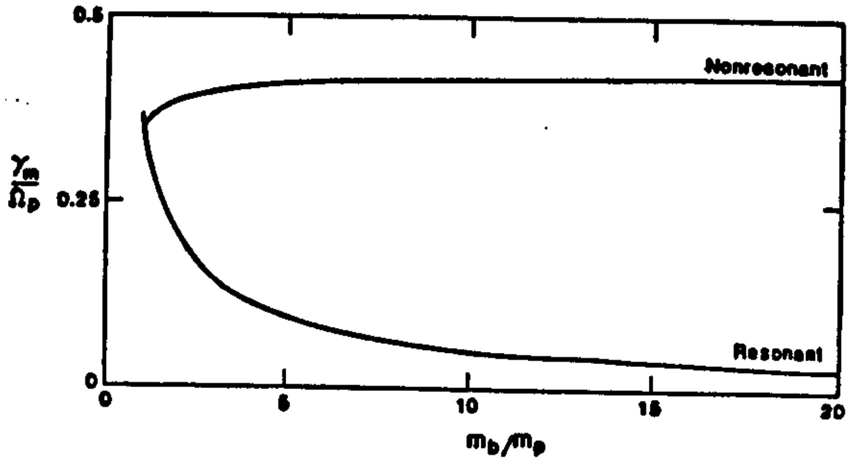
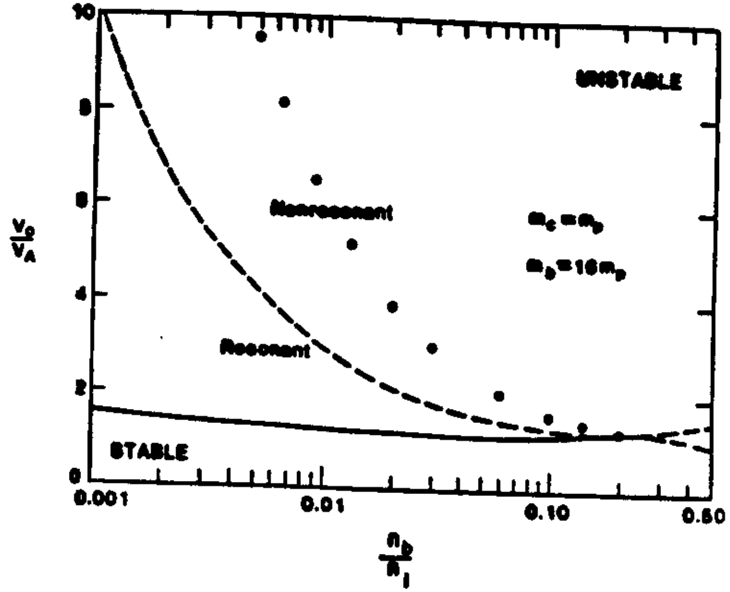


Figure 2



From Winske and Gary 1986

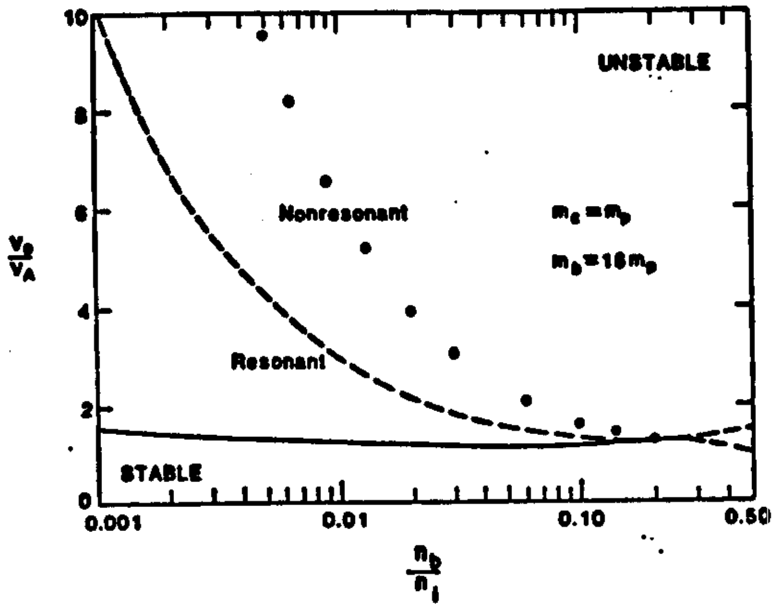
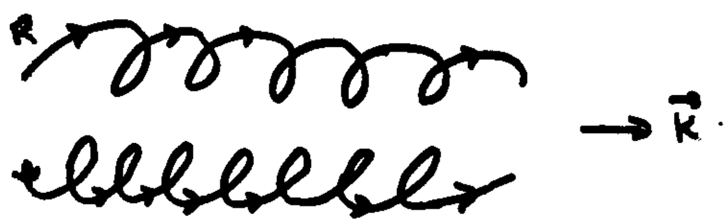


Figure 4

Wave Properties

1. Both modes are right-hand Polarized
2. resonant mode has left-handed helecity while the nonresonant mode has right-handed helecity



3. Both modes are circularly Polarized and noncompressional when Propagating along the field line, and elliptically Polarized and compressional at oblique angles.

Hybrid Simulation model

(Particle ion, massless fluid electrons)

ions : $m \frac{d\vec{V}}{dt} = e\left(\vec{E} + \frac{\vec{V}\times\vec{B}}{c}\right)$

electrons :

$n_e m_e \frac{d\vec{V}_e}{dt} = 0 = -e n_e\left(\vec{E} + \frac{\vec{V}_e\times\vec{B}}{c}\right) - \frac{\partial}{\partial x} P_e$ mom. eq.

$\frac{3}{2}\frac{\partial}{\partial t}(n_e T_e) + \frac{\partial}{\partial x}\left(\frac{3}{2} n_e T_e V_{ex}\right) + n_e T_e \frac{\partial V_{ex}}{\partial x} = Q$ energy eq.

Charge neutrality : $n_e = n_i$

Zero current in x-direction : $V_{ex} = \bar{V}_{ix}$

Maxwell's eqs. :

$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$

$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{J}$

$\nabla\cdot\vec{B} = 0$

obtain E_x from electron momentum eq.

ions : $m \frac{d\vec{V}}{dt} = e\left(\vec{E} + \frac{\vec{V}\times\vec{B}}{c}\right)$

electrons :

$n_e m_e \frac{d\vec{V}_e}{dt} = 0 = -e n_e\left(\vec{E} + \frac{\vec{V}_e\times\vec{B}}{c}\right) - \frac{\partial}{\partial x} P_e \hat{x}$ mom. eq.

$\frac{3}{2}\frac{\partial}{\partial t}(n_e T_e) + \frac{\partial}{\partial x}\left(\frac{3}{2} n_e T_e V_{ex}\right) + n_e T_e \frac{\partial V_{ex}}{\partial x} = Q$ energy eq.

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$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{J}$

$\nabla\cdot\vec{B} = 0$

obtain E_x from electron momentum eq.

trans. comp. of the mom. eq.

$$\vec{J}_T = \eta^{-1}\left(\vec{E} + \frac{\vec{V}_e \times \vec{B}}{c}\right)_T \qquad (1)$$

$$B_y = -\frac{\partial A_z}{\partial x}, \quad B_z = \frac{\partial A_y}{\partial x}, \quad E_{y,z} = -\frac{1}{c}\frac{\partial A_{y,z}}{\partial t}$$

$$\nabla\cdot\vec{B} = 0 \Rightarrow B_x = \text{Const.}$$

$$\nabla\times\vec{B} = \frac{4\pi}{c}J \quad \begin{cases} \dfrac{\partial^2 A_y}{\partial x^2} = -\dfrac{4\pi}{c}J_y \\ \dfrac{\partial^2 A_z}{\partial x^2} = -\dfrac{4\pi}{c}J_z \end{cases} \qquad (2)$$

substitute (2) back into (1), then:

$$\begin{cases} \dfrac{\partial^2 A_y}{\partial x^2} = f_1\left(\dfrac{\partial A_y}{\partial t}, \dfrac{\partial A_z}{\partial t}, \dfrac{\partial A_y}{\partial x}, \dfrac{\partial A_z}{\partial x}\right) \\ \dfrac{\partial^2 A_z}{\partial x^2} = f_2(\text{-} \quad \text{--} \quad \text{--} \quad \text{--}) \end{cases} \qquad (3)$$

$$u_j^n = A_y \qquad w_j^n = A_z \qquad \begin{array}{l} n = \text{time step} \\ j = \text{cell } \# \end{array}$$

$$\frac{\partial^2 A_y}{\partial x^2} = \frac{u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}}{\Delta x^2}, \quad \frac{\partial A_y}{\partial x} = \frac{u_{j+1}^{n+1} - u_{j-1}^{n+1}}{2\Delta x}$$

$$\frac{\partial A_y}{\partial x} = \frac{u_{j+1}^{n+1} - u_{j+1}^{n}}{\Delta t}$$

now eqs. (3) can be written as

$$\tilde{\tilde{A}}_j \cdot \tilde{X}_{j+1} + \tilde{\tilde{B}}_j \cdot \tilde{X}_j + \tilde{\tilde{C}}_j \cdot \tilde{X}_{j-1} = \tilde{D}_j$$

where $\tilde{X}_j = \begin{pmatrix} u_j^{n+1} \\ u_j^{n+1} \end{pmatrix}$

and A_j, B_j, C_j are 2x2 matrices with known elements. similarly D_j is a known vector.

$$\tilde{X}_j = \tilde{\tilde{E}}_j \cdot \tilde{X}_{j+1} + \tilde{F}_j \qquad j = 0, 1, \cdots, N$$

$$\tilde{\tilde{E}}_j = -\left[\tilde{\tilde{B}}_j + \tilde{\tilde{C}}_j \cdot \tilde{\tilde{E}}_{j-1}\right]^{-1} \tilde{\tilde{A}}_j$$

$$\tilde{F}_j = \left[\tilde{\tilde{B}}_j + \tilde{\tilde{C}}_j \cdot \tilde{\tilde{E}}_{j-1}\right]^{-1} \cdot \left[\tilde{D}_j - \tilde{\tilde{C}}_j \cdot \tilde{F}_{j-1}\right]$$

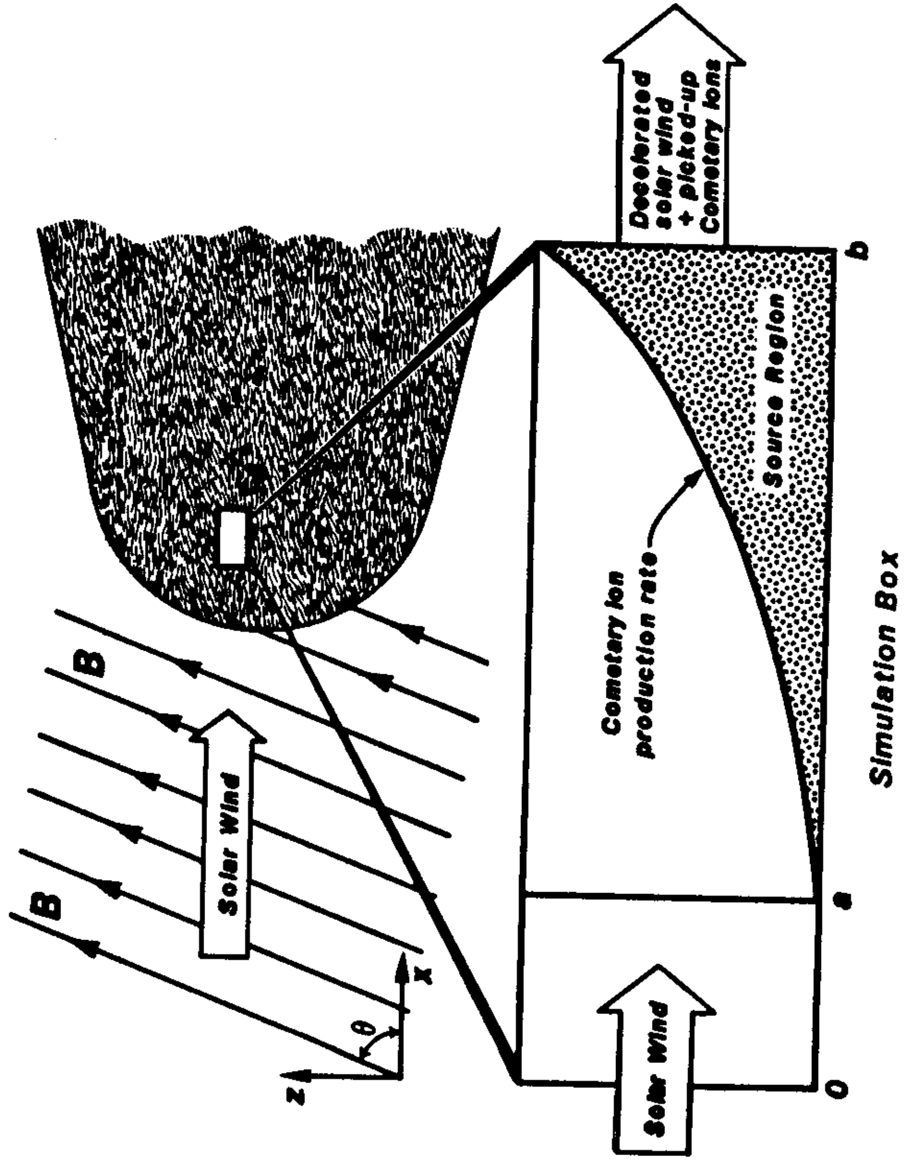
Thus the vector potential and the transverse components of the $\vec{E}$ and $\vec{B}$ are found.

Use the vector potential to find the current using eq. (2) which in turn gives $\vec{V}_e$ since $\vec{V}_i$ are known.

Use electron energy eq. to Calcute T_e or P_e

then the longitudinal electric field

$$E_x = -\frac{1}{c}(\vec{V}_e \times B_x) - \frac{1}{n_e}\frac{\partial P_e}{\partial x}$$



Model Parameters

Solar wind
$$\begin{cases} V_{sw} \equiv V_0 = 6\,V_A \\ \dfrac{\omega_{pi}}{\Omega_p} = 4000 \\ \beta_p = \beta_e = 1 \end{cases}$$

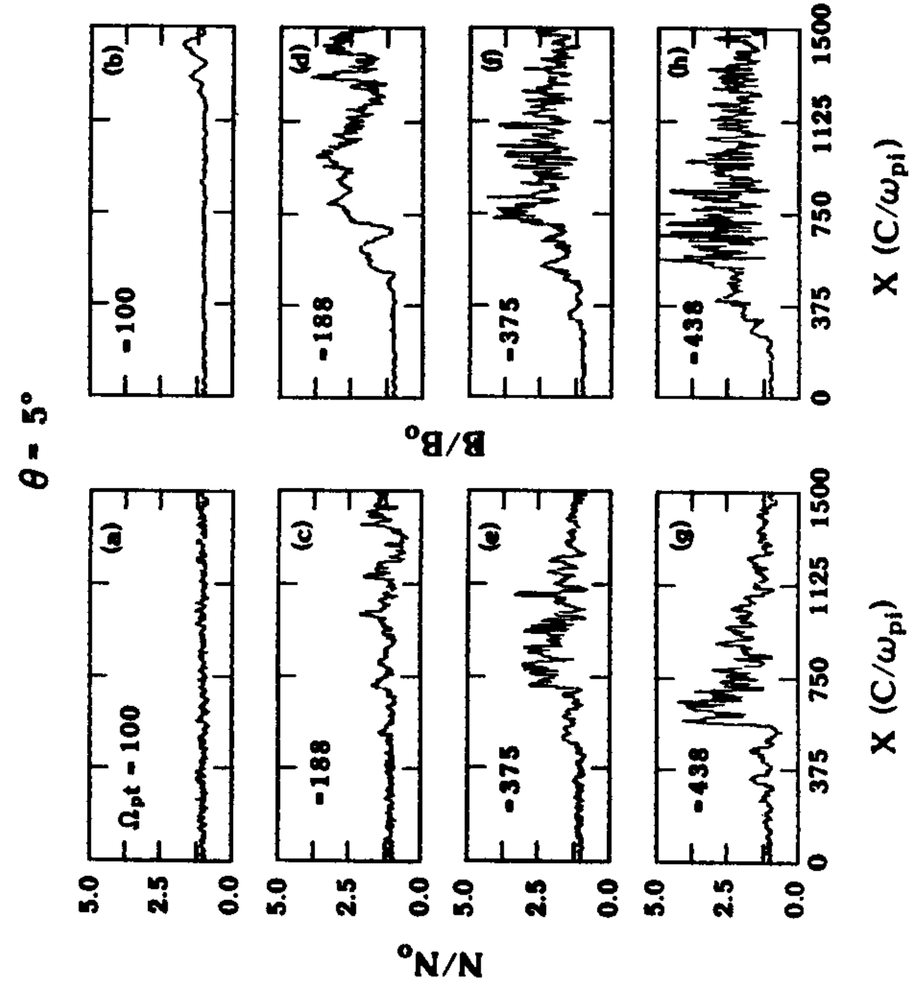
Simulation Param.
$$\begin{cases} L \equiv \text{syst. size} = 1500\ c/\omega_{pi} \\ \#\ \text{Part.} \sim 3\times10^5 \\ \#\ \text{cells} = 500 \\ \Delta t = 0.125\,\Omega_p^{-1} \end{cases}$$

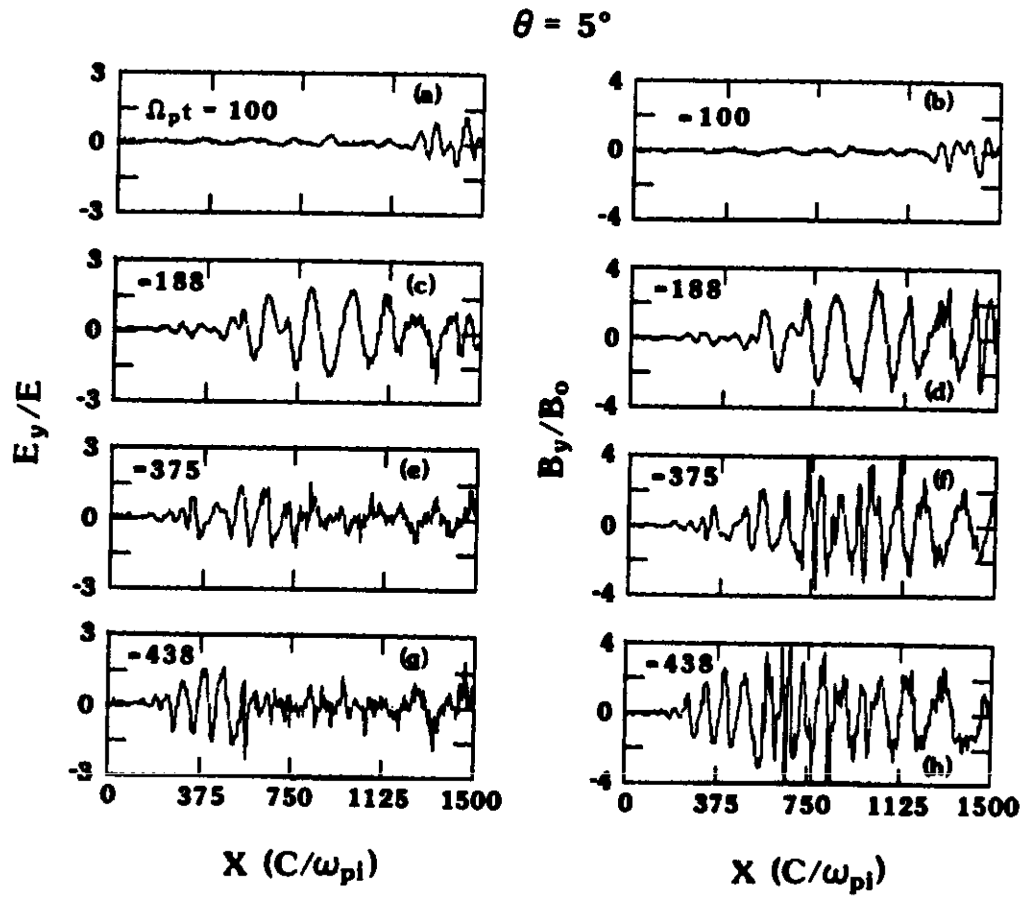
Cometary ions
$$\begin{cases} \text{Represented by: } O^+ \\ x_a = 75\ c/\omega_{pi} \\ x_b = 1500\ c/\omega_{pi} \end{cases}$$

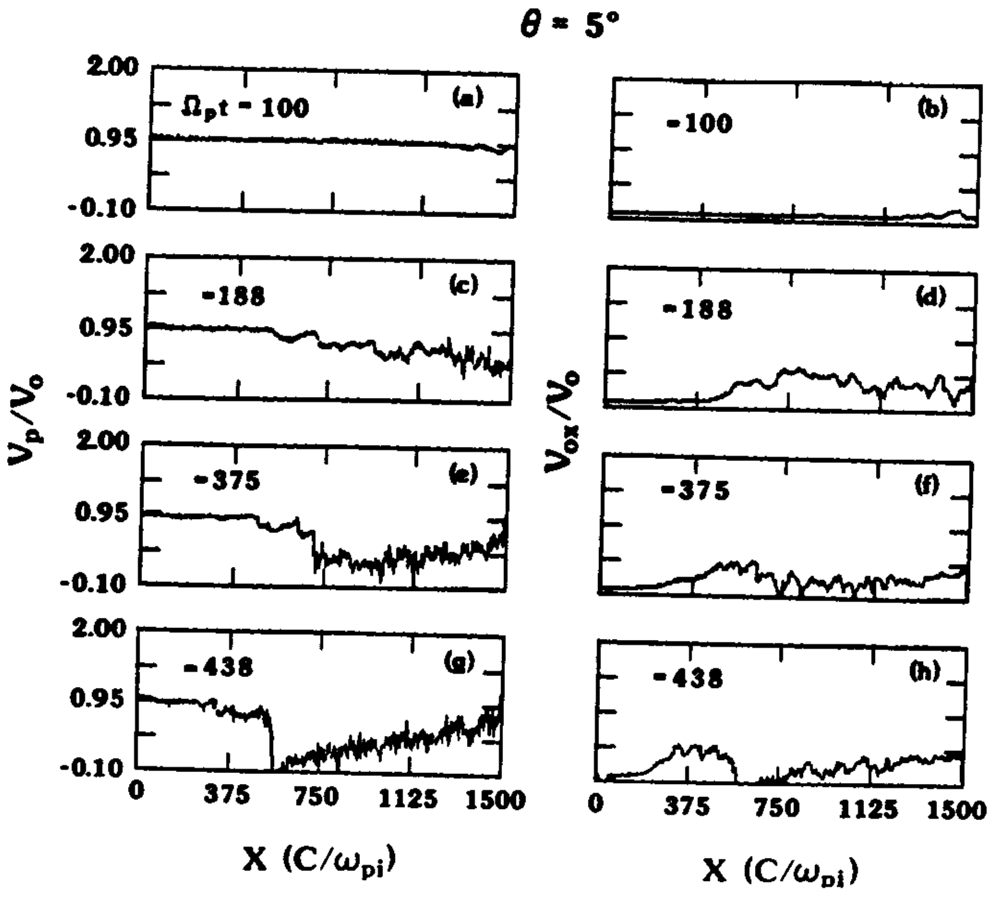
Assuming $n_p = 10\ cm^{-3}$ & $G = 10^{30}$ mol/s

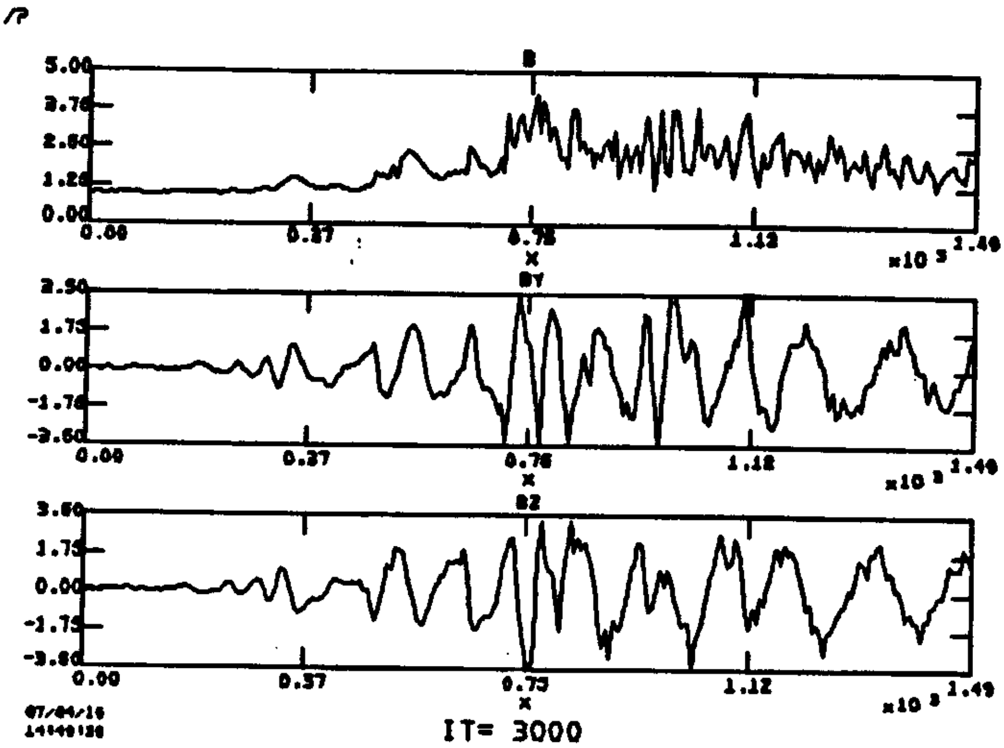
$L \sim 10^5$ Km

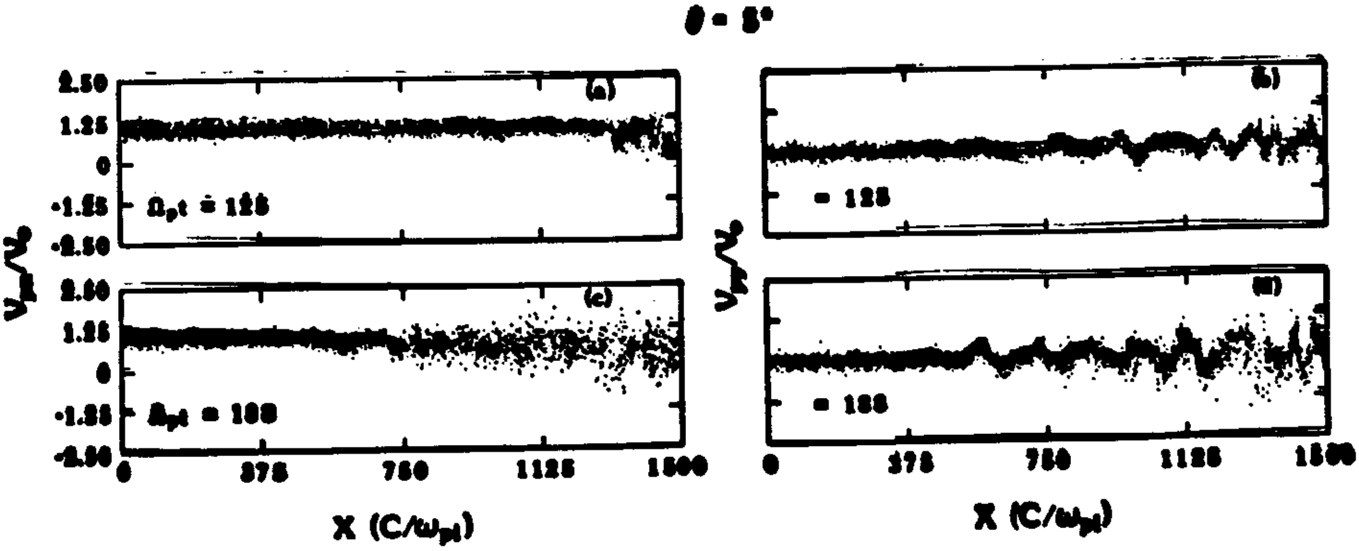
$R_a \sim 2\times10^5$ Km



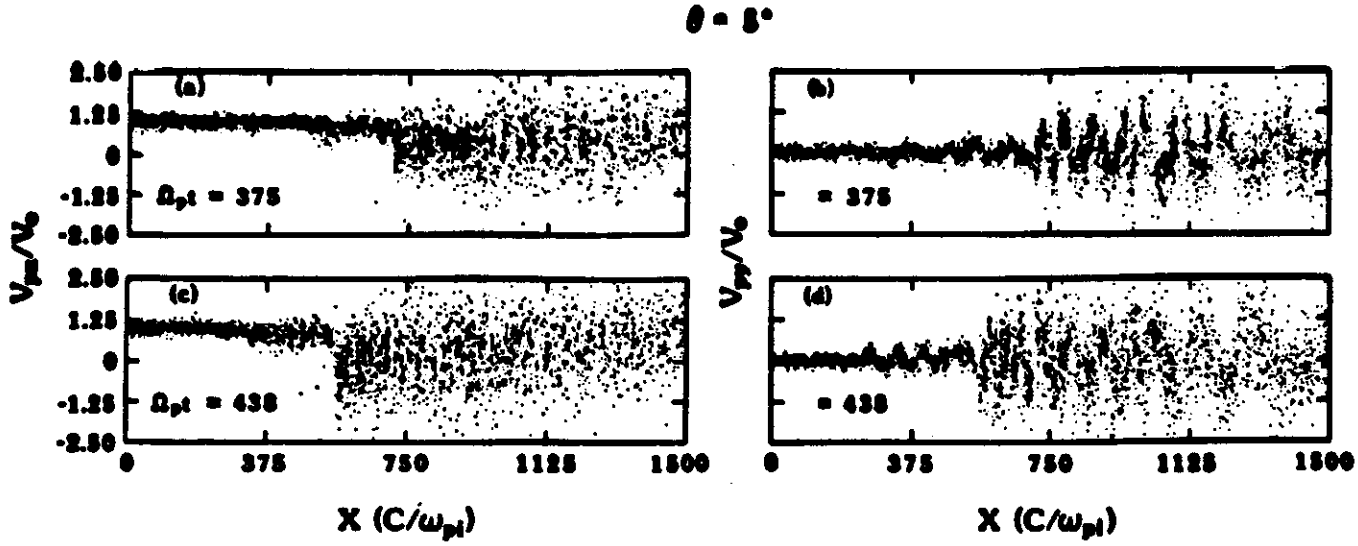

θ = 5°
E_y/E
B_y/B_o
Ω_pt = 100
=100
=188
=375
=438
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
X (C/ω_pi)


θ = 5°
V_p/V_o
V_ox/V_o
Ω_pt = 100
=100
=188
=375
=438
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
X (C/ω_pi)

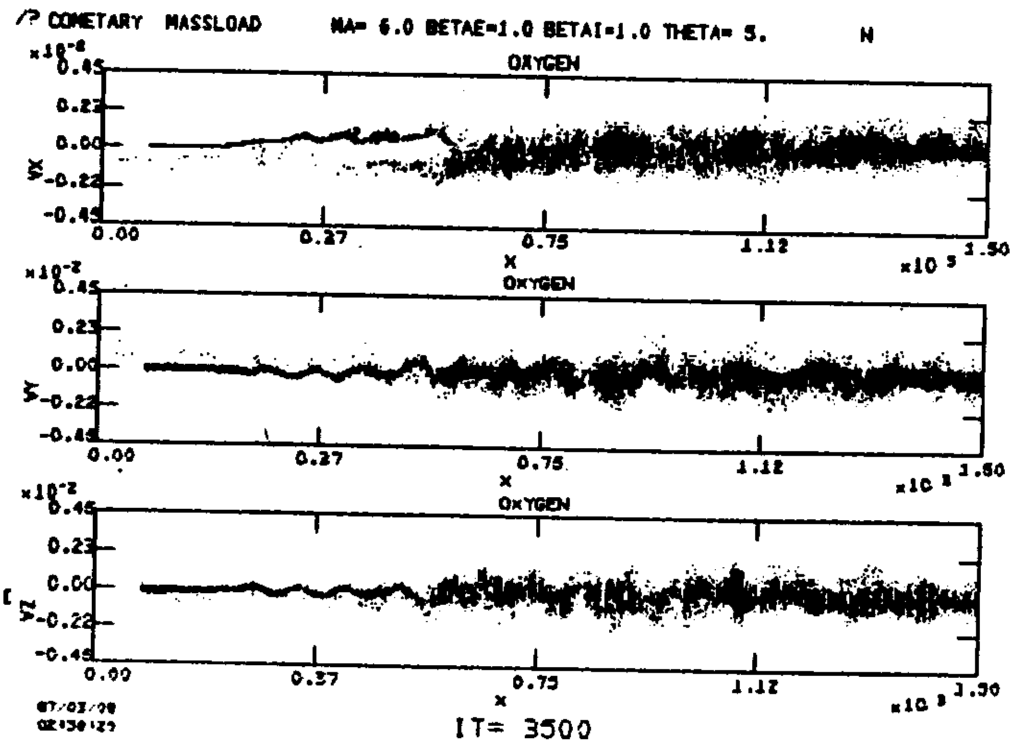

B
BY
BZ
X
x10 3
IT= 3000


X (C/ω_pi)
X (C/ω_pi)

52


θ = 8°
(a)
Ω_{pt} = 375
(b)
= 375
(c)
Ω_{pt} = 438
(d)
= 438
V_{px}/V_o
V_{py}/V_o
2.50
1.25
0
-1.25
-2.50
0
375
750
1125
1500
X (C/ω_{pi})
X (C/ω_{pi})

53


/? COMETARY MASSLOAD
MA= 6.0 BETAE=1.0 BETAI=1.0 THETA= 5.
N
OXYGEN
VX
VY
VZ
x10^-2
0.45
0.22
0.00
-0.22
-0.45
0.00
0.37
0.75
1.12
1.50
x10^3
X
IT= 3500

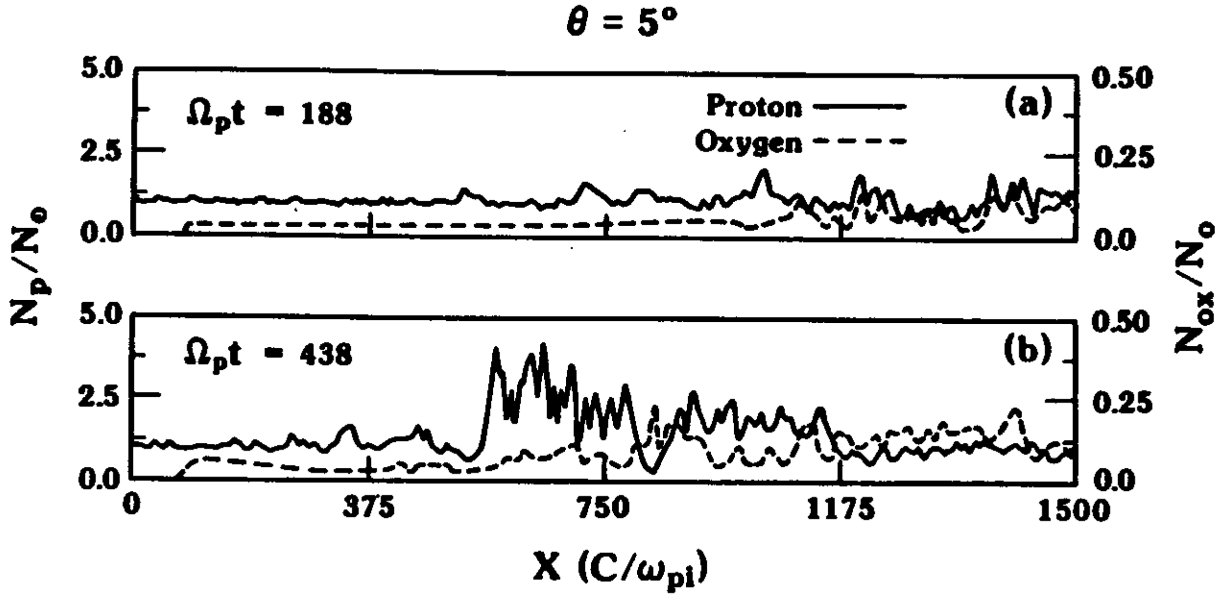
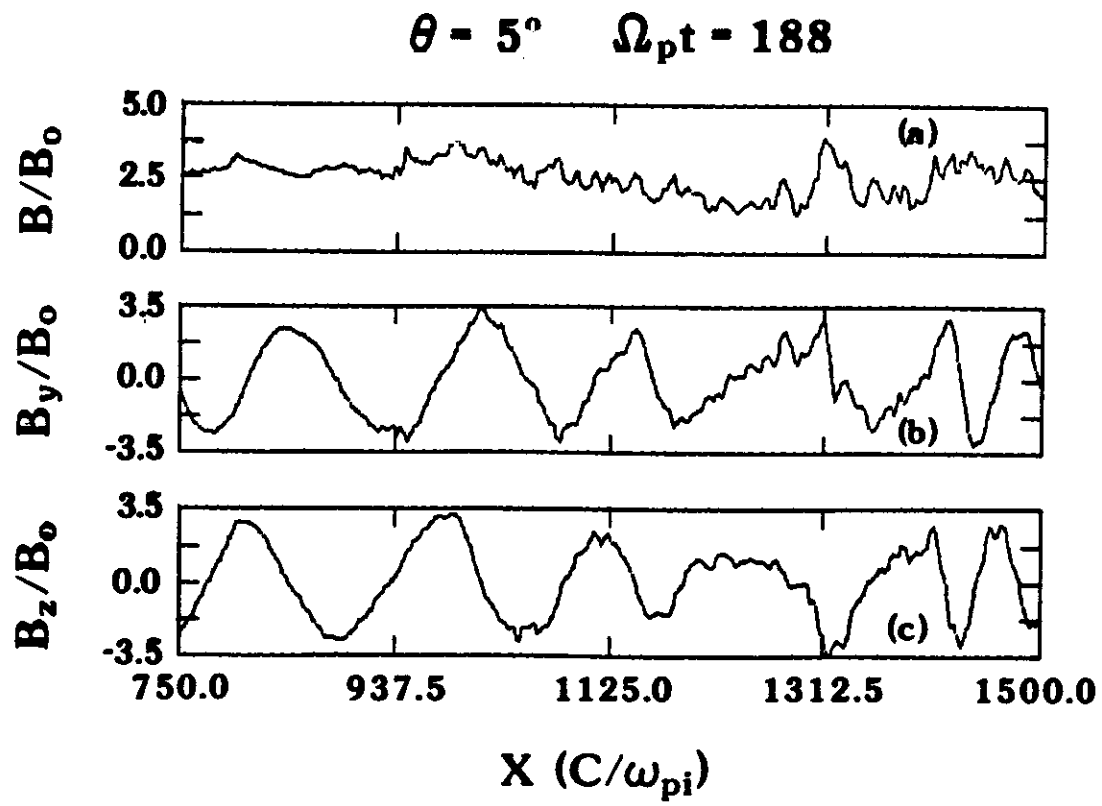
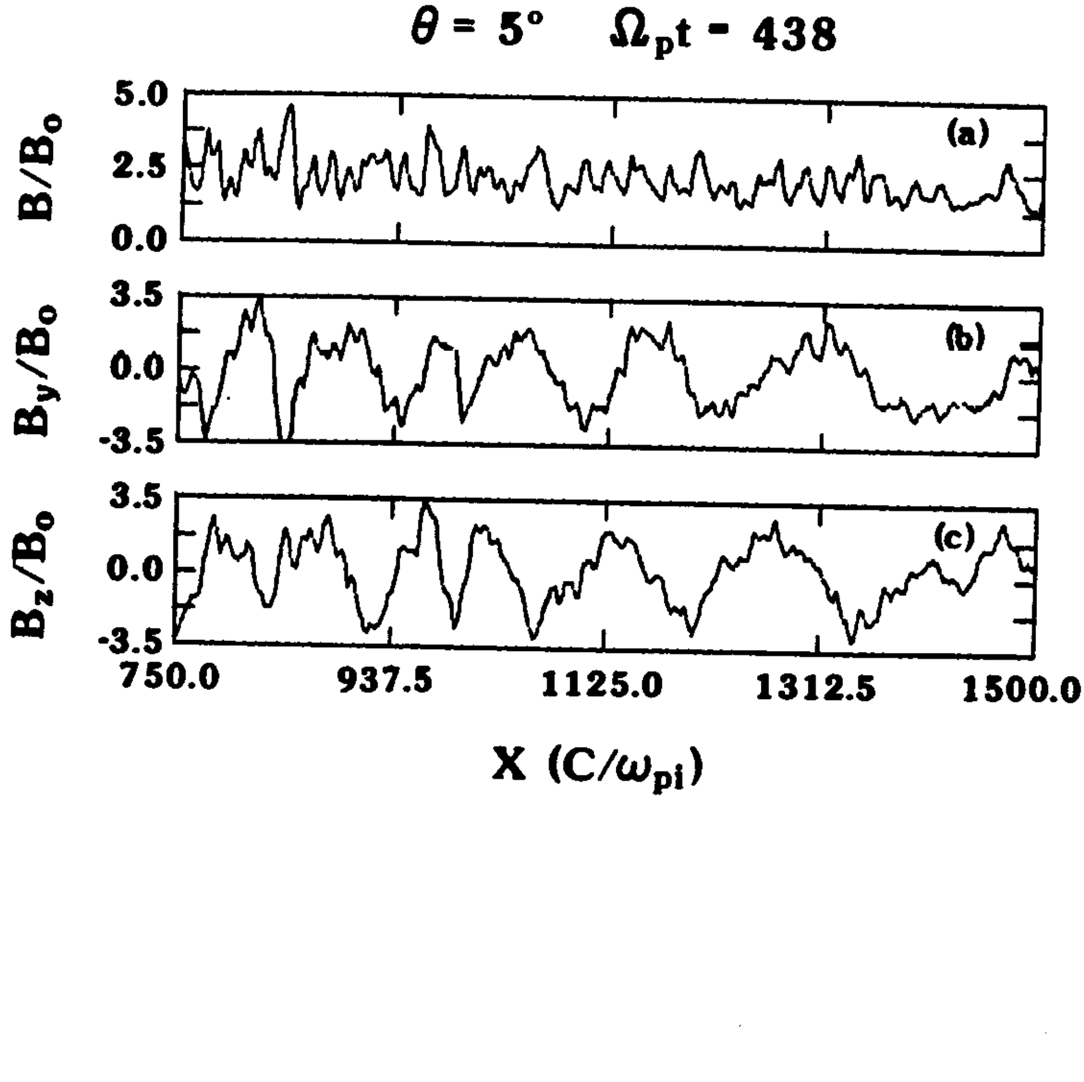

θ = 5°
Ωpt = 188
Proton
Oxygen
(a)
Ωpt = 438
(b)
Np/No
Nox/No
5.0
2.5
0.0
0.50
0.25
0.0
0
375
750
1175
1500
X (C/ωpi)

Figure 22


θ = 5° Ωpt = 188
B/Bo
By/Bo
Bz/Bo
5.0
2.5
0.0
3.5
0.0
-3.5
(a)
(b)
(c)
750.0
937.5
1125.0
1312.5
1500.0
X (C/ωpi)

56



θ = 5° Ωpt - 438
B/Bo
5.0
2.5
0.0
(a)
By/Bo
3.5
0.0
-3.5
(b)
Bz/Bo
3.5
0.0
-3.5
(c)
750.0
937.5
1125.0
1312.5
1500.0
X (C/ωpi)