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SPRING COLLEGE ON PLASMA PHYSICS

(25 May - 19 June 1987)

COLLECTIVE INSTABILITIES IN

COMETARY ENVIRONMENT

K. Hizanidis

University of Maryland
College Park
U.S.A.

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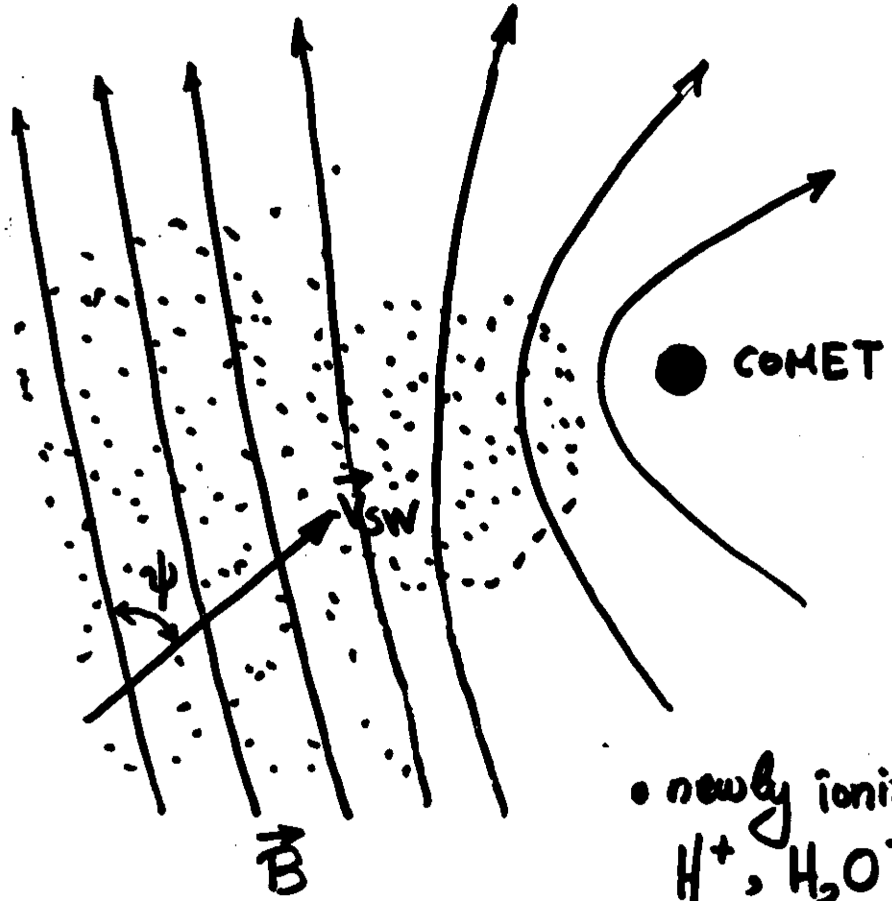
Kyriakos Hizanidis
University of Maryland, College Park
USA.



CHARGE EXCHANGE

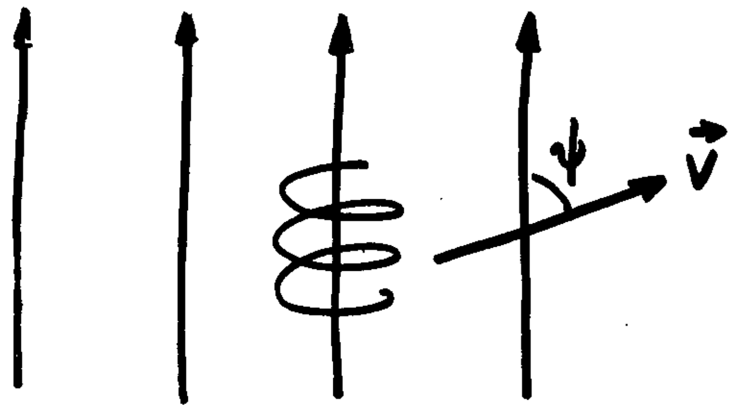
PHOTOIONIZATION BY UV SOLAR RADIATION

- newly ionized H^+, H_2O^+
- solar wind plasma, H^+, e^-

BRAKING OF SW

- Mass Loading ($L >> L_{\text{Bow Shock}}$)
- [illegible]URBULENT COUPLING

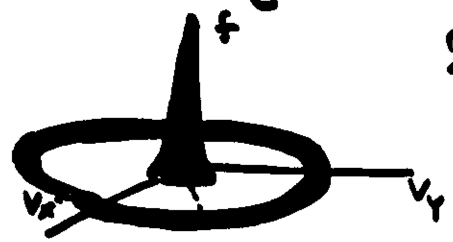
LOCALLY IN SW-FRAME



IONS ARE "PICKED UP" BY SW $\vec{E}$-FIELD

$$\vec{E} = -\frac{\vec{v} \times \vec{B}}{c}$$

$\psi = \frac{\pi}{2}$



RING

$u = V_{SW} \sin\psi = V_{SW}$

$\psi = 0$

FIELD-ALIGNED BEAM

LINEAR RESPONSE

PLASMA IS INHERENTLY CAPABLE OF SUSTAINING RICH CLASSES OF E-M PHENOMENA

- COLLECTIVE RESPONSE TO EXTERNAL DISTURBANCES
- INSTABILITIES
- ADJUSTMENT

$\vec{E}(\vec{r},t) \longrightarrow \vec{J}(\vec{r},t)$

LINEAR RESPONSE ASSUMPTION

- Weak disturbances

$$\vec{J}(\vec{r},t) = \int_{VOL} d\vec{r}' \int_{-\infty}^{t} dt' \overleftrightarrow{\sigma}(\vec{r},t;\vec{r}',t') \cdot \vec{E}(\vec{r}',t')$$

CAUSALITY

LINEAR RESPONSE FUNCTION INDEPENDENT OF $\vec{E}$

- UNPERTURBED STATE Homogeneous & Stationary

TRANSLATIONAL INVARIANCE

$$\vec{J}(\vec{k},\omega) = \overleftrightarrow{\sigma}(\vec{k},\omega) \cdot \vec{E}(\vec{k},\omega)$$

MAXWELL EQUATIONS

↓

DISPERSION RELATION

DISPERSION RELATION

DIELECTRIC TENSOR $\overleftrightarrow{\epsilon}(\vec{k},\omega)$

CONDUCTIVITY TENSOR $\overleftrightarrow{\sigma}(\vec{k},\omega)$

DISPERSION TENSOR $\overleftrightarrow{D}(\vec{k},\omega)$

$$\overleftrightarrow{\epsilon} = \overleftrightarrow{I} + 4\pi i\omega \overleftrightarrow{\sigma}$$

$$\overleftrightarrow{\sigma}(\vec{r},t) = \frac{1}{2\pi V} \sum_{\vec{k}} \int_{c} d\omega\, \overleftrightarrow{\sigma}(\vec{k},\omega)\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\overleftrightarrow{D}(\vec{k},\omega) = \overleftrightarrow{\epsilon}(\vec{k},\omega) - \eta^2\left(\overleftrightarrow{I} - \frac{\vec{k}\vec{k}}{k^2}\right) \; ; \; \eta = \frac{kc}{\omega}$$

$$\overleftrightarrow{D} \cdot \vec{E} = -4\pi i\omega \vec{J}_{EXT}$$

WITH NO EXTERNAL AGITATION THE PLASMA CAN ONLY SUPPORT COLLECTIVE MODES THAT SATISFY ($\vec{J}_{EXT} = \vec{0}$ THEN)

$$\boxed{\det\left|\overleftrightarrow{D}(\vec{k},\omega)\right| = 0}$$

HOMOGENEOUS, ANISOTROPIC NONEQUILIBRIUM PLASMAS

- EXTERNAL HOMOGENEOUS MAGNETIC FIELD $\vec{B}_0 = B_0 \vec{e}_z$
- NO EXTERNAL ELECTRIC FIELDS
- COLLISIONS NEGLECTED

$$f_{0\alpha} = f_{0\alpha}(v_\perp, v_z)$$

BEAMS, COLD RINGS, ANISOTROPIC T ETC,...

$$\vec{v} \times \vec{B}_0 \cdot \frac{\partial f_{0\alpha}}{\partial \vec{v}} = 0$$

$$\left\{\vec{E}(\vec{r},t), \vec{B}(\vec{r},t)\right\} \rightleftarrows \delta f_\alpha$$

$$\boxed{\frac{d}{dt}\delta f_\alpha = -\frac{e_\alpha}{m_\alpha}\left[\vec{E} + \frac{1}{c}(\vec{v}\times\vec{B})\right]\cdot\frac{\partial f_{0\alpha}}{\partial \vec{v}}}$$

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial \vec{r}} + \frac{e_\alpha}{m_\alpha}\frac{\vec{v}\times\vec{B}_0}{c}\cdot\frac{\partial}{\partial \vec{v}}$$

$$\vec{J} = \sum_\alpha \vec{J}_\alpha = \sum_\alpha e_\alpha \int d^3v\, \vec{v}\, \delta f_\alpha$$

$$\boxed{\overleftrightarrow{\epsilon}(\vec{k},\omega) = \mathrm{funct}\left(\frac{\partial f_{0\alpha}}{\partial \vec{v}}, \alpha, B_0\right)}$$

DIELECTRIC TENSOR OF A MAGNETOACTIVE PLASMA (USE INDICES)

$$\epsilon_{ij}(\omega,\vec{k}) = \delta_{ij} + i\sum_\alpha \frac{\omega_\alpha^2}{\omega^2}\int d^3v\, v_i \int_{-\infty}^{\psi} d\psi' \frac{\partial f_{0\alpha}}{\partial v_\ell}\left(\frac{\omega - \vec{k}\cdot\vec{v}}{\Omega_\alpha}\delta_{\ell j} + \frac{k_\ell v_j}{\Omega_\alpha}\right)$$

$$\times \exp\left[i\,\frac{(\omega - k_z v_z)(\psi' - \psi) - k_\perp v_\perp(\sin\psi' - \sin\psi)}{\Omega_\alpha}\right]$$

UNPERTURBED ORBITS

$\psi = \Omega_\alpha t$ $\quad(\delta f_\alpha \to 0$ as $t \to -\infty)$ ADIABATIC SWITCHING

$\Omega_\alpha \equiv e_\alpha B_0 / m_\alpha c \quad , \quad \omega_\alpha^2 = 4\pi n_\alpha e_\alpha^2 / m_\alpha$

$$f_{0\alpha} = f_{0\alpha}(v_\parallel, v_\perp)$$

$$\boxed{\epsilon_{ij}(\omega,\vec{k}) = \delta_{ij} + \sum_\alpha \frac{\omega_\alpha^2}{\omega^2}\int d^3v \sum_n \frac{\frac{\omega - k_z v_z}{v_\perp}\frac{\partial f_{0\alpha}}{\partial v_\perp} + \frac{k_z v_z}{v_\perp}\frac{\partial f_{0\alpha}}{\partial v_\parallel}}{\omega - k_z v_z - n\Omega_\alpha}\Pi_{ij}^{(n)}}$$

$$\Pi_{ij}^{(n)} \equiv F_{\alpha i} F_{\alpha j}^* \quad , \quad \xi_\alpha = \frac{k_\perp v_\perp}{\Omega_\alpha}$$

$$\vec{F}_\alpha \equiv \left(\frac{n}{\xi_\alpha} v_\perp J_n(\xi_\alpha), -i v_\perp J_n'(\xi_\alpha), v_z J_n(\xi_\alpha)\right)$$

Frame Transformations

- METHOD FOR CALCULATING D.T. IN COMPLEX SITUATIONS ON THE BASIS OF ITS (KNOWN) FORM IN A MOVING FRAME (INERTIAL)

* NOT APPLICABLE FOR ROTATING ($P_{\perp 0} \neq 0$) DISTRIBUTION FUNCTIONS $f_{\alpha 0}$.

- GOOD FOR BEAMS ($\vec{v}_{0\alpha}$)

$$\epsilon_{ij}(\omega,\vec{k}) = \delta_{ij} + \sum_{\alpha} \frac{\omega_\alpha'}{\omega}\alpha_{ik}\left[\epsilon_{kl}(\omega_\alpha', k_\alpha') - \delta_{kl}\right]\beta_{lj}$$

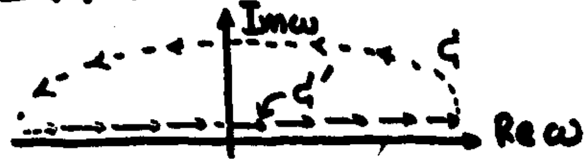
$$\left.\begin{aligned}
\omega_\alpha' &= \omega - \vec{k}\cdot\vec{v}_{0\alpha} \\
\vec{k}_\alpha' &= \vec{k} \\
\alpha_{ij} &= \delta_{ij} + \frac{k_j v_{0\alpha i}}{\omega_\alpha'} \\
\beta_{ij} &= \frac{\omega_\alpha'}{\omega}\delta_{ij} + \frac{k_i v_{0\alpha j}}{\omega}
\end{aligned}\right\}\ \text{GENERALIZES FOR RELATIVISTIC CASES}$$

UNSTABLE MODES $(\vec{k},\omega)$

SOLUTIONS TO $\det|\overleftrightarrow{D}(\vec{k},\omega)| = 0$
WITH $\mathrm{Im}\,\omega > 0$

[THE NUMBER OF UNSTABLE SOLUTIONS TO THE DISPERSION RELATION IS GIVEN BY

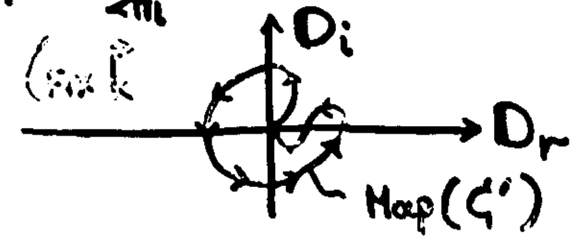
$$N_{UNST} = \int_{\varsigma} \frac{d\omega}{\mathbb{D}(\vec{k},\omega)} \frac{\partial}{\partial\omega}\mathbb{D}(\vec{k},\omega) \quad , \quad \mathbb{D} = \det|\overleftrightarrow{D}|$$



FOR $|\omega| \to \infty$ $\quad \overleftrightarrow{D} \to \overleftrightarrow{\epsilon} \to \overleftrightarrow{I}$ (VACUUM-LIKE)
THEREFORE ONLY ς'-PORTION CONTRIBUTES.
THEN :

$$N_{UNST} = \frac{1}{2\pi i} \ln \mathbb{D}(\vec{k}, \mathrm{Re}\,\omega = \infty)$$

, CHOOSING $1 = \mathbb{D}(\vec{k}, \mathrm{Re}\,\omega = -\infty)$



LONGITUDINAL AND TRANSVERSE WAVES IN ANISOTROPIC CASES (like, beams, rings)

THE DIRECTIONAL DEPENDENCE OF $\overleftrightarrow{\epsilon}$ IS ONLY DUE TO $\vec{k}$, FOR ISOTROPIC (ROTATIONALLY INVARIANT PLASMA, IN VELOCITY SPACE). THEN (AND ONLY THEN)

$$\overleftrightarrow{\epsilon} = \epsilon_L \overleftrightarrow{I}_L + \epsilon_T \overleftrightarrow{I}_T$$

$$\overleftrightarrow{I}_L \equiv \vec{k}\vec{k}/k^2 \,, \quad \overleftrightarrow{I}_T \equiv \overleftrightarrow{I} - \overleftrightarrow{I}_L$$

$$\overleftrightarrow{\epsilon}_L = \frac{\vec{k}\cdot\overleftrightarrow{\epsilon}\cdot\vec{k}}{k^2} \,, \quad \overleftrightarrow{\epsilon}_T = \frac{\mathrm{Tr}\,\overleftrightarrow{\epsilon} - \epsilon_L}{2}$$

and

$$D = 0 \implies \begin{cases} \epsilon_L = 0 \\ \epsilon_T = \eta^2 \end{cases}$$

LONGITUDINAL DISPERSION RELAT. $E \leftrightarrow \rightarrow k$

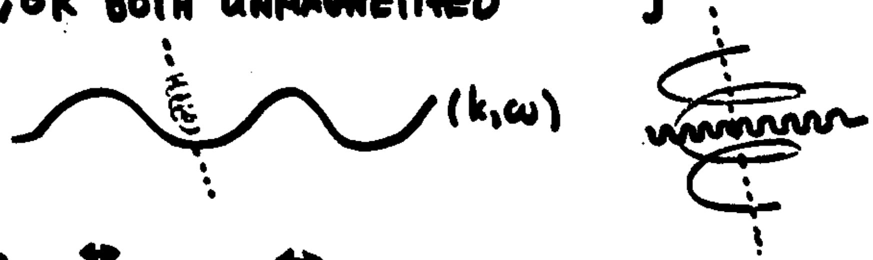
$E \updownarrow \rightarrow k$ TRANSVERSE MODE DISPERSION RELAT.

FOR OUR CASE (BEAMS, RINGS) SPLITTING IMPOSSIBLE $\Rightarrow$ E-M WAVES NOT PURELY LONGITUDINAL OR PURELY TRANSVERSE.

APPROXIMATE WHENEVER POSSIBLE

"REAL LIFE" SITUATIONS WHERE APPROXIMATIONS CAN BE MADE

- MULTISPECIES (TWO)
  MULTICOMPONENT (THREE)
- ONE SPECIES CAN BE TREATED AS MAGNETIZED, THE OTHER AS UNMAGNETIZED, OR BOTH UNMAGNETIZED (DEPENDS!)



$$\overleftrightarrow{\epsilon} = \overleftrightarrow{I} + \sum_\alpha \overleftrightarrow{\chi}_\alpha \leftarrow \text{LINEAR SUSCEPTIBILITIES}$$

IF A SPECIES IS UNMAGNETISED THEN

$$\overleftrightarrow{\chi}_\alpha(\text{UNM}) \simeq \overleftrightarrow{\chi}_\alpha(\text{MAGN}, B_0 \to 0) \quad \text{HARD}$$

$$\boxed{\overleftrightarrow{\chi}_\alpha(\text{UNM}) = \frac{\omega_\alpha^2}{\omega^2}\int d^3v \, \frac{\vec{v}\,\frac{\partial f_{0\alpha}}{\partial \vec{v}}\cdot\left[(\omega - \vec{k}\cdot\vec{v})\overleftrightarrow{I} + \vec{k}\vec{v}\right]}{\omega - \vec{k}\cdot\vec{v}}}$$

$k_\perp = 0$ MODES

$$\mathbb{D} = 0 \Rightarrow \boxed{\begin{array}{ll} \epsilon_{zz} = 0 & \text{LONGIT. MODES} \\ \epsilon_{xx} \pm i\epsilon_{xy} = \eta^2 & \left\{ \begin{array}{l} \text{ORDIN.} \\ \text{EXTRAORD.} \end{array} \right. \end{array}}$$

IT CAN BE SHOWN THAT FOR ANISOTROPIC PLASMAS LONGITUDINAL MODES <u>WITH $k_\perp = 0$</u> ARE ALWAYS <u>STABLE</u>. THEY ARE <u>ALWAYS</u> STABLE FOR <u>ANY $\vec{k}$</u> FOR PLASMAS WHERE <u>ALL SPECIES</u> CAN BE TREATED AS <u>UNMAGNETIZED</u>

(FOR <u>LONGITUDINAL OSCILLATIONS</u> STRONGY MAGNETIZED MEANS: $\Omega_\alpha^2 \gg \omega_\alpha^2$ [illegible] TIZED: $\Omega_\alpha^2 \ll \omega_\alpha^2$)

$$\boxed{\text{FOR } k_\perp = 0 \text{ WE FOCUS ONLY ON} \quad \epsilon_{xx} \pm i\epsilon_{xy} = \eta^2}$$

DRIFTING COLD RING

PIONEERED BY WU, DAVIDSON, HARTLIE

NEWLY IONIZED ATOMS CAN INDUCE GENERATION OF E-M WAVES BECAUSE THE POSSES HIGHLY ANISOTROPIC DISTRIBUTIONS (FREE ENERGY)

- NO SPREAD, PEAKED AT $v_\perp = v_{0\perp} = |(\vec{V}_{sw} - \vec{V}_N) \times \vec{b}|$ AND $v_{0\parallel} = |(\vec{V}_{sw} - \vec{V}_N) \cdot \vec{b}|$ AT SW-FRAME
- SW COLD (AT SW-FRAME)
- SPATIALLY HOMOGENEITY
- $(v_p^{sw})^2 \ll v_{0\parallel}^2, v_{0\perp}^2 \ll (v_e^{sw})^2$
- $k_\perp = 0$ (ONLY TRANSVERSE MODES ARE OF INTEREST)
- $v_\alpha^{sw} \ll \left| \dfrac{\omega \pm \Omega_\alpha}{k} \right|$, $|\omega| \ll |\Omega_e|$
- $n_i \ll n_p$
- NEWLY GENERATED ELECTRONS ARE NEGLECTED

SW: SOLAR WIND
e : ELECTRONS (SW)
p : PROTONS (SW)
i : NEWLY GENERATED IONS

$$\boxed{\begin{array}{l} f_{0i} = \dfrac{n_i}{2\pi v_\perp}\,\delta(v_{0\parallel} - v_\parallel)\,\delta(v_{0\perp} - v_\perp) \\ \qquad\qquad\qquad f_{0\alpha} = \dfrac{n_\alpha}{2\pi v_\perp}\,\delta(v_\parallel)\,\delta(v_\perp) \\ \epsilon_{xx} \pm i\,\epsilon_{xy} = \eta^2 \qquad\qquad \alpha : e, p \end{array}}$$

$$0 = D_\pm(k,\omega) \equiv \epsilon_{xx} \pm i\epsilon_{xy} - \eta^2 \qquad\qquad k \equiv k_\parallel$$

$$= 1 - \frac{k^2c^2}{\omega^2} - \frac{\omega_e^2}{\omega(\omega \pm \Omega_e)} - \frac{\omega_p^2}{\omega(\omega \pm \Omega_p)}$$

$$- \frac{\omega_i^2}{\omega^2}\left[\frac{\omega - k v_{0\parallel}}{\omega - k v_{0\parallel} \pm \Omega_i} + \frac{k^2 v_{0\perp}^2}{2(\omega - k v_{0\parallel} \pm \Omega_i)^2}\right]$$

I: $|\Omega_p| \ll |\omega| \ll |\Omega_e|$

- $\omega \pm \Omega_e \approx \pm\Omega_e$
- $\omega \pm \Omega_p \approx \omega$
- $|\omega/k|^2 \ll c^2$

$\tilde\omega = (\omega - k v_{0\parallel} \pm \Omega_i)/(\pm\Omega_i)$ [illegible]

$$\boxed{0 = \tilde\omega^3 - \tilde\omega^2\,\frac{c^2k^2 + \omega_p^2 + \omega_i^2 \pm (-k v_{0\parallel} \pm \Omega_i)\,\omega_p^2/\Omega_p}{\omega_p^2\,\Omega_i/\Omega_p} + \varepsilon\,\frac{\Omega_i}{\Omega_p}\,\tilde\omega - \varepsilon\,\frac{k^2 v_{0\perp}^2}{2\Omega_i\Omega_p}}$$

DISPERSION RELATION

$$\left.\begin{array}{l} ① + ② \approx 0 \\ \tilde\omega \sim O(1) \end{array}\right\} \quad \text{WHISTLER}, \ \mathrm{Im}\,\tilde\omega = \mathrm{Im}\,\omega = 0$$

$$\left.\begin{array}{l} ② + ④ \approx 0 \\ \tilde\omega \sim O(\varepsilon) \end{array}\right\} \quad \text{FOR} \quad M_{A\parallel}^2 \equiv \frac{v_{0\parallel}^2}{v_A^2} < \left|\frac{\omega_r}{\Omega_p}\right| \ll \frac{m_p}{m_e}$$

$$\mathrm{Im}\,\omega > 0, \quad \omega_r = k v_{0\parallel} \mp \Omega_i$$

$$\boxed{\gamma_{k\to\infty} \approx \frac{\omega_i}{\sqrt{2}}\,\frac{v_{0\perp}}{c} = \frac{M_{A\perp}}{\sqrt{2}}\,\Omega_i}$$

$$\boxed{k:\ |\Omega_p| \ll |k v_{0\parallel} \mp \Omega_i| \ll |\Omega_e|}$$

THERMAL EFFECTS : $\gamma_{k \sim 1/\sqrt{T_{\parallel i}}} \approx 0$

$M_{A\perp}$	$\approx$	$1{,}5 \div 3$
protons	$\gamma_{k\to\infty}^{-1} \approx$	$1 \div 2$ sec
H_2O^+	$\gamma_{k\to\infty}^{-1} \approx$	$20 \div 40$ sec

II : $|\omega| \ll |\Omega_p|$

- $\omega \pm \Omega_e \approx \pm\Omega_e$
- $\omega \pm \Omega_p \approx \pm\Omega_p$
- $\omega_e^2/\Omega_e + \omega_p^2/\Omega_p = 0$ CHARGE NEUTR.

$$\boxed{\begin{aligned}&\tilde\omega^4 \pm \frac{2(kv_{0\parallel}\mp\Omega_i)}{\Omega_i}\tilde\omega^3 - \frac{c^2k^2-(1+c^2/V_A^2)(kv_{0\parallel}\mp\Omega_i)^2+\omega_i^2}{(1+c^2/V_A^2)\Omega_i^2}\tilde\omega^2\\&+\varepsilon\frac{\omega_p^2/\Omega_p^2}{(1+c^2/V_A^2)}\tilde\omega - \varepsilon\frac{k^2v_{0\perp}^2}{2\Omega_i^2}\frac{\omega_p^2/\Omega_p^2}{(1+c^2/V_A^2)} = 0\end{aligned}}$$

$$\left.\begin{aligned}&\tilde\omega \sim O(\varepsilon^{1/2})\\&③+⑤ \simeq 0\end{aligned}\right\} \quad \begin{aligned}&\text{FOR } M_{A\parallel}^2 < \Omega_i^2/\omega^2\\&\omega_r = kv_{0\parallel} \mp \Omega_i\end{aligned}$$

$0 < \mathrm{Im}\,\omega$

$$\boxed{\gamma_k \approx \frac{\omega_i}{\sqrt2}\frac{v_{0\perp}}{c} = \frac{M_{A\perp}}{\sqrt2}\Omega_i}$$

$$\boxed{k: |kv_{0\parallel} \mp \Omega_i| \ll |\omega_r|}$$

III : $k^2v_{0\perp}^2/2\Omega_i \ll |\omega - kv_{0\parallel}| \ll \Omega_i$

$$\omega_r \approx \varepsilon k v_{0\parallel}, \quad \mathrm{Im}\,\omega = kv_A\sqrt{1+\frac{v_{0\perp}^2-2v_{0\parallel}^2}{2v_A^2}}$$

INSTABILITIES ASSOCIATED WITH LONGITUDINAL OSCILLATIONS (E-S)

$$\boxed{k_\perp = 0}$$

SINCE ALL LONGITUDINAL OSCILLATIONS ASSOCIATED WITH ANISOTROPIC DISTRIBUTIONS (NON-DRIFTING) ARE STABLE

WE EXPECT THE SOURCE OF ENERGY WHICH CAN DRIVE THIS INSTABILITY TO BE THE DRIFT MOTION $v_{0\parallel}$.

- $v_i < v_{0\parallel} \ll v_e$
- UNMAGNETIZED SPECIES $r_{Lc} \gg \lambda_{De}$
  $r_{Lc} \gg \lambda = \frac{2\pi}{k_\parallel}$ $\quad \lambda_D \equiv v_e/\omega_e$
- FREQUENCIES : $v_r \ll |\omega| \ll$
- DRIFTING ION RING

$\epsilon_L = 0 \Rightarrow$

$$0 = 1 + \frac{1}{k^2\lambda_{De}^2}\left[1 - I_+\left(\frac{\omega - kv_{0\parallel}}{kv_e}\right)\right] - \frac{\omega_i^2}{\dots}$$

DEFINITION $I_+(x) = x e^{-\frac{x^2}{2}} \int_{i\infty}^{x} d\tau\, e^{\frac{\tau^2}{2}}$

USEFUL ASYMPTOTIC EXPANSIONS

$$I_+(x) \approx \begin{cases} -ix\sqrt{\frac{\pi}{2}} & \text{FOR } |x| \ll 1 \\ -ix\sqrt{2\pi}\, e^{-x^2/2} & \text{FOR } |x| \gg 1,\ |\mathrm{Im}\, x| \gg |\mathrm{Re}\, x| \\ & \mathrm{Im}\, x < 0 \\ 1 + \frac{1}{x^2} + \frac{3}{x^4} + \cdots - i\sqrt{\frac{\pi}{2}}\, x e^{-x^2/2} & \text{FOR } |x| \gg 1 \\ & |\mathrm{Re}\, x| \gg |\mathrm{Im}\, x| \\ & \mathrm{Im}\, x < 0 \end{cases}$$

NOTE

$$\epsilon_L = 0 \approx> \quad \omega_r' = \frac{\omega_i\, k\lambda_{De}}{\sqrt{1 + k^2\lambda_{De}^2}} + k v_{0\parallel}$$

$$\frac{\gamma}{\omega_r} \simeq \frac{\sqrt{\pi}}{2} \frac{(k v_{0\parallel} - \omega_r)^3}{|k| v_e\, \omega_i^2} \frac{1}{k^2\lambda_{De}^2} > 0$$

[illegible]

DRIFTING DISTRIBUTIONS WITH SPREAD (BEAM-LIKE)

(GARY, WINSKIE, SMITH, LEE, GOLDSTEIN FORSLUD, ETC...)

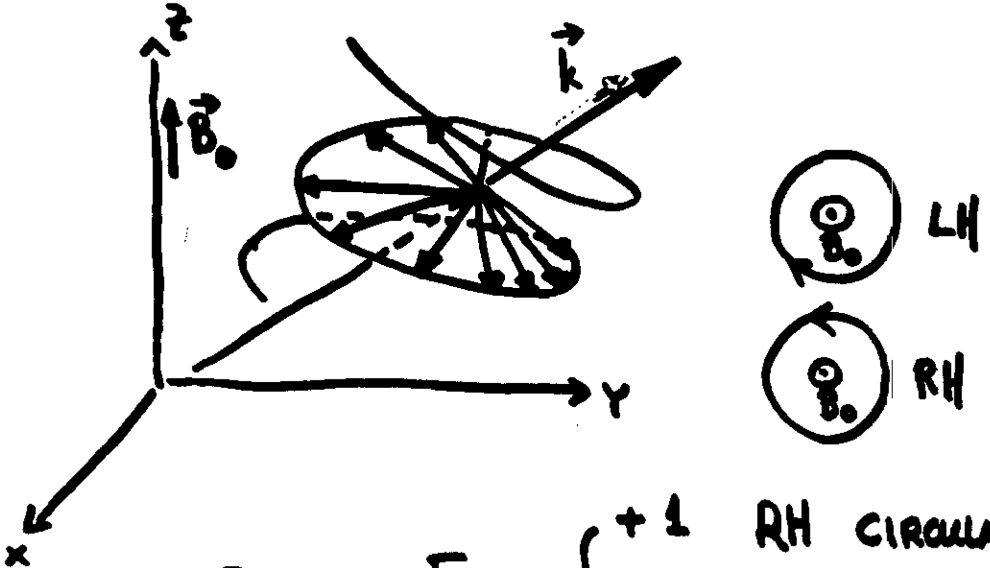
- DRIFT ALONG $\vec{B}_0$
- ANISOTROPIC (BOTH SW and KEWLY-BORN IONS)
- $\max(T_e, T_p) \lesssim T_i$
- $|\omega_r| \lesssim \Omega_p$

THERE IS EVIDENCE THAT INSTABILITIES [illegible] WITH ION-[illegible] INSTABILITIES ARE THE SOURCE OF LARGE AMPLITUDE WAVES

(ICE OBSERVATIONS OF G-Z COMET)

THE PUZZLE, HOWEVER, IS THAT THE OBSERVATIONS SHOW <u>LINEARLY</u> <u>POLARIZED</u> WAVES

POLARIZATION



$$P \equiv \frac{E_Y}{i E_X} \begin{cases} +1 & \text{RH CIRCULAR} \\ -1 & \text{LH CIRCULAR} \\ >0 & \text{RH ELLIPTIC} \\ <0 & \text{LH ELLIPTIC} \end{cases}$$

$k_\perp = 0$ BEAM-LIKE DISTRIBUTIONS

TRANSVERSE MODES (E-M)

$$0 = \mathbb{D}_\pm \equiv \epsilon_{xx} \pm i \epsilon_{xy} - n^2$$

- $|\omega_r| \lesssim |\Omega_p| \ll |\Omega_e|$

EXPANSION OF THE DENOMINATORS $\omega - k v_z + \Omega_p$ (p: protons) IN POWERS OF ω_r / Ω_p.

$$\omega - k v_z + \Omega_e \simeq \Omega_e$$

NOTE: FOR $k_\perp = 0$ ([illegible] $= 0$) ONLY $n = 0, \pm 1$ CONTRIBUTE [illegible]

$$\boxed{0 \approx 1 - \frac{k^2 c^2}{\omega^2} + \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2} \int d^3v \left[(\omega - k v_z) \frac{\partial f_{0\alpha}}{\partial v_\perp} + k v_\perp \frac{\partial f_{0\alpha}}{\partial v_z} \right] \times \frac{1}{2} \left[\frac{1}{(-k v_z \pm \Omega_\alpha)} - \frac{\omega}{(-k v_z \pm \Omega_\alpha)^2} \right]}$$

- $kr_{Le}, kr_{Lp} < 1$
- $\langle v_z \rangle_e = \frac{1}{n_e} \int d^3v \, v_z f_{oe} = 0$
- $V_A^2 \ll c^2 \quad (\sim 10^{-8})$

$$D_\pm = 0 \Rightarrow \left\{ \begin{array}{l} \mathrm{Im}\, D_\pm = 0 \\ \mathrm{Re}\, D_\pm = 0 \end{array} \right\} \quad (\omega = \omega_r + i\gamma)$$

$$\boxed{\begin{array}{l} \gamma = \dfrac{a(k)}{\omega_r + c(k)} \\ 0 = \omega_r^2 - \dfrac{a^2(k)}{[\omega_r + c(k)]^2} - 2b(k) + 2\omega_r c(k) - \dfrac{d(k)\,a(k)}{\omega_r + c(k)} \end{array}}$$

- $c(k) = \frac{\pi \Omega_p^2}{2n_p} \mathrm{Re}\, \mathrm{I}$
- $d(k) = \frac{\pi \Omega_p^2}{2n_p} \mathrm{Im}\, \mathrm{I}$

$$\mathrm{I} = \int dv_z \int dv_\perp v_\perp (-kv_z \pm \Omega_p)^{-2} \times \left[-kv_\perp^2 \frac{\partial f_{oi}}{\partial v_z} \mp 2\Omega_p f_{oi} + 2(2kv_z \pm \Omega_p)(-kv_z \pm \Omega_p)^2 \Omega_p^{-2} f_{oi} \right]$$

APPROXIMATE SOLUTIONS

$$\left(\begin{array}{l} \omega_r = -c \pm \sqrt{b + \sqrt{b^2 + a^2}} \\ \gamma = \pm \dfrac{a}{\sqrt{b + \sqrt{b^2 + a^2}}} \\ P = \pm\, \omega_r / |\omega_r| \end{array} \right)$$

* a : CYCLOTRON RESONANCE TERM

* b : FIRE-HOSE TERM $\left(\langle v_z^2 \rangle_{o,i} - \frac{\langle v_\perp^2 \rangle_{o,i}}{2} \right)$

c : LESS IMPORTANT CYCLOTRON RESONANCE

DISCUSSION

- $n_p \sim n_e \sim 10^2 n_i$
- $T_p \sim T_e \sim T_i/10$
- $\beta_p \sim 1$
- $c/V_A \sim 10^4$
- $T_\perp \sim T_\parallel$ FOR ALL THREE SPECIES

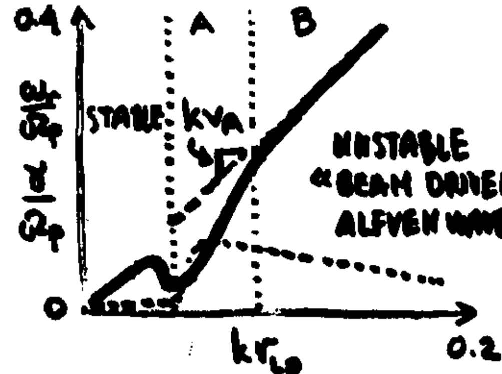
i : protons

$$v_0 \equiv |v_{0i} - v_{0p}|$$

i, p: DRIFTING BI-MAXWELL.
e: BI-MAXWELL.
Note: $a \propto v_0$

b>0 : FIRE HOSE STABLE

$v_0 = 7 v_A$



IT IS A RESONANT INSTABIL.

at γ_{max}

$\omega_r \approx k v_{0i} - \Omega_i$

$$\omega_r \approx \begin{cases} kv_A \text{ in B} \\ < kv_A \text{ in A} \end{cases}$$

$$\gamma_{max} \approx \Omega_p \left(\frac{n_i}{2n_p}\right)^{1/3} \left(\frac{n_p}{n_i}\right)^{2/3}$$

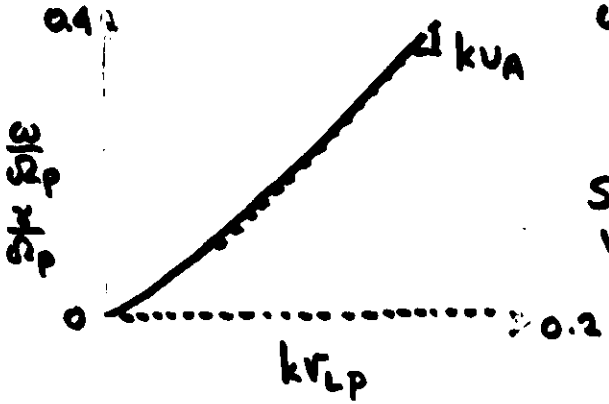
$P = 1$

CASE $D_+ = 0$ $\quad \omega_r = -c \pm [b + (b^2 + a^2)^{\frac{1}{2}}]^{\frac{1}{2}}$ $\quad \gamma = a[b + (b^2 + a^2)^{\frac{1}{2}}]^{-\frac{1}{2}}$

$P = \text{sign}(\omega_r)$ ONLY + GIVES UNSTABLE

b>0: FIRE-HOSE STABLE

$v_0 = 0$



$\omega_r \approx \sqrt{2b} \approx kv_A$

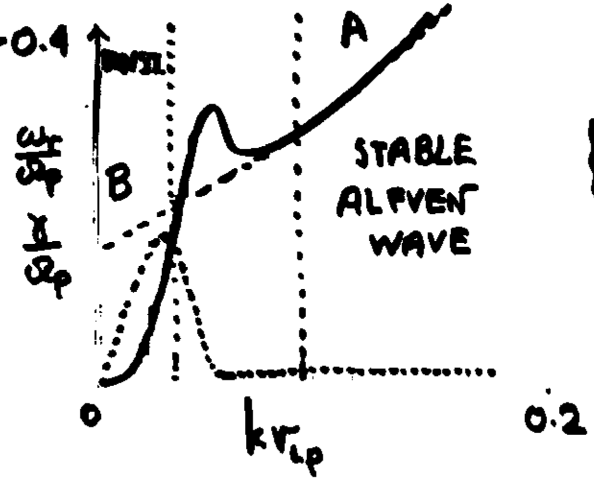
$\gamma = 0$

$P = 1$

STABLE ALFVEN WAVE

$v_0 = 21 v_A$

at small k . b<0 : pressure anisotropy



at γ_{max}

$\omega_r \neq k v_{0i} - \Omega_i$

$$\omega_r \approx \begin{cases} kv_A \text{ in A} \\ < kv_A \text{ in B} \end{cases}$$

$$\gamma_{max} \approx \Omega_p \left(\frac{n_i}{2n_p}\right)\left(\frac{v_0}{v_A}\right)$$

$P = 1$

IT IS A NON-RESONANT INSTABILITY

NOTE

ALFVEN WAVE

$$(\omega_r + i\gamma)^2 = -\frac{k^2 V_A^2}{1+V_A^2/c^2}\left(\frac{P_{\parallel}-P_{\perp}}{B_0^2/4\pi} - 1\right)$$

$$P_{\parallel} \equiv \sum_{SW} n T_{\parallel}$$

$$P_{\perp} \equiv \sum_{SW} n T_{\perp}$$

FOR $P_{\parallel} = P_{\perp} \rightarrow$ STABLE ALFVEN WAVE

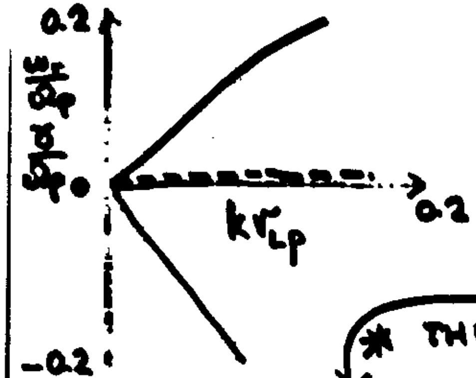
FOR $P_{\parallel} > P_{\perp} + \frac{B_0^2}{4\pi} \rightarrow$ UNSTABLE AT ZERO ω_r

- IN OUR CASE WE HAVE INSTABILITY AT ZERO $\omega_r(k)$ EVEN WITH $P_{\parallel} = P_{\perp}$
- BEAM DRIVES IT.

CASE $\boxed{D}=0$ $\quad \omega_r = -c \pm \left[b + (b^2+a^2)^{\frac{1}{2}}\right]^{\frac{1}{2}}$ $\quad \gamma = \pm a\left[b+(b^2+a^2)^{\frac{1}{2}}\right]^{-\frac{1}{2}}$

$P = -\mathrm{sign}(\omega_r)$

$U_0 \approx 0 \div 7 V_A$



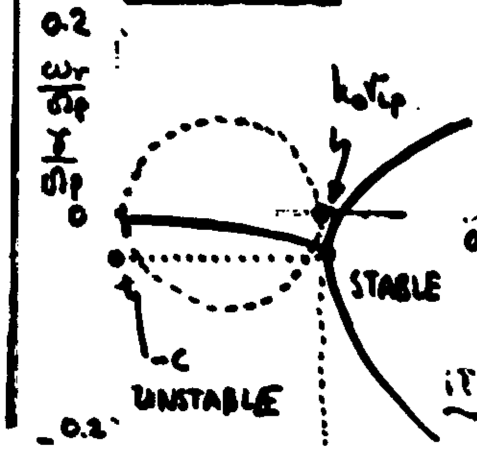
$b > 0$: FIRE-HOSE STABLE

$\omega_r \approx \pm\sqrt{2b}$

$\gamma = 0$

* THRESHOLD $v_0 \approx \sqrt{\frac{n_p}{n_i}\frac{m_p}{m_i}}\, v_A$

$U_0 \approx 21 V_A$ *



0.2 $k r_{Lp}$

- FIRE-HOSE UNSTABLE
  $k r_{Lp} < k_0 r_{Lp}$ $\quad b < 0$
- STABLE
  $k r_{Lp} > k_0 r_{Lp}$ $\quad b > 0$

$\omega_r \geq -c$

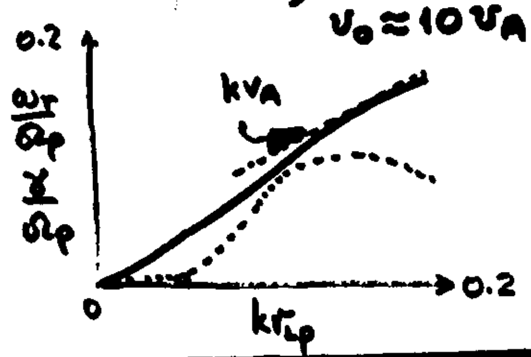
$\gamma_{max} \approx \Omega_p \left(\frac{n_i}{2 n_p}\right)\left(\frac{v_0}{v_A}\right)$

$P = +1$

IT IS A NON-RESONANT INSTAB.

SO FAR RH-POLARIZED RESONANT AND NON-RESONANT INSTABILITIES (FOR $T_i/T_p \approx 10$)

FOR HIGHER VALUES OF T_i/T_p (~ 100) ONE CAN GET A LH-POLARIZED RESONANT INSTABILITY,

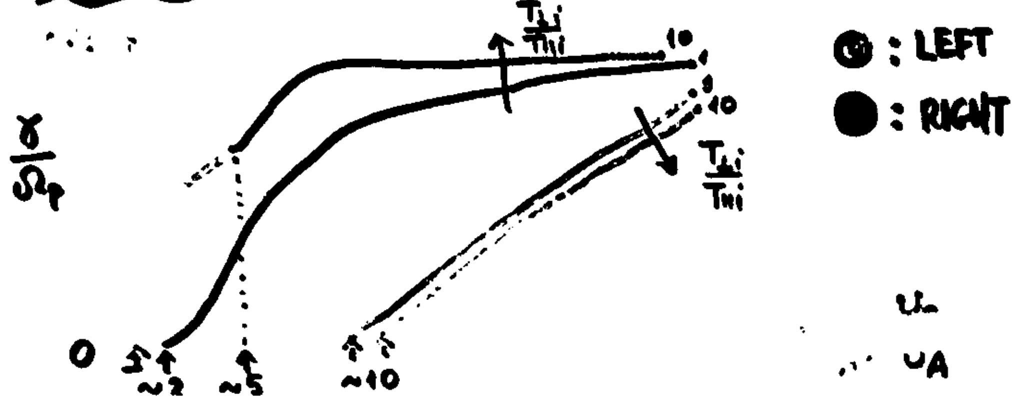


$$\omega_r \approx k v_A$$

$$\gamma_{max} \approx \left(\frac{n_i}{2n_p}\right)^{1/3} \left(\frac{m_p}{m_i}\right)^{2/3} \Omega_p$$

$$P = -1$$

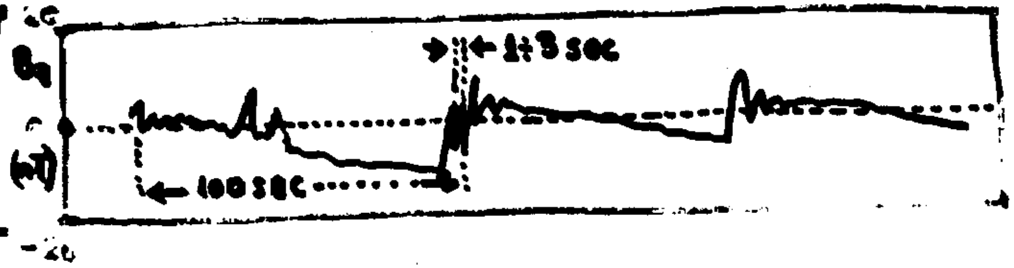
ION CYCLOTRON ANISOTROPY INSTABILITY



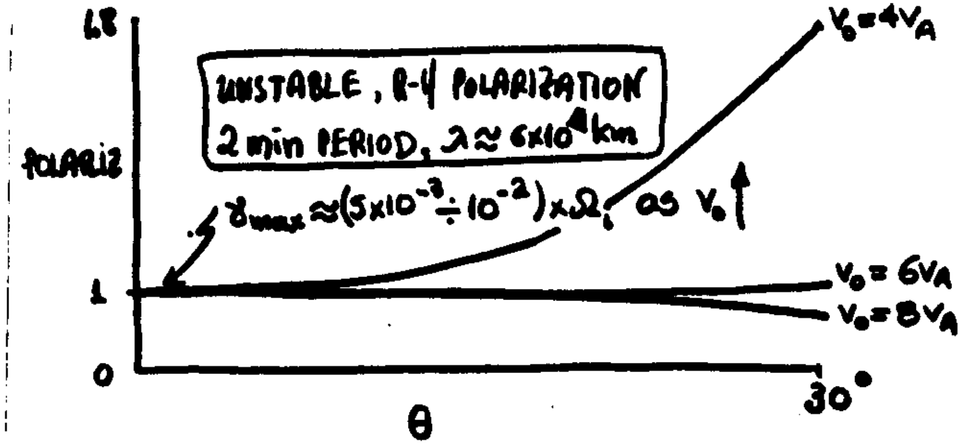
REMARKS - ICE. OBSERVATIONS

(TSURUTANI, SMITH, GARY, WINSKEY)

TURBULENCE G-Z COMET IS CHARACTERIZED BY: (a) SMALL k, LINEARLY POLARIZED WAVES OF LOW FREQUENCY ($10^{-2}\,sec^{-1}$) AND (b) SMALL k, CIRCULARLY POLARIZED WAVES OF HIGHER FREQUENCY ($0.3\,sec^{-1}$).

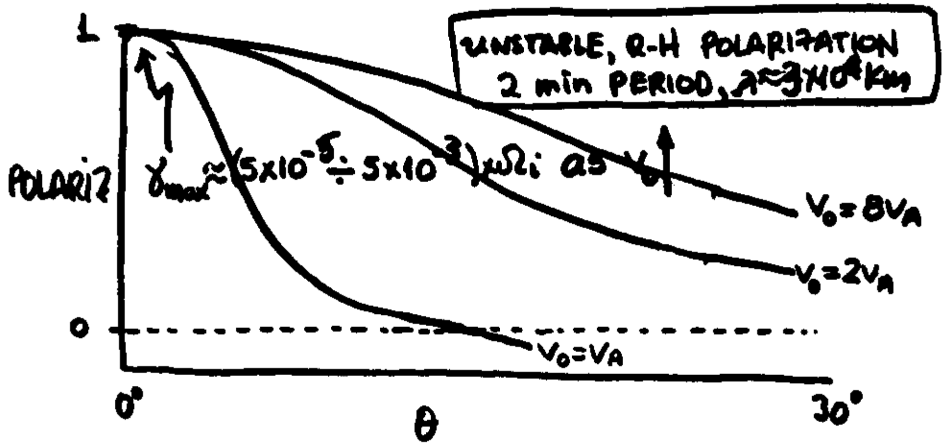


* NUMERICAL EVALUATIONS FOR OBLIQUE PROPAGATION SHOW:



i: WATER PICK-UP IONS, $v_A/c = 2\times10^{-4}$, $T_e \approx 30\,eV$, $T_p \approx 15\,eV$

$n_{sw} \approx 5\,cm^{-3}$, $n_i \approx 10^{-2}\,cm^{-3}$, $T_i = 100\,T_p$, $V_{sw} \approx 500\,km/s$



i: WATER PICK-UP IONS, $v_A/c = 2\times10^{-4}$, $T_e = 30\,eV$, $T_p = 15\,eV$

$n_{sw} \approx 10\,cm^{-3}$, $n_i \approx 10^{-4}\,cm^{-3}$, $T_i = 500\,T_p$, $V_{sw} \approx$ [illegible] km/s

$$\text{POLARIZ} \equiv \mathrm{Re}\, \frac{iB_x}{B_y}\frac{\omega_r}{|\omega_r|}$$

RING INSTABILITIES (AKIMOTO, HIZANIDIS, ...)

RELEVANT FOR THE QUASI-PERPENDICULAR PORTION OF THE COMETARY MAGNETOSPHERE $\left(\widehat{(\vec{V}_{comet}, \vec{V}_{sw})} > 45^\circ\right)$. THE COMETARY PICK-UP IONS FORM RING DISTRIBUTIONS WITH SMALL FIELD ALIGNED BEAM COMPONENT

ASSUMPTIONS : (SOLAR WIND FRAME)

- SOLAR WIND (e, p) COLD
- e: STRONGLY MAGNETIZED
  p: UNMAGNETIZED
- i: COLD RING, UNMAGNETIZED (p or H_2O^+)
- $kc \gg \omega \gg k v_e$, $\Omega_p \ll \omega \ll \Omega_e$
- $v_A/c \approx 10^{-4} \Rightarrow \omega_e/\Omega_e \approx$ [illegible]
- $\alpha \equiv \dfrac{m_p}{m_i}\dfrac{n_i}{n_p}$, $\vec{k} = (k_x, 0, k_z)$, $k_z = k\cos\theta$

  $\alpha = 10^{-3} \div 10^{-2}$

* DO NOT CONTR. IN THE DISP. REL.

DISPERSION RELATION

ASSUME $\alpha \ll 1$

$$\overleftrightarrow{\chi}_a = \frac{\omega_a^2}{\omega^2}\int d^3v\, \vec{v}\, \frac{\partial f_{oa}}{\partial \vec{v}} \cdot \left(\overleftrightarrow{I} + \frac{\vec{k}\vec{v}}{\omega - \vec{k}\cdot\vec{v}}\right)$$

$a: i, p$ $\quad f_{oi} = \dfrac{\delta(v_\parallel)\,\delta(v_\perp - v_{\perp 0})}{2\pi v_\perp}$

$$f_{op} = \frac{\delta(v_\parallel)\,\delta(v_\perp)}{2\pi v_\perp}$$

$$\overleftrightarrow{\chi}_e = \frac{\omega_e^2}{\omega^2}\int d^3v \sum_n \left(\frac{\omega - k_z v_z}{v_\perp}\frac{\partial f_{oe}}{\partial v_\perp} + \frac{k_z v_z}{v_\perp}\frac{\partial f_{oe}}{\partial v_\parallel}\right) \times (\omega - k_z v_z - n\Omega_e)^{-1}\, \vec{F}_e \vec{F}_e^{\,*}$$

$$f_{oe} = \frac{\delta(v_\parallel)\,\delta(v_\perp)}{2\pi v_\perp}$$

$$0 = 1 + \frac{\omega_e^2}{\Omega_e^2}\left(1 + \frac{\omega_e^2}{k^2c^2}\right) - \frac{\omega_p^2}{\omega^2} - \frac{\omega_e^2}{\omega^2}\,\frac{\cos^2\theta}{\sin^2\theta + \frac{\omega_e^2}{k^2c^2}} - \frac{\alpha\,\omega_p^2\,\omega}{(\omega^2 - k_\perp^2 v_{\perp 0}^2)^{3/2}}\ * \qquad k^2 = k_\parallel^2 + k_\perp^2$$

LONGITUDINAL & TRANSVERSE COUPLED

* CHARACTERIZES THE TWO-DIMENSIONALITY OF THE RING DISTRIBUTION AS OPPOSED TO THE BEAM

* IT IS ALSO DERIVABLE FROM THE MAGNETIZED SUSCEPTIBILITY FOR THE RING IONS :
(LOMINADZE + STEPANOV)

PICK UP IONS

$$\overleftrightarrow{\epsilon} = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ -\epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & -\epsilon_{23} & \epsilon_{33} \end{pmatrix} \qquad K_{\parallel} = 0 \text{ (FLUTE MODES)}$$

$$\epsilon_{11} = -\alpha \frac{\omega_p^2}{\omega^2}\left(1 + \frac{2}{\zeta}\sum_n \frac{n^3 J_n J_n'}{\frac{\omega}{\Omega_i} - n}\right) = 1 + \chi_{11}$$

$$\epsilon_{22} = -\alpha \frac{\omega_p^2}{\omega^2}\left[1 + \frac{1}{\zeta}\left(\zeta^2 \sum_n \frac{n J_n'^2}{\frac{\omega}{\Omega_i} - n}\right)'\right] = 1 + \chi_{22}$$

$$\epsilon_{33} = -\alpha \frac{\omega_p^2}{\omega^2}\left(1 + \underbrace{2\zeta \frac{v_{0\parallel}^2}{v_{0\perp}^2}\sum_n \frac{n J_n J_n'}{\frac{\omega}{\Omega_i} - n}}_{\to 0}\right) = 1 + \chi_{33}$$

$$\epsilon_{12} = -i\alpha \frac{\omega_p^2}{\omega^2}\sum_n \frac{n^2 J_n J_n'}{\frac{\omega}{\Omega_i} - n} = \chi_{12} \qquad v_{\parallel 0} = 0 \Rightarrow \text{DECOUPLING}$$

$$\epsilon_{13} = -\alpha \frac{\omega_p^2}{\omega^2}\, 2\frac{v_{0\parallel}}{v_{0\perp}}\sum_n \frac{n^2 J_n J_n'}{\frac{\omega}{\Omega_i} - n} = \chi_{13} \to 0$$

$$\epsilon_{23} = i\alpha \frac{\omega_p^2}{\omega^2}\frac{v_{0\parallel}}{v_{0\perp}}\zeta\sum_n \frac{n J_n J_n'}{\frac{\omega}{\Omega_i} - n} = \chi_{23} \to 0$$

AT RESONANCE $\omega_r = k v_{\perp 0}$ WE EXPECT THE STRONGEST INTERACTION.

SINCE $\Omega_i \ll |\omega|$ AND $\zeta \gg 1$ WE MUST SUM UP A LARGE NUMBER OF TERMS.

UTILIZE THE IDENTITIES:

$$R_1 \equiv \sum_n \frac{J_n^2(x)}{Y - n} = -i\int_0^\infty e^{iYz} J_0\left(2x\sin\frac{z}{2}\right) dz$$

$$R_2 \equiv \sum_n \frac{[J_n'(x)]^2}{Y - n} = -\frac{i}{2}\int_0^\infty e^{iYz}\left[\cos z\, J_0\left(2x\sin\frac{z}{2}\right) + J_2\left(2x\sin\frac{z}{2}\right)\right] dz$$

SINCE $|Y| \gg 1$ AND $|x| \gg 1$, ONLY $z \ll 1$ ARE IMPORTANT IN R_1, R_2; THEREFORE, APPROXIMATE $\sin\frac{z}{2} \approx \frac{z}{2}$

FOR $\omega \approx k_\perp v_{\perp 0}$ WE OBTAIN

$$\boxed{\begin{aligned} \chi_{11} &\approx -\alpha \frac{\omega_p^2 \omega}{(\omega^2 - k_\perp^2 v_{\perp 0}^2)^{3/2}} - 1 \\ \chi_{22} &\approx -\frac{\alpha}{4} \frac{\omega_p^2 \omega}{(\omega^2 - k_\perp^2 v_{\perp 0}^2)^{3/2}} - 1 \\ \chi_{12} &\approx -i\frac{3\alpha}{2} \frac{\omega_p^2 \omega^3}{(\omega^2 - k_\perp^2 v_{\perp 0}^2)^{5/2}} \end{aligned}}$$

NON-RESONANT INSTABILITIES

$$\omega = \omega_r + i\gamma \qquad \frac{\gamma}{\omega_r} \ll 1 \quad (\text{DROP } \gamma^2)$$

DISPERSION RELATION $\Rightarrow$

$$\omega_r^2 = \omega_H^2 \frac{1 + \dfrac{(m_i/m_e)\cos^2\theta}{1 + \omega_e^2/k^2c^2}}{1 + \omega_e^2/k^2c^2}$$

$$\omega_H^2 \equiv \Omega_p \Omega_e$$

$*$ $\gamma_{NR} \propto \alpha$ HIGHLY DISPERSIVE (DEPENDS UPON k) NEGLIGIBLE FOR SMALL α

RESONANT INSTABILITIES

$$\omega = \omega_r + i\gamma \qquad \omega_r = k v_{\perp 0}$$

DISPERSION RELATION $\Rightarrow$

$\omega_r =$ SAME AS FOR THE NON RESONANT ONES

$*$ $\gamma_R \approx \omega_r \frac{\alpha^{2/5}}{2} \gg \gamma_{NR}$: WEAKLY DISPER.

RESONANT RING INSTABILITIES

ELECTRON LANDAU DAMPING.

IF ONE RELAXES THE ASSUMPTION FOR COLD SW-ELECTRONS, THEN FOR MODES WITH PHASE VELOCITIES ω/k_z COMPARABLE WITH THE ELECTRON THERMAL SPEED v_e ONE SHOULD EXPECT CONSIDERABLE DUMPING OF THE WAVES, I.E. THE ELECTRON SUCSESSIBILITY χ_e BECOMES SUCH AS TO CONSIDERABLY REDUCE ANY GROWING RESONANT MODE $\omega \approx k_z v_e$

[illegible] LANDAU DAMPING. IN THE [illegible] FINITE k_z AND v_e WE [illegible] :

$$\frac{\omega_r}{k_z} \gg v_e \Rightarrow \boxed{M_A^2 \sin^2\psi \gg \frac{m_p}{m_e} \beta_e \cot^2\theta}$$

$$\beta_e \equiv n_e T_e / (B^2/8\pi)$$

SINCE $\omega_r \gg \Omega_i$, ONE HAS TO SEEK FOR AN ADDITIONAL RESTRICTION ON β_e IN ORDER TO HAVE THE LINEAR THEORY VALID.

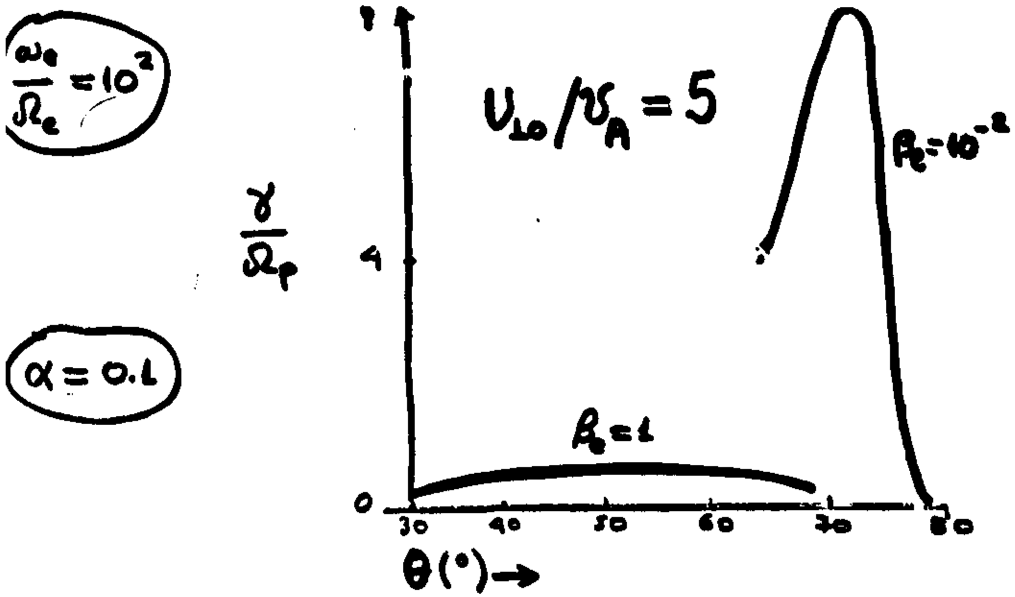
$$\left\{ \omega_r/k_z \gg v_e \text{ AND } \omega_r \gg \Omega_p \right\} \Rightarrow$$

$$\left\{ \begin{array}{l} M_A^2 \sin^2\psi \gg \frac{m_i}{m_e} \beta_e \cot^2\theta \\ M_A^2 \sin^2\psi \ll \frac{m_i}{m_e} \frac{\cot^2\theta}{4} \end{array} \right\} \Rightarrow \boxed{\beta_e \ll 0.25}$$

* FOR THE SOLAR WIND CASE $\beta_e \sim 0.5$: THE [illegible] ELECTRON LANDAU DAMPING WILL AFFECT THE RESONANT INSTABILITY.

* <u>ONE CAN AVOID IT FOR $k_z \to 0$</u>

ONE SHOULD EXPECT DRASTIC CHANGES GOING TO THE $k_z = 0$ CASE



LANDAU DAMPING SUPPRESSES $\gamma < \Omega_p$

$\theta = \pi/2$ INSTABILITY

e^- LANDAU DAMPING IS NOT IMPORTANT

COLD DISPERSION RELATION $\Rightarrow$

$$\omega_r^2 \simeq \omega_H^2 \frac{1 - M_A^2 \sin^2\psi}{1 + \omega_H^2/\omega_p^2}$$

FOR FINITE β_e :

$$\omega_r^2 \simeq \omega_H^2 \frac{1 + \beta_e - M_A^2 \sin^2\psi}{1 + \omega_H^2/\omega_p^2}$$

IT CAN BE ONLY TRIGERED WHEN :

$$M_A^2 < \frac{1 + \beta_e}{\sin^2\psi}$$

AND $\gamma > \Omega_p$ AT :

$$\alpha > \alpha_{CRITICAL} \approx 2^{3/2} \left(\frac{m_e}{m_i}\right)^{3/4} \left(\frac{1 + \beta_e}{1 + \beta_e - M_A^2 \sin^2\psi}\right)^{3/4}$$

- ESTIMATES FROM GIOTTO DATA SHOW $\alpha \approx 2\times10^{-3} \div 7\times10^{-3}$ FOR $d = 1.2 \div 2.2 \times 10^{6}$ Km FROM THE COMET. SINCE $\alpha_{CRITICAL} \approx 5\times10^{-4}$ UNSTABLE ($\gamma > \Omega_p$ TOO) LOWER HYBRID MODES CAN EXIST (FLUTE - $\theta = \pi/2$ - MODES)
- FREQUENCIES $1 \div 1.5$ sec^{-1} (LOWER HYBRID) HAVE ACTUALLY BEEN MEASURED (VEGA-1 MISSION TO COMET HALLEY)
- AN OTHER PIECE OF INFORMATION FROM VEGA-1 IS THE MEASURED MAGNETIC AND ELECTRIC FIELD STRENGTHS (SO, THEIR RATIO).

(E/B) AND (TRANSVERSE/LONGITUDINAL) RATIOS FOR $\theta = \pi/2$ RING INSTABILITIES.

$$\overleftrightarrow{D} \cdot \vec{E} = \vec{0} \quad \Rightarrow \quad \det|\overleftrightarrow{D}| = 0$$

ONE CAN DETERMINE THE RELATIVE MAGNITUDE OF THE COMPONENTS E_x, E_y, E_z FROM THE RATIO OF THE COFACTORS OF THEIR COEFFICIENTS.

$$\frac{|\vec{k}\times\vec{E}|}{|\vec{k}\cdot\vec{E}|} \simeq \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \frac{M_A^2 \sin^2\psi}{(1 - M_A^2\sin^2\psi + \beta_e)^{1/2}}$$

WHICH CAN BECOME SIGNIFICANTLY LARGE AS THE SOLAR-WIND DECELERATES : THE INSTABILITY BECOMES ALMOST LONGITUDINAL ("ELECTROSTATIC").

SIMILARLY:

$$\frac{|\vec{E}|}{|\vec{B}|} \simeq \frac{\Omega_e}{\omega_e} \frac{\left(1+\beta_e - M_A^2 \sin^2\psi\right)^{1/2}}{M_A \sin\psi}$$

$M_A \approx 1$, $\beta_e \approx 0.5$, $\psi \simeq 60°$, AND

$n \simeq 12\ \mathrm{cm}^{-3}$; $B \simeq 11 \times 10^{-4}$ Gauss (VEGA 1)

$$\frac{|\vec{E}|}{|\vec{B}|} = 10^{-2}$$ IN GOOD AGREEMENT

WITH THE OBSERVATIONS.

A FINAL NOTE

HARTLE & WU INVESTIGATED ELECTROSTATIC INSTABILITIES AT $\Omega_p \ll \omega_- \ll \omega_e$ AT $\theta = \pi/2$ FOR COLD ELECTRONS, COLD [illegible] UNMAGN. PICK-UP IONS AND WARM PROTONS (SW) THAT IS $kv_p \gtrsim \Omega_p$. THEY FOUND

$$\omega_r \simeq k u_{10} \simeq \omega_H$$

$$\gamma \simeq \frac{\alpha^{2/5}}{2} \omega_H \left(\frac{2}{\pi}\right)^{1/5} \times \left(\frac{m_e}{m_p}\right)^{1/5} \left(\frac{v_p}{V_{aL}}\right)^{2/5}$$

$$< \frac{\alpha^{2/5}}{2} \omega_H$$

UNLESS $v_p \gtrsim v_{aL}$

PARAMETERS OF INTEREST

- $B_0 \sim 10^{-4}$ Gauss
- $n_{sw} \sim 10 \text{ cm}^{-3}$
- $r_{L\,protons} \sim 50$ km
- $\begin{pmatrix} \Omega_{protons} \sim 10 \text{ sec}^{-1} \\ \tau_{g\,protons} \sim 1 \text{ sec} \end{pmatrix}$
- $T_e \sim T_p \sim T_{sw} \sim 30$ eV
- $V_A \sim 30$ km/sec
- $\frac{\omega_e}{\Omega_e} \sim 250$