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SPRING COLLEGE ON PLASMA PHYSICS

(25 May - 19 June 1987)

SIMULATION OF MASS FLOWS IN DRIVEN SYSTEMS

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Lecture II: Simulation of mass Flows in Driven Systems

Outline

(1) Introductory Concepts

(2) MHD-particle Description of Plasma

(3) Basic Algorithm and Numerical methods

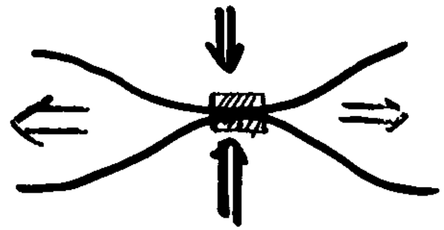
(4) Fluctuation Spectrum and Wave Propagation

(5) Applications

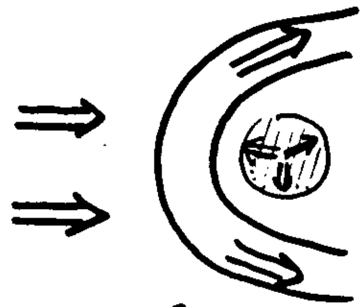
- Artificial Plasma Releases
- K-H Instability
- gravitational Instability

(1) Introductory Concepts

- consider problem of mass flows in space plasmas. Examples include



reconnection process



Flow around an obstacle

- External source maintained → driven system → 'quasi'-steady state if source maintained. Question is, how does dissipation occur to maintain state?

- **If** dissipation occurs at small scales this means that models we use must correctly handle this behavior. For example, finite ion inertia could be an important shielding effect at small scales in the mass flow problem.

- This has motivated us to construct new models of the plasma which contain the basic particulate nature of the plasma but is subject to MHD forces. In this lecture one such approach is described and applied.

(2) MHD-particle Description

- Two methods traditionally used
  - → Lagrangian schemes – cells move with fluid elements. In 2 or 3-D fluid can tangle up and multi-valuedness occurs.
  - → Eulerian schemes – mesh of cells fixed in frame of reference at rest and fluid streams by across mesh. Advective terms difficult to handle in continuity eq. ($\rightarrow n<0$)

- Alternative method to solving Lagrangian MHD equations:
  - → let finite-size particles to represent elements of neutral fluid. Rather than charge cloud as in conventional code, mass distribution. Spatial mesh introduced to accumulate fluid quantities.

(Leboeuf et. al., J. Comp. Phys, 1980)

- Field quantities, such as B, are only assigned at the mesh points. magnetic forces are computed through stress tensor evaluated at the grid points and particle pressure gradient computed from density in Fourier space.

- Particles carry quantities associated with fluid elements across the fixed background mesh.

- price we pay is that we now need a lot of particles (to keep noise low) and field quantities. Severely constrains us for certain problems.

- the MHD-particle model correctly simulates the dynamics of the particles which make up the fluid. MHD forces are employed rather than conventional Lorentz force.

- ideal MHD scheme (Lebouef et. al. 1980)

Lagrangian form

$$\frac{d\underset{\sim}{r}_i}{dt} = \underset{\sim}{v}_i$$

$$\frac{d\underset{\sim}{v}_i}{dt} = \frac{-1}{\rho}\underset{\sim}{\nabla} p - \frac{1}{8\pi\rho}\underset{\sim}{\nabla} B^2 + \frac{1}{4\pi\rho}\underset{\sim}{\nabla}\cdot(\underset{\sim}{B}\,\underset{\sim}{B})$$

$$\frac{\partial \underset{\sim}{B}}{\partial t} = \underset{\sim}{\nabla} \times (\langle \underset{\sim}{v} \rangle \times \underset{\sim}{B})$$ Eulerian Form

- NB) $\rightarrow$ Since magnetic force in momentum conservative form $\Rightarrow \underset{\sim}{\nabla}\cdot\underset{\sim}{B} = 0$

$$\nabla p = T \nabla n \quad \text{(isothermal case)}$$

2 further extensions

(i) Inclusion of Hall terms

ions: $$n m_i \frac{d v_i}{dt} = n|e| \left(\underset{\sim}{E} + \frac{\underset{\sim}{v}_i \times \underset{\sim}{B}}{c} \right) - \nabla p_i$$

electrons: $$n m_e \frac{d v_e}{dt} = 0 = -n|e| \left(\underset{\sim}{E} + \frac{\underset{\sim}{v}_e \times \underset{\sim}{B}}{c} \right) - \nabla p_e$$

continuity: $$\frac{dn}{dt} + n \underset{\sim}{\nabla} \cdot \underset{\sim}{v}_i = 0 \quad , \quad n_e \approx n_i$$

$$\underset{\sim}{J} = n|e| \left(\underset{\sim}{v}_i - \underset{\sim}{v}_e \right)$$

Since $\underset{\sim}{J} = \frac{c}{4\pi} \underset{\sim}{\nabla} \times \underset{\sim}{B} \implies \underset{\sim}{\nabla} \cdot \underset{\sim}{J} = 0 \rightarrow n_e = n_i$

$$\underset{\sim}{\nabla} \times \underset{\sim}{E} = -\frac{1}{c} \frac{\partial \underset{\sim}{B}}{\partial t}$$

From electrons: $\underset{\sim}{E} = -\frac{\underset{\sim}{v}_e \times \underset{\sim}{B}}{c} - \frac{1}{ne} \underset{\sim}{\nabla} p_e$

which gives

$$n m_i \frac{d \underset{\sim}{v}_i}{dt} = \frac{ne}{c} (\underset{\sim}{v}_i - \underset{\sim}{v}_e) \times \underset{\sim}{B} - \underset{\sim}{\nabla}(p_e + p_i)$$

$$= \frac{\underset{\sim}{J} \times \underset{\sim}{B}}{c} - \underset{\sim}{\nabla}(p_e + p_i)$$

$$= -\frac{\underset{\sim}{B} \times (\underset{\sim}{\nabla} \times \underset{\sim}{B})}{4\pi} - \underset{\sim}{\nabla}(p_e + p_i)$$

Since $$\frac{\partial \underset{\sim}{B}}{\partial t} = -c \, \underset{\sim}{\nabla} \times \underset{\sim}{E}$$

$$\underset{\sim}{E} = \frac{(\underset{\sim}{v}_i - \underset{\sim}{v}_e) \times \underset{\sim}{B}}{c} - \frac{\underset{\sim}{v}_i \times \underset{\sim}{B}}{c} - \frac{\underset{\sim}{\nabla} p_e}{ne}$$

$$= c \, \underset{\sim}{\nabla} \times \left[\frac{\underset{\sim}{B} \times (\underset{\sim}{\nabla} \times \underset{\sim}{B})}{4\pi n e} + \frac{\underset{\sim}{v}_i \times \underset{\sim}{B}}{c} + \frac{\underset{\sim}{\nabla} p_e}{ne} \right]$$

- Final equations: *(with Hall current)*

$$\frac{d \underset{\sim}{r}_i}{dt} = \underset{\sim}{v}_i$$

$$\frac{d \underset{\sim}{v}_i}{dt} = -\frac{\underset{\sim}{B} \times (\underset{\sim}{\nabla} \times \underset{\sim}{B})}{4\pi n m_i} - \frac{1}{n m_i} \underset{\sim}{\nabla} (p_e + p_i)$$

Hall term

$$\frac{\partial \underset{\sim}{B}}{\partial t} = \underset{\sim}{\nabla} \times (\underset{\sim}{v}_i \times \underset{\sim}{B}) + \underset{\sim}{\nabla} \times \left[\frac{c}{4\pi n e} \underset{\sim}{B} \times (\underset{\sim}{\nabla} \times \underset{\sim}{B}) + \frac{c}{ne} \underset{\sim}{\nabla} p_e \right]$$

- closure by assuming equation of state.

• dispersion characteristics

$$-i\omega \underset{\sim}{v} = \frac{1}{\rho}\left[i(\underset{\sim}{k}\times\underset{\sim}{\hat{B}})\times\underset{\sim}{B}_0\right]$$

$$-i\omega \underset{\sim}{\hat{B}} = i\underset{\sim}{k}\times(\underset{\sim}{v}\times\underset{\sim}{B}_0) + \frac{c}{4\pi en}\underset{\sim}{k}\times\left[(\underset{\sim}{k}\times\underset{\sim}{\hat{B}})\times\underset{\sim}{B}_0\right]$$

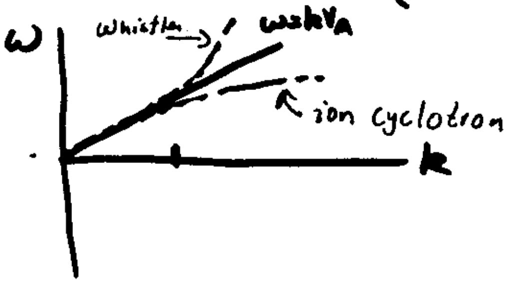
$\underset{\sim}{k} \parallel \underset{\sim}{B}_0$ (linearly polarized)

$$(\omega - k^2V_A^2)\,\hat{B} - \frac{ic}{4\pi en}\,\omega\,(\underset{\sim}{k}\cdot\underset{\sim}{B}_0)(\underset{\sim}{k}\times\underset{\sim}{\hat{B}}) = 0$$

$$\omega = \pm\left[\left(\frac{k^2V_A^2}{2\Omega_i}\right) \pm kV_A\left(1+\frac{k^2V_A^2}{4\Omega_i^2}\right)^{1/2}\right]$$

$$\frac{kV_A}{\Omega_i} < 1 \implies \omega \sim kV_A\left(1 \pm \frac{kV_A}{2\Omega_i}\right)$$

$$\frac{kV_A}{\Omega_i} > 1 \implies \omega \sim \begin{cases}\Omega_i & \text{ICW} \\ k^2V_A^2/\Omega_i & \text{whistler}\end{cases}$$



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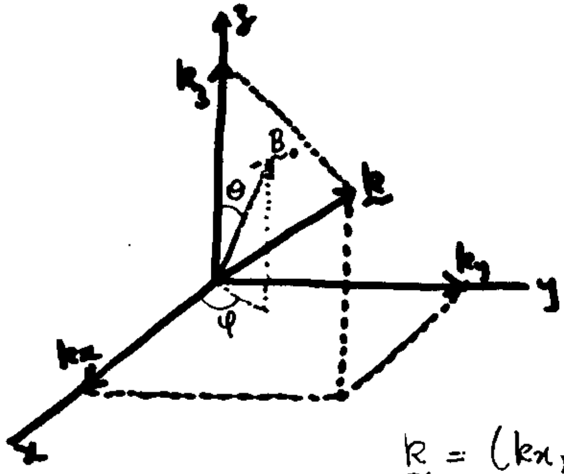
• general case (3-D)

$$-i\omega\hat{B} = i\left\{\frac{\underset{\sim}{k}\cdot\underset{\sim}{B}_0}{4\pi\rho\omega}\left[\underset{\sim}{k}(\underset{\sim}{B}_0\cdot\hat{B}) - \hat{B}(\underset{\sim}{k}\cdot\underset{\sim}{B}_0)\right]\right.$$

$$\left. + \frac{(\underset{\sim}{B}_0\cdot\hat{B})\left[(\underset{\sim}{k}\cdot\underset{\sim}{B}_0)\frac{k^2c_s^2}{\omega^2}\underset{\sim}{k} - k^2\underset{\sim}{B}_0\right]}{4\pi\rho\omega(1-k^2c_s^2/\omega^2)}\right\} + \frac{c}{4\pi ne}(\underset{\sim}{k}\times\hat{B})(\underset{\sim}{k}\cdot\underset{\sim}{B}_0)$$

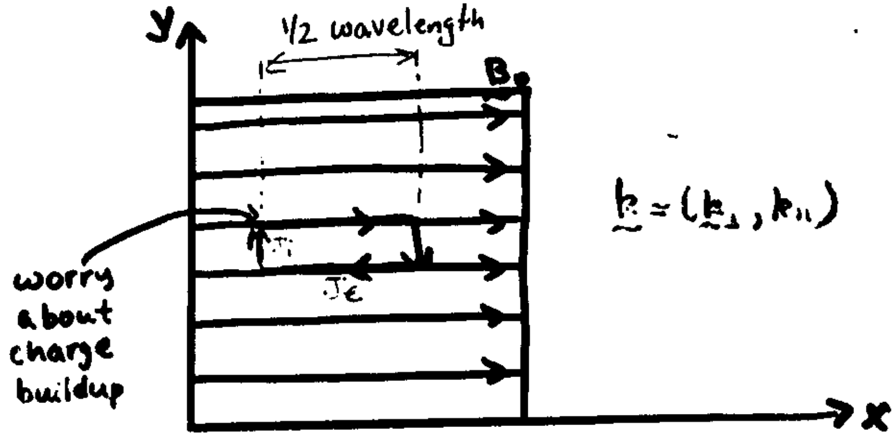
$$\left(c_s^2 = \frac{T_e+\gamma T_i}{m_i}\right)$$



$$\underset{\sim}{k} = (k_x, k_y, k_z)$$

$$\underset{\sim}{B}_0 = (B_0\sin\theta\cos\varphi,\ B_0\sin\theta\sin\varphi,\ B_0\cos\theta)$$

10



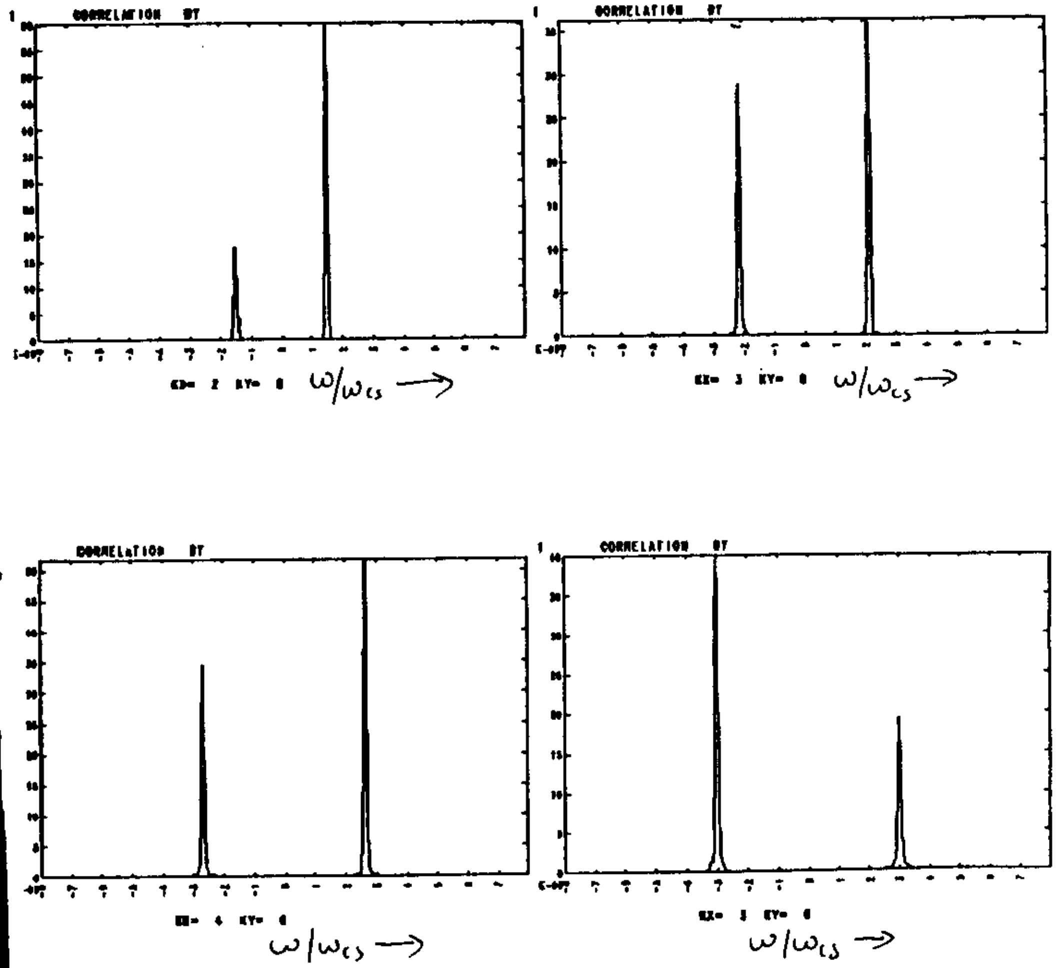
$\underset{\sim}{k} = (\underset{\sim}{k}_\perp, k_\parallel)$

- Due to density fluctuations $\underset{\sim}{J}_i$'s could be different at half wavelength $\Rightarrow$ a charge buildup in rectangle $\Rightarrow \underline{\nabla}\cdot\underline{J} \neq 0$

This would violate $\underline{\nabla}\times\underline{B} = \frac{4\pi}{c}\underline{J}$ and results would be wrong.

Power Spectrum for the case without the Hall term. ③

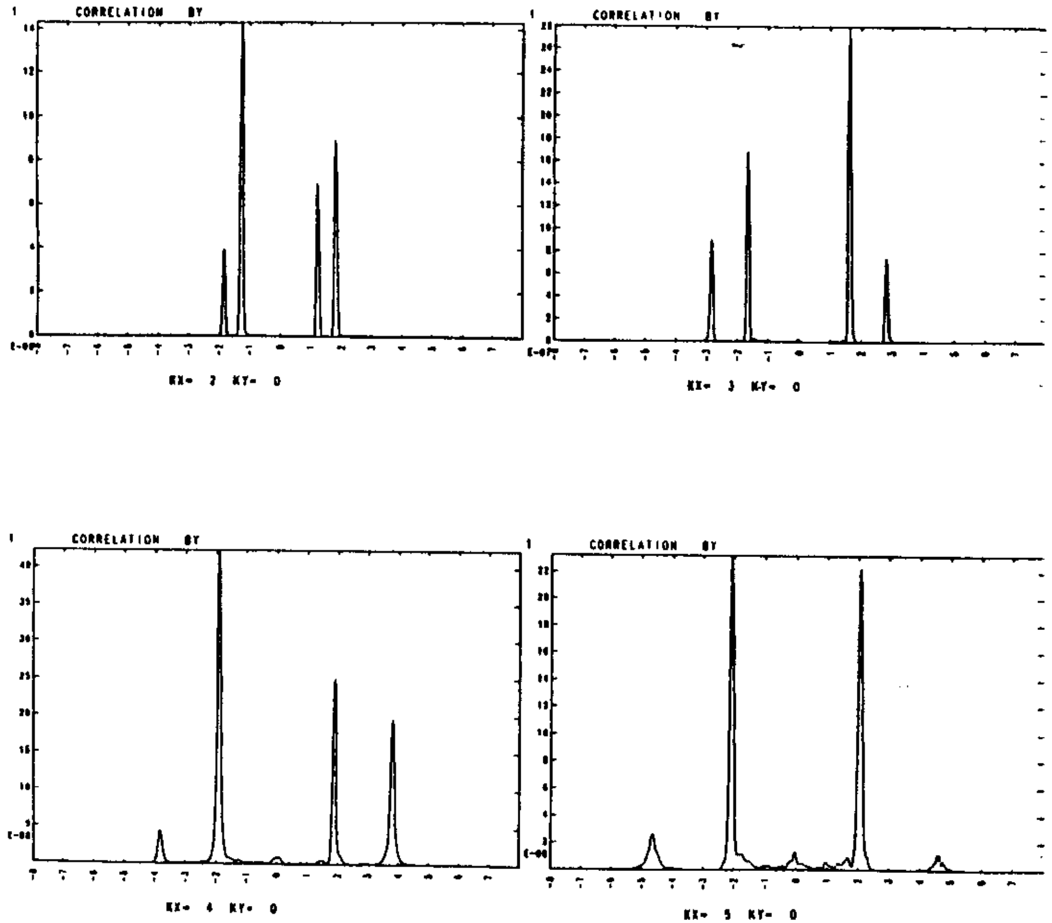
$\underline{k} \parallel \underline{B}$, $\theta = 0°$



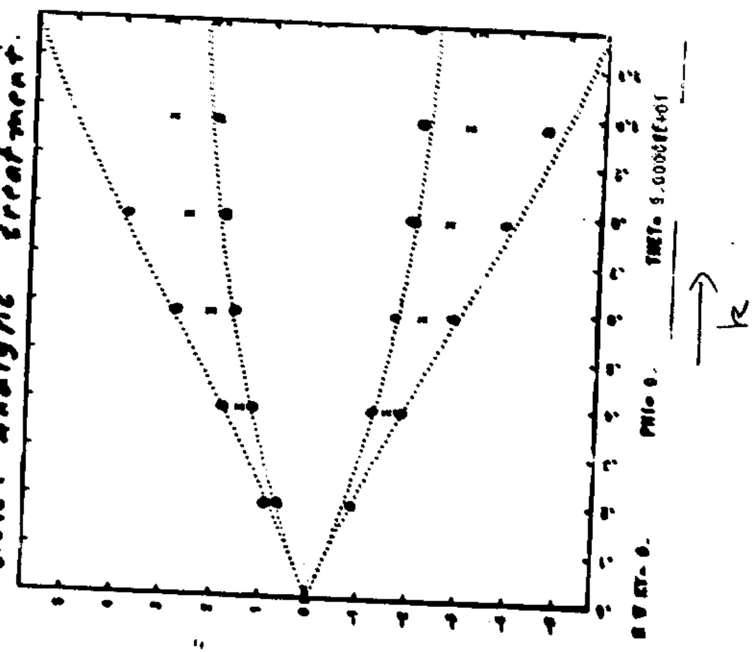
$\underset{\sim}{B}(k_x, k_y)$, $k_x = 2\pi m/L_x$, $m = 0, \pm 1, \dots, \pm L_x/2 - 1$

$k_y = 2\pi n/L_y$, $n = 0, \pm 1, \dots, \pm L_y/2 - 1$

with the Hall term. (10)



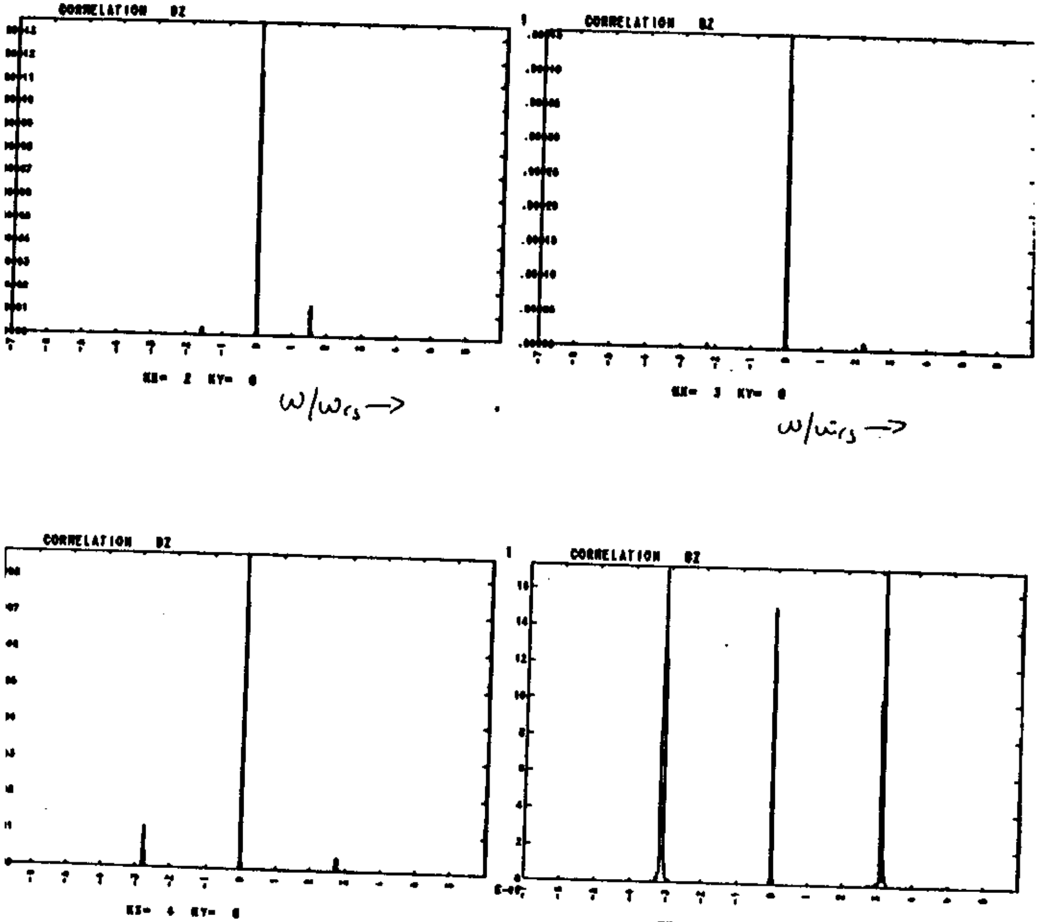
Simulation on analytic linear theory. // propagation $\theta = 0$.
×: results without the Hall term
•: Ion cyclotron (Hall term $\neq 0$)
•: whistler (Hall term $\neq 0$)
dots: Analytic treatment.



3|3

Power spectrum for the BZ Autocorrelation. Case of $B_0 = B_0 \hat{z}$ $\theta = 0$, $\varphi = 45°$

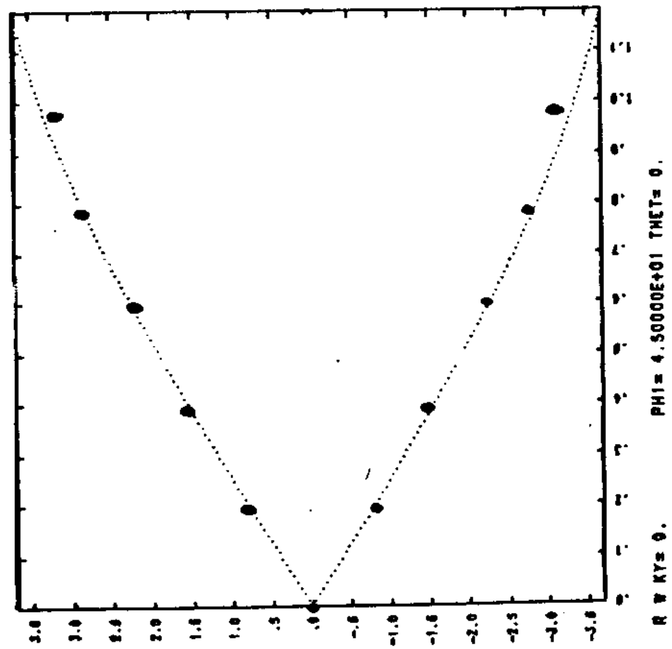
(12)



• : Magnetosonic waves.

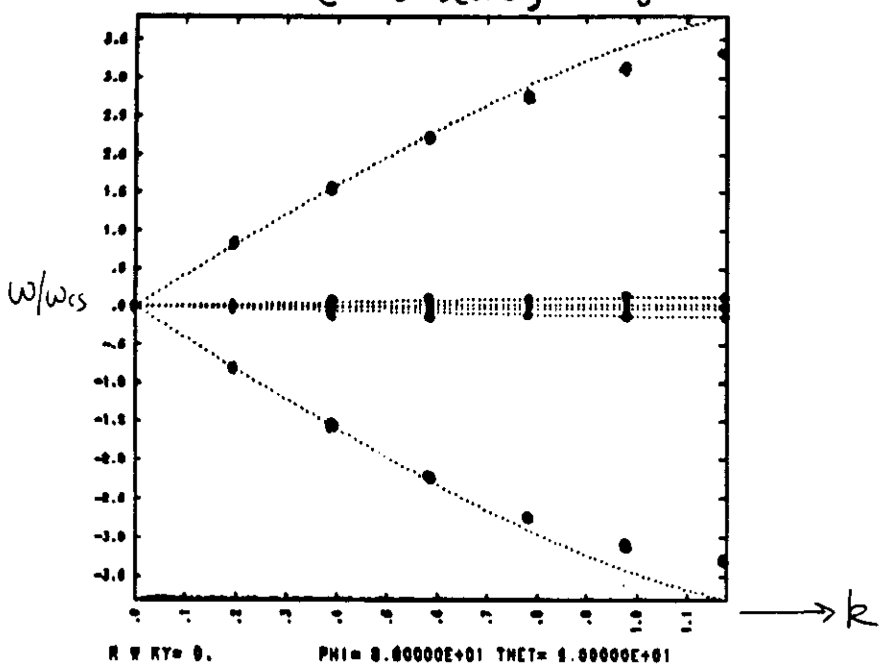
⋮ : data : Analytic results.

$\theta = 0°$, $\varphi = 45°$



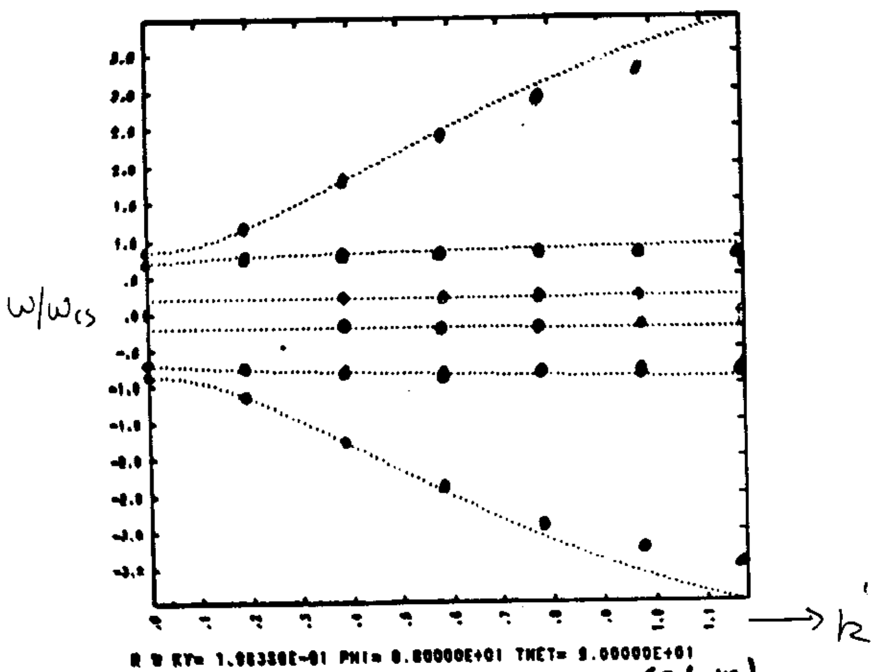
(51)

Bz, By correlations
on analytic theory.
$\theta = 90°$, $\varphi = 88$
(large scale) $Ky = 0$



•: Ion cycltron.
•: Fast wave (magnetosonic).
dots: Analytic Calculations.

Bz, By correlations
on analytic theory.
$Ky = 1$, $\theta = 90°$, $\varphi = 88°$.



•: Ion cycltron (slow).
•: whistler (fast)
•: Ion acoustic.
dots: analytic treatment.

VII Inclusion of Temperature Eq.

- particles can also be assigned a temperature and non ideal effects such as resistivity and heat conductivity can be included
- From second moment of kinetic equation

$$\frac{dT_i}{dt} = \frac{-1}{n}\left[(\underline{p}\cdot\nabla)\cdot\underline{v} - \underline{J}\cdot\left(\underline{E}+\frac{1}{c}\underline{v}\times\underline{B}\right) + \nabla\cdot\underline{W}\right]$$

$$\underline{E} = -\frac{\underline{v}\times\underline{B}}{c} + \eta\,\underline{J}$$

$$\underline{W} = -\underline{\underline{\kappa}}\cdot\nabla T$$ Heat flux due to collisions

$$\therefore \left\{\begin{aligned} \frac{dT_i}{dt} &= \frac{-1}{n}(\underline{p}\cdot\nabla)\cdot\underline{v} + \frac{1}{n}\left(\nabla\cdot\underline{\underline{\kappa}}\cdot\nabla T\right) + \frac{c^2}{(4\pi)^2 n}\left[(\nabla\times\underline{B})\cdot\underline{\underline{\eta}}\cdot(\nabla\times\underline{B})\right] \\ \frac{dx_i}{dt} &= \underline{v}_i \end{aligned}\right.$$

$$\frac{\partial \underline{B}}{\partial t} = \nabla\times(\underline{v}\times\underline{B}) - \frac{c^2}{4\pi}\nabla\times\underline{\underline{\eta}}\cdot(\nabla\times\underline{B})$$

(3) Basic Algorithm & Numerical methods

- **System of equations**

$$\frac{d\underline{x}_i}{dt} = \underline{v}_i$$

$$\frac{d\underline{v}_i}{dt} = F_v(T, B, n)$$

$$\frac{dT_i}{dt} = F_T(T, B, n, v)$$

$$\dot{B} = F_B(B, v, n)$$

- **Time-centered scheme (2^{nd} order acc.)**

$$x_i^{n+1} - x_i^n = \Delta t\, v_i^{n+1/2}$$

$$v_i^{n+1/2} - v_i^{n-1/2} = \Delta t\, F_v(T^{n*}, B^{n*}, n^{n*})$$

$$T_i^{n+1/2} - T_i^{n-1/2} = \Delta t\, F_T(T^{n*}, B^{n*}, n^{n}, v^{n*})$$

$$B^{n+1/2} - B^{n-1/2} = \Delta t\, F_B(B^{n*}, v^{n*}, n^{n*})$$

where
$$n^{n*} = \frac{3}{2} n^{n-1/2} - \frac{1}{2} n^{n-3/2}$$
$$T^{n*} = \frac{3}{2} T^{n-1/2} - \frac{1}{2} T^{n-3/2} \quad \text{etc.}$$

• Just as in particle codes, we express density

$$n(\underline{x}) = \sum_i S(x - x_i)$$

Gaussian shape

$$S(x-x_i) = \frac{1}{(2\pi a^2)^{3/2}} e^{-(\underline{x}-x_i)^2/2a^2}$$

$$\longrightarrow n(\underline{x}) = \sum_g S(\underline{x}-\underline{x}_g) \underbrace{\rho(g)}_{(NGP)(CIC)} = \sum_{i\in g} 1$$

(Nearest grid value)

$$\langle v \rangle^n = \frac{1}{n_g} \sum_{i \in g} v_i^n$$

$$\langle T \rangle^n = \frac{1}{n_g^n} \sum_i T_i^n$$

(5) Applications

(i) K-H Instability

(Shear flow)

(ii) gravitational instability

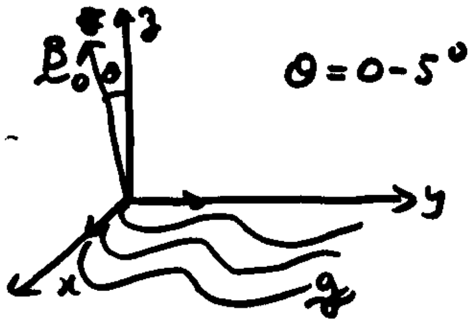
$$\underset{\sim}{g} = g_x \hat{x} + g_y \hat{y}$$

$$g_x = g_0 \frac{L_y}{L_x} \cos(k_x x) \sin(k_y y)$$

$$g_y = g_0 \sin(k_x x) \cos(k_y y)$$

$$\frac{L_{cx}\sin(\theta)}{2} = L_c = \frac{2\pi}{\gamma} C_A$$

$$\gamma \sim \sqrt{k_\perp g}$$



$\theta = 0-5°$

K-H Instability

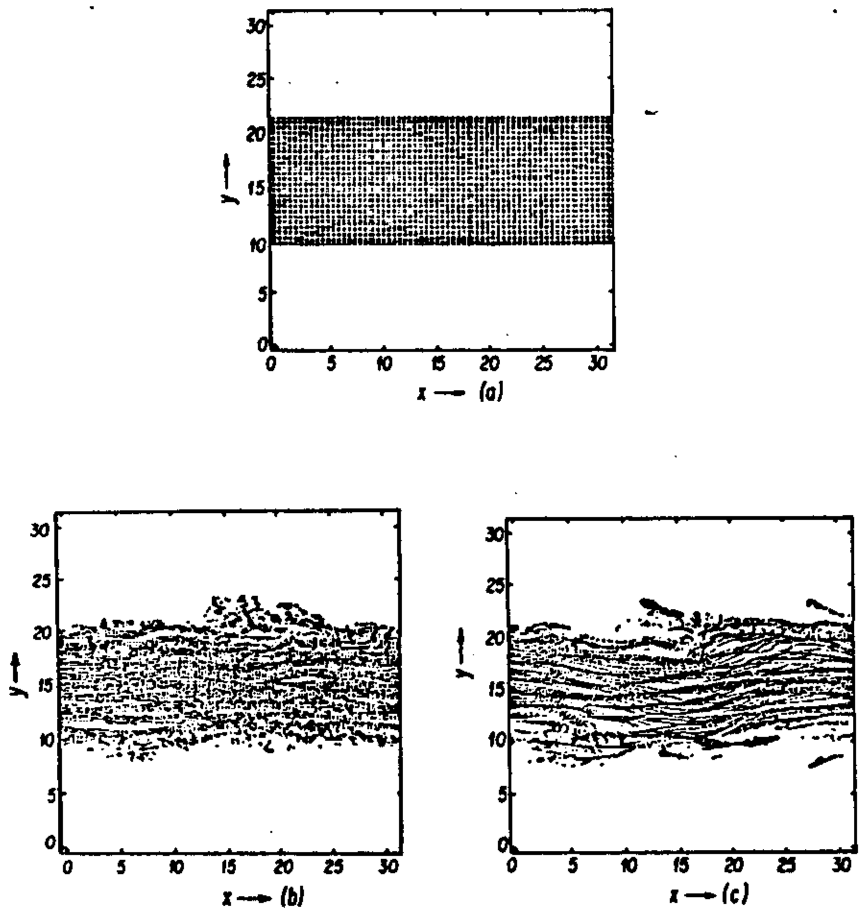


Fig. 8

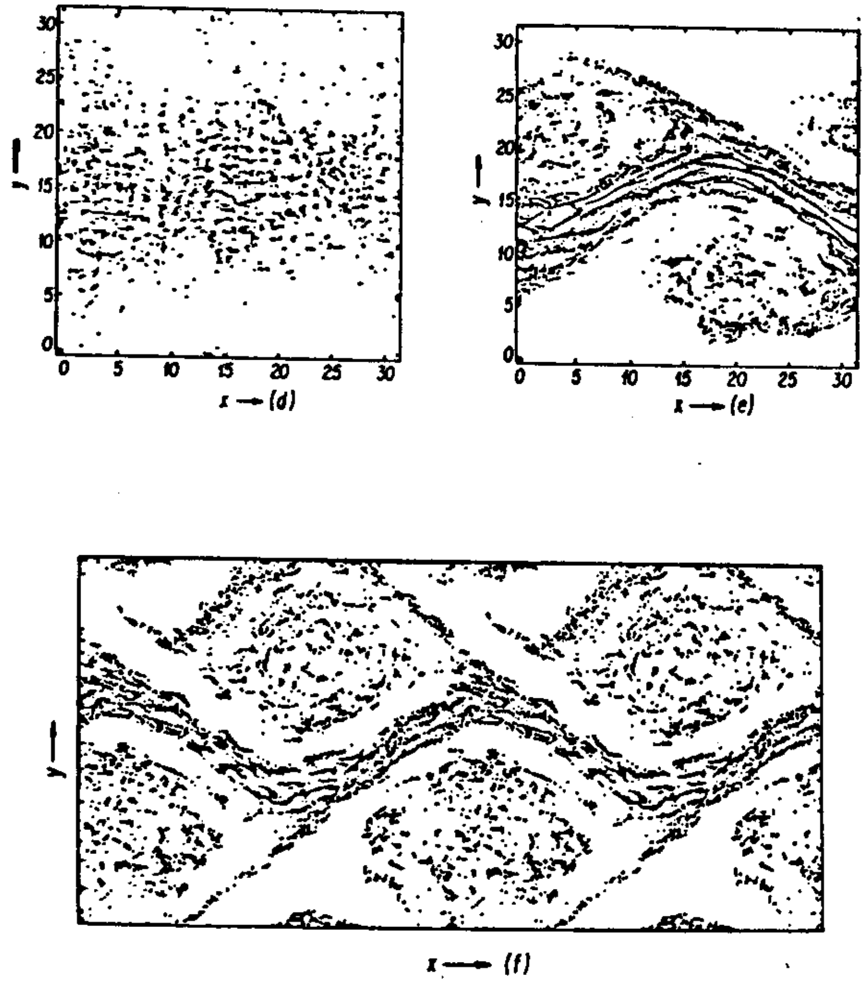
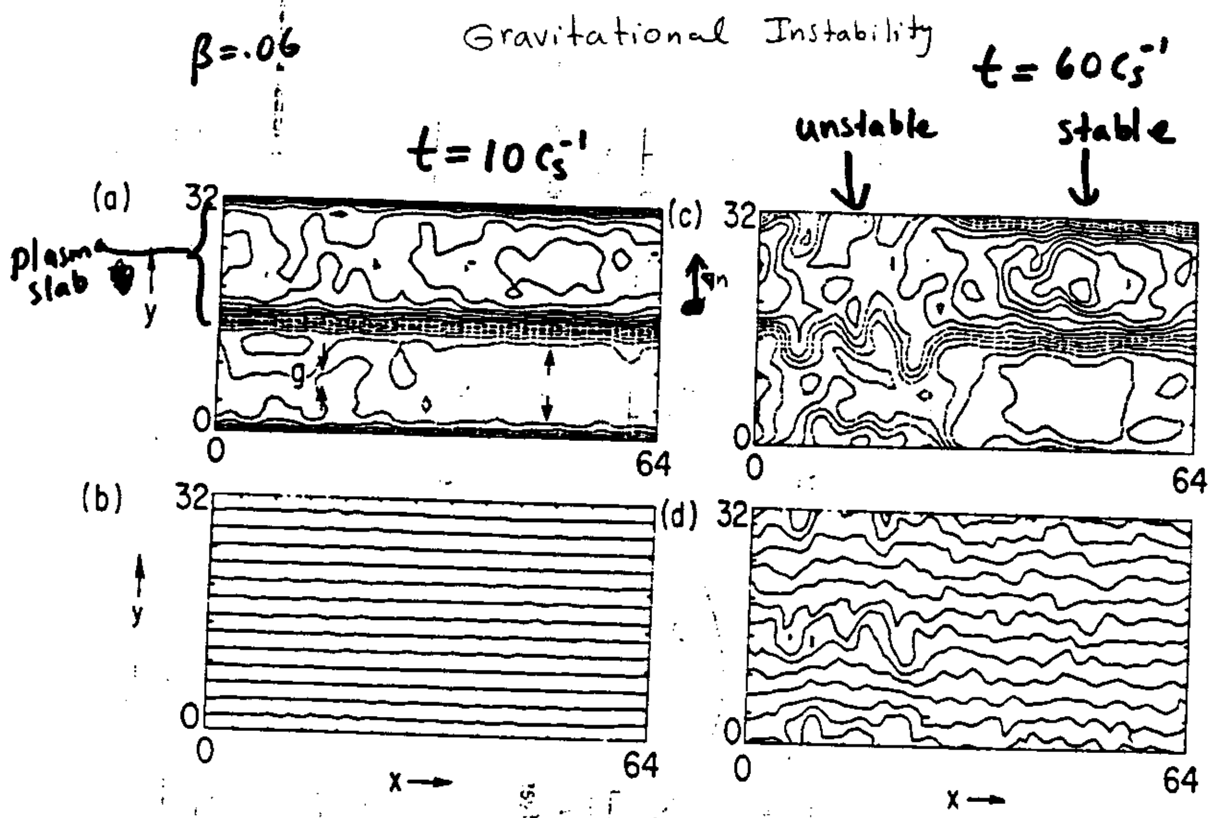
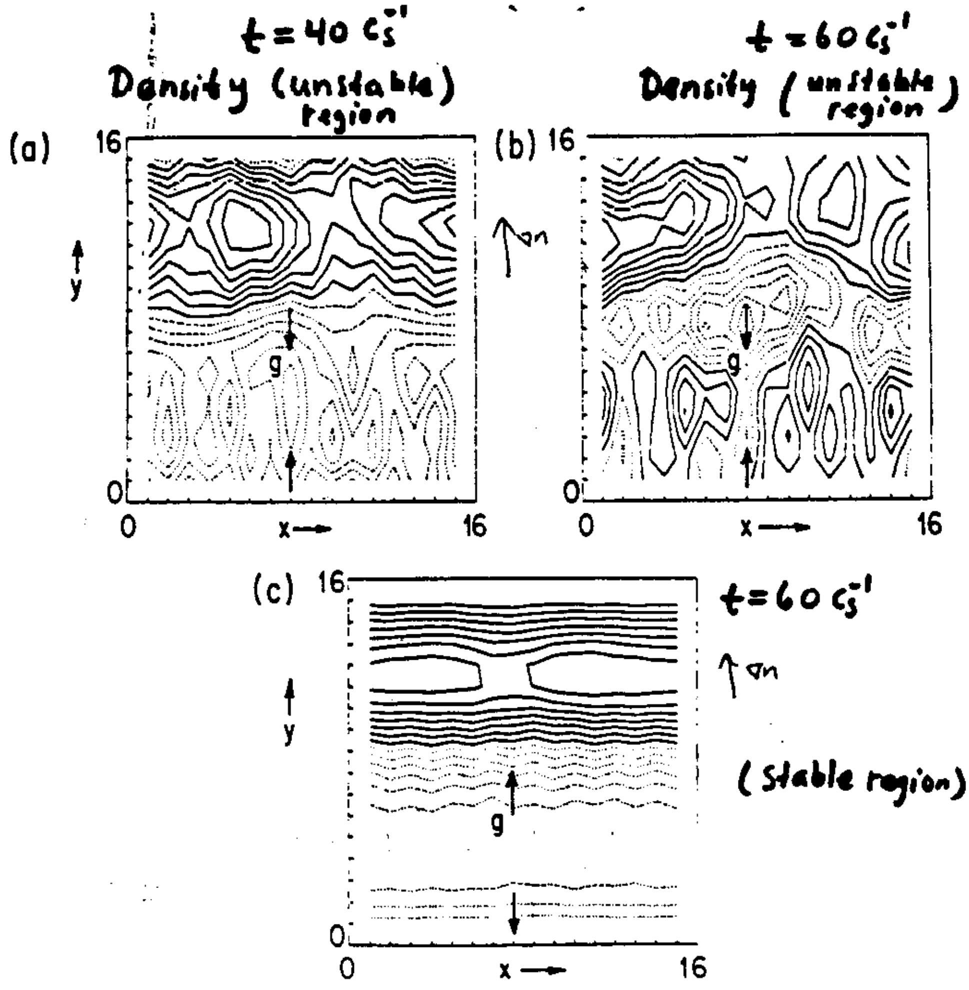


Fig. 8



β=.06
Gravitational Instability
t = 10 c_s^{-1}
t = 60 c_s^{-1}
unstable
stable
(a)
(b)
(c)
(d)
plasma slab
y
g
∇n
x →
32
0
64

FIG. 2



t = 40 c_s^{-1}
Density (unstable region)
t = 60 c_s^{-1}
Density (unstable region)
(a)
(b)
(c)
t = 60 c_s^{-1}
(stable region)
∇n
y
g
x →
16
0

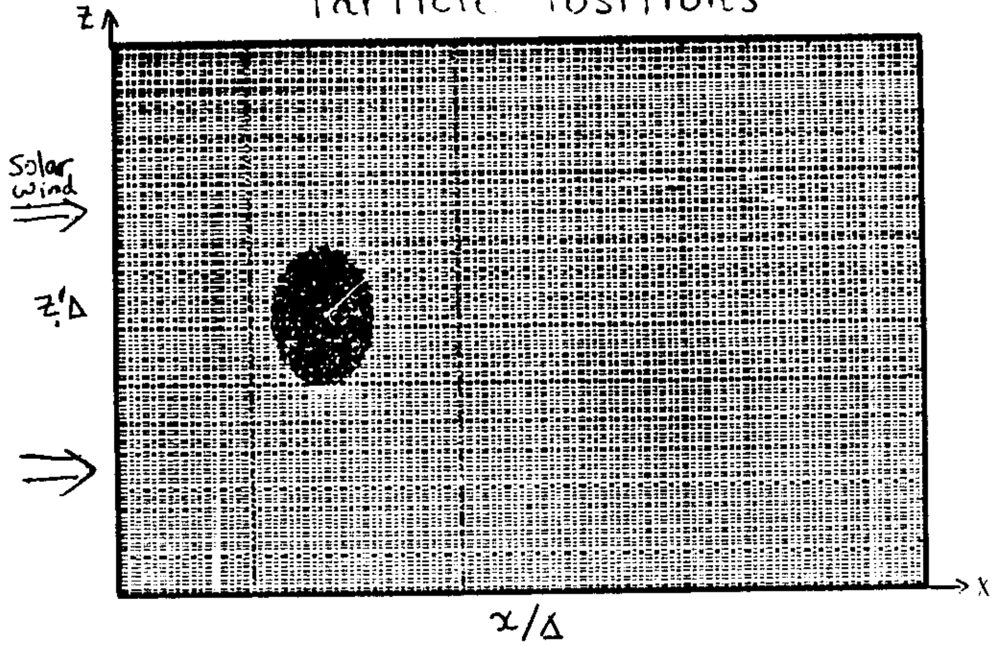
FIG. 3

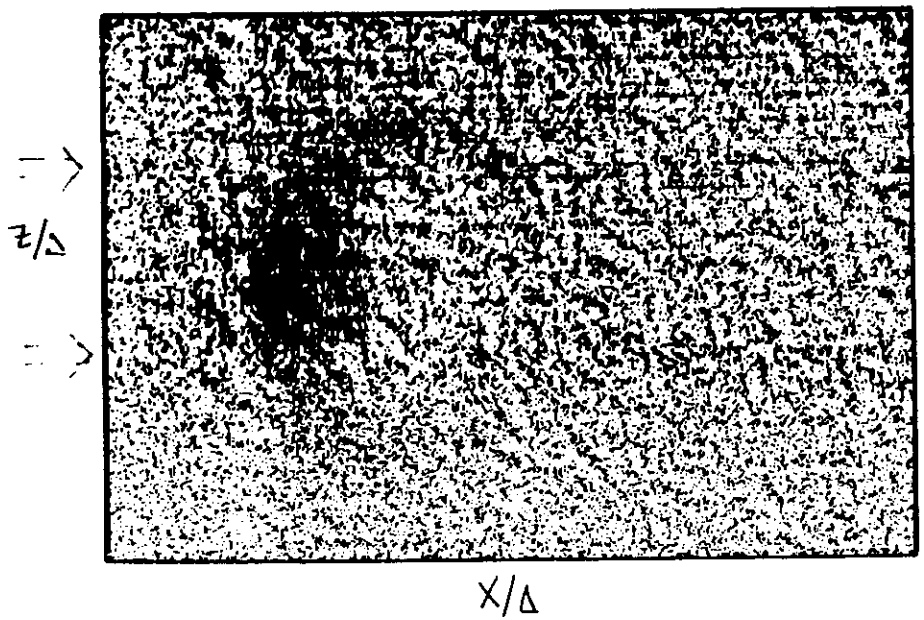
(iii) plasma releases & comets

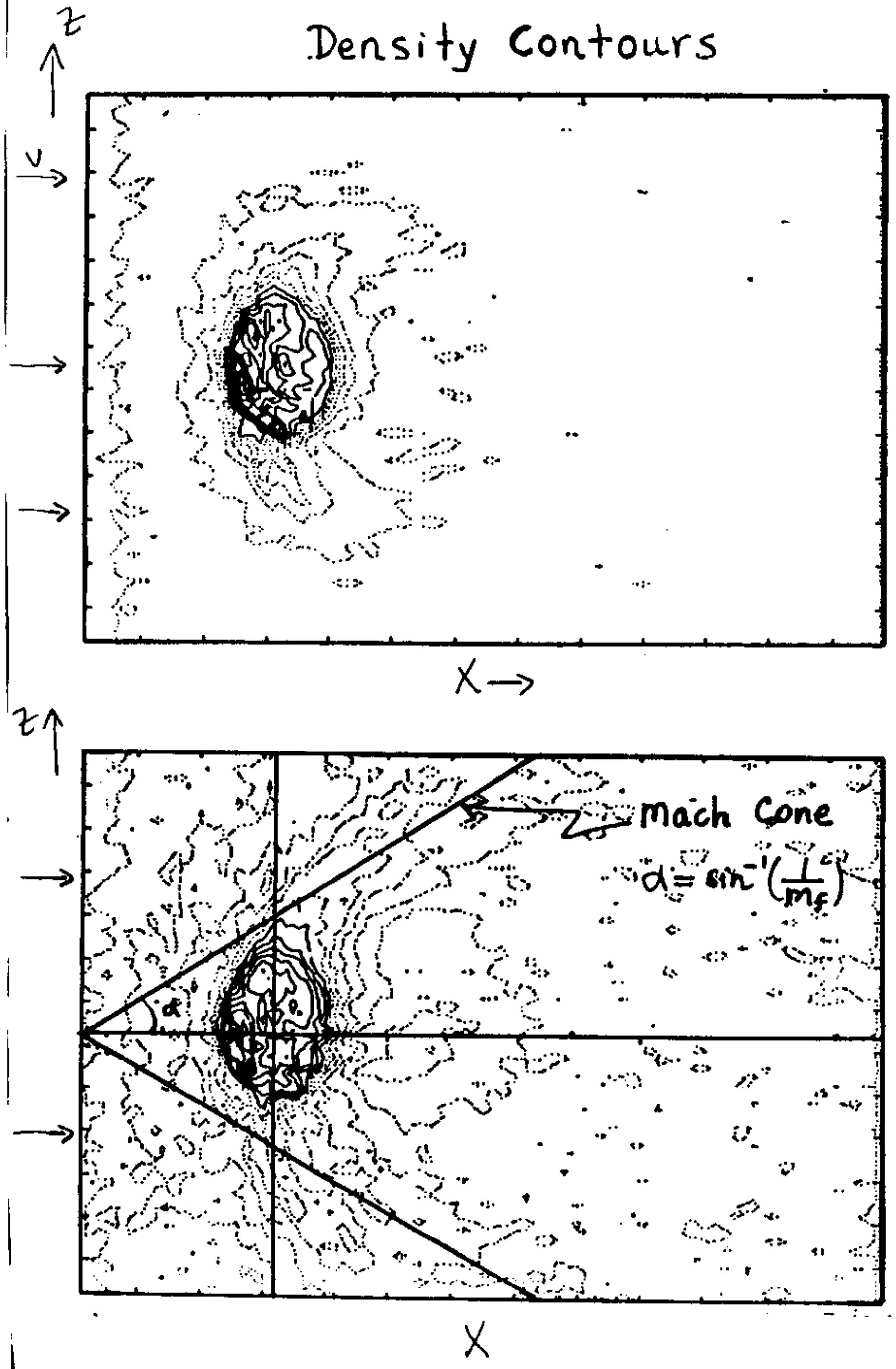
$\underbrace{\text{plasma releases}}_{\text{Impulsive}}$ & $\underbrace{\text{comets}}_{\text{Steady state}}$

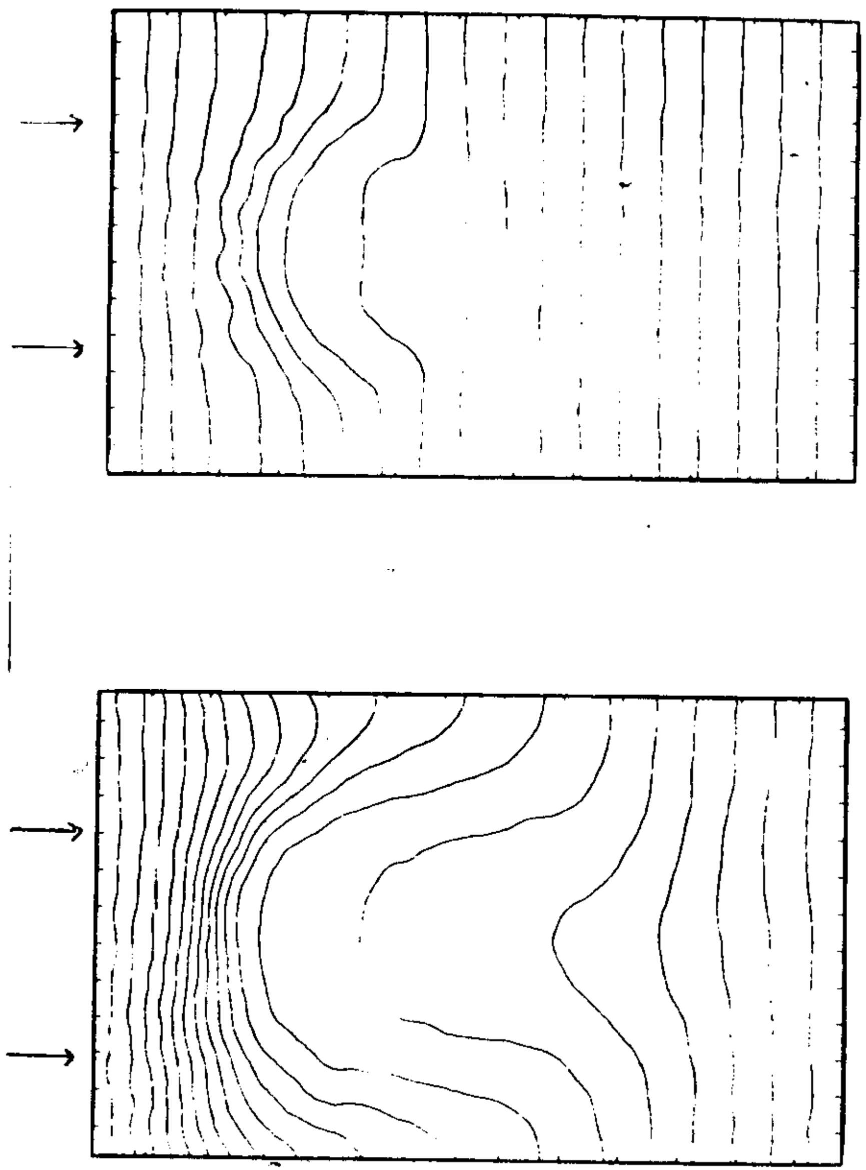
- basic problem of flow past an obstacle.

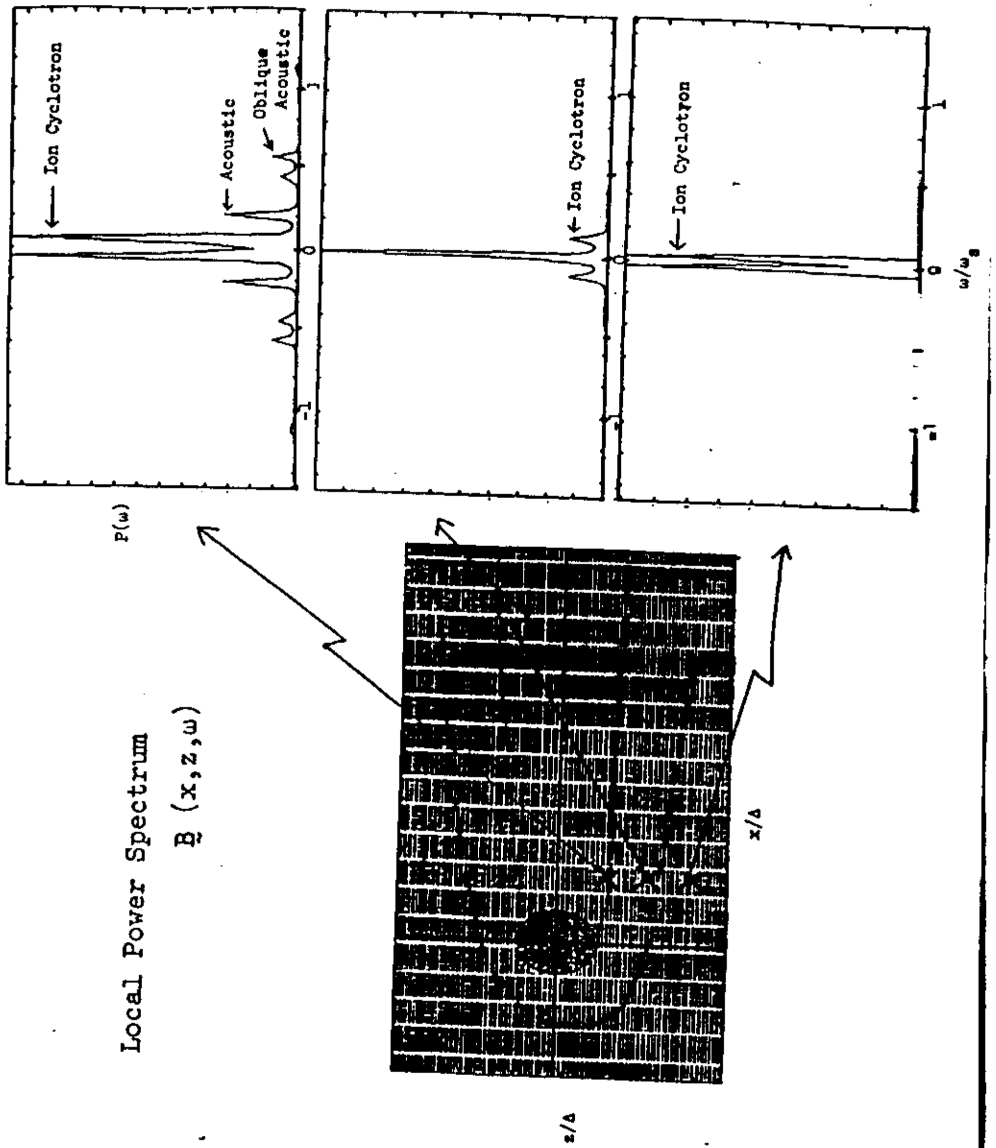
Particle Positions






Density Contours
z
v
X →
z
Mach Cone
$\alpha = \sin^{-1}\left(\frac{1}{M_f}\right)$
α
X





Local Power Spectrum
$\underset{\sim}{B}(x, z, \omega)$

Linear Analysis