INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
9 JUN 1987
LIBRARY

INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) - P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONE: 2240-1
CABLE: CENTRATOM - TELEX 460392 - I

H4.SMR/210 - 31

SPRING COLLEGE ON PLASMA PHYSICS

(25 May - 19 June 1987)

DIFFUSIVE ACCELERATION BY E-M WAVES

K. Hizanidis

University of Maryland
U.S.A.

DIFFUSIVE ACCELERATION BY E-M WAVES

Kyriakos Hizanidis
University of Maryland

- SINGLE PARTICLE <u>ORBIT</u> INTEGRATION

- VELOCITY (MOMENTUM, ACTION) SPACE EVOLUTION OF <u>DISTRIBUTION</u>

- INITIAL CONDITIONS
- CONSTANTS OF THE UNPERTURBED MOTION

DEMONSTRATION BY A PROTOTYPE PROBLEM:

ELECTRONS, RELATIVISTIC, E-M WAVES WITH $\omega/k > c$.

SOLAR PHYSICS

PULSAR PHYSICS

ACCELERATOR PHYSICS

COHERENT WAVES *

<u>ELECTROSTATIC</u> : Smith & Kaufman

(LIMITED ACCELERATION)

- PARALLEL PROPAGATION - VACUUM

$$\omega - k_z v_z = \frac{\Omega}{\gamma} \quad , \quad \frac{c k_z}{\omega} = 1$$

$$\underline{\gamma (1 - \beta_z) = K \equiv \frac{\Omega}{\omega}}$$

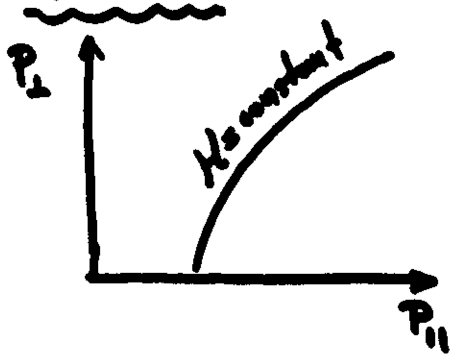
- OBLIQUE PROPAGATION

$$\underline{\omega - k_z v_z = \ell \frac{\Omega}{\gamma}}$$

OVERLAPPING RESONANCES

- Kolomenskii and Lebedev 1963
- Sharma et al 1982 electron cyclotron maser instabilities driven by loss cone distribution in solar flare accelerated electrons
- Menyuk et al, 1987 • Karimabadi et al, 1986

$\frac{\omega}{k} > c$, RELATIVISTIC EFFECTS



OPEN LINES OF CONSTANT $\mathcal{H}$

$$\boxed{H_0 = mc^2\gamma_0 - c\,\frac{P_h}{n_h}}$$

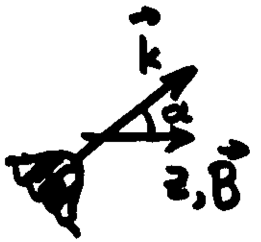
$$\gamma_0 = \left[1 + \frac{(P_\alpha + P_h)^2}{m^2c^2} + \frac{2\omega_{ce} J}{mc^2}\right]^{1/2}$$

$$\boxed{H_1 = \sum_{i,l} h_{il} \sin\psi_{il}}$$

HAMILTONIAN FORMULATION

$$\vec{B} = B_0 \vec{e}_z$$

$$\vec{A} = \sum_{i=1}^{N} A_{0i}\left(\vec{e}_x \cos\alpha \sin\phi_i + \vec{e}_y \cos\phi_i - \vec{e}_z \sin\alpha \sin\phi_i\right) + x B_0 \vec{e}_y$$



$$\phi_i = k_{xi} x + k_{zi} z - \omega_i t$$

$$\mathcal{H} = c\left[\left(\vec{p} + \frac{e\vec{A}}{c}\right)^2 + m^2c^2\right]^{1/2}$$

$$\psi_{il} = k_{zi} z_\alpha \left(1 - \frac{n_h}{n_{zi}}\right) + k_{zi} z_h \frac{n_h}{n_{zi}} + l\theta$$

$$h_{il} = h_{il}(\alpha_i, \gamma_0, l, u_z, \rho)$$

$$u_z = \frac{P_\alpha + P_h}{mc}$$

$$\rho = \left(\frac{2cJ}{eB_0}\right)^{1/2}$$

CANONICAL TRANSFORMATIONS
ELIMINATION OF Y and t.

$$H = c\Big[\big(p_x + mc\cos\alpha \sum_i \alpha_i \sin\psi_i\big)^2 + \big(\frac{exB_0}{c} + mc\sum_i \alpha_i \cos\psi_i\big)^2 + \big(\sum_i p_{zi} - mc\sin\alpha \sum_i \alpha_i \sin\psi_i\big)^2 + m^2c^2\Big]^{1/2} - c\sum_i \frac{p_{zi}}{n_{zi}}$$

$$n_{zi} \equiv ck_{zi}/\omega_i \qquad \psi_i = k_{xi}x + k_{zi}z_i$$

$$\alpha_i \equiv ck_{zi}/\omega_i$$

$$(x, p_x),\ \{(z_i, p_{zi});\ i = 1 \div N\}$$

$$F = z_a \sum_i \Big(1 - \frac{n_h}{n_{zi}}\Big) P_{zi} + z_h \sum_i \Big(\frac{n_h}{n_{zi}}\Big) P_{zi}$$

$$\frac{N}{n_h} \equiv \sum_{i=1}^{N} \frac{1}{n_{zi}}$$

$$\boxed{\begin{aligned} z_i &= z_a\Big(1 + \frac{n_h}{n_{zi}}\Big) + \frac{n_h}{n_{zi}} z_h \\ p_a &= \sum_i \Big(1 - \frac{n_h}{n_{zi}}\Big) P_{zi} \\ p_h &= n_h \sum_i \frac{P_{zi}}{n_{zi}} \end{aligned}}$$

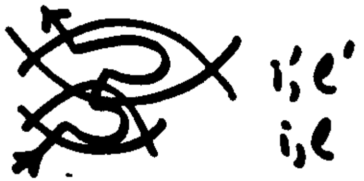
EXPANSION

$$(x, p_x) \rightarrow (\theta, J) \qquad p_x = \Big(\frac{2eB_0J}{c}\Big)^{1/2}\cos\theta \qquad x = \Big(\frac{2cJ}{eB_0}\Big)^{1/2}\sin\theta$$

$\alpha_i \ll 1$ ALL i

$$H \simeq H_0 + H_1 + \cdots$$

$p_h+\Delta p_h,\ p_a+\Delta p_a,\ J+\Delta J,\ z_h+\Delta z_h,\ z_a+\Delta z_a,\ \theta+\Delta\theta$



$p_h,\ p_a,\ J,\ z_h,\ z_a,\ \theta$

- τ_2 : INTERACTION TIME
- τ_1 : RELAXATION TIME

$$\tau_2 \ll \tau_1$$

FPK : τ

$$\tau_2 \ll \tau \ll \tau_1$$

CHAPMAN-KOLMOGOROV

$$f(\vec{X},t)=\int d\vec{X}'\,P(\vec{X},t\,|\,\vec{X}',t') \times f(\vec{X}',t')$$

$$\vec{X}\equiv\{J, p_a, p_h, \theta, z_a, z_h\}$$

ASSUME $(\theta, \{k_{zi} z_a\}, \{k_{zi} z_h\})$ CHANGE IN MUCH SHORTER TIME SCALE

$P \rightarrow \bar{P}$ INDEPENDENT OF PHASES

$$f(J,p_a,p_h)=\int dJ'\int dp_a'\int dp_h'\,\bar{P}(J,p_a,p_h\,|\,J',p_a',p_h')\,f(J',p_a',p_h')$$

Molvig and Hizanidis 1984

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial \vec{X}}\cdot(\vec{F}f) + \frac{1}{2}\frac{\partial^2}{\partial\vec{X}\partial\vec{X}}:(\overleftrightarrow{D}f)$$

$\downarrow$ Hamiltonian Systems
Landau 1947

$$\frac{\partial f}{\partial t} = \frac{1}{2}\frac{\partial}{\partial \vec{X}}\cdot\overleftrightarrow{D}\,\frac{\partial f}{\partial \vec{X}}$$

$$\overleftrightarrow{D} = \lim_{\tau\to 0^+}\tau^{-1}\int d\Delta\vec{X}\,\Delta\vec{X}\Delta\vec{X}\,\bar{P}(\vec{X}+\Delta\vec{X}, t+\tau\,|\,\vec{X}, t)$$

$$\lim_{\tau\to 0^+}\tau^{-1}\langle\Delta\vec{X}\Delta\vec{X}\rangle$$

$$\langle\Delta\vec{X}\Delta\vec{X}\rangle = \int_0^{\tau} d\tau' \int_{-\tau'}^{\tau-\tau'} d\tau''\,\overleftrightarrow{C}(t+\tau', t+\tau'+\tau'')$$

$$\overleftrightarrow{C}(t, t+\tau) = \langle\dot{\vec{X}}(t+\tau)\dot{\vec{X}}(t)\rangle$$

$\Downarrow$ EVALUATION UP TO ZEROTH ORDER IN α_i's.

$$\bar{P} \sim \prod_i \delta(\theta - \theta(t;\theta_0)) \times \delta(k_{zi}z_a - k_{zi}z_a(t;z_{a0})) \times \delta(k_{zi}z_h - k_{zi}z_h(t;z_{h0}))$$

(REVERSIBLE PROCESS)

$$\langle a(t,t')\rangle = \int d\theta_0 \int \{dk_{zi}\,z_{a0}\} \int \{dk_{zi}\,z_{h0}\} \times a(t,t';\theta_0, z_{a0}, z_{h0})$$

EVALUATE $\overleftrightarrow{C}$ $\rightsquigarrow$ DEPEND UPON $|t-t'|$ ONLY

$$\langle\Delta\vec{X}\Delta\vec{X}\rangle = 2\int_0^{\tau} d\tau'(\tau-\tau')\,\overleftrightarrow{C}(\tau')$$

HAMILTON EQUATIONS

$$\dot{P}_a = -\sum_{i,l} h_{il} k_{zi}\left(1-\frac{n_h}{n_{zi}}\right)\cos\psi_{il}$$

$$\dot{P}_h = -\sum_{i,l} h_{il} k_{zi}\frac{n_h}{n_{zi}}\cos\psi_{il}$$

$$\dot{J} = -\sum_{i,l} h_{il}\, l\cos\psi_{il}$$

$$\dot{z}_a = \frac{P_a+P_h}{m\gamma_0} + \sum_{i,l}\frac{\partial h_{il}}{\partial P_a}\sin\psi_{il}$$

$$\dot{z}_h = \frac{P_a+P_h}{m\gamma_0} - \frac{c}{n_h} + \sum_{i,l}\frac{\partial h_{il}}{\partial P_h}\sin\psi_{il}$$

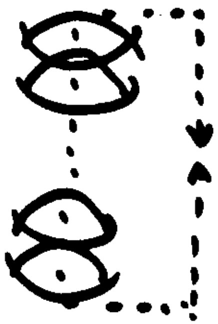
$$\dot{\theta} = \frac{\omega_{ce}}{\gamma_0} + \sum_{i,l}\frac{\partial h_{il}}{\partial J}\sin\psi_{il}$$

$$C_{hh}(t,t') = \frac{n_h^2}{2c^2}\sum_{i,l} h_{il}^2\omega_i^2 \times \cos\left[\left(k_{zi}v_z - \omega_i + \frac{l\omega_{ce}}{\gamma_0}\right)(t-t')\right]$$

$$\begin{bmatrix} D_{hh} & D_{hJ} & D_{ha} \\ D_{Jh} & D_{JJ} & D_{Ja} \\ D_{ah} & D_{aJ} & D_{aa}\end{bmatrix} = \pi\sum h_{il}^2\,\delta\left(k_{zi}v_z - \omega_i + \frac{l\omega_{ce}}{\gamma_0}\right) \times \begin{bmatrix} n_h^2\omega_i^2/c^2 & n_h l\omega_i/c & k_{zi}^2(1-n_h/n_{zi}) \\ n_h l\omega_i/c & l^2 & k_{zi}l(1-n_h/n_{zi}) \\ k_{zi}^2(1-n_h/n_{zi}) & k_{zi}l(1-n_h/n_{zi}) & k_{zi}^2(1-n_h/n_{zi})^2 \end{bmatrix}$$

$$v_z \equiv \frac{P_a+P_h}{m\gamma_0}$$

OVERLAPPING $k_{zi} V_z - \omega_i + \frac{l\omega_{ce}}{\gamma_0} = 0$



$\Delta p_h = c \Delta u_h$

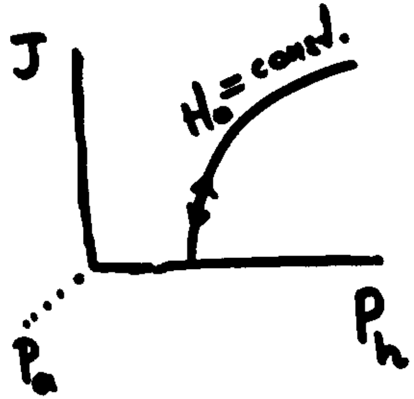
$$\begin{bmatrix} d_{hh} & d_{hJ} & d_{ha} \\ d_{Jh} & d_{JJ} & d_{Ja} \\ d_{ah} & d_{aJ} & d_{aa} \end{bmatrix} = \frac{\pi}{\Delta u_h} \sum_{i,l} \alpha_i^2 \cdots$$

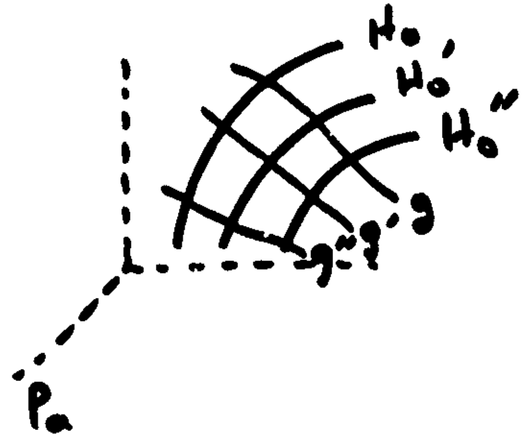
$$\cdots \times \begin{bmatrix} \boxed{\quad 0 \quad} & \boxed{1} \\ \boxed{\quad L \quad} & \boxed{2} \end{bmatrix}$$

in $(n_{zi} - n_h)$

$\overleftrightarrow{d}$: NORMALIZED DIFFUSION COEFFICIENT
$(m^2 c^A \omega_{ce}^B)$

H_0 : CONSTANT OF THE UNPERTURBED MOTION





$$(J, p_a, p_h) \rightarrow (H_0, g, p_a)$$

- ORTHOGONALITY CONDITION
- CHOICE OF THE JACOBIAN (h) OF THE TRANSFORMATION

EVALUATION OF $\frac{\partial g}{\partial p_h}, \frac{\partial g}{\partial J}$

TRANSFORMED (FPK)

$$\frac{1}{h}\frac{\partial f}{\partial t} = \frac{1}{2}\frac{\partial}{\partial H_0}\left(\bar{D}_{HH}\frac{1}{h}\frac{\partial f}{\partial H_0}\right) + \cdots$$

$$\bar{D}_{ij} = \bar{D}_{pq}\frac{\partial i}{\partial p}\frac{\partial j}{\partial q}$$

$$(i,j) : (H_0, g, p_a)$$

$$(p,q) : (J, p_h, p_a)$$

$$\begin{pmatrix} d_{gg} & d_{ga} & d_{gH} \\ d_{ag} & d_{aa} & d_{aH} \\ d_{Hg} & d_{Ha} & d_{HH} \end{pmatrix} = \frac{\pi}{\Delta\nu_n}\sum_{res}\frac{\omega_i}{\omega_{ce}}\frac{\alpha_i^2}{\gamma_0}\frac{n_h}{n_i^2}$$

$$\times \cdots\cdots$$

$$\times \begin{pmatrix} \nu_{hi}^2\gamma_0^2 & \nu_{zi}\gamma_0(n_{zi}-n_h) & \nu_{hi}\gamma_0(n_h-n_{zi})\beta_z \\ \nu_{zi}\gamma_0(n_{zi}-n_h) & (n_{zi}-n_h)^2 & -(n_{zi}-n_h)^2/n_h \\ \nu_{hi}\gamma_0(n_h-n_{zi})\beta_z & -(n_{zi}-n_h)^2/n_h & \beta_z^2(n_{zi}-n_h)^2 \end{pmatrix}$$

EVALUATE : d_{HH}, d_{gg}, d_{gH}, d_{ag}, d_{aH}

$$\overleftrightarrow{d} = \overleftrightarrow{d}_0 + \overleftrightarrow{d}_1 + \overleftrightarrow{d}_2$$

$$= \begin{pmatrix} \boxed{0} & \boxed{1} \\ \boxed{1} & \boxed{2} \end{pmatrix}$$

SEPARATION OF TIME SCALES

$$C(f) = C_0(f) + C_1(f) + C_2(f)$$

$$= \frac{\partial}{\partial \vec{x}} \cdot \vec{D}_0 \frac{\partial f}{\partial \vec{x}} + \frac{\partial}{\partial \vec{x}} \cdot \vec{D}_1 \cdot \frac{\partial f}{\partial \vec{x}}$$

$$+ \frac{\partial}{\partial \vec{x}} \cdot \vec{D}_2 \cdot \frac{\partial f}{\partial \vec{x}}$$

$$\varepsilon = \left(\frac{\tau_{gg}}{\tau_{HH}} \right)^{1/2}$$

$$f = f_0 + f_1 + f_2 \cdots$$

$$\boxed{\begin{aligned} C_0(f_0) &= 0 \quad * \\ C_0(f_1) &= -C_1(f_0) \\ C_0(f_2) &= \frac{\partial f_0}{\partial t} - C_2(f_0) - C_1(f_1) \end{aligned}}$$

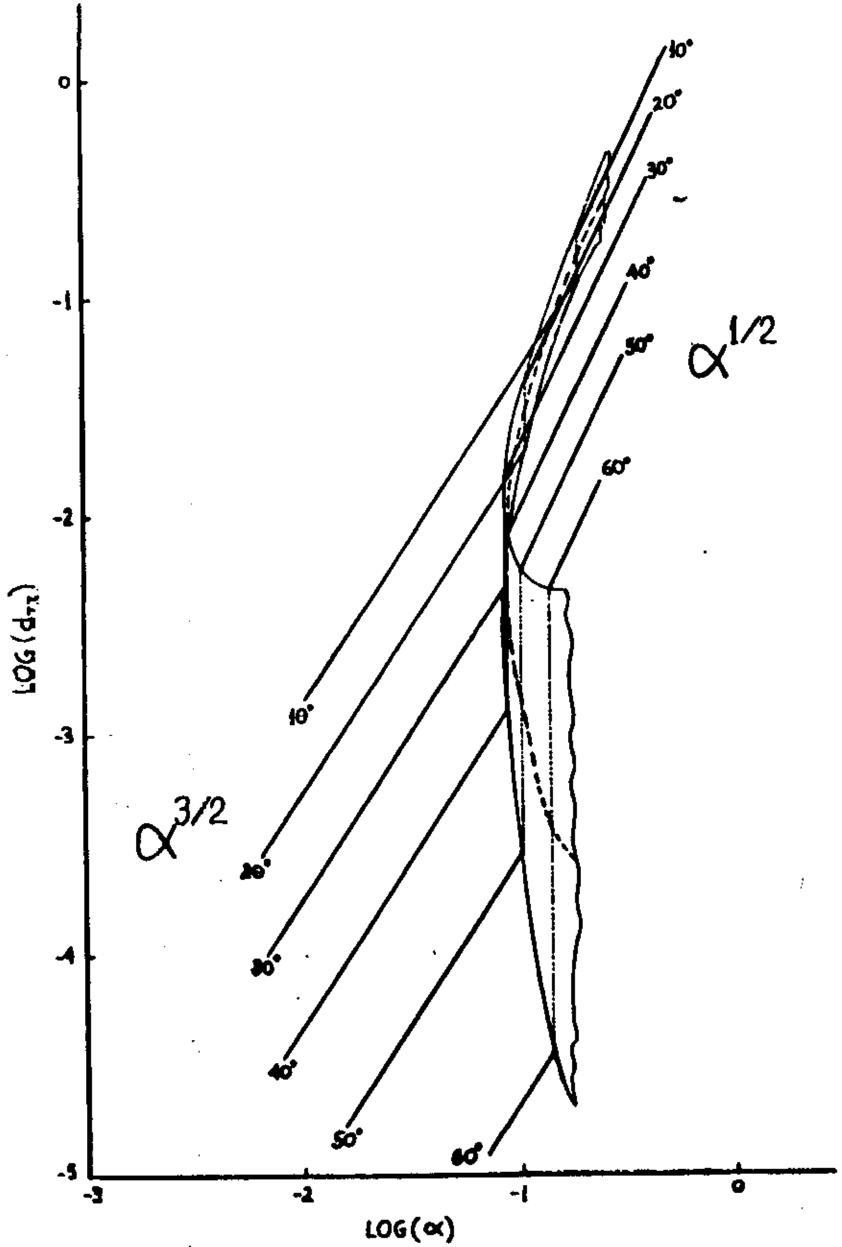


FIG. 5

20

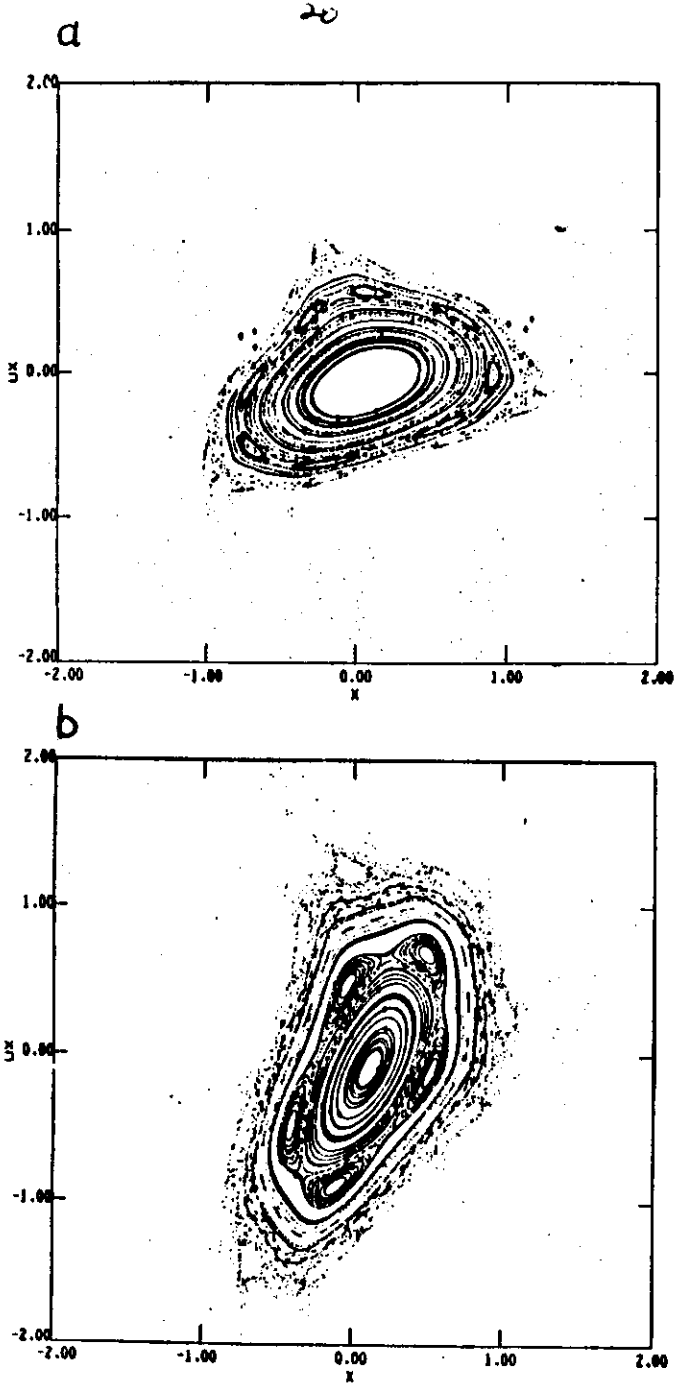


FIG. 3

21

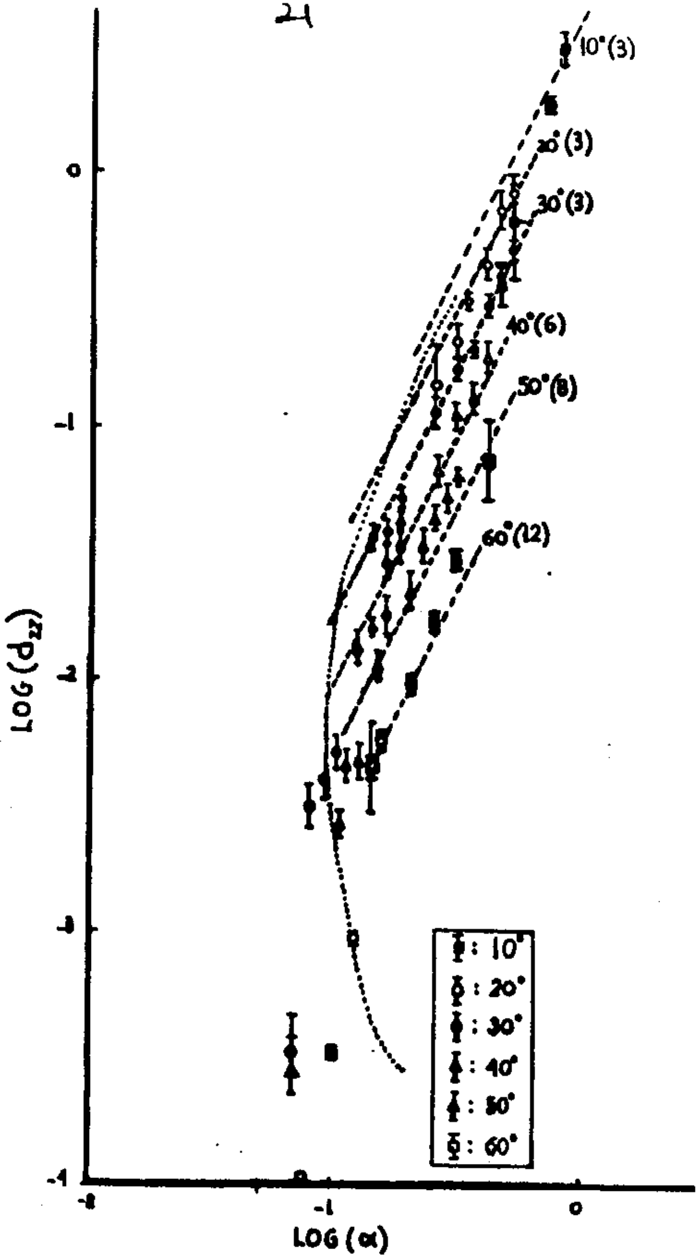


FIG. 10

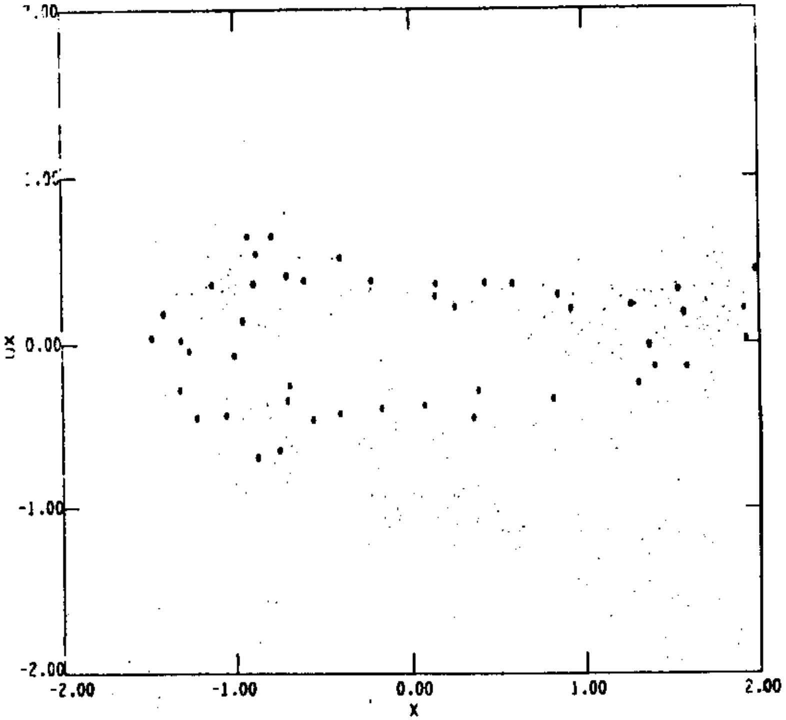

UX
X
-2.00
-1.00
0.00
1.00
2.00

FIG.4

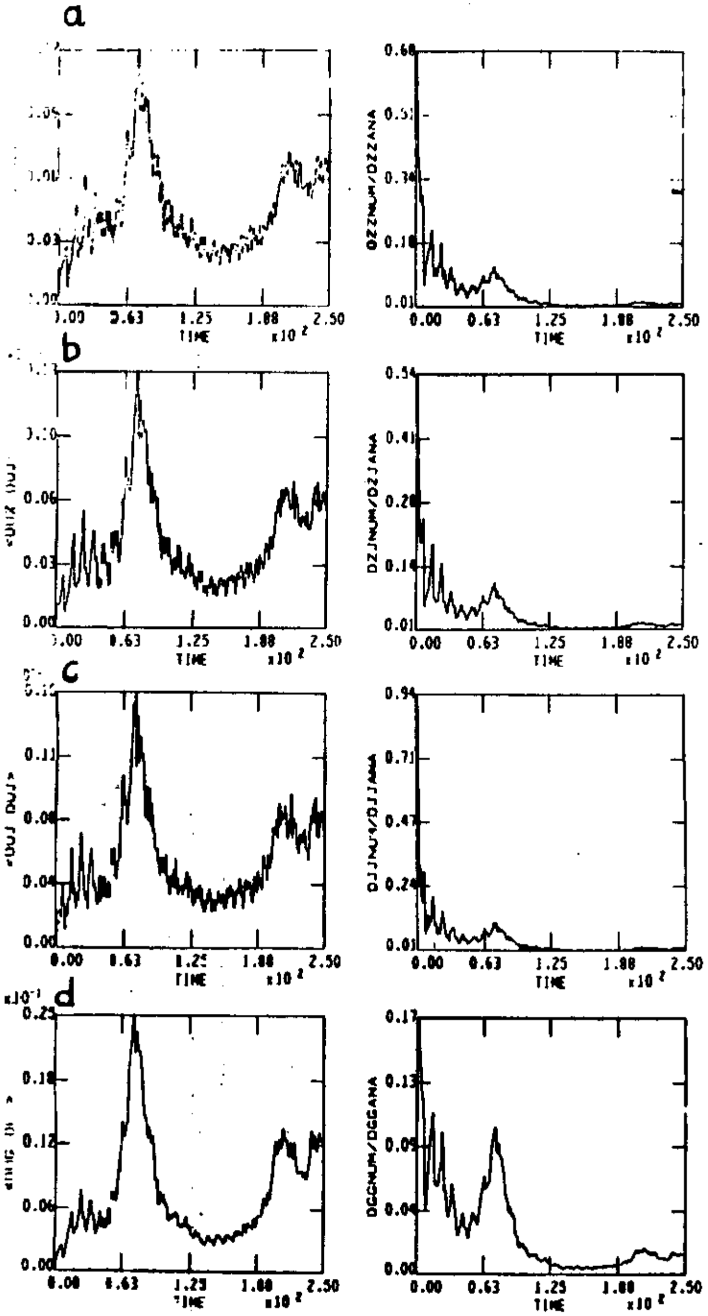

a
b
c
d
TIME
DZZNUM/DZZANA
DZJNUM/DZJANA
DGGNUM/DGGANA
0.00
0.63
1.25
1.88
2.50

FIG.8

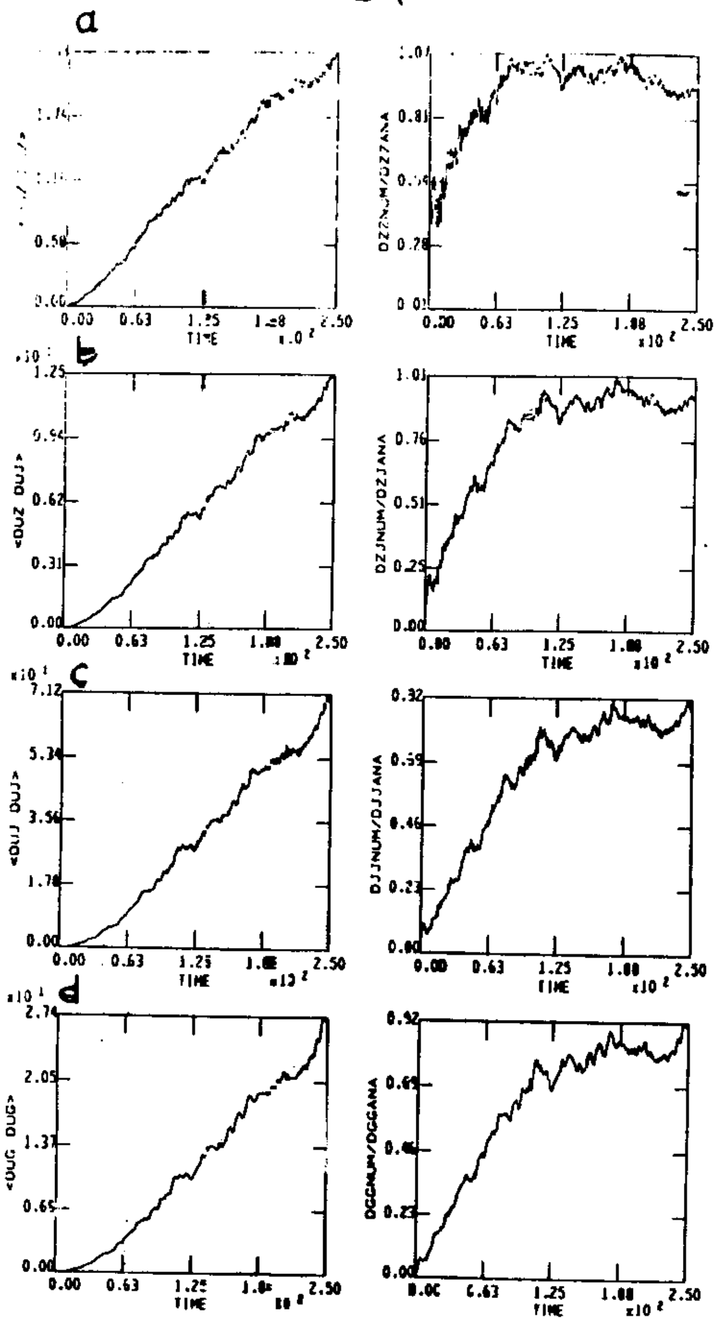

a
0.50
0.00
0.00
0.63
1.25
2.50
TIME
0.81
0.28
0.00
0.63
1.25
1.88
2.50
TIME
x10 2
b
x10 1
1.25
0.94
0.62
0.31
0.00
<DUZ DUJ>
0.00
0.63
1.25
1.88
2.50
TIME
1.01
0.76
0.51
0.25
0.00
DZJNUM/DZJANA
0.00
0.63
1.25
1.88
2.50
TIME
x10 2
c
x10 1
7.12
5.34
3.56
1.78
0.00
<DUJ DUJ>
0.00
0.63
1.25
1.88
2.50
TIME
0.92
0.69
0.46
0.23
0.00
DJJNUM/DJJANA
0.00
0.63
1.25
1.88
2.50
TIME
x10 2
d
x10 1
2.74
2.05
1.37
0.65
0.00
<DUG DUG>
0.00
0.63
1.25
2.50
TIME
0.92
0.69
0.23
0.00
DGGNUM/DGGANA
0.63
1.25
1.88
2.50
TIME
x10 2

FIG. 7