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NONLINEAR ABSORPTION IN WAVE DRIVEN PLASMAS

RAX Jean-Marcel
J.E.T. Joint Undertaking
Oxon, U.K.

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RAX Jean-Marcel *
DRFC Centre d' Etudes Nucleaire de cadarache
13108 Saint paul lez Durances CEDEX FRANCE

ABSTRACT: We study the saturation of the wave absorption in a radio frequency driven plasma. A critical energy above which it is useless to work is introduced. The influence of an electric field and practical consequences are then considered.

1-Introduction

In wave driven plasmas electromagnetic energy is absorbed by suprathermal resonant electrons. There are now a large number of theoretical results on this process [1-7], but mainly on the electron current response. In this lecture we shall study the saturation of the wave absorption in the lower hybrid range of frequencies and introduce a critical energy above which it is useless to work. In order to write the equation which governs this saturation we shall express the deformation of the electrons distribution in term of the absorbed power. Then in section three we shall introduce the concept of critical energy. The influence of an electric field is studied in section four and practical consequences of this non-linear behaviour are then considered.

During all the paper, rather than the M-K-S system, we shall use the cτ-m-τ system, c is the velocity of light, m the electron mass and τ the collision time $\tau=m^2c^3/(4\pi ne^4 Ln(\Lambda))$ (n is the electron density). The relevant parameters of the study will be Ze the ion charge and θ the temperature. ϵ will denote the Heaviside function and δ the Dirac distribution.

Let us begin with the following simple model: a single fast electron with momentum p in contact with a cold plasma can be considered as a dissipative system which is able to absorb a certain amount of power W(p). The excitation relaxation process is described by:

$$\frac{dp^2}{dt} = -p^{-1}+2W(p)$$

The maximum power W* that can be absorbed at the point p in the momentum space is ≈ n(p)/2p where n is the number of electrons. So if we go back to the usual units we obtain:

$$W^*(p)=[n(p)/p]\ 2\pi ne^4 Ln(\Lambda)$$

This simple model gives an order of magnitude but does not take into account the right stochastic dynamic of the electrons, so we shall consider F(**p**) the distribution function of the electron population.

2-Deformation of the electron distribution

We shall decompose the wave into a set of incoherent photons (R.P.A) and we shall study the response of the electron population to this set of photons with frequency ω and index **N**. Each electron with momentum **p'** will suffer a quantum kick:

$$\delta p'=\hbar\omega v'^{-1}, \qquad \delta\mu'=\hbar\omega p'^{-1}v'^{-1}\mu'^{-1}[1-\mu'^2]$$

So if we call F^0 the equilibrium distribution, the plasma response to the set of kicks will be $F(\mathbf{p})=F^0(\mathbf{p})+f(\mathbf{p})$ where f is the solution of the following equation :

$$-\mathbf{C}(\mathbf{p})f=\int d\mathbf{p}'[-\delta(\mathbf{p}-\mathbf{p}')+\delta(\mathbf{p}-(\mathbf{p}'+\delta\mathbf{p}'))]\,W(\mathbf{p}')/\hbar\omega$$

The right hand side term is a source term which describe the set of kicks. **C** is the (linearized) Landau collision operator and $W(\mathbf{p}')$ is the power absorbed at the point $\mathbf{p}'$ in momentum space.

If we introduce the operator:

$$\partial' = v'^{-1}\partial_{p'} + v'^{-2}\mu'^{-1}[1-\mu'^2]\partial_{\mu'}$$

In the classical limit the electron distribution function becomes:

$$F(\mathbf{p}) = F^0(\mathbf{p}) + \int d\mathbf{p}'\; W(\mathbf{p}')\,\partial' G(\mathbf{p},\mathbf{p}')$$

Where the Green function G is the solution of :

$$-\mathbf{C}(\mathbf{p}).G(\mathbf{p},\mathbf{p}')=\delta(\mathbf{p}-\mathbf{p}')$$

Thus we are able to express the deformation of the distribution function if we are able to find the Green function G: let us now consider this problem. The Landau operator is given by:

$$\mathbf{C} = p^{-2}\partial_p p^2 e^{-p^2/2\theta} a\, \partial_p e^{p^2/2\theta} + \left[(b+Z)/2p^3\right]\partial_\mu(1-\mu^2)\partial_\mu$$

$$a=\int dp'\, e^{-p'^2/2\theta}\left[\epsilon(p-p')v'^2v^{-3} + \epsilon(p'-p)v'^{-1}\right] \Big/ 3\int dp' e^{-p'^2/2\theta}$$

$$b=\int dp'\, e^{-p'^2/2\theta}\left[\epsilon(p-p')(3v^2-v'^2)pv^{-3} + \epsilon(p'-p)2pv'^{-1}\right] \Big/ 3\int dp' e^{-p'^2/2\theta}$$

As the resonant electrons are suprathermal we can perform the multipolar expansion of the Maxwell distribution .

$$F^0(\mathbf{p}) = [e^{-p^2/2\theta}]/(2\pi\theta)^{3/2} \approx \delta(p)/4\pi p^2$$

and we obtain:

$$\mathbf{C}= p^{-2}\partial_p + ((Z+1)/2)\,p^{-3}\partial_\mu(1-\mu^2)\partial_\mu$$

The first term describes energy losses and the second one pitch angle scattering which are the dominant processes for these fast electrons. In order to find G the following representation of $\delta(\mathbf{p}-\mathbf{p}')$ is the suitable one because it is a sum over the **C** eigenfunctions.

$$\sum_l \int dk(2l+1/4\pi)(1/2\pi)e^{ik[-p^3/3]}e^{-ik[-p'^3/3]}[p/p']^{l(l+1)(Z+1)/2}P_l(\mu)P_l(\mu')$$

P_l are the Legendre polynomials and we obtain the Green function:

$$G(\mathbf{p},\mathbf{p}')=\sum_l(2l+1/4\pi)\epsilon(p'-p)p^{l(l+1)(Z+1)/2}\;P_l(\mu)p'^{-l(l+1)(Z+1)/2}\;P_l(\mu')$$

The Heaviside function comes from energy loss and the next term gives the coupling with pitch angle scattering.

The control parameter of the proposed problem is $U(N_\parallel)$ the injected density of electromagnetic energy at $\omega, N_\parallel$. As we are working in the R.P.A the probability of interaction between the electron $[p,\mu]$ and the photon $[\mathbf{N},\omega]$ is given by the Einstein coefficients. Let us call $2\pi\hbar^{-1}\sigma\delta(p\mu N_\parallel-1)$ the absorption cross section and $2\pi\hbar^{-1}\sigma^*\delta(p\mu N_\parallel-1)$ the induced emission cross section.

$$2\pi\sigma/p\mu=A=8\pi^2e^2v\mu(\mathbf{b}.\mathbf{F}.\mathbf{b})/\omega\partial_\omega(\mathbf{D}:\mathbf{F})$$

$$\mathbf{D}=\omega^2(\mathbf{NN}-N^2\mathbf{I}+\epsilon_\perp\mathbf{I}+(\epsilon_\parallel-\epsilon_\perp)\mathbf{bb}+i\epsilon_H\mathbf{b}\times\mathbf{I})$$

$$\mathbf{F}=\omega^4\left[(\epsilon_\perp-N^2)\epsilon_\parallel\mathbf{I}-(\epsilon_\perp-N^2)\mathbf{NN}-((\epsilon_\perp-N^2)(\epsilon_\parallel-\epsilon_\perp)+\epsilon_H^2)\mathbf{bb}+i\epsilon_H(\mathbf{N}\times(\mathbf{b}\times\mathbf{N}))\times\mathbf{I}\right]$$

b is the direction of the magnetic field, ($\epsilon_\parallel,\epsilon_\perp,\epsilon_H$) are the element of the cold dielectric tensor [8-10] and the dispersion relation gives $\mathbf{N}(N_\parallel,\omega)$. The balance between emission and absorption can be written:

$$W(\mathbf{p})d\mathbf{p}=2\pi\hbar^{-1}\int dN_\parallel U(N_\parallel)\left[F(\mathbf{p})\sigma\delta(p\mu N_\parallel-1)d\mathbf{p}-F(\mathbf{p}+\delta\mathbf{p})\sigma^*\delta((p+\delta p)(\mu+\delta\mu)N_\parallel-1)d(\mathbf{p}+\delta\mathbf{p})\right]$$

If we use the Einstein relations and the expression of F in term of W we obtain :

$$W(\mathbf{p})=U(1/p\mu)AF^0(\mathbf{p})/\theta-U(1/p\mu)A\int d\mathbf{p}'W(\mathbf{p}')\partial\partial' G(\mathbf{p},\mathbf{p}')$$

The physical interpretation of this equation is straightforward, the first term on the right hand sides describes the equilibrium (thermal) absorption and the second one shows that when power is absorbed at **p'** it modify the power absorbed at **p**.

3-Critical energy

The last equation of the previous section shows that the dynamics of the power absobed at the point **p** in the momentum space is given by an integral

equation. The Neumann solution of this integral equation is an U expansion of W. The first term of this expansion is the usual linear absorption and the others are nonlinear processes:

$$W(\mathbf{p})=U(1/p\mu)AF^0(\mathbf{p})/\theta-U(1/p\mu)A\int d\mathbf{p}'(U(1/p'\mu')A'F^0(\mathbf{p}')/\theta)\partial\partial' G(\mathbf{p},\mathbf{p}')+O(U^3)$$

Let us look at the figure -1-, the absorbed power W is a nonlinear functionnal of U and when $U\longrightarrow\infty$ it saturates.

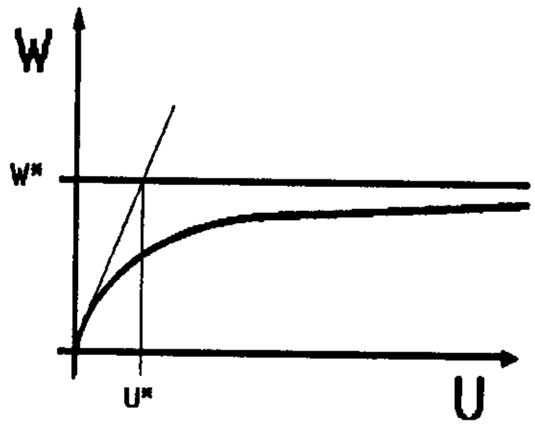

Fig-1-

If we consider the simplest Pade approximant of the previous Neumann series: $W\propto U/(1+U/U^*)$
this relationship as the right physical properties when $U\longrightarrow 0$ and $U\longrightarrow\infty$ and the second term of the previuos expansion gives us the critical energy U^*:

$$U^* = U(1/p\mu)F^0(\mathbf{p}) \Big/ \int d\mathbf{p}' U(1/p'\mu')A'F^0(\mathbf{p}')\partial\partial' G(\mathbf{p},\mathbf{p}')$$

This critical energy gives us the threshold for nonlinear absorption and for typical low density experiments the order of magnitude of the maximum absorbed power is $1 W/cm^3$.

4-Influence of an electric field

In this section we shall study the influence of an additional electric field . A D.C field will influence the power absorption because it will increase the number of resonant electrons and decrease the slope of the distribution function. The plasma response to the infinitesimal electric field :

$$E = E\ dt\ \delta(t-t')\ \mathbf{b}$$

is a summ of kicks: $\delta p = eE\mu dt$ $\delta\mu = eE(1-\mu^2)dt/p$

If we introduce the local Ohmic power deposition $O(\mathbf{p}) = eEv\mu\ F(\mathbf{p})$ and if $E < E_{Dreicer}$ the steady state response is:

$$F(\mathbf{p}) = F^0(\mathbf{p}) + \int d\mathbf{p}'\ O(\mathbf{p}')\ \partial' G(\mathbf{p},\mathbf{p}')$$

Now we switch on the wave and we have the following response to both wave and inductive power .

$$F(\mathbf{p}) = F^0(\mathbf{p}) + \int d\mathbf{p}'\ O(\mathbf{p}')\ \partial' G(\mathbf{p},\mathbf{p}') + \int d\mathbf{p}'\ W(\mathbf{p}')\ \partial' G(\mathbf{p},\mathbf{p}')$$

This equation shows that the inductive and the lower hybrid responses are the same but O and W are different. Now if we write the equation which governs W we obtain the O/W coupling:

$$W(\mathbf{p}) = U(1/p\mu)A\theta^{-1}F^0(\mathbf{p}) - U(1/p\mu)A\int d\mathbf{p}' W(\mathbf{p}')\partial\partial' G(\mathbf{p},\mathbf{p}')$$

$$- U(1/p\mu)A\int d\mathbf{p}' O(\mathbf{p}')\partial\partial' G(\mathbf{p},\mathbf{p}')$$

The physical interpretation of this last equation is clear, the sign of E can increase or decrease the absorbed power. A critical energy which depend on E can be introduce $U^*(E) = U^*(0) + O(E)$ [11]. Rather than detailing this influence of an electric field, let us consider some consequences of the nonlinear absorption.

5-discussion

What are the practical consequences of this saturation? The first one is the existence of an upper bound for all the responses of the plasma. An interesting point to note is that these responses are nonlinear functionnals of the injected energy but are linear functionnals of the absorbed power, so W is the key quantity to interpret the nonthermal responses. For example the moments of order n and m of $F(\mathbf{p})$ are given by:

$$\langle p^m\mu^n\rangle = \langle p^m\mu^n\rangle_0 + W\sum_{0\le l\le n}(2l+1)2^{l+1}n!(n/2+1/2)!/(l+n+1)!(n/2-1/2)!$$

$$\left[\left[(m+3)/(l(l+1)(Z+1)+2m+6)\right]N_{\parallel}^{-(m+1)}\right)$$

$\langle p^m\mu^n\rangle_0$ is the equilibrium moment and we have considered a narrow spectrum $W(\mathbf{p}) \propto W\delta(p-N_{\parallel}^{-1})\delta(\mu-1)$ (if m=1 and n=1we obtain the usual Fisch efficiency). If we are able to measure one of these responses we are then able to predict the others on the basis of W, so the fact that the absorption is nonlinear and that there exist a critical energy limits the performances but is not an obstacle from the diagnostic point of view. For example electron cyclotron emission (ece) can be used to study $W \approx W_{ece} / \Omega^2 \omega_p^{-2} N_{\parallel}^{-3} \mathrm{Ln}(\Lambda)^{-1}$ and the results clearly show the expected saturation[12-14].

*Present address: **JET Joint Undertaking**
Abingdon
Oxfordshire OX14 3EA ENGLAND

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