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SPRING COLLEGE ON PLASMA PHYSICS

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ROTATIONAL INSTABILITIES IN THE FIELD - REVERSED CONFIGURATION

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Rotational Instabilities in the Field-Reversed Configuration

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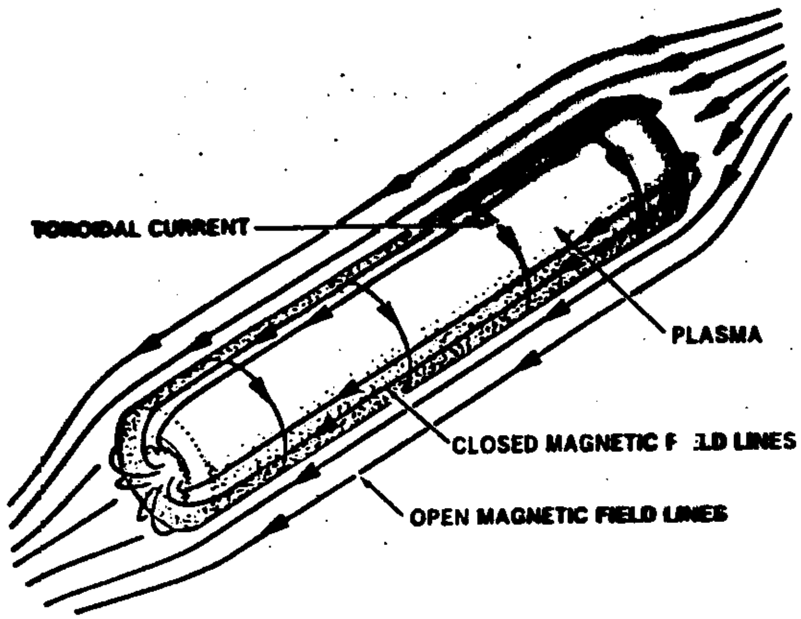
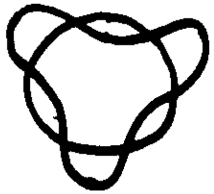
Acknowledgment: FRX experimental group, Los Alamos Natl. Lab.

Field-Reversed Configuration (FRC)



Experiments are generally observed to:

- begin rotating
- are disrupted by n=2 rotational instability (without multipole stabilization)

n=2   n=3

MHD Theory predicts:

- instability for any amount of rotation
- large n grows fastest, all n unstable $\gamma = \sqrt{n-1}\,\Omega$

(Taylor, 1962)

Define: $\alpha = \dfrac{-\Omega_i}{\Omega_e - \Omega_i} = \dfrac{-\Omega_i}{\Omega_*}$

Finite Larmor radius theory predicts:

- $n>2$ modes stable
- $n=2$ instability for $\alpha > 1.6$

(Freidberg + Pearlstein, 1978)

But, experiments observe instability onset at $\alpha = 0.4$ to 1.0

FLR theory:

- good approximation for non-reversed theta pinch
- poor approximation for FRC due to field null

Need kinetic model for ions

∴ use quasineutral hybrid computational model

QUASINEUTRAL HYBRID MODEL

(Byers et al, 1978; Harned, 1982)

- Assume quasineutrality
- Neglect transverse displacement current (Darwin)
- Ion particles
- Electrons - inertialess fluid
- Fully nonlinear

EQUATIONS:

Ampere's law and electron momentum eq.:

$$\bar{E} = \frac{1}{4\pi n_i e}(\nabla\times\bar{B})\times\bar{B} - \frac{1}{n_i e c}\bar{J}_i\times\bar{B} - \frac{1}{n_i e}\nabla(n_i T_e) + \frac{c\eta}{4\pi}\nabla\times\bar{B}$$

Faraday's law:

$$\frac{\partial\bar{B}}{\partial t} = -c\,\nabla\times\bar{E}$$

Particle equations of motion:

$$\frac{d\bar{v}}{dt} = \frac{q}{m}\left(\bar{E} + \frac{\bar{v}}{c}\times\bar{B}\right)$$

$$\frac{d\bar{x}}{dt} = \bar{v}$$

- Forces on particles and source terms from linear weighting (PIC)

- Advance fields with predictor-corrector method

- Vacuum fields matched across a transition region

- $2\frac{1}{2}d$, $\frac{\partial}{\partial z} = 0$

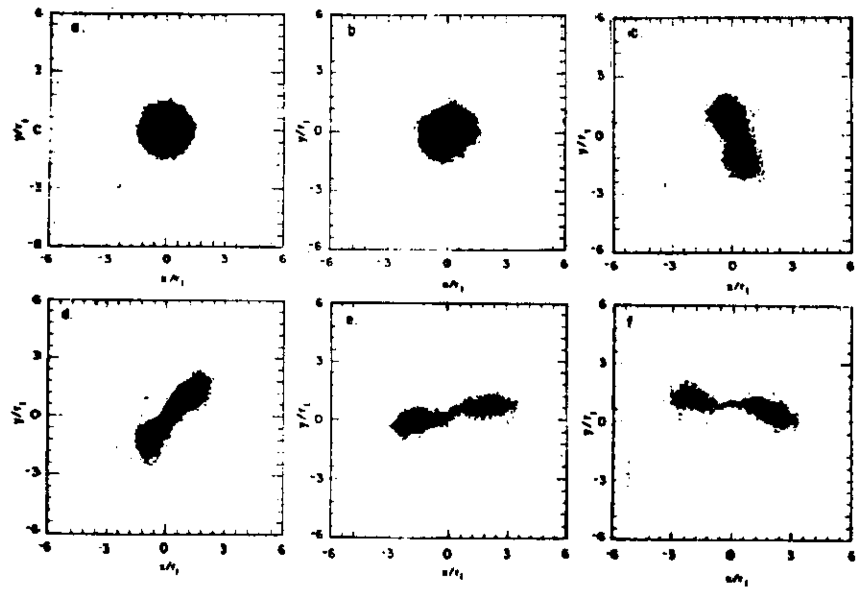
EXAMPLE:

$\alpha = 1.0$, $\beta_0 = 0.49$, $\Omega_* = 0.05\,\omega_{ci}$

$S = a/\rho_0 = 8.3$ (like FRX-B)

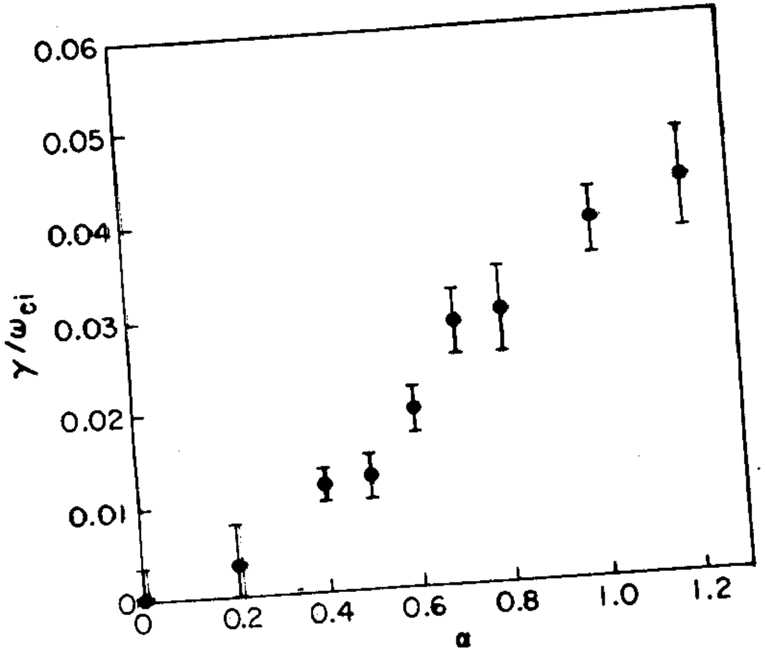
Should be stable from FLR theory

50,000 particles, 100x100 grid



Growth rates

- slightly less than MHD
- $n>2$ stable

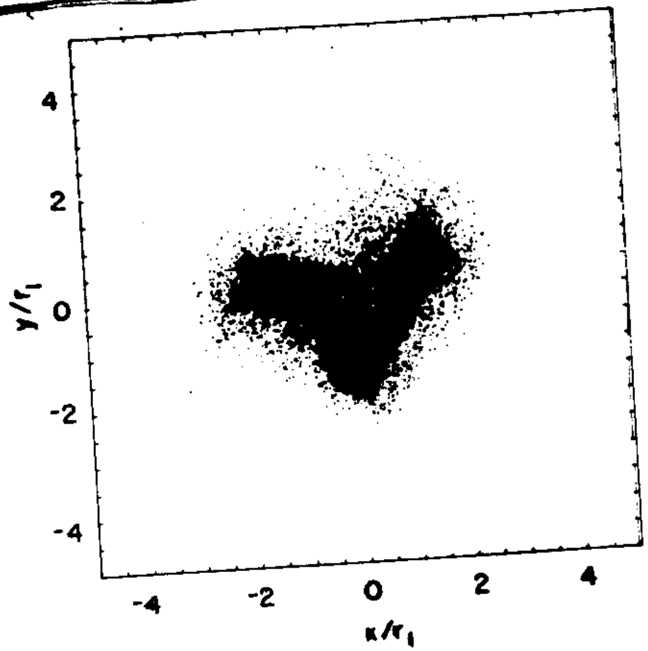


- No threshold observed as in FLR

For strongly reversed cases higher n modes can be unstable

$\Omega_* = 0.05\,\omega_{ci}$ $\beta_0 = 0.23$ $\alpha = 1.1$

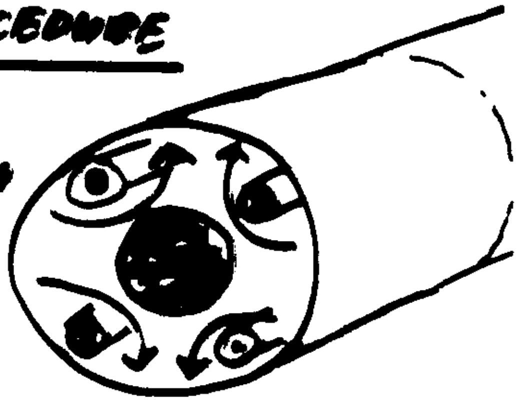
n=3 instability



MULTIPOLE FIELDS

Can stabilize $n=2$ mode
(Ohi et al, 1983; Hoffman et al, 1983; Siemon et al, 1986)

EXPERIMENTAL PROCEDURE



- Form FRC plasma
- Apply multipole fields, which are excluded from the main part of the plasma on experimental timescales
- $n=2$ instability is not observed if the multipole field strength exceeds a threshold value

THEORY

MHD – (Ishimura, 1984)

- assumed circular cross-section
- ignored mode coupling

- Predicts stability if quadrupole field at plasma edge satisfies

$$B_q^2 > r_s^2 \Omega^2 \mu_0 \rho_0$$

- B_q is a factor of ≈ 4 larger than experimental observation

MUST CONSIDER:

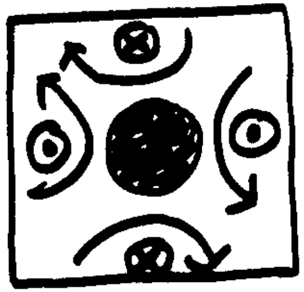
1.) Modification of equilibrium due to quadrupole field
   - should be stabilizing (Spencer + Tuszewski, 1985)

2.) Kinetic treatment of ions
   - substantially changes mode and growth rates (Horned, 1983)

SIMULATION PROCEDURE

1.) t=0, initialize rotating truncated rigid rotor distribution (ρ→0 outside r_s) with no quadrupole field

2.) Turn on Ioffe bars, increasing linearly in time until $B_{quad} = B_q$ at the plasma edge



3.) Quadrupole field does not penetrate appreciably inside separatrix because plasma is highly conducting on these time scales

FRX-C CROSS-SECTION WITH QUADRUPOLE FIELDS (End-on holography, K. McKenna)

Equilibrium has a large amplitude n=4 distortion

FRX-C/T Experimental Parameters: (Rej et al, 198[illegible])

$B_0 = 4kG$, $\beta_s = 0.31$, $x_s = 0.6$, $S = \frac{R}{\rho_i} = 13.1$

$\alpha = \frac{-\Omega_i}{\Omega_*} = 0.81$

Simulations use:

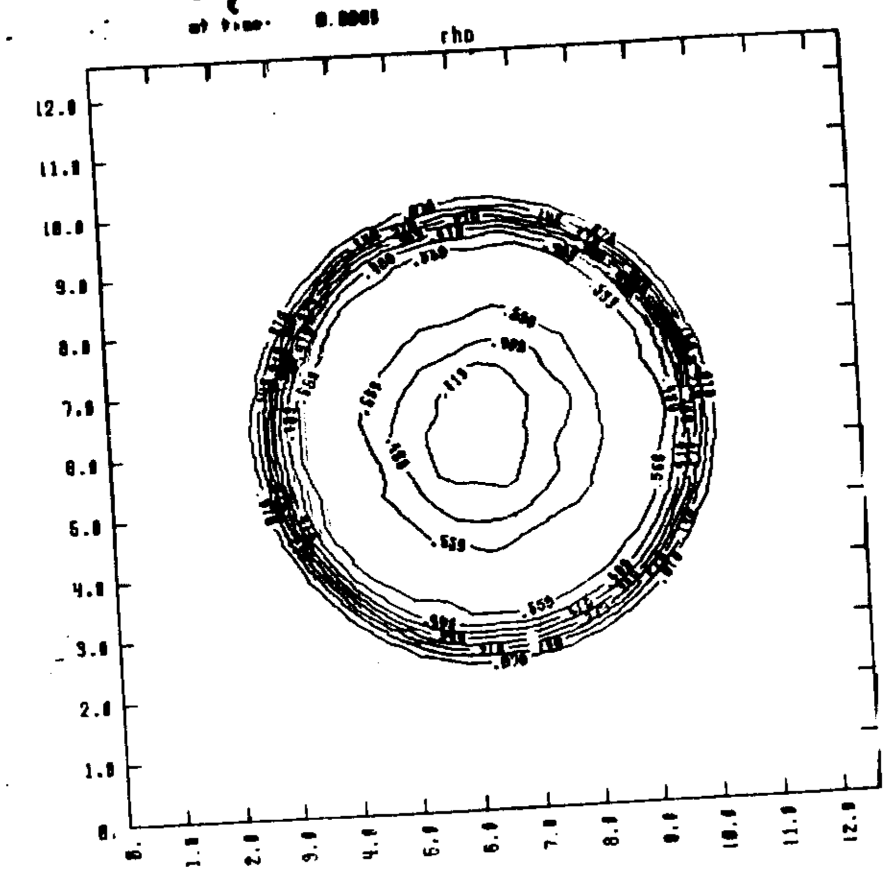
50,000 - 100,000 particles

80×80 grids

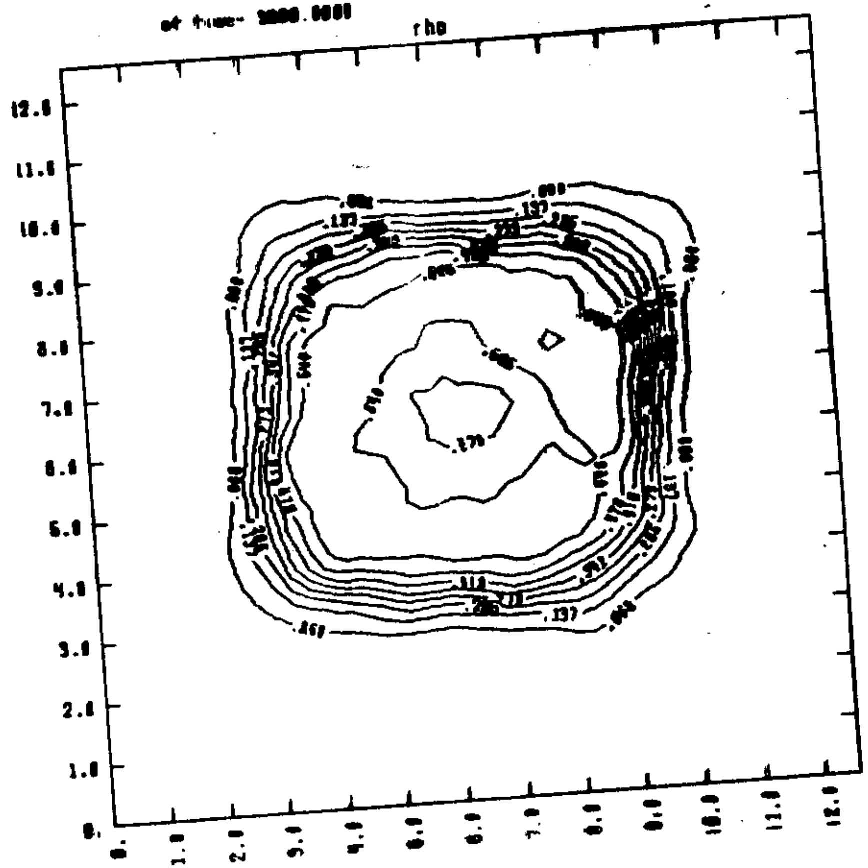
* Vary quadrupole field strength, B_q, applied to this axisymmetric equilibrium

EXAMPLE:

Density contours, initial axisymmetric equilibrium



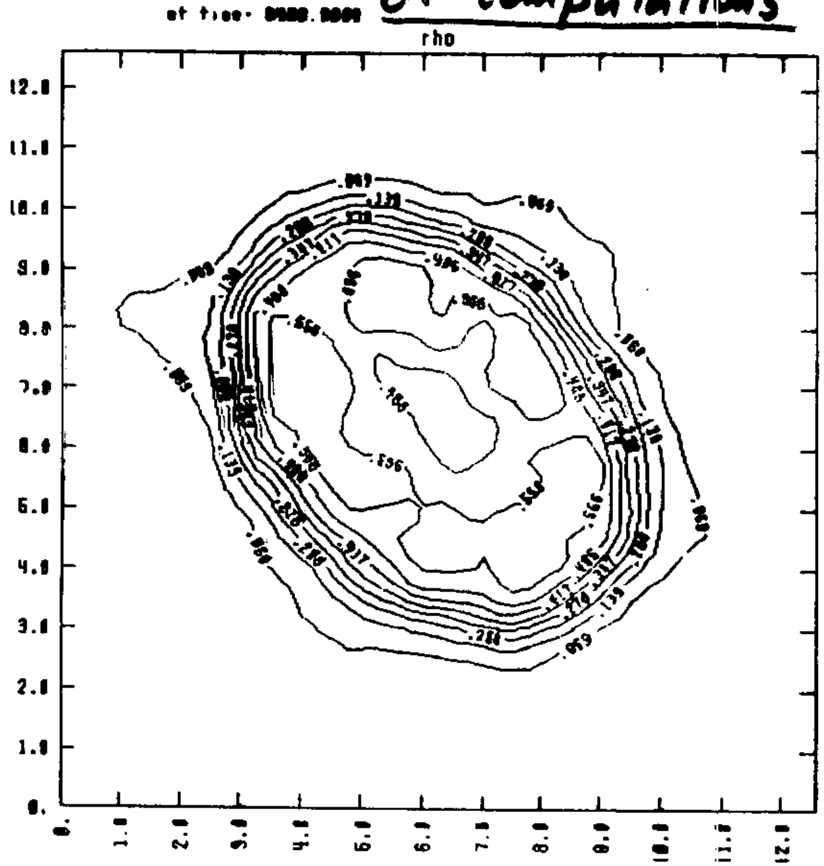
- Quadrupole field has been applied
- Equilibrium develops a large amplitude n=4 distortion



(Note: coils have been rotated 45° from experimental positions)

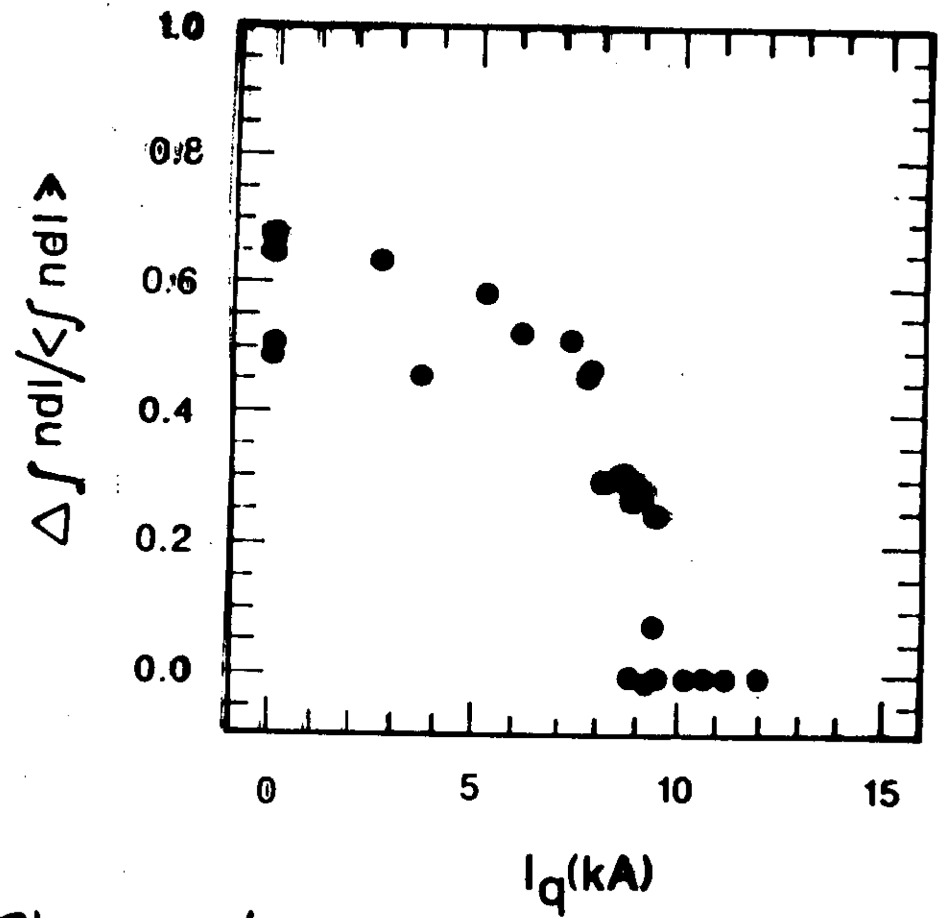
$B_q / B_0 = 0.13$

— just below stability threshold of computations



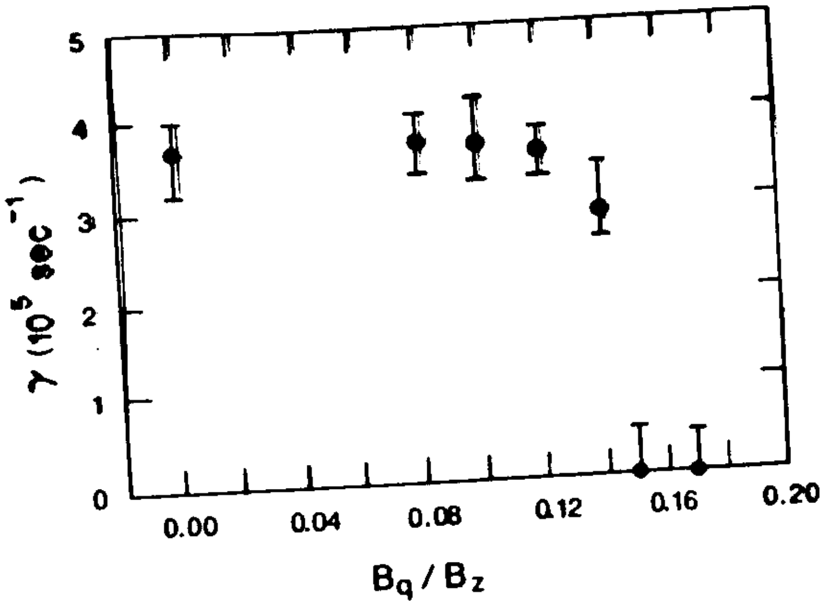
n=2 instability appears

FRX-C/T Integrated line density measurements



Shows sharp threshold for quadrupole field to stabilize the n=2 mode

Growth rates from hybrid simulations



BOTH EXPERIMENT AND SIMULATIONS SHOW:

- Sharp threshold for stabilization of n=2 mode, well below axial field strength
- Mode grows essentially unaffected by quadrupole field for B_q below threshold
- Equilibria develop large amplitude n=4 distortions

DIFFERENCE:

- Experimental threshold $B_q \approx .08\, B_0$
- Simulation threshold $B_q \approx .15\, B_0$

POSSIBLE SOURCES OF DISCREPANCY

1.) Equilibrium in expt. is probably not a rigid rotor, since $\beta_s = 0.31$ an $X_s = 0.6$ do not satisfy the avg. beta condition, $\langle\beta\rangle = 1 - X_s^2/2$ for a rigid rotor

2.) Equilibrium in simulation should have more time to relax to the presence of the quadrupole field

CONCLUSIONS

1.) Kinetic effects are essential to model FRC rotational instability (FLR is NOT sufficient)

2.) No rotation threshold for n=2 instability

3.) Simulations show large amplitude n=4 equilibrium distortions due to quadrupole field

4.) Simulations show stabilization of n=2 by quadrupole field above threshold

($B_q > 0.15 B_0$ for FRX-C/T-like parameters)