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ABSTRACT

The linear stability code ERATO is applied to MHD equilibria of low and moderate aspect ractio to determine the operational diagrams imposed by n = 1 external kink modes in the q_0-q_s domain. An unstable wedge between two stable windows appears at low aspect ratio, where strong non-resonant m = 1 instabilities are driven by toroidal coupling effects.

INTRODUCTION

Ideal magnetohydrodynamic (MHD) theory constitutes the simplest model to describe the dynamics of magnetic plasma confinement systems. Nonetheless, the very simplicity of the dynamics permits this model to tackle problems in devices with very complicated geometries. Linear MHD stability codes such as PEST[1] and ERATO[2] have become very important tools in the design of Tokamaks. They help map out the regions of stable operation and identify those configurations that maximise the parameter beta (β) which is the ratio of the volume average pressure to the volume average magnetic field energy density.

We briefly describe the ERATO stability code and discuss the input profiles that are used to generate MHD equilibria. The domains of stable operation imposed by n = 1 external kink modes for Tokamaks with low and moderate aspect ratios are compared. The eigenstructure patterns of unstable equilibria with similar values of β and safety factor profiles in each one of these domains are also investigated.

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PHYSICS BACKGROUND OF ERATO

The ERATO code determines the linear ideal MHD stability properties of axisymmetric plasma configurations by examining the energy principle[3] that is obtained from the linearised MHD equations, namely,

$$\delta L = \delta W_p + \delta W_v - \omega^2 \delta W_k = 0 . \tag{1}$$

The potential energy of the plasma is expressed as (2)

$$\delta W_p = \frac{1}{2}\int d^3x \left\{ \left[\underline{\nabla}\times(\underline{\xi}\times\underline{B}) + (\underline{\nu}\cdot\underline{\xi})(\underline{j}\times\underline{\xi})\right]^2 + \gamma p (\underline{\nabla}\cdot\underline{\xi})^2 - 2(\underline{\nu}\cdot\underline{\xi})^2(\underline{j}\times\underline{\nu})\cdot(\underline{B}\cdot\underline{\nabla})\underline{\nu} \right\} , \tag{2}$$

where $\underline{\nu} \equiv \underline{\nabla}\psi/|\underline{\nabla}\psi|$ is the unit vector normal to the flux surfaces. The equilibrium magnetic and current density fields are $\underline{B}$ and $\underline{j}$, respectively, and the perturbed displacement vector is $\underline{\xi}$. The equilibrium plasma pressure is p and γ is the adiabatic index. The contribution of the vacuum fields to the energy of the system is given by

$$\delta W_v = \frac{1}{2}\int d^3x \, (\underline{\nabla}\times\underline{A})^2 , \tag{3}$$

where $\underline{A}$ is the perturbed magnetic vector potential, and the kinetic energy of the system is

$$\delta W_K = \frac{1}{2}\int d^3x \, \rho \xi^2 , \tag{4}$$

where ρ is the mass density. A plasma equilibrium state is unstable if the eigenvalue $\omega^2 > 0$. The value obtained for ω corresponds to the growth rate of the instability.

Furthermore, the perturbation is expanded as[4]

$$\underline{\xi} = \underline{\eta}(\psi,\theta)\exp\left[in(\phi - q\theta)\right] \tag{5}$$

when nq > 10, where ϕ is the geometric toroidal angle and θ is the poloidal angle in a straight magnetic field line coordinate system (ψ,θ,ϕ). In systems with axisymmety, instabilities with different

different toroidal mode numbers n are decoupled one from another and can be therefore investigated independently. The safety factor q, which is the derivative of the toroidal magnetic flux with respect to the poloidal magnetic flux, corresponds to the number of toroidal transits a magnetic field line makes per poloidal transit. This quantity is one of the critical variables of Tokamak MHD stability analysis because instabilities tend to concentrate in regions where q has an integer value. To increase the accuracy of the calculations, the ERATO code is constructed with a variable radial mesh that permits packing around the rational q surfaces. The perturbed vector amplitudes $\underline{\eta}(\psi,\theta)$ are expanded further using finite hybrid elements. A detailed description of the numerical scheme of the ERATO code can be found in Reference 2.

PARAMETRISATION OF THE EQUILIBRIA

The plasma-vacuum interface for the MHD equilibrium models under consideration is described by

$$r = R + a\cos(\theta + \delta\sin\theta) \qquad (6)$$

and

$$z = E a \sin\theta , \qquad (7)$$

where r is the distance of any boundary point from the major axis and z is its distance from the midplane. The other parameters are the major radius R, the minor radius a, the elongation E and the triangularity δ.

The average toroidal current density flowing within a poloidal magnetic flux surface ψ is defined as

$$J(\psi) = \frac{2\pi}{V'}\int_0^{2\pi} d\theta \sqrt{g}\,(\underline{j}\cdot\underline{\nabla}\phi) , \qquad (8)$$

where $V'(\psi) \equiv 2\pi\int_0^{2\pi} d\theta\sqrt{g}$ is the differential volume and $\sqrt{g}$ is the Jacobian of the transformation from the cylindrical coordinates

(r,ϕ,z) to the magnetic coordinates (ψ,θ,ϕ). The prime indicates the derivative of a flux surface quantity with respect to ψ. The total current is

$$I = \frac{1}{2\pi}\int_{\psi_0}^{\psi_s} d\psi\, V'(\psi)\, J(\psi) . \qquad (9)$$

We prescribe the $p'(\psi)$ and $J(\psi)$ profiles with functional forms that have continuous piecewise smooth radial derivatives, namely

$$p'(\psi) = \begin{cases} 0 & \psi_0 \le \psi < \psi_c \\ \text{cubic function} & \psi_c \le \psi < \psi_d \\ \text{quadratic function} & \psi_d \le \psi \le \psi_s \end{cases} \qquad (10)$$

and

$$J(\psi) = \begin{cases} \text{quadratic function} & \psi_0 \le \psi < \psi_a \\ \text{cubic function} & \psi_a \le \psi < \psi_b \\ 0 & \psi_b \le \psi \le \psi_s \end{cases} \qquad (11)$$

Typically we choose $p'(\psi_s) = 0$, $p'(\psi_d)$ to be a maximum, and $\psi_d = \psi_b$ so that the peak of the pressure gradient matches the point at which J vanishes. A detailed description of the specific forms and the philosophy behind the choice of these profiles can be found in Reference 5.

NUMERICAL RESULTS

We concentrate here on a configuration with low aspect ratio. Although design and engineering constraints pose some severe problems, the physics aspects indicate the possibility of operating at very high values of β which makes this an attractive type of device to consider.[6] The plasma boundary parameters are given by $E = 1.68$, $A \equiv R/a = 1.67$ and $\delta = 0.3$. The operational diagram in the q_0-q_s space determined by n = 1 external kink instabilities is obtained with fixed $\beta_I = 8\pi\int p\,ds/\mu_0 I^2 = 0.35$ and shown in Fig. 1. The q_0-q_s

domain is scanned by varying the current, the current profile and the total pressure. What we find for $q_s < 4$ are two stable bands localised about $q_o = 1.0$ and $q_o = 1.1$, with an unstable band wedged in between. This type of structure differs considerably from that obtained in a more conventional Tokamak with moderate aspect ratio. As an illustration, we consider a configuration with A = 3.7, E = 2.0 and $\delta = 0.4$. The corresponding operational diagram that is shown in Fig. 2 for a case with $\beta_I = 0.95$ has only a single stable band. It is also useful to compare the eigenstructures of two unstable equilibria that lie in the operating diagrams presented and have similar monotonic q profiles and β values. The instability flow pattern for an A = 1.67 equilibrium that lies in the unstable wedge of Fig. 1 with $q_o = 1.065$, $q_s = 3.517$ and $\beta = 8\%$ appears in Fig. 3. The pattern reveals noticeable m = 2 and m = 3 activity about the q = 2 and q = 3 surfaces, respectively. Throughout the bulk of the plasma a significant non-resonant m = 1 structure is clearly visible that is driven unstable by the strong toroidal coupling with the m = 2 and m = 3 external modes at this low aspect ratio. The instability flow pattern for an A = 3.7 equilibrium with $q_o = 1.06$, $q_s = 2.89$ and $\beta = 7.4\%$ appears in Fig. 4. A dominant external m = 2 mode is localised about the q = 2 surface. Here a non-resonant m = 1 mode is also apparent, but it is much weaker than the one shown in Fig. 3 because the toroidal coupling effects at A = 3.7 are not as strong.

SUMMARY AND DISCUSSION

A brief description of the ERATO stability code has been presented and the profiles and boundaries that are used to prescribe MHD equilibria have been discussed. We choose to employ $J(\psi)$ as the input profile in order to have control over the total plasma current and the current profile as well as to avoid peeling instabilities that arise from current density discontinuities at the plasma-vacuum interface when for example the $q(\psi)$ profile is prescribed and maintained in a flux conserving manner.

We have examined the stability of a low aspect ratio configuration to n = 1 external kink modes and found that they were two stable operating windows with an unstable wedge in between localised near $q_o \approx 1.05$ in the $q_o - q_s$ domain. In a more conventional moderate aspect ratio configuration, only a single stable operating band is observed in this domain. The eigenstructure in a low A equilibrium in the unstable wedge has significant non-resonant m = 1 activity throughout the bulk of the plasma driven by the strong toroidal coupling with m > 2 instabilities. A comparable moderate A unstable equilibrium displays negligible non-resonant m = 1 activity because the toroidal coupling effects with the dominant m > 2 modes is correspondingly weaker. These equilibria we have computed are not optimal for ballooning stability. However, because the low n-kink modes tend to be sensitive to the plasma current, the current profile and the global value of β, but relatively independent of the details of the pressure profile, the stability to ballooning modes can be achieved with appropriate retailoring of the p' profile without significantly altering the kink mode stability properties of the configurations under consideration.

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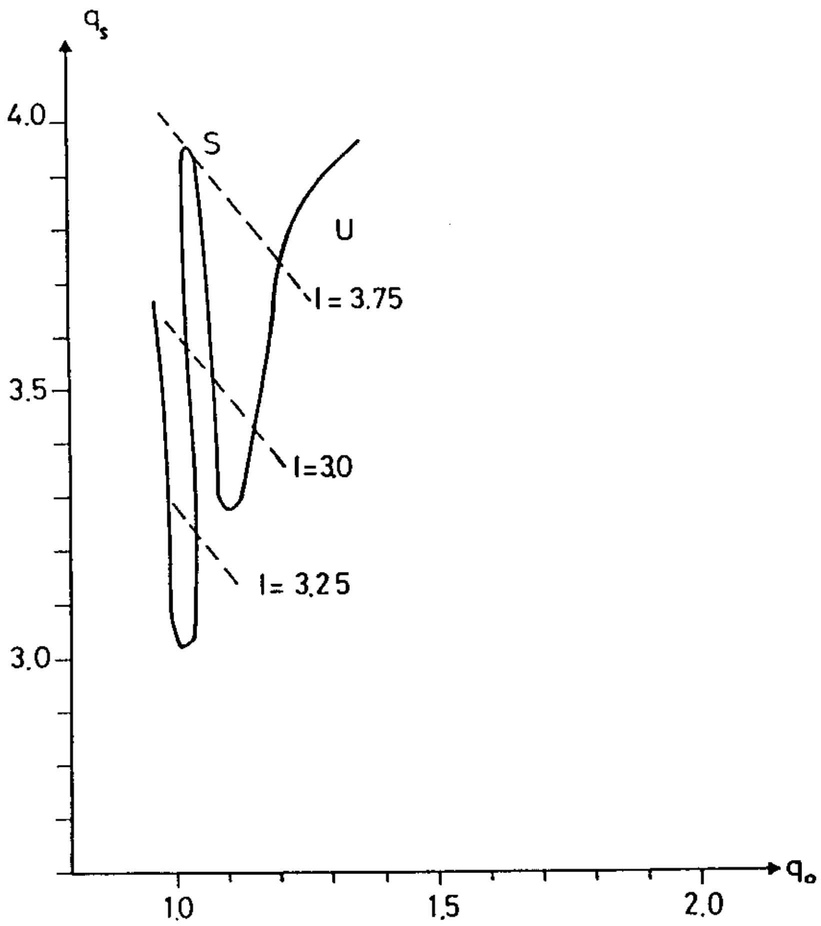
FIGURE CAPTIONS

Fig. 1: The $\beta_I = 0.35$ stability boundaries with $q_o - q_s$ space for a configuration with $A = 1.67$, $E = 1.68$ and $\delta = 0.3$ (solid curve). The dashed lines connect points of constant normalised current $\mu_0 I/RB_o$.

Fig. 2: The $\beta_I = 0.95$ stability boundaries in the $q_o - q_s$ space for a configuration with $A = 3.7$, $E = 2.0$ and $\delta = 0.4$ (solid curve). The dashed lines connect point of constant normalised current.

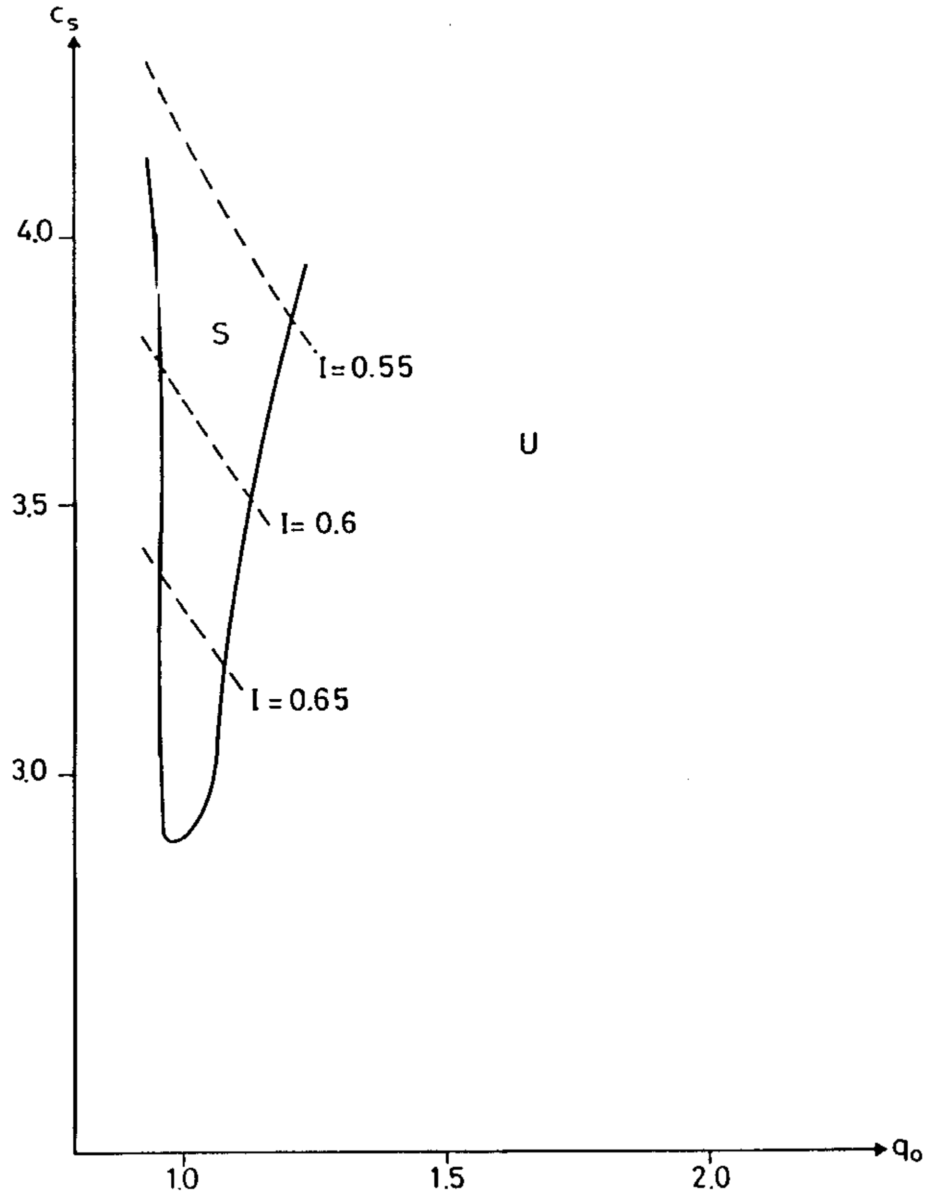
Fig. 3: The instability flow pattern in a configuration with $A = 1.67$, $E = 1.68$, $\delta = 0.3$ and $\beta = 8\%$. The equilibrium state has $q_o = 1.065$ and $q_s = 3.517$.

Fig. 4: The instability flow pattern in a configuration with $A = 3.7$, $E = 2.0$, $\delta = 0.4$ and $\beta = 7.4\%$. The equilibrium state has $q_o = 1.06$ and $q_s = 2.89$.



q_s
4.0
3.5
3.0
S
U
I = 3.75
I=3.0
I= 3.25
1.0
1.5
2.0
q_o

FIG. 1



c_s
4.0
3.5
3.0
S
U
I=0.55
I= 0.6
I=0.65
1.0
1.5
2.0
q_o

FIG. 2

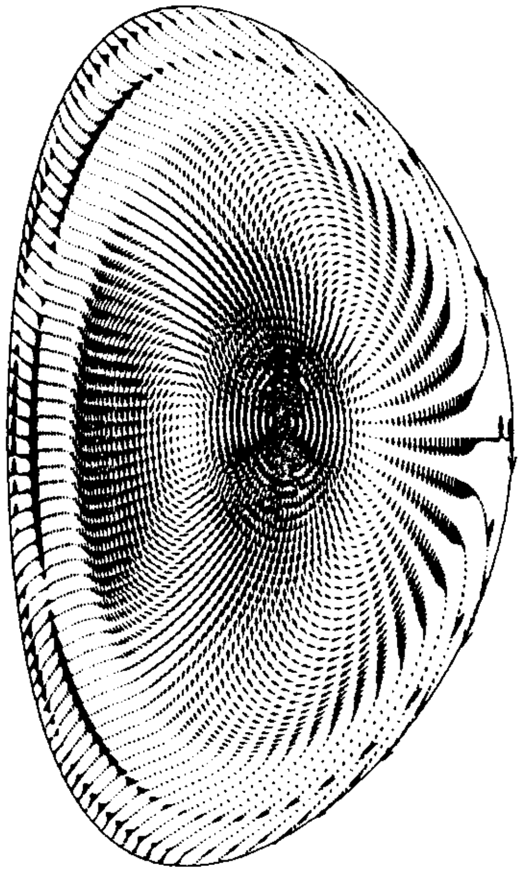


FIG. 3

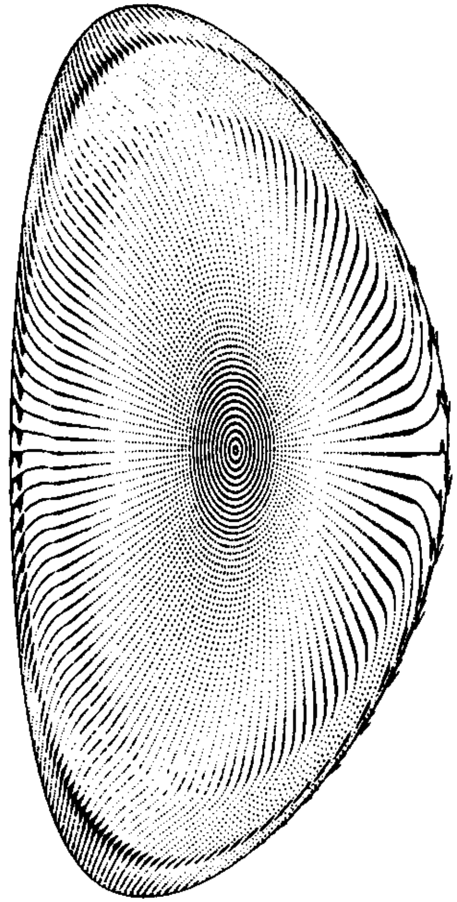


FIG. 4