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PRINCIPLES OF BEAT-WAVE ACCELERATORS

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1. INTRODUCTION

One of the most efficient ways of accelerating charged particles is to exploit the very large electric field associated with a longitudinal electrostatic wave.

Having the phase velocity and the accelerating field the same direction, it is possible at least in the principle, to "phase-lock" a beam of injected particles and to accelerate them to very high energies.

To sustain a longitudinal electrostatic wave a "plasma" is necessary, so that the crucial point is how to produce large amplitude electrostatic waves in a plasma. The large electric fields associated with laser beams offer an immediate candidate but there are two problems to be considered.

1) Electromagnetic waves are transverse;
2) A plasma is not transparent to signals below the plasma frequency ω_p: while this is a natural resonance frequency, it is also a cut off frequency for any transverse electromagnetic perturbation. In nature ionosphere offers an example of this phenomenon. It is known that the index of refraction $N(\omega)$ is given by (unmagnetised, isotropic, cold electron plasma)

$$N(\omega) = \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} \quad ; \quad \omega_p^2 = \frac{4\pi e^2 n_o}{m_e} \tag{1}$$

where n_o is the density of free electrons and e, m_e the electron charge and mass. For the ionosphere $n_o \simeq 10^4 - 10^6$ cm^{-3} corrisponding to $\omega_p \simeq 6 \cdot 10^6 - 6 \cdot 10^7$ rad $\cdot s^{-1}$. The presence in the ionosphere of several layers of plasma with varying density makes also N dependent on height. We can study the electron density at various heights by means of the pulses of radiation transmitted vertically upwards. According to the dependence of electron density on height, a pulse of a given frequency ω_1 enters the layer without reflection if $\omega_1 > \omega_p(h_L)$. When the density is large enough to make $\omega_1 \simeq \omega_p(h_L)$, the index of reflection vanishes and the pulse is reflected.

As a conclusion, in order to achieve a longitudinal wave excitation in plasmas we must first of all penetrate the plasma with e.m. transversal waves and subsequently transform them into longitudinal wave. This is achieved in the Beat Wave Accelerator.

At this point the following question rises spontaneously: why a plasma accelerator? Well, let us immagine to create an electric field by means of a dielectric filled plane capacitor to accelerate charged particles. The acceleration increases as the potential between plates increases. But an intrinsic limit exists because of the dielectric rigidity.

If the electric field overcomes dielectric rigidity breakdown and "perforation" of dielectric occurs. On the contrary, breakdown can not occur in a fully ionised plasma: it can support ultra-high fields. For a plasma a limit to the field also exists, as shown by Dawson (1959). Its expression follows from Poisson's equation:

$$\overline{\nabla}\cdot\overline{E} \sim kE = 4\pi e n_1 \qquad (2)$$

where n_1 is the density perturbation.

The maximum field in a cold plasma obtains when $n_1 \sim n_o$, so that: $(k \sim \omega_p/c)$

$$eE^{max} \sim \frac{4\pi n_o e^2}{\omega_p/c} = m\omega_p c \simeq \sqrt{n_o} \quad (eV/cm) \qquad (3)$$

For $n_o \simeq 10^{18} cm^{-3}$, we have $eE^{max} \sim 1GeV/cm$. In conventional accelerators we have lower acceleration gratients ($eE^{max} \sim 20MeV/m$) so that with plasma based accelerators high energies can be reached in much shorter distances. In the following we shall give a fluid description (forced oscillator equations) of the plasma accelerator based on the concept of "ponderomotive force" which we introduce first for completeness.

2. THE PONDEROMOTIVE FORCE

A charged particle in an electromagnetic wave essentially oscillates in the direction of the $\overline{E}$ field (figure 8 motion). If however the field envelope has a spatial gradient it can exert a longitudinal (direction of wave propagation) force on the particle known as the ponderomotive force.

Consider a cold, collisionless electron-ion plasma. For this system we write the following Lorentz-Maxwell equations:

$$m_\alpha \frac{d\overline{v}_\alpha}{dt} = q_\alpha \overline{E} + \frac{q_\alpha}{c}\overline{v}_\alpha \mathrm{x} \overline{B} \quad \text{(motion)}$$

$$\frac{\partial n_\alpha}{\partial t} + \overline{\nabla}\cdot(n_\alpha \overline{v}_\alpha) = 0 \quad \text{(continuity)}$$

$$\overline{\nabla}\cdot\overline{E} = 4\pi\sum_\alpha q_\alpha n_\alpha \ , \quad \overline{\nabla}\cdot\overline{B} = 0 \qquad (4)$$

$$\overline{\nabla}\cdot\overline{E} = -\frac{1}{c}\frac{\partial\overline{B}}{\partial t}$$

$$\overline{\nabla}\mathrm{x}\overline{B} = \frac{4\pi}{c}\sum_\alpha q_\alpha n_\alpha v_\alpha + \frac{1}{c}\frac{\partial\overline{E}}{\partial t}$$

where the α index varies on the fluid species (α=e,i) and $\overline{v}_\alpha(\overline{r},t)$ is the velocity field, $\overline{E}$ and $\overline{B}$ are electric and magnetic fields, n_α is the number density field, q_α is the electric charge. Here we used the fluid derivative $\frac{d}{dt} = \frac{\partial}{\partial t} + \overline{v}\cdot\overline{\nabla}$.

Consider the propagation in the plasma of an electromagnetic wave with a space- and time-dependent amplitude. Assuming z as the direction of propagation we consider

the one-dimensional case, that is all quantities only depend on z,t and not on x,y (i.e. $\overline{\nabla} = \hat{z}\frac{\partial}{\partial z}$). Then, writing the vector potential of the wave as:

$$\overline{A}(z,t) = \overline{A}_s(z,t)exp[i(kz-\omega t)] \qquad (5)$$

where $\overline{A} = (A_x, A_y, 0)$ and $\overline{A}_s(z,t)$ is a complex amplitude and separating the equation of motion into longitudinal (z) and transverse components we have:

$$m_\alpha\frac{dv_{\alpha z}}{dt} = -q_\alpha\frac{\partial\phi}{\partial z} + \frac{q_\alpha}{c}[\overline{v}_\alpha \mathrm{x}(\overline{\nabla}\mathrm{x}\overline{A})]_z \qquad (6)$$

$$m_\alpha\frac{d\overline{v}_{\alpha\perp}}{dt} = -\frac{q_\alpha}{c}\frac{\partial\overline{A}}{\partial t} - \frac{q_\alpha}{c}[\overline{v}_\alpha \mathrm{x}(\overline{\nabla}\mathrm{x}\overline{A})]_\perp \qquad (7)$$

In this geometry it is:

$$[\overline{v}_\alpha \mathrm{x}(\overline{\nabla}\mathrm{x}\overline{A})]_z = \overline{v}_{\alpha\perp}\cdot\frac{\partial\overline{A}}{\partial z} \ , \quad [\overline{v}_\alpha \mathrm{x}(\overline{\nabla}\mathrm{x}\overline{A})]_\perp = -(\overline{v}_\alpha\cdot\overline{\nabla})\overline{A}$$

so that eq.n(7) gives:

$$\frac{d\overline{v}_{\alpha\perp}}{dt} = -\frac{q_\alpha}{m_\alpha c}\frac{d\overline{A}}{dt} \qquad (8)$$

which is immediately integrated into:

$$\overline{v}_{\alpha\perp} = -\frac{q_\alpha}{m_\alpha c}\overline{A} \qquad (9)$$

so that the longitudinal equation becomes:

$$m_\alpha\frac{dv_{\alpha z}}{dt} = -q_\alpha\frac{\partial\phi}{\partial z} - \frac{q_\alpha^2}{2m_\alpha c^2}\frac{\partial}{\partial z}|\overline{A}|^2 \equiv -q_\alpha\frac{\partial\phi}{\partial z} + |\overline{f}_\alpha| \qquad (10)$$

where

$$\overline{f}_\alpha = -\hat{z}\frac{q_\alpha^2}{2m_\alpha c^2}\frac{\partial}{\partial z}|\overline{A}|^2$$

The concept of ponderomotive force is conveniently introduced for cases when the wave amplitude $\overline{A}_s$ is "slowly varying", that is when

$$\frac{1}{|A_s|}\frac{\partial|A_s|}{\partial t} << \omega$$

so that the wave field is a low frequency envelope modulating a high frequency (ω) oscillation. Then the ponderomotive force $\overline{f}_{NL}$ is defined as the average value of $\overline{f}$ in a wave period $T = 2\pi/\omega$:

$$\overline{f}^{(\alpha)}_{NL} = <\overline{f}_\alpha> = \frac{1}{T}\int_0^T \overline{f}_\alpha(z,t)dt = -\frac{q_\alpha^2}{4m_\alpha\omega^2c^2}\frac{\partial}{\partial z}|\overline{A}_s|^2\hat{z} \qquad (11)$$

Only for the case of a circularly polarized wave it is $\overline{f}_{NL} = \overline{f}$. Some considerations follow from (11).

1) Given the inverse dependence on the particle mass the ponderomotive force on ions is much smaller than on electrons. Consequently on the electron timescale (ω_{pe}^{-1}) the ions can be considered immobile (cold plasma): only the electrons are initially

displaced by $\overline{f}_{NL}$ and a longitudinal electric field is consequently generated by the charge separation. At later times ($\sim \omega_{pi}^{-1}$) ion motion has also to be taken into account.

2) We remark that we used the following method to find an expression of the ponderomotive force. Initially, a transverse e.m. wave field only exists: the plasma density is unperturbed ($n = n_o$) and the z-component of both electric field and fluid velocity are zero. At this order of approximation the net force on the plasma is absent: in fact, the oscillating factor in eq.(5) gives a zero-average force on the plasma particles. At the next order the effect related to the slow-varying term $A_s(z,t)$ is introduced, so that a radiation pressure acts on the plasma. As a consequence, the plasma is perturbed. That this nonlinear force has a non- zero-average is easily seen from (11) where the square of A_s is present. Thus, we note that there are two time-scales in this description: a fast-time scale ($t \sim \omega^{-1}$) and a slow-time scale ($t > \omega^{-1}$). Evidently, ponderomotive force, longitudinal plasma motion and scalar potential appear on the slow-time scale. This is referred to as "low-frequency plasma response".

3. It is possible to write the ponderomotive force in terms of a "ponderomotive potential" ϕ_{NL} as $\overline{f}_{NL} = \overline{\nabla}\phi_{NL}$ with

$$\phi_{NL} = -\frac{\omega_p^2}{\omega^2}\frac{< E^2 >}{8\pi} \qquad (12)$$

where E is the transverse wave electric field ($E \sim \frac{\omega}{c}A$).

3. BEAT WAVE MECHANISM

In the Beat Wave Accelerator (BWA), as proposed by Tajima and Dawson (1979), the longitudinal field associated with the plasma wave is generated "beating" two e.m. waves (hereafter referred to as pumps) of frequencies and wave-vectors

$$\omega_1 - \omega_2 = \omega_p \ , \quad k_1 - k_2 = k_p = \frac{\omega_p}{c} \qquad (13)$$

(resonance conditions)

where (ω_p, k_p) are the frequency and wave-number of the excited plasma wave.

This resonant excitation of a plasma wave is a three-wave process (Raman forward decay) where eq.s(13) can be considered (multiplied by $\hbar$) as the laws of energy and momentum conservation for the two incoming photons and the generated plasmon. This process is best understood in terms of the ponderomotive force associated with the beat wave and a simple fluid model for the plasma.

Introducing the isotropic plasma dielectric constant (from (1))

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2} \qquad (14)$$

and taking into account (11) and (12) we get:

$$\overline{f}_{NL} = (\epsilon - 1)\overline{\nabla}\frac{< E^2 >}{8\pi} = (\epsilon - 1)\hat{z}\frac{\partial}{\partial z}\frac{< E^2 >}{8\pi} \qquad (15)$$

This (nonlinear) force moves the electron in the longitudinal (beat propagation) direction opposite to the gradient ($\epsilon < 1$) until the restoring force due to the immobile ($\omega_{pi} >> \omega_{pe}$) ions equals the ponderomotive force, thus establishing plasma oscillations, i.e. density perturbations δn with an associated longitudinal electric field E_p. The maximum amplitude of such a field in a cold plasma is given by (section 1):

$$E_p^{max} = \frac{m_e c}{e}\omega_p \qquad (16)$$

and wave-breaking occurs for $E_p > E_p^{max}$.

The phase velocity v_ϕ of the plasma wave is:

$$v_\phi \equiv \frac{\omega_p}{k_p} = \frac{\omega_1 - \omega_2}{k_1 - k_2} \simeq \frac{d\omega}{dk} \equiv v_g \qquad (17)$$

where v_g is the group velocity of the pump and we used the conditions $\omega_1 \simeq \omega_2$, $k_1 \simeq k_2$ (i.e.: $\omega_p << \omega_1, \omega_2$; $k_p << k_1, k_2$). For propagation in an unmagnetised plasma it is:

$$v_g = c(1 - \omega_p^2/\omega_o^2)^{1/2} \ , \quad \omega_o \equiv \frac{\omega_1 + \omega_2}{2} \qquad (18)$$

and then

$$v_\phi \simeq v_g \simeq c \quad \text{for} \quad \omega_p/\omega_o << 1 \qquad (19)$$

The condition (19) has to be associated to the resonance conditions (13) for efficient acceleration: in this case in fact the beat wave and the plasma wave move in synchronism through the plasma and the plasma field E_p can thus trap and accelerate a beam of particles injected (at the end of the beat pulse) with velocity v_ϕ i.e. with an initial energy:

$$m_e c^2 \gamma_{particle} = m_e c^2 (1 - v_\phi^2/c^2)^{-1/2} = \frac{1}{2}\frac{\omega_o}{\omega_p}(MeV) \qquad (20)$$

4. BWA THEORY IN FLUID DESCRIPTION

a). *Equations and solutions (linear treatment).*

We assume a cold isotropic plasma of classical electrons and an immobile, neutralising ion background and we look for the plasma response to an incoming wave (pump) on a timescale that ignores ion motion. Assume two linearly polarised pumps of amplitude E_{o1} and E_{o2} propagating in the z-direction:

$$\overline{E}_j(z,t) = \overline{E}_{oj}\cos(k_j z - \omega_j t) \quad (j = 1,2) \qquad (21)$$

The sum of the two is the "beat wave":

$$\overline{E}(z,t) = \overline{E}_{o1}\cos(k_1 z - \omega_1 t) + \overline{E}_{o2}\cos(k_2 z - \omega_2 t)$$

Consequently:

$$< \overline{E}^2 > = \frac{E_{o1}^2 + E_{o2}^2}{2} + E_{o1}E_{o2}\cos(k_s z - \omega_s t) \qquad (22)$$

and

$$\phi_{NL} = -\frac{\omega_p^2}{\omega^2}\left[\frac{E_{oT}^2}{16\pi} + \frac{E_{o1}E_{o2}}{8\pi}\cos(k_s z - \omega_s t)\right] \qquad (23)$$

where the average was made on $T = 2\pi/\omega$ with $\omega = (\omega_1 + \omega_2)/2$, $E_{OT}^2 \equiv E_{o1}^2 + E_{o2}^2$, and $\omega_s = \omega_1 - \omega_2$, $k_s = k_1 - k_2$, with $\omega_s << \omega_1, \omega_2$ and $k_s << k_1, k_2$.

From (15) we obtain:

$$F_{NL} = \frac{\nu_{o1}\nu_{o2}}{2} eE_p^{max} \sin(k_s z - \omega_s t) \tag{24}$$

where we have used $\omega_1\omega_2 = \omega^2 - \omega_s^2 \simeq \omega^2$ and we have introduced the quiver velocity:

$$\nu_{oj} = \frac{eE_{oj}}{m_e \omega c} \quad (j = 1, 2) \tag{25}$$

Denoting with ϕ the ambipolar potential associated with the charge separation due to the ponderomotive effect on electrons, i.e. $\overline{E}_p = -\overline{\nabla}\phi$, the fluid model equations are (low-frequency plasma response):

$$m_e \left(\frac{\partial}{\partial t} + v_e \frac{\partial}{\partial z} \right) v_e = -eE_p + F_{NL} \quad \text{"motion"} \tag{26}$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial z}(n_e v_e) = 0 \quad \text{"continuity"} \tag{27}$$

$$\overline{\nabla} \cdot \overline{E}_p = 4\pi e(n_o - n_e) \quad \text{"Poisson"} \tag{28}$$

where v_e is the longitudinal electron velocity and n_e the perturbed electron density. Linearizing the equations for:

$$n_e(z,t) = n_o + n(z,t) \quad ; \quad v_e(z,t) = v(z,t) \tag{29}$$

we have the system:

$$m_e \frac{\partial v}{\partial t} = -eE_p + F_{NL} \tag{30}$$

$$\frac{\partial n}{\partial t} + n_o \frac{\partial v}{\partial z} = 0 \tag{31}$$

$$\frac{\partial E_p}{\partial z} = -4\pi e n \tag{32}$$

Making use of (24) this system can be written as:

$$\frac{\partial^2 v}{\partial t^2} + \omega_p^2 v = \frac{-\nu_{o1}\nu_{o2}}{2} (eE_p^{max}) \frac{\omega_s}{m_e} \cos(k_s z - \omega_s t) \tag{33}$$

$$\frac{\partial^2 n}{\partial t^2} + \omega_p^2 n = -\frac{\nu_{o1}\nu_{o2}}{2} (eE_p^{max}) \frac{k_s n_o}{m_e} \cos(k_s z - \omega_s t) \tag{34}$$

$$\frac{\partial^2 \epsilon}{\partial t^2} + \omega_p^2 \epsilon = \frac{\nu_{o1}\nu_{o2}}{2} \omega_p^2 \sin(k_s z - \omega_s t) \tag{35}$$

where:

$$\epsilon = E_p / E_p^{max} \tag{36}$$

The resonant condition of these "forced harmonic oscillator" equations are exactly the conditions (13) ($\omega_s = \omega_p, k_s = k_p$) corrisponding to the solution:

$$v(z,t) = -\frac{\nu_{o1}\nu_{o2}}{4} \frac{eE_p^{max}}{m_e \omega_p} \sin k_p z \sin \omega_p t - \frac{\nu_{o1}\nu_{o2}}{4} \frac{eE_p^{max}}{m_e} t \sin(k_p z - \omega_p t) \tag{37}$$

$$n(z,t) = -\frac{\nu_{o1}\nu_{o2}}{4} \frac{eE_p^{max}}{m_e} \frac{k_p n_o}{\omega_p^2} \sin k_p z \sin \omega_p t - \frac{\nu_{o1}\nu_{o2}}{4} \frac{eE_p^{max}}{m_e} \frac{k_p n_o}{\omega_p} t \sin(k_p z - \omega_p t) \tag{38}$$

$$\epsilon(z,t) = \frac{\nu_{o1}\nu_{o2}}{4} \cos k_p z \sin \omega_p t + \frac{\nu_{o1}\nu_{o2}}{4} \omega_p t \cos(k_p z - \omega_p t) \tag{39}$$

These represent the sum of a stationary wave and of a plasma wave of growing amplitude. The ratio of the stationary to the growing wave amplitude is $(\omega_p t)^{-1}$ and the plasma wave can therefore be taken as the "secular" solution to the problem for $\omega_p t >> 1$.

b . *Relativistic saturation.*

A nonlinear calculation of the growth rate including the effect of relativistic mass variation (Rosembluth and Liu, 1972) shows that ϵ cannot increase indefinitely as shown by eq.(39) but, because of relativistic detuning (ω_p changes) of the harmonic oscillator equation (35), saturates at

$$\epsilon_{max} = \left(\frac{16}{3} \nu_{o1}\nu_{o2} \right)^{1/3} \tag{40}$$

The plasma wave growth occurs in the pulse of the beat wave: the breaking limit (40) combined with eq.(39) gives therefore a limit for the pulse duration, that is:

$$\tau_{pump} \leq (0.32\omega_p)^{-1} \left(\frac{\nu_{o1}\nu_{o2}}{4} \right)^{-2/3} \sec \tag{41}$$

which is in the nanosecond region per laser pumps. For $t \sim \tau_{pump}$ the longitudinal field has reached its maximum amplitude and particles injected at this time with the right energy (eq.(20)) can be accelerated. The maximum energy gain ($\epsilon \simeq \epsilon_{max}$) in the BWA scheme is (Tajima and Dawson, 1979):

$$W_{max} = 2\epsilon_{max} \left(\frac{\omega_o}{\omega_p} \right)^2 (MeV) \tag{42}$$

with the time and length to reach this energy given by:

$$t_{eff} \simeq \frac{W_{max}}{ecE_p} \simeq \frac{4}{\omega_p} (\omega_o/\omega_p)^2 \tag{43}$$

$$l_{eff} \simeq ct_{eff} = \frac{4c}{\omega_p} (\omega_o/\omega_p)^2 \tag{44}$$

Since $W_{max} \sim n_o^{-1}$ and $W_{max}/l_{eff} \sim \sqrt{n_o}$, the higher the final energy desired, the slower the gradient in the simple BWA Scheme (Beat Wave Dilemma).

This intrinsic difficulty can be solved in the surfatron scheme (Katsouleas and Dawson, 1983).

As a conclusion we give the estimates for Conventional, BWA and Surfatron Accelerators ($\lambda = 1$ micron, $n_o = 10^{18}$ cm^{-3}, $B = 50$ KG, $\epsilon = 0.5$):

CONVENTIONAL	1 TeV in 50 km
BWA	1 TeV in 600 m
SURFATRON	1 TeV in 60 cm x 20 m

5. SOME IMPORTANT PARAMETRIC PROCESSES RELATED TO BWA (INSTABILITIES)

One of the approximations in the model presented so far has been to consider the transverse field (beating "pumps") as unchanged as it travels through the plasma. A more complete theory would have to consider the effects of plasma motion on the field such as Raman cascading on plasma density perturbations: put in other words in a plasma the fields have to be self- consistent solutions of the Lorents-Maxwell system and a "given" pump can only be taken as an initial or boundary value for the final solution.

The excited plasma wave, when it reaches large enough amplitudes, can also drive other plasma instabilities. The model presented can therefore only be considered as a sero-order solution to the problem: other effects, such as the three-dimensional nature of the beam entering the plasma (Fedele et al. 1986), the optimisation of the frequency mismatch $\omega_s = \delta\omega_p$ (Tang et al. 1985, Noble 1985), the role of plasma noise and ion motion on wave saturation (Horton and Tajima 1985) as well as the role of pump-driven and plasma-driven parametric instabilities (Fried et al. 1976) have to be considered for a better understanding of the BWA scheme.

Here we shall give a brief summary of some parametric instabilities thought to be relevant to the BWA.

Consider a large amplitude wave (electromagnetic or electrostatic) of wavelength λ_o, incoming in the plasma, as a pump, toward a density perturbation. For instance, suppose that the last has wavelength $\lambda_1 = \frac{1}{2}\lambda_o$. This perturbation will be a unidimensional lattice for the pump. So it will produce a partial reflection and the incoming and reflected waves will generate a stationary wave with maxima corrisponding to minima of perturbation. Consequently a ponderomotive force will appear because of the interference envelope. It will push the electrons toward lower intensity field regions, corrisponding to higher densities. Thus the density perturbation will be enhanched where it was already higher and this fact will produce, in turn, an increasing backscattering which, in turn, will intensificate more and more the density perturbation and so forth. This phenomenon is enhanced when the density perturbation corrisponds to a plasma eigenmode (plasma oscillations).

We will give some examples about these kind of processes:

a). *Stimulated Brillouin Backscattering Instability (SBBI).*

In the SBBI processes a large amplitude e.m. wave (ω_o, k_o) decays in another e.m. wave (ω_2, k_2) and a iono-acoustic wave (ω_1, k_1). This instability has the following growth rate:

$$\gamma_g \sim 1.6\mathrm{x}10^{14}\left(\frac{2\alpha_o}{\gamma_\phi}\right)^{2/3} s^{-1} \quad (45)$$

where α_o is the normalised pump amplitude (quiver velocity) and $\gamma_\phi = \omega_o/\omega_p$. For $\alpha_o \sim 0.1$ and $\gamma_\phi \simeq 100$:

$$\gamma_g = 2.5x10^{12} s^{-1}.$$

The influence on the beat wave mechanism is not yet well known.

b). *Stimulated Raman Backscattering Instability (SRBI).*

In the SRBI processes the e.m. pump (ω_o, k_o) decays in another e.m. wave (ω_2, k_2) and an electron plasma wave (ω_1, k_1). The growth rate is:

$$\gamma_g \sim \frac{10^{16}}{\lambda_o}\frac{\alpha_o}{\gamma_\phi^{1/2}} \quad (46)$$

For $\alpha_o \sim 0.1$, $\gamma_\phi \sim 100$:

$$\gamma_g \sim 10^{13} s^{-1}.$$

c). *Plasma Decay Instability (PDI).*

In the PDI processes a large amplitude electron plasma wave (ω_o, k_o) decays in another electron wave (ω_2, k_2) and a iono-acoustic wave (ω_1, k_1). These processes represent excitation mechanisms for iono-acoustic waves. The growth rate is:

$$\gamma_g = \frac{\alpha_o^{2/3}}{\gamma_\phi}\omega_o \quad (47)$$

For $\alpha_o \sim 0.1$, $\gamma_\phi \sim 100$, $\lambda_o \sim 1\mu m$:

$$\gamma_g \sim 10^{13} s^{-1}$$

d). *Filamentation Instability (FI).*

To describe this process it will be useful to refer to an example. Consider in a plasma an ion density perturbation perpendicular to the direction of propagation of an e.m. wave incoming in the plasma. The latter will propagate along constant density lines. But the index of refraction is larger in the depressions of density and smaller where the density is higher. On the other hand, it is known that the e.m. radiation prefers to propagate in the regions with high index of refraction. Thus it will concentrate greatly in the density-depressed regions (filamentary structure). The filamentation leads to an instability. In fact, the filaments introduce a ponderomotive force (like in the self-focussing of beam laser of finite diameter) that acts in such a way that there is a self-focussing for each of them. So that a further new radiation remains trapped which in turn produces an additional ponderomotive effect and so forth.

The growth rate is given by:

$$\gamma_g \simeq \frac{\omega_{pi}}{\sqrt{2}}\alpha_o \quad (48)$$

For $\gamma_\phi = 100$, $\lambda_o = 1\mu m$:

$$\gamma_g \sim 5x10^{10} s^{-1}$$

6. MICROWAVE DRIVEN RESONANT EXCITATION OF PLASMA WAVES

The difficulties associated with the use of high intensity laser pumps (in terms of cost, time, technology and competing instabilities) have stimulated the proposal for different schemes for plasma accelerators. Chen et al. (1985), for instance, suggest the use of electron beams to excite the plasma wave and Formisano et al. (1986) suggest the use of microwave (μw) pumps instead of lasers for scaled experiments based on recent results by de Angelis et al. (1986) who have shown that resonant excitation of large amplitude plasma waves ($\geq 10^4 Volt/cm$) is possible through the beating of two microwaves of nearly equal frequency in an "open resonator" filled with plasma of subcritical density.

Given the lower power ($\sim MW$) and larger pulse widths ($\sim \mu$sec) of the present generation of μw generators with respect to lasers an immediate consequence of the scaling will be a less efficient acceleration; the advantage on the other hand is to make experiments more affordable (in terms of cost, time and technology) so that, while the ultimate goal for media accelerators rests with high intensity laser beams, it could be worth to test the basic theories with scaled experiments. It should also be mentioned however, that the μw range can also offer possibilities that are out of the laser range and that μw technology is in rapid expansion so that a full investigation of schemes in the millimetric wavelength could prove useful not only for scaled experiments but also for alternative solutions to some of the problems in media accelerators.

As an example we mention the possibility of power flux confinement. Consider the pump intensity $I_o = P/A$ where P is the power in the pump and $A = \pi R^2$ is the focussing area: for lasers typically it is $R \sim 10\lambda_p$ where λ_p is the plasmon wavelength whereas for μw it is possible to scale at $R < \lambda_p$ and regain part of the loss due to the lower power. In fact, if λ_o is the pump wavelength ($\sim$ 1 mm), it is possible to take

$$R \sim \lambda_o = \lambda_p \frac{\omega_o}{\omega_p} \tag{49}$$

and since $\omega_o/\omega_p >> 1$ is a necessary condition for efficient acceleration we can have $R << \lambda_p$ with a corrisponding gain in pump intensity. All the effects of strong focussing can be generated and studied in the μw regime much better than in the laser regime. One of the effects is the generation of radial fields: a recent calculation (Fedele et al., 1986) in cylindrical coordinates for a gaussian pump profile gives (see Appendix):

$$E_r(r,z,t) \sim \delta \frac{r}{R^2} \exp(-2r^2/R^2)[1 - \omega_p t \sin(k_p z - \omega_p t)] \tag{50}$$

$$E_z(r,z,t) \sim \exp(-2r^2/R^2)\delta\omega_p t \cos(k_p z - \omega_p t) \tag{51}$$

where:

$$\delta = \frac{1}{4}\nu_{01}\nu_{02} \tag{52}$$

for the radial and longitudinal electric field components.

Notice that as R is made smaller the radial field increases: this could be advantageous as particles are pushed towards the axis where the accelerating field E_z is higher. The problem of betatron oscillations and of strong focussing in general can be carefully investigated in the μw range.

In conclusion, we report for completeness the main results from the cited work of de Angelis et al. (1986) on microwave driven plasma waves.

Consider (Fig.1) an open resonator of length $2l$ with spherical copper mirrors of radius R_o, enclosed in a plasma chamber transparent to microwaves with plasma of sub-critical density for the incoming radiation: microwaves from a generator coupled to the left mirror (grid). The field distribution in the cavity has the typical caustic shape with maximum focusing at the center

$$\pi W_o^2 = \lambda[l(R_o - l)]^{1/2} \quad \text{"focusing area"} \tag{53}$$

The field is fairly uniform between $-z_R$ and $+z_R$ (Raileigh distance) where

$$z_R = \pi w_o^2/\lambda \quad \text{"Raileigh distance"} \tag{54}$$

In (53) and (54) λ is the pump wavelength, $\lambda = \frac{1}{2}(\lambda_1 + \lambda_2) \equiv \lambda_f$ for the two beating pumps case.

The beat amplitude grows (because of multiple reflections in the cavity) and so does the resulting ponderomotive force on electrons so that the driven plasma wave will also grow until relativistic saturation or collisional saturation, depending on the pump amplitude.

There are two relevant timescales in the problem: the resonator saturation time τ_R (time for the pump field to reach its maximum amplitude) and the plasma wave saturation time τ_s.

These depend on the resonator dimensions, on the generator characteristics and coupling and on the plasma parameters and are determined as:

$$\tau_R = \frac{8\pi^2\mu^2\nu}{\alpha\omega_f} \quad , \quad \tau_s = \tau_R(63/K)^{1/7} \tag{55}$$

where

$$\mu = w_o/\lambda_f \quad , \quad \nu = l/z_R \quad , \quad \omega_f = \frac{1}{2}(\omega_1 + \omega_2) \tag{56}$$

$$\alpha = \frac{1}{2}[0.2(\omega_f/10^{16})^{1/2} + |T|^2] \tag{57}$$

where $|T|^2$ is the transmission coefficient of the mirror grid and

$$K = 1.3\text{x}10^{-11}(P_{KW}|T|^2)^2\nu^3\mu^2\alpha^{-7} \tag{58}$$

where P_{KW} is the generator power in kilowatts. The above results are given for a choice $\Delta\lambda_f/\lambda_1 = 0.1$, that is $\omega_f/\omega_p = 10$.

The generated plasma field has a radial and a longitudinal (resonator axis) component and its maximum amplitude will be $\epsilon(\tau_s)$.

An estimate of the maximum field on axis ($r = 0$) gives:

$$\epsilon_{max} = 4.7\frac{\nu}{\alpha^2}\frac{P_{KW}}{10^6}|T|^2[x + \frac{1}{2} - 2(1 - \frac{1}{2}e^{-x})^2] \tag{59}$$

where $x = \tau_s/\tau_R$.

As an example consider the NRL gyrotron (Read 1985) with f_1=120 GHz and $P_{KW} = 10^2$ coupled to a resonator with l=10 cm and $w_o = 2\lambda_f$ (i.e. μ =2, ν =2.8) and a transmission coefficient $|T|^2$=0.1. This gives:

f_2= 109 GHz, ω_f=7.2x10^{11} sec^{-1}, λ_f=2.6 mm,
ω_p=6.9x10^{10} sec^{-1}, n_o=1.4x10^{12} cm^{-3}, z_R=3.6 cm,
τ_R=26 ns, τ_s=27 ns, ϵ_{max}=0.01

corresponding to a maximum longitudinal field (on axis) of 1.3x10^4 volt/cm, which is an interesting level for scaled experiments on the physics of the BWA scheme as suggested in a forthcoming paper (de Angelis et al. 1987). Coupling to high power microwave devices such as the cyclotron autoresonance maser ($P_{KW} = 10^4$) or free electron lasers ($P_{KW} \simeq 10^5$) would of course produce even higher field levels.

In Fig. 2 & 3 the results are given, for the gyrotron and Carm generators, for different geometries and coupling.

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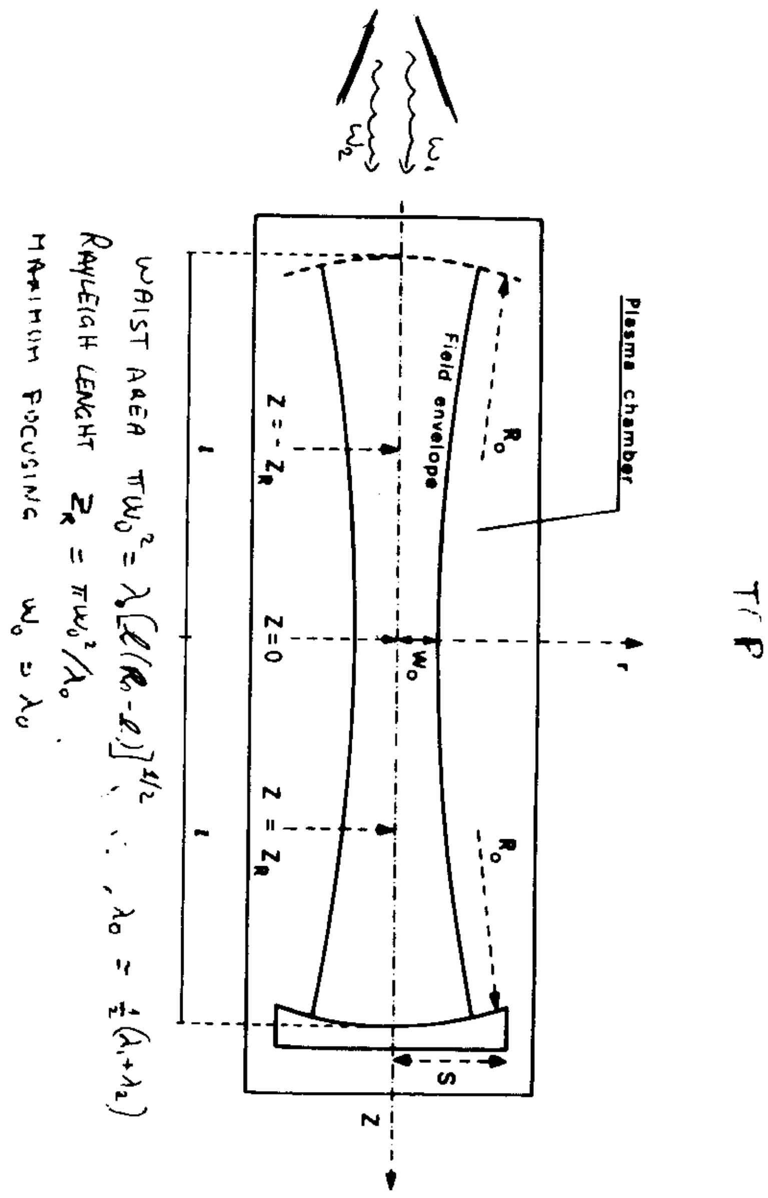
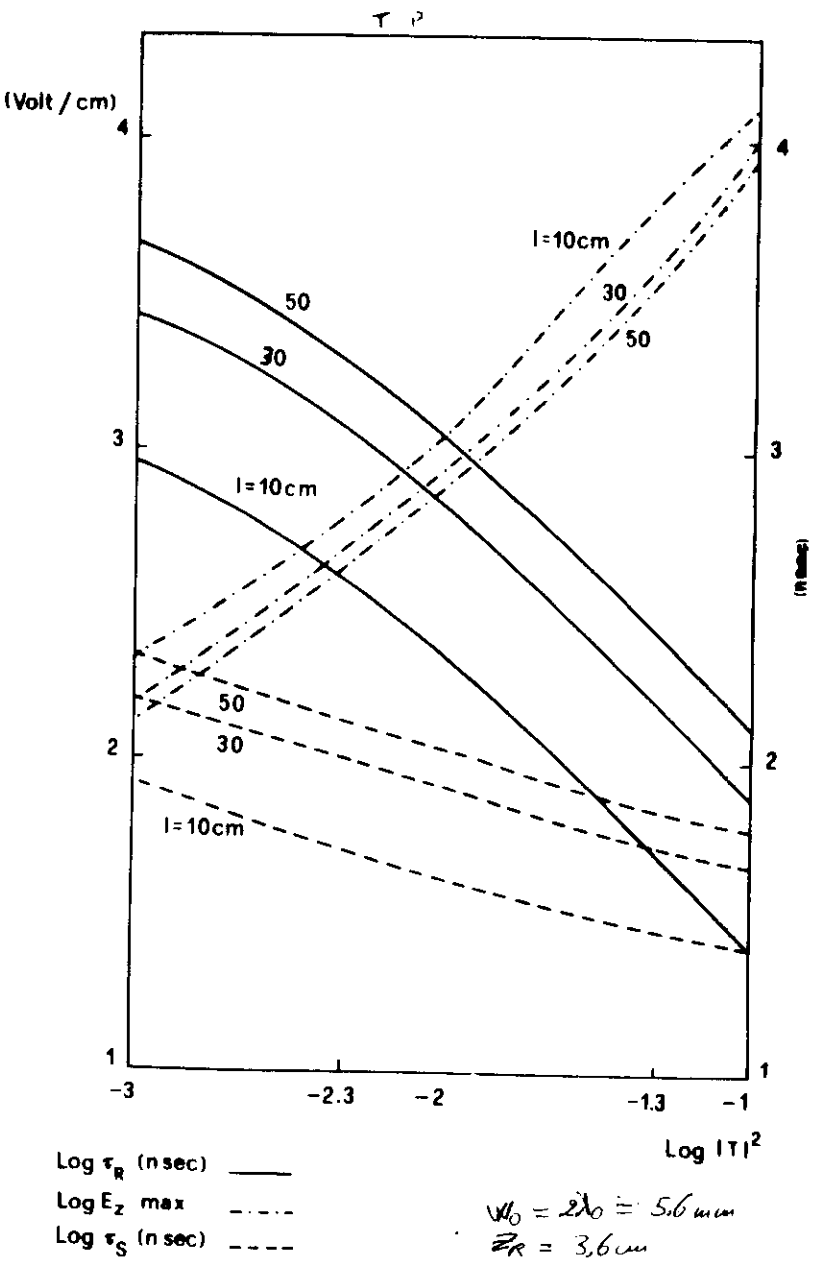


FIG. 1

NRL GYROTRON

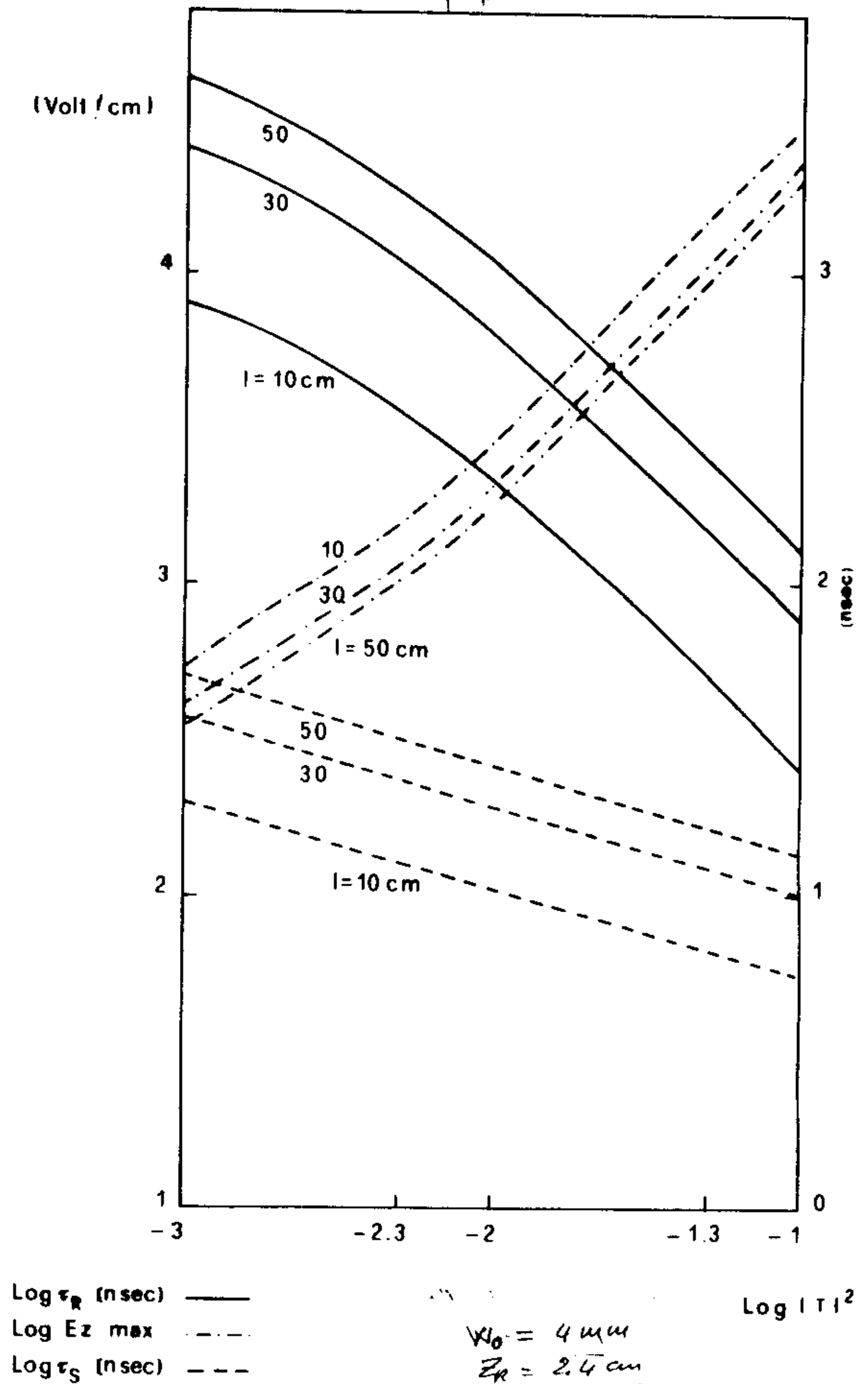
P = 100 kW, f_1 = 120 GHz



$w_0 = 2\lambda_0 = 5.6$ mm
$z_R = 3.6$ cm
$\omega_p = 7 \times 10^{10}$ sec^{-1}

FIG. 2

P = 10 MW, f_1 = 150 GHz



$w_0 = 4$ mm
$z_R = 2.4$ cm
$\omega_p = 9 \times 10^{10}$ sec^{-1}

FIG. 3

GENERATION OF RADIAL FIELDS FOR GAUSSIAN PUMP PROFILE

Linearized equations are:

$$m_e \frac{\partial \underset{\sim}{v}}{\partial t} = \underset{\sim}{\nabla} (e\phi + \phi_{NL}) \qquad (1)$$

$$\frac{\partial n}{\partial t} + n_o \underset{\sim}{\nabla} \cdot \underset{\sim}{v} = 0 \qquad (2)$$

$$\nabla^2 \phi = 4\pi en \qquad (3)$$

where $\underset{\sim}{v}$ and n are the (first-order) electron velocity and density perturbation, n_o the unperturbed density, ϕ the ambipolar potential due to charge separation, $\underset{\sim}{E} = -\underset{\sim}{\nabla}\phi$, and ϕ_{NL} is the ponderomotive potential associated with the ponderomotive force[7] on electrons:

$$\underset{\sim}{f}_{NL} \equiv \underset{\sim}{\nabla}\phi_{NL} = \frac{\epsilon - 1}{n_o} \underset{\sim}{\nabla} \left(\frac{E_p^2}{8\pi}\right) \qquad (4)$$

where E_p is the beat-wave field and ϵ the plasma dielectric. Equations 1-3 can be written as a set of forced-oscillator equations:

$$\frac{\partial^2 n}{\partial t^2} + \omega_p^2 n = -\frac{n_o}{m} \nabla^2 \phi_{NL} \qquad (5)$$

$$\frac{\partial^2 \underset{\sim}{E}}{\partial t^2} + \omega_p^2 \underset{\sim}{E} = \frac{\omega_p^2}{e} \underset{\sim}{\nabla}\phi_{NL} \qquad (6)$$

$$\frac{\partial^2 \underset{\sim}{v}}{\partial t^2} + \omega_p^2 \underset{\sim}{v} = \frac{1}{m} \frac{\partial (\underset{\sim}{\nabla}\phi_{NL})}{\partial t} \qquad (7)$$

Equations 5-7 are valid in any coordinate system. We solve them in cylindrical coordinates (r,z,ϕ) with z along the direction of beat propagation.

Assuming for the two pumps:

$$\underset{\sim}{E}_j(r,z,t) = \underset{\sim}{E}_o e^{-r^2/R^2} \cos(k_j z - \omega_j t) \qquad (8)$$

$$(j = 1,2)$$

the resulting field (beat-wave) is:

$$\underset{\sim}{E}_p(r,z,t) = 2\underset{\sim}{E}_o e^{-r^2/R^2} \cos(z\Delta k - t\Delta\omega)\cos(k_o z - \omega_o t)$$

$$\equiv \underset{\sim}{E}_s(r,z,t)\cos(k_o z - \omega_o t) \quad , \qquad (9)$$

where $\Delta k = (k_1 - k_2)/2$, $\Delta\omega = (\omega_1 - \omega_2)/2$,

$$k_o = (k_1 + k_2)/2 \ , \quad \omega_o = (\omega_1 + \omega_2)/2 \ . \qquad (10)$$

For $\omega_1 \sim \omega_2$, $k_1 \sim k_2$ (a necessary condition in the BWA scheme[1]) the "amplitude" E_s of the high frequency wave (ω_o) is "slowly" varying in space and time and we can therefore average on the fast time scale ω_o^{-1} to get the ponderomotive force on the time-scale of the envelope E_s:

$$\langle \underset{\sim}{f}_{NL} \rangle = \frac{\epsilon - 1}{n_o} \underset{\sim}{\nabla} \left(\frac{\langle E_p^2 \rangle}{8\pi}\right) = \frac{1}{2} \frac{\epsilon - 1}{n_o} \underset{\sim}{\nabla} \left(\frac{E_s^2}{8\pi}\right) \qquad (11)$$

where, for the unmagnetized case, $\epsilon = 1 - \omega_p^2/\omega_o^2$. Thus:

$$\phi_{NL} = -\frac{1}{2} \frac{e^2}{m_e \omega_o^2} E_s^2 = -\frac{e^2 E_o^2}{m_e \omega_o^2} e^{-2r^2/R^2} \cos^2(z\Delta k - t\Delta\omega) \qquad (12)$$

and, in cylindrical coordinates:

$$\nabla\phi_{NL} = \frac{\partial\phi_{NL}}{\partial r}\hat{r} + \frac{\partial\phi_{NL}}{\partial z}\hat{z} \; ;$$

$$\nabla^2\phi_{NL} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi_{NL}}{\partial r}\right) + \frac{\partial^2\phi_{NL}}{\partial z^2} \qquad (13)$$

This gives, for the density perturbation equation:

$$\frac{\partial^2 n}{\partial t^2} + \omega_p^2 n = A(r) + B(r)\cos(k_d z - \omega_d t) \qquad (14)$$

with:

$$A(r) = -\frac{8f_o}{(Rk_d)^2}\left(1 - \frac{2r^2}{R^2}\right)e^{-2r^2/R^2} \qquad (15)$$

$$B(r) = -f_o\left[1 + \left(\frac{8}{(Rk_d)^2}\right)\left(1 - \frac{2r^2}{R^2}\right)\right]e^{-2r^2/R^2} \qquad (16)$$

$$k_d = k_1 - k_2 \, , \quad \omega_d = \omega_1 - \omega_2 \, , \quad f_o = \frac{\omega_p^2}{\omega_o^2}\frac{E_o^2}{8\pi}\frac{k_d^2}{m_e} \, . \qquad (17)$$

The "resonant" solution $\omega_d = \omega_p$, $k_d = k_p$ is the driven plasma oscillation:

$$n(r,z,t) = \frac{A(r)}{\omega_p^2}(1-\cos\omega_p t) + \frac{B(r)}{2\omega_p^2}\sin k_p z \sin\omega_p t - \frac{B(r)}{2\omega_p}t\sin(k_p z - \omega_p t) \, . \qquad (18)$$

Note that the radial density profile is not simply the profile of the pump intensity.

The equations for electric field components decouple:

$$\frac{\partial^2 E_r}{\partial t^2} + \omega_p^2 E_r = \left(\frac{8m_e c^2\omega_p^2}{e}\right)\alpha\frac{r}{R^2}e^{-2r^2/R^2}\left[1 + \cos(k_d z - \omega_d t)\right] \qquad (19)$$

$$\frac{\partial^2 E_z}{\partial t^2} + \omega_p^2 E_z = \left(\frac{2m_e c^2\omega_p^2 k_d}{e}\right)\alpha\, e^{-2r^2/R^2}\sin(k_d z - \omega_d t) \qquad (20)$$

where α is related to the square of the quiver velocity $4\alpha = (e^2E_o^2/m_e^2\omega_o^2c^2)$. The resonant solutions are:

$$E_r(r,z,t) = -4\left(\frac{m_e c\omega_p}{e}\right)\alpha\frac{r}{k_p R^2}e^{-2r^2/R^2}\left[\omega_p t\sin(k_p z - \omega_p t) - 2(1 - \cos\omega_p t) + \sin k_p z\sin\omega_p t\right] \qquad (21)$$

$$E_z(r,z,t) = -\left(\frac{m_e c\omega_p}{e}\right)\alpha e^{-2r^2/R^2}\left[\omega_p t\cos(k_p z - \omega_p t) + \cos k_p z\sin\omega_p t\right] \qquad (22)$$

We notice that the linear growth rate for the longitudinal field, as given by this linear fluid model, is in complete agreement with a previous[8] nonlinear, relativistic calculation up to saturation which, of course, is not in the linear theory.

The presence of radial focusing or defocusing field components can be of particular relevance to experiments with microwave pumps recently proposed[9] since the normalized cross sectional area, $\pi(k_p R)^2$, where the pump energy is distributed can be made much smaller in the microwave regime than in the laser regime.

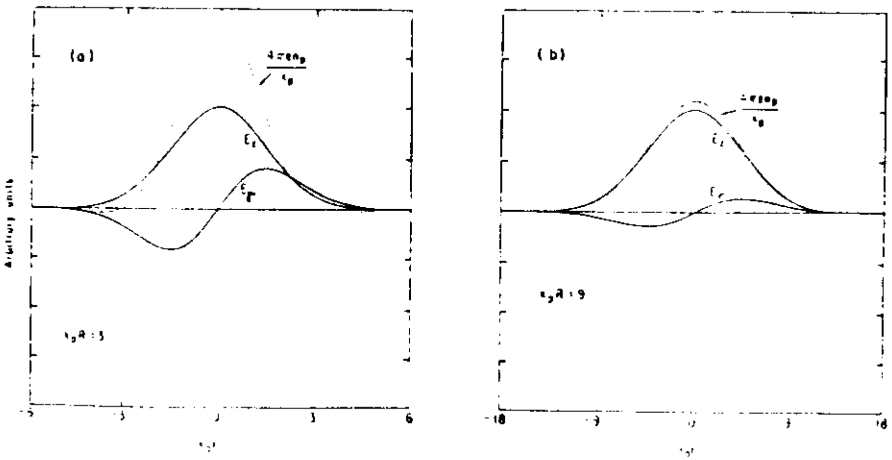


FIG. 1. Amplitudes of n, E_z and E_r from the secular terms of Eqs. (18), (20), and (21) for (a) $k_pR = 3$ and (b) $k_pR = 9$ [corresponding to the parameters of the recent UCLA experiment (Ref. 10)].

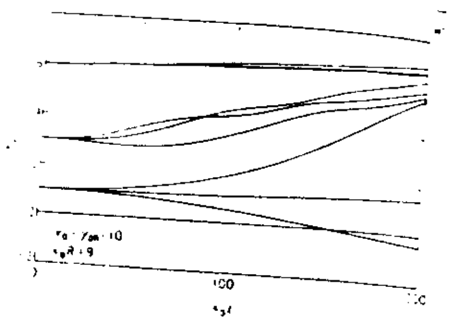


FIG. 2. Particle trajectories in the fields corresponding to Fig. 1(b). $(eE_z/m\omega_pc)_{max} = 0.05$, $\omega_p/k_pc = v_{ph}/c = 0.995$. Three particles were initialized at each of four radial positions $(k_pr = 0,1,3,6)$ with phases $[k_pz(t=0) = -\pi/4, 0, \pi/4]$ correspond to focusing and accelerating, defocusing and accelerating, and maximum accelerating (no focusing) plasma-wave fields.

FIG. 4