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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

THIRD ANNUAL ICTP SUMMER WORKSHOP ON PARTICLE PHYSICS

20 June - 31 July 1983

(COLLECTED LECTURES)

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

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and
United Nations Educational Scientific and Cultural Organization
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PREFACE

Between June 20th and July 31st 1983, the International Centre for Theoretical Physics held its third consecutive Summer Workshop on Elementary Particle Physics. The purpose of the Workshop is to provide a forum of discussion on the latest developments in high energy physics for the Centre's associate scientists and young physicists from all over the world and in particular from developing countries. What follows is a set of lecture notes and reading material which formed the basis of the discussions.

In addition to the lectures listed in the table of contents, there were four introductory lectures on supersymmetry given by Profs. J. Gates ([illegible]) and M. [illegible] (Trieste) and a special invited talk on quark [illegible] by Prof. S. Adler (Princeton). The corresponding notes or reading material for these lectures can be requested directly from the above contributors.

THIRD ANNUAL ICTP SUMMER WORKSHOP ON PARTICLE PHYSICS

TABLE OF CONTENTS

I. APPLIED SUPERGRAVITY/UNIFIED THEORIES

APPLIED N=1 SUPERGRAVITY

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super-Higgs need not necessarily arise at the tree level ("tree breaking (T.B) models"). The SU(2)XU(1) breaking can also arise dynamically from renormalization group loop corrections [D5,D6,D16-D18] ("renormalization group (R.G.) models"). The essential difference structurally between the T.B. models and the R.G. models is the following: In the T.B. models, the low energy effective potential [D4,D10,D11], in the Higgs' sector contains a pair of Higgs doublets H^{α} and H'_{α}, $\alpha = 1,2$ and a singlet field U. The singlet plays a crucial role in SU(2)XU(1) breaking in the T.B. models. For the case of the R.G. models one discards the singlet and retains only the pair of Higgs doublets. SU(2)XU(1) is broken in the R.G. models due to the large Yukawa couplings of a heavy top quark. Typically one needs $m_t \gtrsim 100$-200 GeV. More recently model independent analyses of the low energy domain have been given which can accomodate the T.B. and the R.G. models as well as a whole class of other variations in between [E7-E9].

At the phenomenological level, Supergravity unified theories make some unique predictions and are free from the usual defects of the corresponding global SUSY theories. Thus for example, unlike the global supersymmetry theory [B4] there are no light scalar bosons in these theories. Indeed in the low energy domain the scalar bosons (that do not become superheavy) acquire a characteristic mass $O(m_g)$ [E10,E1]. Unlike the case of the SUSY theories, in supergravity theories, one can at least eliminate the cosmological constant by ...ameter (fine tuning) in the superpotential (see Sec. III ...uppression of the

not arranged by hand as in global theories but arise naturally as a consequence of the supergravity models.

The fermionic partners of the photon and the gluons, i.e. the photino and the gluinos, are massless at the tree level. However, the photino and the gluinos grow massess at the loop level [E6,D17].

$$\tilde{m}_{\gamma} = \frac{8}{3}\frac{\alpha}{4\pi}\bar{C}\,\tilde{m}_{g}\ ;\ \tilde{m}_{g} = \frac{\alpha_3}{4\pi}\bar{C}m_{g} \simeq 8\tilde{m}_{\gamma}\quad, \tag{1.3}$$

where $\bar{C}$ is proportional to the Casimir of the (heavy) multiplet exchanged in the loop (See Eq. 5.13). For normal size heavy multiplets, one expects $\tilde{m}_{\gamma}$ to lie in the range of (1-10) GeV and the gluino mass in the range (5-80) GeV from Eq. (1.3).

The fermionic partners of the W and Z bosons also grow masses at the tree level. In fact as discussed by Weinberg [E4] and the authors [E5] there appear in theories of the type discussed above, relatively light gauge fermions. Thus in the charged sector one has a Wino (Supersymmetric partner of the W boson), the $\tilde{W}_{(-)}$, lying below the W boson and in the neutral sector one has a Zino (Supersymmetric partner of the Z boson), the $\tilde{Z}_{(-)}$, lying below the Z boson. In each of these sectors there also exist additional supersymmetric partners, i.e. a Wino, $\tilde{W}_{(+)}$, lying above the W boson and a Zino, $Z_{(+)}$, lying above the Z boson, in Supergravity models [E5] as well as other neutral Zinos [E7-E9].

The existence of $\tilde{W}_{(-)}$ and $\tilde{Z}_{(-)}$ are clearly very exciting from an ...point since one has the possiblity of detecting these ... now been experimentally confirmed [F2,F3]. The decays of the W and Z which are universal are for the W [E4]

$$W^{\pm} \rightarrow \tilde{W}_{(-)}{}^{\pm} + \tilde{\gamma} \qquad (1.4)$$

and for the Z [E5]

$$Z \rightarrow \tilde{W}_{(-)}{}^{+} + \tilde{W}_{(-)}{}^{-} \qquad (1.5)$$

since they occur in both the T.B. and the R.G. models. A remarkable feature of the decay of Eq. (1.5) is that it is of "Industrial Strength" i.e. the branching ratio of $Z \rightarrow \tilde{W}^{+} + \tilde{W}^{-}$ relative to $Z \rightarrow e^{+}e^{-}$ is characteristically of size O(5). The T.B. and the R.G. models each possess additional decays which are unique to them. Thus in the T.B. model one has the decay [E3,E9]

$$W^{\pm} \rightarrow \tilde{W}_{(-)}{}^{\pm} + \tilde{Z}_{(-)} \qquad (1.6)$$

if $M_W > (\tilde{m}_W + \tilde{m}_Z)$. In R.G. models, the process of Eq. (1.6) is kinematically disallowed. However, in the R.G. models there exists a new light neutral Higgsino $\tilde{Z}_{(3)}$ (we call it the "Twilight Zino" due to its very weak coupling with ordinary matter) which couples with normal strength with the Z allowing for the following decay [E8,E9]

$$Z \rightarrow \tilde{Z}_{(3)} + \tilde{Z}_{(3)} \qquad (1.7)$$

Each of the decays of Eqs. (1.4)-(1.7) have their own characteristic signal. Thus the decay of Eq. (1.4) would lead to jets in one direction balanced by an

Unidentified Fermionic Object (UFO), the photino, in the opposite direction. These UFO events would each consist of a single jet with unbalanced momentum. There are similar characteristic UFO signals for other processes of Eqs. (1.5)-(1.7). Further, there exists the possibility of testing the T.B. vs the R.G. models through Eqs. (1.6) and (1.7) in the decays of the W and Z. The W and Z decays thus provide a possible test of supersymmetry at the $\bar{p}p$ collider. A search for gluinos can also be carried out at the $\bar{p}p$ collider [E11,E5]. Other possible tests of supersymmetry could come through $e^{+}e^{-}$ collisions in the processes $e^{+}e^{-} \rightarrow \tilde{\gamma}\tilde{\gamma}\gamma$ [E13,E14]. (See Fig. (1)) This experiment can be carried out at the current energies at PEP and PETRA. In most models the production of the selectron (the supersymmetric partner of the electron) would require larger energies such as those contemplated at LEP (though the model of [E14] can accomodate a light selectron). The process $e^{+}e^{-} \rightarrow \tilde{\gamma}\tilde{\gamma}e^{+}e^{-}$ [E15] offers another possibility, though its cross-section is expected to be quite small (see Fig. (2)).

There are cosmological constraints on some of the "ino" masses [E16,E17] though in these lectures we shall not discuss these here in any detail.

In our analysis we shall use N=1 Supergrvity with the minimal set of auxiliary fields [C1–C4]. Here the field content consists of the spin2, spin3/2 fields $e_{a\mu}$, φ_μ and the auxiliary fields $S, P, A\mu$. The Supergravity Lagrangian invariant (up to a total divergence) under local supersymmetry transformations is then [C1, C2]

$$L_{S.G.} = -\frac{e}{2k^2}R(e,\omega) - \frac{e}{3}|u|^2 + \frac{e}{3}A_\mu A^\mu - \frac{1}{2}\bar{\varphi}_\mu R^\mu \tag{2.1}$$

where

$$u = S - iP \tag{2.2}$$

$$R_{\mu\nu}{}^{rs} = \partial_\mu \omega_\nu{}^{rs} + \omega_\mu{}^{rp}\omega_{\nu p}{}^{s} - \mu \leftrightarrow \nu \tag{2.3}$$

$$R^\mu = \varepsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu D_\rho(\omega)\varphi_\sigma \tag{2.4}$$

$$R = e_r{}^\mu\, e_s{}^\nu R_{\mu\nu}{}^{rs} \tag{2.5}$$

$$D_\mu = \partial_\mu + (1/2)\ \omega_{\mu rs}\sigma^{rs} \tag{2.6}$$

$$\omega_{\mu rs} = \omega_{\mu rs}\ (e) + K_{\mu rs}\ (e,\varphi_\mu) \tag{2.7}$$

$$K_{\mu rs}(e,\varphi_\mu) = (k^2/4)(\bar{\varphi}_\mu\gamma_r\varphi_s - \bar{\varphi}_\mu\gamma_s\varphi_r + \bar{\varphi}_r\gamma_\mu\varphi_s) \tag{2.8}$$

and e is the determinant of the vierbein.

In the construction of the GUT models one needs couplings of N=1 Supergravity with matter. Matter consists of left-handed chiral (F-type) multiplets

$$\Sigma^a = (Z^a, \chi^a{}_L, h^a) \quad , \tag{2.9}$$

where $Z^a = A^a + iB^a$ are complex scalar fields, $\chi^a{}_L$ are left-handed Weyl spinors and h^a are complex auxiliary fields. The index a in Σ^a is an internal symmetry index such that Σ^a belongs to a reducible representation of a gauge group G. In addition, matter contains a vector (D-type) multiplet $V(V=V^\dagger)$ which has components

$$V = (C, \xi, H, K, V_\mu, \lambda, D) \tag{2.10}$$

and is in the adjoint representation of the gauge group G. In (2.9) ξ, λ are Majorana spinors, C,H,K are scalars while D is an auxiliary scalar field. The vector multiplet is reduced significantly in the Wess-Zumino guage [A1] and one has

$$V = (V_\mu, \lambda, D) \quad . \tag{2.11}$$

The rules of coupling a single F-type multiplet and a single D-type multiplet with Supergrvity are known. For the F-type multiplet one has [C2, C3]

$$e^{-1}L_F = \mathrm{Re}[h + uZ + \bar{\varphi}_\mu \gamma^\mu X + \bar{\varphi}_\mu \sigma^{\mu\nu} \varphi_{\nu R} Z] \tag{2.12}$$

and for the D-type multiplet one has [C2]

$$\begin{aligned} e^{-1}L_D = {} & D - \frac{ik}{2}\bar{\varphi}_\mu \gamma^5 \gamma^\mu \lambda - \frac{2}{3}(SK - PH) \\ & + \frac{2}{3}kV_\mu (A^\mu + \frac{3}{8}ie^{-1}\varepsilon^{\mu\rho\sigma\tau}\ \bar{\varphi}_\rho \gamma_\tau \varphi_\sigma) \\ & + i\frac{k}{3}e^{-1}\bar{\xi}\gamma_5\gamma_\mu R^\mu + \frac{ik^2}{8}\varepsilon^{\mu\nu\rho\sigma}\bar{\varphi}_\mu \gamma_\nu \varphi_\rho \bar{\xi}\varphi_\sigma \\ & - \frac{2}{3}k^2 C\ e^{-1}L_{S.G.} \end{aligned} \tag{2.13}$$

One may contrast the results on Eqs. (2.12) and (2.13) with the corresponding situtíon for global supersymmetry where only the F and D terms are admissible in the Lagrangian. For the case of supergravity all elements of the F and D multiplets enter the Lagrangian to preserve the local supersymmetry gauge invariance.

Cremmer et. al [C7] have given the most general coupling of a single chiral multiplet with supergravity. This scheme exhibits the possibility of spontaneous breakdown of the Supergravity gauge invariance and a mass growth for the gravitino through a minimization of its true effective potential. However, the existence of only a single chiral multiplet in the coupling scheme does not allow one to formulate supergravity GUT theories. This led the authors to a generalization of these results to couple N=1 Supergravity to an arbitrary number of chiral multiplets belonging to a reducible

representation of a grand unified gauge group G and simultaneously to a gauge multiplet belonging to the adjoint representation of the gauge group [C8]. Equivalent formulations have been given by other authors [C9–C13]. The details of construction are presented in Appendix A and in this section we shall only outline the general procedure and state results for the relevant parts of the Lagrangian we need.

The procedure consists in forming the most general F and D multiplets out of Eqs. (2.9) and (2.11) using the rules of tensor calculus and maintaining the invariance under the gauge group G. Next one couples these F and D multiplets to supergravity using Eqs. (2.12) and (2.13). The resulting Lagrangian is expressed most conveniently in terms of functions $g(Z^a)$, $\phi(Z^a, Z_a)$ and $f_{\alpha\beta}(z^a)\cdot g(z)$ is the familiar superpotential which is the lowest element of the most general gauge singlet F-multiplet formed out of the chiral multiplets of Eq. (2.9) i.e.

$$g(z^a) = \sum B_{a_1 \ldots a_m} Z^{a_1} \ldots Z^{a_m} \tag{2.14}$$

$\phi(Z^a, Z_a)$ represents the lowest element of the most general gauge singlet D multiplet formed out of the chiral multiplet $\sum^a$ of Eq. (2.9) and its hermitian conjugate, i.e.,

$$\phi(Z^a, Z_a) = \sum A^{a_1 \ldots a_m}{}_{b_1 \ldots b_n} Z_{a_1} \ldots Z_{a_m} Z^{b_1} \ldots Z^{b_n} \tag{2.15}$$

The co-efficients $B^{a_1 \ldots a_m}$ and $A^{a_1 \ldots b_n}$ in Eqs. (2.14) and (2.15) are arbitrary parameters except that they are chosen to maintain invariance under G. A convenient procedure in implmenting the coupling scheme is to first couple the gauge multiplet V of Eq. (2.11) to the chiral multiplets Σ^a and next couple the resultant structure to supergravity.

To obtain the Lagrangian in a useful form one must first carry out an elimination of all the auxiliary fields in the theory (both in the matter and the supergrvity sector). In addition one needs to make point transformations to put the kinetic energy of the dynamical fields into canonical form. The details of the resulting Lagrangian are given in Appendix A.

The Lagrangian is determined in terms of two arbitrary functions $\mathcal{G}(Z^a, Z_a)$ and $f_{\alpha\beta}(Z^a)$ The function $\mathcal{G}(Z^a, Z_a)$ is a special combination of ϕ and g:

$$\mathcal{G}(Z^a, Z_a) = -\frac{k^2}{2} \; d(Z, Z^\dagger) - \ln\left(\frac{k^6}{4}|g(Z^a)|^2\right) \tag{2.16}$$

and exhibits the invariance

$$g \rightarrow g \, e^{f(z)} \tag{2.17}$$

$$d \rightarrow d - \frac{2}{k^2}(f(z) + f^\dagger(z)) \tag{2.18}$$

where

$$d = -(6/k^2)\ln\left(-\frac{k^2}{3}\phi\right) \quad . \tag{2.19}$$

Eqs. (2.17) and (2.18) along with the fact that the kinectic energies of the scalar fields are proportional to $\mathcal{G},^a{}_b \equiv \partial^2\mathcal{G}/\partial Z^b \partial Z_a$ makes $\mathcal{G},^a{}_b$ as the metric in the Kähler manifold [C14] defined by coordinates Z^a, Z_b.

The arbitrary function $f_{\alpha\beta}(Z)$ enters in the Yang-Mills sector [C9]

$$e^{-1}L(F_{\alpha\beta}) = \frac{1}{2} f_{\alpha\beta}\left(-\frac{1}{4}F^\alpha_{\mu\nu}F^{\mu\nu\beta} - \frac{1}{2}\bar{\lambda}^\alpha \not{D}\lambda^\beta + \frac{1}{2}D^\alpha D^\beta + \frac{1}{4}F^\alpha_{\mu\nu}\tilde{F}^\beta_{\mu\nu} - \frac{1}{2} D_\mu(\bar{\lambda}^\alpha \gamma^\mu \lambda_R{}^\beta)\right) + \text{h.c.} \tag{2.20}$$

There is no theory to determine $f_{\alpha\beta}(z)$ in the current framework. However, we know that in the absence of gravitational interactions renormalizability requires that one have $f_{\alpha\beta} = \delta_{\alpha\beta}$. Thus if the quantum supergravity theory was appropriately controlled in the ultra-violet domain such as through the phenomena of "asymptotic safety" [F4], the deviations of $f_{\alpha\beta}$ from the global limit should be only due to gravitational loop effects. Weinberg [E4] has argued that these loops obey to a good approximation, a U(n) symmetry among the n chiral multiplets, and as a consequence, deviations of $f_{\alpha\beta}$ from $\delta_{\alpha\beta}$ should be very small. We shall thus assume in our analysis

$$f_{\alpha\beta}(Z) = \delta_{\alpha\beta} \quad . \tag{2.21}$$

Under the assumption of Eq. (2.21) the Bose part of the Lagrangian then takes the form

$$L_B = -(e/2k^2)R(e,\omega) + (e/k^2)\mathcal{G},^a{}_b \mathcal{D}_\mu Z_a \mathcal{D}^\mu Z^b$$

$$+ (e/k^4)\exp(-\mathcal{G})[3 + (\mathcal{G}^{-1})^a{}_b \, \mathcal{G},_a \, \mathcal{G},^b]$$

It is useful to express Eq. (2.22) in an alternate form using the function $d(Z,Z^\dagger)$ of Eq. (2.19). One has then L_B in the form

$$e^{-1}L_B = -\frac{e}{2k^2}R(\omega(e)) - \frac{1}{2}d,^a{}_b \mathcal{D}_\mu Z_a \mathcal{D}_\mu Z^b - \frac{1}{4}F^\alpha_{\mu\nu}F^{\mu\nu\alpha} - e^{-1}V \tag{2.23}$$

where V is the potential of the scalar fields and is given by

$$V = \frac{e}{2}\exp(\frac{k^2}{2}d)[(d^{-1})^a{}_b G_a G^b - \frac{3}{2}k^2|g|^2] \tag{2.24}$$

where G_a is defined by

$$G_a \equiv \frac{\partial g}{\partial Z^a} + \frac{k^2}{2}d,_a g \tag{2.25}$$

and $(d^{-1})^a{}_b$ is the inverse of the matrix $(d),^b{}_a \equiv \partial^2 d/\partial Z^a \partial Z_b$.

In our new notation, $d(Z,Z^\dagger)$ acts as the potential in the Kähler manifold. The choice

$$d = Z_a Z^a \tag{2.26}$$

corresponds to a flat Kähler manifold with the matrix $d,_a{}^b = \delta_a{}^b$ and leads to a normalized kinetic energy for the scalar fields in Eq. (2.23). The choice of Eq. (2.24) may be too restrictive and the gravitational loop corrections may possibly modify Eq. (2.24). However, the gravitational loop corrections to a good approximation are still expected to preserve the U(n) symmetry among the n-chiral fields and a more general choice for an effective Kähler potential d should be a general function of $Z_a Z^a$. For simplicity, however, we shall carry out most of the analysis in the following sections under the assumption of a flat Kähler manifold, Eq. (2.26).

III. SPONTANEOUS SYMMETRY BREAKING AND SUPER-HIGGS EFFECT

For the flat Kähler manifold the extrema equations arising from Eq. (2.24) have the form

$$(\frac{\partial G_b}{\partial Z_a} + \frac{k^2}{2}Z^a G_b)G^b - k^2 g\, G = 0 \quad . \tag{3.1}$$

On the real manifold of VEVS, Eq. (3.1) reduces down to

$$T_{ab}\, G_b = 0 \tag{3.2a}$$

where G_b is given by Eq. (2.25) and T_{ab} is defined by

$$T_{ab} = \frac{\partial^2 g}{\partial Z^a\, \partial Z^b} + \frac{k^2}{2}(Z_a\frac{\partial g}{\partial Z^b} + Z_b\frac{\partial g}{\partial Z^a}) + \frac{k^4}{4}Z_a Z_b g - k^2\delta_{ab}g \tag{3.2b}$$

Eq. (3.2) may be satisfied through the vanishing of all the G_a i.e.

$$G_a = 0 \tag{3.3}$$

Under this circumstance one has that supersymmetry is unbroken though the gauge symmetry of the theory may be broken. An example of such a breaking is provided by the superpotential

$$g = \lambda\,(\frac{1}{3}\mathrm{Tr}\Sigma^3 + \frac{1}{2}M\mathrm{Tr}\Sigma^2) \quad , \tag{3.4}$$

Where $\Sigma_y{}^x$ is the adjoint representation 24 of SU(5). Satisfaction of Eq. (3.3) shows that a minimum of the potential exists when $(\Sigma_y{}^x)_{diag}$ possesses one of the following vacuum expectation values;

$$\text{(i) } 0 \quad \text{(ii) } \frac{1}{3}M(1,1,1,1,-4) \quad \text{(iii) } M(2,2,2,-3,-3) \tag{3.5a}$$

and the VEVS of all other components of $\Sigma_y^{\,x}$ are zero. Solution in Eq. (3.5a) does not break supersymmetry. However, solutions (ii) and (iii) of Eq. (3.5a) break the gauge group. Thus solution (ii) breaks SU(5) into SU(4)XU(1) while solution (iii) breaks SU(5) into SU(3)XSU(2)XU(1). A second example is provided by the superpotential [B2]

$$g = \lambda_o X(M^2 - \mathrm{Tr}\Sigma^2) + \lambda_1 \mathrm{Tr}\Sigma^2\Delta + \lambda'_1 Y \mathrm{Tr}\Sigma\Delta \tag{3.5b}$$

Here $\Delta_y^{\,x}$ is a second $\underline{24}$ representation of SU(5). Satisfaction of Eq. (3.3) shows that a minimum exists when $X = \Delta_y^{\,x} = 0$ and $\Sigma_y^{\,x}$ and Y have one of the following two solutions;

$$\text{(i)} \quad \Sigma_y^{\,x} = \frac{M}{\sqrt{20}}(\delta_y^x - 5\,\delta_5^x\,\delta_y^5), \; Y = \frac{3M}{\sqrt{20}} \tag{3.6}$$

$$\text{(ii)} \quad \Sigma_y^{\,x} = \frac{M}{\sqrt{30}}\,[2\delta_y^{\,x} - 5(\delta_4^{\,x}\delta_y^{\,4} + \delta_5^{\,x}\delta_y^{\,5})], \; Y = \frac{M}{\sqrt{30}}\frac{\lambda_1}{\lambda'_1} \tag{3.7}$$

Again solutions of Eqs. (3.6) and (3.7) preserve supersymmetry but the gauge symmetry is broken. Solution (ii) corresponds to a residual symmetry of SU(4)XU(1) while (iii) corresponds to the residual symmetry SU(3)XSU(2)XU(1).

The symmetry breaking solutions of the type Eq. (3.5) which preserve

supersymmetry exhibit an interesting phenomena. Substitution of Eq. (3.3) in the tree effective potential gives

$$V_{min}(Z_o^{\,a}, Z_{oa}) = -\frac{3k^2}{4}|g(Z_o^{\,a})|^2 \exp(\tfrac{1}{2}k^2 Z_{oa}\, Z_o^{\,a}) \tag{3.8}$$

Eq. (3.8) implies that the degeneracy of the vacuum solutions encountered in global supersymmetry is removed due to the $O(k^2)$ corrections to the vacuum energy [D1,D22,D23]. From Eq. (3.8) one finds that if any one of the minimum solutions is chosen to be Minkowskian by the adjustment of an additive constant to the superpotential, then all other solutions would necessarily be of anti-deSitter nature and would have vacuum energies which are negative. Normally a situation where the Minkowskian vacuum arises in association with anti-deSitter vacuum would seem to require that Minkowskian vacuum would be unstable. However, the presence of gravitation can help restore stability [D24]. In fact for situations where supersymmetry is preserved, Weinberg [D22] has argued that the Minkowskian vacuum would actually be stable for any finite size perturbation even though it does not have the lowest energy.

One may notice that for the potential of Eq. (3.5b), both solutions (3.6) and (3.7) give a vanishing $g(Z_o^{\,a})$ which implies that for this case gravitation does not lift the degeneracy of models of the type Eq. (3.5b) [D2]. Thus there exists the possibility in these models of realizing a vacuum structure where the Minkowskian vacuum is the lowest state of energy when supersymmetry is broken [D2].

We consider next the case of broken supersymmetry. On the mass-shell supersymmetry transformations for the spin 3/2 and spin 1/2 Weyl fields are (see Appendix A)

$$\delta\chi^a = (\gamma^\mu \varepsilon_R)\hat{\mathcal{D}}_\mu Z^a - \frac{1}{2}(\mathcal{G}_{,b}\bar{\chi}^b - \mathcal{G}_{,}{}^b\bar{\chi}_b)\chi^a + (\mathcal{G}^{-1})^a{}_b \mathcal{G}_{,}{}^b{}_{cd}\,\bar{\chi}^c\chi^d\varepsilon_L$$

$$- k^{-1}\exp(-\frac{\mathcal{G}}{2})(\mathcal{G}^{-1})^a{}_b\mathcal{G}^b\varepsilon_L \qquad (3.9)$$

$$\delta\psi_{\mu L} = 2k^{-1}D_\mu(\varepsilon,\psi_\mu)\varepsilon_L + k^{-2}\exp(-\frac{\mathcal{G}}{2})\gamma_\mu\varepsilon_R + \frac{1}{2}(\mathcal{G}_{,a}\bar{\chi}^a - \mathcal{G}_{,}{}^a\bar{\chi}_a\varepsilon)\psi_{\mu L}$$

$$+ k^{-1}\sigma_{\mu\nu}\varepsilon_L\mathcal{G}_{,}{}^a{}_b\,\bar{\chi}^a\gamma^\nu\chi^b - \frac{k}{8}(\delta_\mu^\nu + \gamma^\nu\gamma_\mu)\varepsilon_L\bar{\lambda}^\alpha\gamma_\nu\gamma_5\lambda^\alpha - \frac{k^{-1}}{2}(\mathcal{G}_{,a}\mathcal{D}_\mu Z^a - \mathcal{G}_{,}{}^a\mathcal{D}_\mu Z_a)\varepsilon_L \quad (3.10)$$

From Eqs. (3.9) and (3.10) one finds for the vacuum expectation values (for a flat Kähler manifold)

$$\langle\delta\chi^a\rangle_o = -\exp(\frac{k^2}{4}Z_o{}^a Z_{oa})\mathcal{G}_a(Z_o)\varepsilon_L \qquad (3.11)$$

$$\langle\delta\psi_{\mu L}\rangle_o = \frac{k}{2}\exp(\frac{k^2}{4}Z_o^a Z_{oa})g(Z_o)\gamma_\mu\varepsilon_R + 2k^{-1}\partial_\mu\varepsilon_L \qquad (3.12)$$

Thus a necessary requirement for the breaking of supersymmetry is that at least one G_a is non-zero. In the representation where T_{ab} is diagonal this implies at least one non-vanishing eigenvalue for T_{ab}. In the unitary gauge where the spin1/2 Goldstino is absorbed by the gravitino, the gravitino mass is given by [C7]

$$m_g = \frac{k^2}{2}g(Z_o)\exp(\frac{k^2}{4}Z_o^\dagger Z_o) \qquad (3.13)$$

The simplest example of the super-Higgs effect occurs when one couples a single chiral multiplet $\Sigma = (Z, \chi_L, h)$ with Supergravity and the superpotential is of the form [C6]

$$g_2(Z) = m^2(Z + B) \qquad (3.14)$$

where m^2 and B are constants. Here $G(Z) \neq 0$, and hence Eq. (3.2) requires $T_{zz} = 0$. One finds [C6,C7]

$$kZ_{(-1)} = a(\sqrt{2} - \sqrt{6}),\; kB_{(-)} = -a(2\sqrt{2} - \sqrt{6}),\; a = \pm 1 \quad , \qquad (3.15)$$

where the condition on $B_{(-1)}$ is chosen so that $V_{min} = 0$. From Eq. (3.15) we note that the super-Higgs field Z has VEV ~ $O(M_p)$. Eq. (3.14) represents the simplest possibility and one may consider the more general case of an arbitrary super-Higgs potential of the form

$$g_2(Z) = m^2k^{-1}f_2(kZ) \quad , \qquad (3.16)$$

where $f_2(kZ)$ is an arbitrary dimensionless function of kZ with the expansion

$$f_2(kZ) = f_2{}^0 + kZf_2{}^1 + \ldots \qquad (3.17)$$

The $T_{zz} = 0$ condition for superpotentials of type in Eq. (3.16) would yield supersymmetry breaking solutions which are characteristically of the form

$$\langle Z\rangle \sim O(k^{-1});\ \langle g_2\rangle \sim O(k^{-1}m^2) \quad . \tag{3.18}$$

Eq. (3.18) together with Eq. (3.13) then implies

$$m_g = \frac{k^2}{2} g_2(\langle Z\rangle)\exp(\frac{k^2}{4}\langle Z^\dagger Z\rangle) \sim O(m^2 k) \quad . \tag{3.19}$$

In Sec. V we shall identify the gravitino mass as characteristically of the size of the weak interaction scale i.e. $m_g \sim O(M_W)$ [D1]. Using this correspondence we can identify the mass scale m entering Eq. (3.16) of the super-Higgs potential. One finds from Eq. (3.19) that

$$m \sim (M_W\, M_{Planck})^{1/2} \sim 10^{10}GeV \quad . \tag{3.20}$$

This means that the mass scale entering the super-Higgs effect is an intermediate mass scale and is related by a geometric heirarchy to the weak interaction mass scale and the Planck mass.

We note in passing that there are additional constraints which a super-Higgs potential must satisfy in order that it be an admissible potential for realistic model building. For example, an important requirement is that the particle spectrum of the theory after spontaneous breakdown contains no tacheonic modes. A quadratic form for the superpotential $g_2(z) = m(z^2 + B)$ can be excluded on this basis.

IV. SUPERGRAVITY MODELS

In setting up realistic supergravity models we shall find it convenient to classify the full set of fields Z^A in the matter sector into two categories: the field Z in the super-Higgs sector and the remaining matter fields Z^a. Thus we write

$$Z^A = (Z^a, Z) \quad . \tag{4.1}$$

Our basic supergravity model is then defined by the superpotential [D1]

$$g(Z^A) = g_1(Z^a) + g_2(Z) \quad . \tag{4.2}$$

In the limit k=0, the dynamics of the fields Z^a and the field Z are completely disjoint. However, for k non-zero, the two sectors interact through supergravitational interactions, and the dynamics of each sector is affected. The most dramatic effect occurs in the sector of the fields Z^a due to the influence of the field Z in that one firds the appearance of soft-breaking in the Z^a-sector due to the super-Higgs effect [D1,D4]. We shall illustrate the soft-breaking phenomena by the very simple example where

$$g_1(Z^a) = 0 \quad . \tag{4.3}$$

For the case of Eq. (4.3) one finds that the potential

$$V = \frac{1}{2}\exp\left(\frac{k^2}{2} Z_A Z^A\right)\left[G_A G^A - \frac{3}{2}k|g|^2\right] + \frac{e}{32}|g_\alpha (Z_a, (T^\alpha Z)_a]^2 \qquad (4.4)$$

gives the mass term $\frac{1}{2}m_g^2 Z_a Z^a$ to scalar fields Z^a while for the corresponding fermionic mass matrix (see Appendix A)

$$m_{ab} = \exp\left(\frac{k^2}{4} Z_a Z^a\right)\left[\frac{\partial^2 g}{\partial Z^a \partial Z^b} + \frac{k^2}{2}\left(Z_a \frac{\partial g}{\partial Z^b} + Z_b \frac{\partial g}{\partial Z^a}\right)\right.$$

$$\left. + \frac{k^4}{12} Z_a Z_b g - \frac{2}{3} \frac{\partial g}{\partial Z^a} \frac{\partial g}{\partial Z^b} g^{-1}\right] + h.c. \qquad (4.5)$$

one finds for Eq. (4.3) the result $m_{ab} = 0$. Thus the degeneracy between the bosons and the fermions is lifted and the mass of the gravitino characterises the scale by which the degeneracy is broken. Indeed one notices that the super-Higgs effect generates a common mass which is equal to the gravitino mass in this approximation.

In the general analysis one has $g_1 \neq 0$ which makes the general analysis for the computation of soft breaking more complex. One of the reasons for this complexity is that in general some of the fields in Z^a may be super-heavy involving the GUT mass scale M which is close to the Planck mass k^{-1}. For a SUSY theory $M \sim 3 \times 10^{16}$ GeV and so one has

$$\varepsilon \simeq kM \sim 10^{-2} \qquad (4.6)$$

This means that in the solution to the minimization equations for the determination of VEVS, higher order k corrections of size

$$(kM)M,\ (kM)^2 M, \ldots (kM)^6 M \qquad (4.7)$$

must be controlled in order that the low energy theory be protected from the GUT mass scale M. This is a new heirarchy problem arising only in supergravity models and has no direct analogue in the corresponding global theories. To account for these new complexities of the light and the heavy fields in the matter sector Z^a we classify our fields as follows:

$$Z_a = (\{Z_i\}, \{Z_i'\}, \{Z_\alpha\}) \qquad (4.8)$$

The Z_i are fields with VEVS of O(M) and also of masses of O(M). The Z_i' have vanishing or small $O(m_g)$ VEVs but have masses of O(M). The fields Z_α are light fields with VEVs and mass of $O(m_g)$. Our purpose next is to examine the minimization conditions and establish criteria which would generate the above gauge heirarchy at the tree level. Generation of such a pattern of gauge heirarchy is motivated by our desire for developing GUT models where one needs gauge heirarchies of type Eq. (4.8).

In order to examine the full heirarchy problem at the tree level we develop an expansion solution for the VEVs in powers of k:

$$Z_a = Z_a^{(0)} + Z_a^{(1)} + Z_a^{(2)} + \ldots \qquad (4.9)$$

$$Z_i'^{(0)} = 0,\ Z_\alpha^{(0)} = 0 \qquad (4.10)$$

$$Z = Z^{(-1)} + Z^{(0)} + \ldots \qquad (4.11)$$

where we have that $Z_A^{(n)} \sim O(k^n)$. It is useful to rescale the fields,

$$z_i = M^{-1}Z_i, \; z_\alpha = m_s^{-1}Z_\alpha, \; z = kZ \quad , \qquad (4.12a)$$

where

$$m_s \equiv km^2 \qquad (4.12b)$$

so that z_i, z_α, z etc. are dimensionless and have expansions similar to Eqs. (4.9) - (4.11) but beginning at zeroth order in k. It is also useful to define the dimensionless quantities $\bar{g}$, $\bar{G}_a$ and $\bar{G}_z$ as follows:

$$\bar{g} = (k/m^2)g, \; \bar{G}_z = m^{-2}G_z, \; \bar{G}_a = m_s^{-2}G_a \quad . \qquad (4.13)$$

Now from the extrema equations that determine the VEVs, one can show [D10] that the desired tree gauge heirarchy would be destroyed if $\bar{G}_a$ contained terms of size M/m_s or k^{-1}/m. Indeed one can establish that for a wide class of theories which obey the restriction

$$g_{,\alpha i} \sim O(m_s) \quad , \qquad (4.14)$$

$$g_{,\alpha\beta} \sim O(m_s) \quad , \qquad (4.15)$$

one has $\bar{G}_i$, $\bar{G}_\alpha$ and $\bar{G}_z$ are of order unity so that generally one has at the minimum of the effective potential

$$G_a \sim O(m_s^2) \qquad (4.16)$$

with corrections to the leading order term which are very small i.e. $\varepsilon\delta_s$ and δ_s^2 where ε is defined by Eq. (4.6) and

$$\delta_s \equiv km_s \sim 10^{-16} \quad . \qquad (4.17)$$

Normally one would expect $G_i \sim M^2$ on dimensional grounds and so the result of Eq. (4.16) is quite remarkable. It is Eq. (4.16) which plays the central role in guaranteeing protection of VEVS in the tree level minimization equations. The essential meaning of Eqs. (4.16) is that the effects of the GUT sector characterized by the GUT mass M and of the super Higgs sector characterized by the Planck mass on the low mass sectors is only of size $O(m_s)$ which maintains the mass heirarchy. Thus typically in the low mass sectors the full solution of the extrema equations taking account of the GUT and super Higgs sector would generte the following type series expansion for the light field VEVS:

$$Z_\alpha = m_s z_\alpha = m_s z_\alpha^{(0)} + A_\alpha m_s(kM)(km_s) + B_\alpha m_s(km_s)^2 + \ldots \quad , \qquad (4.18)$$

where $z_\alpha^{(0)}, A_\alpha, B_\alpha, \ldots$ are dimensionless numbers of order unity. This tree level protection holds to arbitrary orders in k.

Eqs. (4.14) and (4.15) act as essential constraints necessary to achieve the tree level gauge heirarchy in the construction of realistic supergravity GUT models. Thus certain types of couplings must either be eliminated or unnaturally suppressed in the superpotential. Thus for example the coupling

$\lambda Z_i Z_j Z_a$ can only appear in the superpotential provided $\lambda \sim m_s/M$ while the coupling $\lambda' Z_i Z_j Z$ can appear provided $\lambda' \sim km_s$.

For model building it is found useful to eliminate the super-Higgs and the heavy fields to achieve a low energy effective potential [D4,D10,D11]. Thus consider the effective potential of the full theory $V(Z_i; Z_a; Z)$ which obeys the extrema equations

$$\frac{\partial V}{\partial Z} = 0; \quad \frac{\partial V}{\partial Z_i} = 0 \tag{4.19}$$

One may solve Eqs. (4.19) to express Z and Z_i in terms of Z_a i.e.

$$Z = Z[Z_a]; \quad Z_i = Z_i[Z_a] \tag{4.20}$$

In practice Eqs. (4.20) would be exhibited in a power series in k. We note that in the low energy domain we are for the present only interested in recovering operators of dimensionality four or less to construct the low energy effective potential (which turns out to imply that the series expansion of Eq. (4.20) need not go beyond order (m_s) corrections). Insertion of Eq. (4.20) into Eq. (4.19) then gives the low energy effective potential $U(Z_a)$

$$U(Z_a) = V(Z_i[Z_a]; Z_a; Z[Z_a]) \tag{4.21}$$

The protection conditions Eqs. (4.19) and (4.22) guarantee that $U(Z_a)$ [illegible]
only $O(m_g^{\ 4})$ and terms $O(Mm_g^{\ 3})$, $O(M^2 m_g^{\ 2})$ etc. cancel [D11]. An alternat[illegible]

procedure is to eliminate the heavy fields and super Higgs fields in the extrema equations of the light sector. Since in the extrema equations, protection of the low energy mass scale has already been achieved, the integration of these equations would yield an effective potential which has the low energy protection already built in. The relevant equations to integrate are Eqs. (3.1) in the light sector. One finds [D10]

$$U(Z_a, Z^a) = \frac{1}{2}\exp\left(\frac{\kappa^2}{2}|Z_a|^2\right)\left[\tilde{g}^{\dagger}_{1,a}\tilde{g}_{1,a} + m_3^2 Z_a Z^a + (\omega + \omega^{\dagger}) + m_s^2 (\tilde{g}_{1,i} G_i^{(0)} + h.c.)\right] \tag{4.22}$$

where

$$\omega = m_2\tilde{g}_1 + m_3 Z^a \tilde{g}_{1,a} \tag{4.23}$$

and

$$\tilde{g}_1(Z_i, Z_a) = g_1(Z_i, Z_a) - g_1(Z_i, 0) - b \tag{4.24}$$

In the deduction of Eq. (4.22) we have used the form of Eq. (4.2) for the superpotential. The three mass parameters m_1, m_2 and m_3 reduce to two when the cosmological constant condition is imposed since then $m_1 = |m_3|$. The constants are

$$m_1^2 = \frac{1}{2}m_s^2[\bar{G}_z^{(0)}\bar{G}_z^{(0)*} - \bar{g}_2^{(0)}\bar{g}_2^{(0)*}] \quad , \qquad (4.25)$$

$$m_2 = \frac{1}{2}m_s[z^{(0)}\bar{G}_z^{(0)} - 3\bar{g}_2^{(0)}] \quad , \qquad (4.26)$$

$$m_3 = \frac{1}{2}m_s\bar{g}_2(0);\ m_1 = |m_3| \quad , \qquad (4.27)$$

In Eq. (4.24), the Z_i appearing are evaluated using the extrema equations for the heavy sectors to zeroth and first order:

$$Z_i = Z_i^{(0)} - \frac{1}{2}m_s\bar{g}_2^{(0)}(M^{-1})_{ij}Z_j^{(0)} \equiv Z_i^{(0)} + Z_i^{(1)} \quad . \qquad (4.28)$$

When one imposes the vanishing of the cosmological constant condition, Eq. (4.22) reduces to the following:

$$U(Z_\alpha, Z_\alpha^\dagger) = \frac{1}{2}\exp(\frac{k^2}{2}|Z_o|^2)[\tilde{g}_{1,\alpha}\tilde{g}^\dagger_{1,\alpha} + m_1^2 Z_\alpha Z^\alpha$$

$$+ (m_2 g'_1 + m_3 Z_\alpha \tilde{g}_{1,\alpha} + \text{h.c.})] \quad , \qquad (4.29)$$

where

$$g'_1 = \tilde{g}_1(Z_i^{(0)} + Z_i^{(1)}, Z^\alpha) - Z_i^{(1)}\tilde{g}_{1,i}(Z_i^{(0)} + Z_i^{(1)}, Z^\alpha) \quad .$$

The results of Eq. (4.29) are equivalent to the analysis of [D11]. The analysis of [D11] is carried out for a general Kähler manifold obeying the U(n) symmetry and involves two additional mass parameters. The analysis of [D4] is limited only to the elimination of the super-Higgs fields and the heavy fields are not integrated out.

V. SU(2)XU(1) BREAKING BY SUPERGRAVITY

A remarkable aspect of supergravity models is that one may induce the breakdown of SU(2)XU(1) gauge invariance through supergravitational interactions [D1] and there exist now many models which contain realization of such a breakdown [see Sec. D of References]. We shall illustrate this aspect of supergravity unified theories in a tree model. We choose for our superpotential g_1 the following form:

$$g_1 = \lambda_1(\frac{1}{3}\mathrm{Tr}\Sigma^3 + \frac{M}{2}\mathrm{Tr}\Sigma^2)$$

$$+ \lambda_2 H^x(\Sigma_x^y + 3M'\delta_x^y)H'_y + \lambda_3 U H'_x H^x$$

$$+ \epsilon_{uvwxy} H^u M^{vw} f_1 M^{xy} + H'_x M^{xy} f_2 M'_y + B_1 \quad . \qquad (5.1)$$

Here Σ_x^y, H^x, H'_x are left-handed chiral fields in the 24, 5 and $\bar{5}$ representations. M'_x and M^{xy} are the matter (quark-lepton) $\bar{5}$ and 10 superfields and f_1 and f_2 are Yukawa coupling constant matrices in the generation space. The superpotential of Eq. (5.1) has the structure of Eq. (3.4) in the GUT sector. In Sec. III we found that in the scheme of Eq. (3.4), one had three inequivalent minima after spontaneous breaking with residual symmetries of (i) SU(5), (ii) SU(4)xU(1) and (iii) SU(3)XSU(2)XU(1) corresponding to the three solutions of Eq. (3.5a). Of course the physically interesting vacua are those corresponding to the case (iii) in Eq. (3.5a) and this is the solution we choose for our analysis. Further, to guarantee that the doublets of Higgs are light we must impose the condition M'=M. On using

Eq. (4.22) one then has the effective potential in the low energy domain in the form [D10]

$$U = \frac{1}{2}E_o[m_3{}^2|U|^2 + m_3{}^2(1+9\lambda^2)(\bar{H}_\alpha H^\alpha + H'_\alpha \bar{H}'^\alpha)$$

$$- 6\lambda m_3{}^2(H'_\alpha H^\alpha + \bar{H}_\alpha \bar{H}'^\alpha) - a\sqrt{\frac{3}{2}}m_3 z^{(0)}\lambda_3 .$$

$$(UH'_\alpha H^\alpha + U^*\bar{H}_\alpha \bar{H}'^\alpha) - 3m_3\lambda_3\lambda(U+U^*)(\bar{H}_\alpha H^\alpha + H'_\alpha \bar{H}'^\alpha) + (\lambda_3)^2|U|^2(\bar{H}_\alpha H^\alpha + H'_\alpha \bar{H}'^\alpha)$$

$$+ (\lambda_3)^2(H'_\alpha H^\alpha)\bar{H}_\beta \bar{H}'^\beta)] + U_M, \qquad \bar{H}^\alpha \equiv H^{\alpha\dagger} \text{ etc.} \qquad (5.2)$$

where $E_o = \exp(4-2\sqrt{3})$. In Eq. (5.2), U_M is the part which involves the Yukawa interactions of the squark and the slepton fields. In the following analysis we shall examine only the minima arising from the Higgs part of the potential in Eq. (5.2) so that the VEVS of the squarks and the slepton fields are assumed to be zero. In the analysis of the extrema equations that govern the VEVs of the Higgs fields appearing in Eq. (5.2), it is convenient to introduce dimensionless parameters x and y defined by

$$U = -axm_s/(\sqrt{2\lambda_3}); \; H^\alpha = \delta_5{}^\alpha y m_s/(\sqrt{2\lambda_3}) \qquad (5.3)$$

One finds for the extrema equations then the following :

$$\xi_1 y^2 + 2xy^2 + x = 0 \qquad (5.4)$$

$$y[x^2 + \xi_1 x + y^2 + \xi_2{}^2] = 0 \qquad (5.5)$$

where

$$\lambda \equiv \lambda_2/\lambda_1; \; \xi_1 = 3-\sqrt{3} - 6\lambda; \; \xi_2 = 1-3\lambda \qquad (5.6)$$

An examination of Eqs. (5.4) – (5.5) show that solutions exist on two branches as shown below:

(i) $\lambda \geq 1.05$: $SU(5) \rightarrow SU(3)XSU(2)XU(1) \rightarrow SU(3)^C XU_\gamma(1)$ (5.7)

(ii) $\lambda \leq -\frac{2}{3}(1.05)$: $SU(5) \rightarrow SU(3)XSU(2)XU(1)$

$\rightarrow SU(2)^C XSU(2)XU(1)$ (5.8)

On branch (i), one finds that for a range of values of the parameter λ, SU(2)XU(1) can break spontaneously to $U_\gamma(1)$. On this branch $SU(3)_C$ is preserved to all orders in k. On branch (ii), for a range of values of λ, SU(2)XU(1) is exactly preserved but $SU(3)_C$ is broken. The physically interesting branch is, of course, given by (i). The scale of breakdown of SU(2)XU(1) is given by m_s of Eq. (4.12b). Thus m_s must be the size of electro-weak mass scale i.e. $O(m_W)$. From Eq. (5.3) one finds on using the experimental value for the Higgs VEV, the result

$$m_s = \xi(246)\text{GeV}; \; \xi \equiv \frac{\lambda_3}{y} \sim O(1) \qquad (5.9)$$

Additional theoretical input is needed to determine ζ which would help define the gravitino mass uniquely. Further, the inclusion of squark and slepton fields into the analysis may generate new minima which may lie lower [D21]. However, from the viewpoint of constructing realistic models what is required is not that our Minkowskian vacuum be the lowest in energy but rather that it be stable against decay into the lower minima or at least its decay life be much larger than the observed life of the universe. Further in certain models of SU(2)XU(1) breaking it is possible to construct vacua where the pathologies discussed above may be circumvented. Thus in Ref. [D2] T.B. models were exhibited where the Minkowskian vacuum may also be made the lowest state of energy and in Ref. [D17] R.G. models are constructed where the inclusion of non-vanishing VEVs of the squark and selectron fields do not generate new minima which lie lower.

The breakdown of SU(2)XU(1) discussed above is truly induced by supergravitational interactions. Thus the VEVs of the Higgs fields in Eq. (5.3) are non-zero for case (i) which breaks SU(2)XU(1) due to supergravitational interactions proportional to k. Thus as $k \to 0$, m_s vanishes and the SU(2)XU(1) symmetry is restored. We note in passing that the model of SU(2)XU(1) breaking proposed in [D4] does not satisfy this criterion i.e. the breakdown of SU(2)XU(1) does not require supergravitational interactions since even as $k \to 0$ one has a breakdown of SU(2)XU(1) in the underlying global theory.

The model of Eq. (5.1) possesses the gauge heirarchy at the tree level to all orders in k. At the one-loop level additional constraints are needed to

guarantee the gauge heirarchy [D3,D7,D26]. As first noted in [D7] the loop gauge heirarchy is destroyed in the model of Eq. (5.1) due to the coupling structure $\lambda_3 UH'H$ [D27]. However, one loop stability criteria may be satisfied by the introduction of additional multiplets as recently discussed in [D20,D19].

Next we turn to the structure of supergravity GUTS. The full analysis of the particle spectrum of supergravity GUTS shall be discussed later in a model independent framework. Here the only mass spectra we shall discuss are those of the photino and the gluinos.

(a) "Direct" Gaugino Masses

The photino and the gluinos are massless at the tree level. It has been suggested [D25] that the gravitational radiative corrections may generate massess for the gluinos due to a term in the supergravity-matter Lagrangian of the type

$$\frac{k^2}{8}\bar{\lambda}\gamma_\mu\sigma^{\nu\rho}\phi^\mu\bar{\phi}_\nu\gamma_\rho\lambda \tag{5.10}$$

Such a term would generate a one loop mass to the gauginos of order $O(m_g)\Lambda^2/M_p{}^2$ where Λ is the ultra-violet cut-off i.e. $\Lambda \approx M_p$ (see Fig. (3a)). However, there exists another part of the Lagrangian which is of the form

$$\frac{k}{4}\bar{\lambda}\gamma_\mu\sigma^{\nu\rho}\psi^\mu F_{\nu\rho} \tag{5.11}$$

and its loop contribution (see Fig. (3b)) cancels the leading $O(m_g)\Lambda^2/M_p^{\,2}$ term arising from Eq. (5.10). Thus the gravitational loop corrections do not appear to generate significant loop gaugino masses. (A similar conclusion appears in Ref. [E18]). Actually a source of significant "direct" gaugino masses is due to the exchange of heavy fields of the GUT sector (see Fig. (4)). The relevant interactions thus involve the couplings of the gauginos with GUT chiral multiplets. The basic interaction is

$$L_{int} = ig_\alpha[\bar{X}^a(\frac{T^\alpha}{2})^a{}_b Z^b \lambda^\alpha - \bar{\lambda}^\alpha Z_a\ (\frac{T^\alpha}{2})^a{}_b X^b] \tag{5.12}$$

The gaugino mass matrix (for the exchange of real representations) is determined by

$$\tilde{m}_\alpha = \frac{g_\alpha^{\,2}}{16\pi^2} m_g\ \bar{C} \tag{5.13}$$

where $\bar{C}$ = CD(R)/D(A), C is the Casimir, D(R) the dimensionality of the representation exchanged and D(A) is the dimensionality of the adjoint representation. (When the exchanged representation is also adjoint e.g. Δ_y^x is a 24 of SU(5) one has $\bar{C}$ = C). Eq. (1.3) follows from Eq. (5.13) [E6,D19,D17]. The exchange of quark and lepton multiplets do not generate any significant contributions to the gaugino masses. Their combined contribuitions are typically less than a GeV [E9,E18].

Photino masses enter importantly in cosmological considertions. The mass density of the universe has an upper limit of of approximately $2X10^{-29}$ g/cm^3.

A lower bound on stable heavy-neutral lepton masses arises because the cosmic-density arising from these particles cannot exceed the current mass density of the universe [E19]. It has recently been pointed out [E17] that Majorana fermion annihilation rate is P-wave suppressed. This effect tends to increase the lower bound on the cosmologically allowed photino masses compared to the conventional lower bounds of 1-2GeV. The lower bound of $\tilde{m}_\gamma$ of 7 GeV was found in [E17]. Similar cosmological considerations can also be carried out for the Twilight Zino showing that it cannot be the lowest lying odd R-parity fermion.

(b). The ρ Parameter

In the electroweak theory the parameter ρ is defined as the ratio of the neutral current to the charge current Fermi couplings. In the SU(2)XU(1) theory with doublets of Higgs ρ=1 at the tree level. However, deviations from unity arise due to the electroweak loop corrections. A significant source of contribution to the ρ-parameter was pointed out by Veltman [E20] as arising from the mass-splittings of the third generation (t,b) quark doublet.

$$\Delta\rho = (\frac{3}{\sqrt{2}}\ \frac{G_F}{8\pi^2})m_t^{\,2} \tag{5.14}$$

which appears to set an upper bound on the top quark mass of ~O(400) GeV due to the experimental bound on ρ [E21]

$$\rho = 1.01 \pm .02\ \text{[Experiment]} \tag{5.15}$$

It is thus interesting to investigate what the status of the ρ parameter is in supergravity GUT theories. A full analysis of ρ for T.B. Supergravity

In order to get a rough idea of what these equations imply we first neglect the gauge coupling terms of Eq. (6.7). Then Eqs. (6.5)-(6.7) can be solved analytically. Thus Eqs. (6.7) and (6.8) yield

$$\alpha_t = \frac{\alpha_o}{1-\xi}\ ;\ A_t = \frac{A_o}{1-\xi}\ ;\ \xi(t) = 3\alpha_o t/\pi \qquad (6.9)$$

where α_o, A_o are the top coupling constants at the GUT mass M. (A_o is determined by the choice of super Higgs potential e.g. $A_o = 3-\sqrt{3}$ for the Polony model.) To solve Eq. (6.5), it is convenient to expand Ψ in terms of the eigenvectors of M i.e. $\Psi = c_1\Psi_1 + C_2\Psi_2 + C_3\Psi_3$ where

$$M\Psi_1 = 6\Psi_1,\ M\Psi_2 = 0 = M\Psi_3,\ \Psi_2 = (1,-2,1),\ \Psi_3 = (1,0,-1) \qquad (6.10)$$

Since at the GUT masses the tree boundary conditons hold, m_H^2, $m_{\tilde t_R}^2$ and $m_{\tilde t_L}^2$ all equal m_g^2 there and hence one finds

$$C_1(\xi) = \frac{1}{2}\frac{m_g^2}{1-\xi}[1 + \frac{1}{3}A_o^2\frac{\xi}{1-\xi}];\ C_2(\xi) = 0;\ C_3(\xi) = -\frac{1}{2}m_g^2 \qquad (6.11)$$

which yields

$$m_H^2 = 3C_1 - \frac{1}{2}m_g^2,\ m_{\tilde t_R}^2 = 2C_1;\ \tilde m_{\tilde t_L}^2 = C_1 + \frac{1}{2}m_g^2 \qquad (6.12)$$

we see that for the physical range of ξ $(-\infty < \xi \leq 0)$ if $C_1(\xi) > 0$ (i.e. A_o^2 is not too large) The squark masses $m_{\tilde t_R}^2$, $m_{\tilde t_L}^2$ can never turn negative. However, the Higgs mass clearly can (for ξ sufficiently negative) signalling the breaking of SU(2)XU(1). Using Eqs. (6.4), (6.11), (6.12) and the fact that at the minimum of the effective potential $h_t = m_t/v$, one finds the condition on the top quark mass m_t to be:

$$\frac{m_t^2}{v^2} = \frac{4\pi^2}{3}\frac{1}{(-t_o)}\left[1 - \frac{2B}{\{(A_o^2+2B-3)^2+4B(3-B)\}^{\frac{1}{2}}+3-A_o^2}\right] \qquad (6.13)$$

where v = 177 GeV, $B = 1-M_z^2/m_g^2$ and $t_o = \ln(\mu_o/M)$. (One has $\mu_o = M_W$ and $M \sim 3\times10^{16}$ GeV.)

It is interesting to trace the origin of the above spontaneous breaking. From Eqs. (6.6) and (6.12) one sees that it is the mixing of the Higgs mass with the squark masses in the β-function matrix M combined with the boundary conditions at the GUT mass that allows m_H^2 to turn negative. <u>These boundary conditions are unique to Supergravity GUT theories</u> and have no known analogue in global SUSY theories. (The boundary conditions relate the gravitino mass to SU(2)XU(1) phenomena. The additional soft breaking term proportional to A_t^2 (also unique to Supergravity GUTS) aids the SU(2)XU(1) breaking, but is not the dominant effect.) [In fact, if A_o is too large $\tilde m_{tR}^2$ turns negative destabilizing the physical vacuum as can be seen from Eqs. (6.11), (6.12)]. We also note from Eq. (6.9), A_t is reduced at the low energy regime from its GUT value A_o, though not dramatically so, and so t-squark soft breaking terms may have interesting physical consequences at low energies.

For the Polony choice $A_o = 3 - \sqrt{3}$, Eq. (6.13) requires that $80\text{GeV} \leq m_t \leq 115$ GeV. The gauge couplings of Eq. (6.7) tend to inhibit the spontaneous breaking and if these are included one finds [D17,D18] (for general A_o) that $100\ \text{GeV} \leq m_t \leq 195$ GeV. Finally, if one includes the direct gaugino mass terms $\tilde m_a$ they tend to aid the breaking of SU(2)XU(1) and one has the lower bound [D16-D18] $m_t \gtrsim 55$GeV (in the limit $\tilde m_a \to \infty$).

VII. MASS SPECTRUM-MODEL INDEPENDENT ANALYSIS

The fact that supersymmetry breaks at a relatively low mass in Supergravity GUTs [i.e. $m_{[illegible]} \sim O(M_W)$] suggests the existence of low lying supersymmetric particles accessible to experiment. This possibility represents one of the most exciting aspects of the theory. As discussed by Weinberg and the authors [E7,E4,E5], there appear in most models relatively light gauge fermions (supersymmetric partners of the SU(3)XSU(2)XU(1) gauge bosons) lying below the W and Z bosons. In the limit where the direct (loop) gaugino masses may be neglected, Weinberg [E4] has shown that if the U(n) symmetry of gravitational loops is valid there will always be at least one charged Wino, $\tilde{W}^{\pm}$ (partner of the W boson) lying <u>below</u> the W boson, and one neutral Zino, $\tilde{Z}^{o}$ (partner of the Z boson) lying below the Z boson as well as a light photino $\tilde{\gamma}$. These may therefore become detectible in such decays as

$$W^{\pm} \rightarrow \tilde{W}^{\pm} + \tilde{\gamma}$$

$$W^{\pm} \rightarrow \tilde{W}^{\pm} + \tilde{Z}$$

$$Z^{o} \rightarrow \tilde{W}^{+} + \tilde{W}^{-} \qquad (7.1)$$

In this section we analyse the mass spectrum of the gaugino and other sectors of the theory and do this in a <u>model independent</u> fashion, [E7, E9, E12] i.e. in a general way that encompasses a wide class of interesting models.

All models currently considered assume the existance of a pair of Higgs doublet superfields $\hat{H}^{a}$ and $\hat{H}'_{a}$, $a = 1,2$. In addition the tree breaking models assume the presence of a singlet superfield $\hat{U}$. In the low energy domain, after integrating out the heavy fields and eliminating the super Higgs field, these fields must interact in a renormalizable way [D11, D10] and thus can be characterized simply by an effective low energy superpotential $g_{eff} = g + g_M$ where

$$g = \mu\, \hat{H}'_{a}\hat{H}^{a} + \lambda'\hat{U}\hat{H}'_{a}\hat{H}^{a} + \frac{1}{6}\lambda''\hat{U}^{3} \qquad (7.2)$$

and g_M contains the Yukawa interactions of the matter multiplets and the Higgs doublets.

In minimizing the low energy effective potential discussed in Sec. IV, both H'_2 and H^2 may develop VEVs (as well as U in the T.B. models). We thus parameterize this breaking by a single angle α [E7-E9] defined by

$$\tan\alpha = w/v;\ v = \langle H^{2}\rangle,\ w = \langle H'_{2}\rangle \qquad (7.3)$$

Hence $M_W = \frac{1}{2}g_2(v^2 + w^2)^{1/2}$ and $M_Z = M_W/\cos\theta_W$. Our general theory thus depends on the parameters α, μ, λ' and λ'' (as well as m_g) and different models can be characterized by different domains of these parameters. Thus for the tree breaking models of Sec. V one has that α is close to 45^{o} and the other parameters are large i.e.

$$\text{(T.B.)} \qquad \alpha \approx 40^{o}\text{-}50^{o};\ \mu \sim m_g;\ \lambda',\ \lambda'' \sim 1\ , \qquad (7.4)$$

while for the renormalization group models of Sec. VI, $\alpha \sim (\mu/m_g)$ is small and λ' and λ'' do not eneter i.e.

$$\text{(R.G.)} \qquad \alpha \sim 10^{\circ}\text{–}25^{\circ};\ \mu/m_g \ll 1;\ \lambda' = 0 = \lambda'' \tag{7.5}$$

(though recently an R.G. model has been proposed [D26] with $\alpha \cong 45^{\circ}$, $\mu \sim m_g$ and $\lambda' = 0 = \lambda''$). Thus the formalism is broad enough to deal with all cases.

We consider first the fermion mass matrices. In the low energy sector, the fermion fields are (a) the SU(3)XSU(2)XU(1) Majorana gauginos $[\lambda_r(x)(r=1\ldots8),\ \lambda^i(x)(i=1,2,3),\ \text{and}\ \lambda^o(x)]$, (b) the l.h. Weyl Higgsinos $[\tilde{H}^{\alpha}(x), \tilde{H}'_{\alpha}(x),\ \alpha=1,2]$ and (c) the neutral Weyl spinor of the $\hat{U}$ multiplet $[\tilde{U}(x)]$. Fermi mass terms arise from three possible sources: (i) From the superpotential [see Eq. 4.5)]:

$$L = -\bar{\chi}_a g_{\text{eff}},{}_{ab}\, \chi^b \quad , \tag{7.6}$$

where χ^a are the Weyl spinor components of the chiral multiplets, (ii) From the gaugino gauge interaction:

$$L_{\lambda\chi} = -\bar{\lambda}^{\alpha} m_{\alpha a} \chi^a + \text{h.c.} \quad , \tag{7.7}$$

where

$$m_{\alpha a} = i g_{\alpha} Z_b \left(\frac{T^{\alpha}}{2}\right)^b{}_a \tag{7.8}$$

T^{α} are the group generators and Z^a are the scalar chiral partners of the χ^a. and (iii) the direct gaugino masses of Eq. (5.13):

$$L_{\lambda} = -\frac{1}{2}\bar{\lambda}^{\alpha} \tilde{m}_{\alpha} \lambda^{\alpha} \quad . \tag{7.9}$$

(a) <u>Charged Gaugino-Higgsino Fermion States</u>

The charged fermion fields, $\lambda \equiv (\lambda' - i\,\lambda^2)/\sqrt{2}$ and the charged Higgsinos $\tilde{H}^1$, $\tilde{H}'_1$ can conveniently be re-expressed in terms of two Dirac fields

$$\varphi_1 = \lambda_R + i\tilde{H}^1;\ \varphi_2 = \lambda_L + i\tilde{H}'_1{}^c \quad , \tag{7.10}$$

where $\lambda_{R,L}$ are the r.h., l.h. components of λ. In the two component space labeled by $\varphi_o = (\varphi_1, \varphi_2)$, the charged mass matrix is

$$M = \mu_+ + \mu_-\tau_3 + \frac{1}{2}(\mu + \tilde{m}_2)\tau_1 + i\tau_2\gamma_5\,\frac{1}{2}(\mu - \tilde{m}_2) \quad , \tag{7.11}$$

where τ_a are Pauli matrices in Φ space,

$$\sqrt{2}\mu_{\pm} = M_W(\cos\alpha \pm \sin\alpha) \tag{7.12}$$

and $\tilde{m}_2 = 2\tilde{m}_\gamma/(8\sin^2\theta_w)$ where $\tilde{m}_\gamma$ is given by Eqs. (1.3) and (5.15).

One may easily diagonalize Eq. (7.11) by an "isotopic" and γ_5 transformation yielding the following mass eigenvalues and physical fields [E9,E7,E8]

$$\tilde{m}_\pm = \frac{1}{2}|[4\mu_+^2 + (\mu - \tilde{m}_2)^2]^{1/2} \pm [4\mu_-^2 + (\mu+\tilde{m}_2)^2]^{1/2}| \qquad (7.13)$$

and

$$\tilde{W}_+ = i\cos\gamma_-\tilde{H}^1 + i\sin\gamma_+ \tilde{H}'^c_1 - \sin\gamma_-\lambda_L + \cos\gamma_+\lambda_R$$

$$\tilde{W}_- = -i\sin\gamma_-\tilde{H}^1 - i\cos\gamma_+ \tilde{H}'^c_1 - \cos\gamma_-\lambda_L + \sin\gamma_+\lambda_R \qquad (7.14)$$

where

$$\tan 2\beta_\pm = (\mu\mp\tilde{m}_2)/(2\mu_\pm); \ \gamma_\pm = \beta_+ \pm \beta_- \qquad (7.15)$$

The equation for $\tilde{W}_-$ holds for $\sin 2\alpha \geq \mu\tilde{m}_2/M_w^2$. For $\sin 2\alpha < \mu\tilde{m}_2/M_w^2$, $\tilde{W}_-$ is replaced by $\gamma_5\tilde{W}_-$.

Note that Eq. (7.13) implies

$$\tilde{m}_+\tilde{m}_- = |\sin 2\alpha M_W^2 - \mu\tilde{m}_2| \qquad (7.16)$$

and thus except when $\mu\tilde{m}_2$ is large, there is always one Wino, $\tilde{W}_-$ with mass $\tilde{m}_- < M_W$. Such a particle may be considerably below the W.

(b) Neutral Gaugino-Higgsino Fermion States

In dealing with the neutral gaugino and Higgsinos, λ^3, λ^0, $\tilde{H}^2$, $\tilde{H}'_2$, $\tilde{U}$ it is convenient to introduce the following Majorana combinations:

$$\lambda^\gamma = \sin\theta_w\,\lambda^3 + \cos\theta_w\lambda^0$$

$$\lambda^z = \sin\theta_w\,\lambda^0 - \cos\theta_w\lambda^3$$

$$\xi = i[\cos\alpha(\tilde{H}^2 - \tilde{H}^{2c}) - \sin\alpha(\tilde{H}'_2 - \tilde{H}'^c_2)]$$

$$\eta = -i[\sin\alpha(\tilde{H}^2 - \tilde{H}'^c_2) + \cos\alpha(\tilde{H}'_2 - \tilde{H}'^c_2)]$$

$$u = i(\tilde{U} - \tilde{U}^c) \qquad (7.17)$$

For this representation the direct gaugino masses are

$$\tilde{m}_z = \cos^2\theta_w\tilde{m}_2 + \sin^2\theta_w\tilde{m}_1 \simeq 1.5\tilde{m}_\gamma$$

$$\tilde{m}_{\gamma z} = \cos\theta_w\sin\theta_w(\tilde{m}_1-\tilde{m}_2) \simeq -0.40\tilde{m}_\gamma \quad . \qquad (7.18)$$

The neutral mass matrix [E7-E9] is in general 5x5 for T.B. models and 4x4 for the R.G. models [which contains no singlet field $\hat{U}(x)$]. In the basis $\varphi = (\lambda^\gamma,\lambda^z,\xi,\eta,u)$ one has

$$M = \begin{pmatrix} \tilde{m}_\gamma & \tilde{m}_{\gamma z} & 0 & 0 & 0 \\ \tilde{m}_{\gamma z} & \tilde{m}_z & M_z & 0 & 0 \\ 0 & M_z & \mu\sin 2\alpha & \mu\cos 2\alpha & 0 \\ 0 & 0 & \mu\cos 2\alpha & -\mu\sin 2\alpha & \mu' \\ 0 & 0 & 0 & \mu' & \mu'' \end{pmatrix} \qquad (7.19)$$

where [from Eq. (7.2)] $\mu' = \lambda' (v^2 + w^2)^{1/2}$ and $\mu'' = \lambda''\langle U\rangle$.

One may of course diagonalize Eq. (7.19) numerically. However, from Eq. (7.18) one sees that the gaugino mixing term $\tilde{m}_{\gamma z}$ is small, and neglecting this effect allows one to separate out the photino eigenfield λ^γ with eigenvalue $\tilde{m}_\gamma$. Further, in currently interesting models,

$$\mu^2 \cos^2 2\alpha \ll M_z^2 \qquad (7.20)$$

(e.g. for T.B. models $\mu \sim M_z$ but $\alpha \approx 45^\circ$ while in R.G. models $\alpha \sim 15^\circ$ but $\mu \sim M_z/5$). In these approximations, Eq. (7.19) has the eigenvalues $\tilde{m}_\lambda$ and [E9]

$$\tilde{\mu}_\pm \cong [M_z^2 + \tfrac{1}{4}(\mu\sin 2\alpha - \tilde{m}_z)^2]^{1/2} \pm \tfrac{1}{2}(\mu\sin 2\alpha + \tilde{m}_z) \qquad (7.21a)$$

$$\tilde{\mu}_{3,4} \cong [\tfrac{1}{4}(\mu\sin 2\alpha + \mu'')^2 + \mu'^2]^{1/2} \pm \tfrac{1}{2}(\mu\sin 2\alpha - \mu'') \qquad (7.21b)$$

(Detailed numerical analysis shows that the above approximations are quite good except in the R.G. model when there is an accidental degeneracy of light masses i.e. when $\mu\sin 2\alpha \cong -\tilde{m}_\gamma$.) The corresponding eigenfields (neglecting $\tilde{m}_{\gamma z}$ but not $\mu\cos 2\alpha$) are λ^γ and $\tilde{Z}_{(k)}$ ($k = +, -, 3, 4,$) where $\varphi_i = (\lambda^z, \xi, \eta, u)$ and

$$\tilde{Z}_{(k)} = (i\gamma_5)^{a_k} \sum \varphi_i N_{ik} D_k^{-1}; \quad a_+ = 0 = a_4,\ a_- = 1 = a_3 \qquad (7.22)$$

Here

$$N_{1k} = M_z \tilde{\lambda}_k, \quad N_{2k} = (\lambda_k - \tilde{m}_z)\tilde{\lambda}_k$$

$$N_{3k} = \mu\cos 2\alpha(\lambda_k - \tilde{m}_z); \quad \tilde{\lambda}_k \equiv \lambda_k + \mu\sin 2\alpha - \mu'^2/(\lambda_k - \mu'')$$

$$N_{4k} = \mu\cos 2\alpha(\lambda_k - \tilde{m}_z)\mu'/(\lambda_k - \mu'')$$

$$D_k = [M_z^2 \tilde{\lambda}_k^2 + (\lambda_k - \tilde{m}_z)^2\{\tilde{\lambda}_k^2 + \mu^2\cos^2 2\alpha(1 + \frac{\mu'^2}{(\lambda_k - \mu'')^2})\}]$$

and

$$\lambda_\pm = \pm\tilde{\mu}_\pm, \quad \lambda_3 = -\tilde{\mu}_3, \quad \lambda_4 = \tilde{\mu}_4 \qquad (7.23)$$

The eigenstates have different properties in the different models. In the T.B. model one sees from Eq. (7.21a) that again one generally has one Zino state, $\tilde{Z}_{(-)}$, lying below the Z boson i.e. $\tilde{\mu}_- < M_z$ and indeed can lie considerably below the Z. Note that this value of $\tilde{\mu}_-$ is indepdendent of the details of model i.e. of μ' and μ''. In general $\tilde{\mu}_{3,4}$ are relatively heavy. In contrast, in the R.G. models (where $\mu' = 0 = \mu''$) μ and α are small e.g. $\mu\sin 2\alpha \approx 10\text{GeV}$ so both $\tilde{\mu}_\pm$ are relatively large and cluster very near (one above, one below) the Z. However, $\tilde{M}_3 \approx |\mu \sin 2\alpha|$ is quite light and well below the Z. We will refer to $\tilde{Z}_{(3)}$ as the "Twilight Zino", as it couples strongly to the Z boson but very weakly to all other matter

(c) Squark and Slepton States

Associated with each Weyl fermion is a complex scalar field e.g. for the first family of quarks and leptons, μ_L, μ_R, d_L, d_R, e_L, e_R, ν_L are the scalar fields $\tilde{u}_L$, $\tilde{u}_R$, $\tilde{d}_L$, $\tilde{d}_R$, $\tilde{e}_L$, $\tilde{e}_R$, $\tilde{\nu}_L$. Neglecting the small Yukawa interactions, these fields are in fact eigenstates of the squark and slepton mass matrices, and one finds for the mass eigenvalues [E9,E12]

$$m_{\tilde{\nu}}^2 = m_g^2 - \tfrac{1}{2}\cos 2\alpha M_z^2$$

$$m_{\tilde{e}_L}^2 = m_g^2 + (\tfrac{1}{2} - \sin^2\theta_w)\cos 2\alpha M_z^2$$

$$m_{\tilde{e}_R}^2 = m_g^2 + \sin^2\theta_w \cos 2\alpha M_z^2$$

$$m_{\tilde{u}_L}^2 = m_g^2 - (\tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta\ )\cos 2\alpha M_z^2$$

$$m_{\tilde{d}_L}^2 = m_g^2 + (\tfrac{1}{2} - \tfrac{1}{3}\sin^2\theta_w)\cos 2\alpha M_z^2$$

$$m_{\tilde{u}_R}^2 = m_g^2 - \tfrac{2}{3}\sin^2\theta_w \cos 2\alpha M_z^2$$

$$m_{\tilde{d}_R}^2 = m_g^2 + \tfrac{1}{3}\sin^2\theta_w \cos 2\alpha M_z^2 \qquad (7.24)$$

which reduce to the results of [D17] for small α. In the R.G. models $\alpha \sim 10^\circ$–25°, and so the factor $\cos 2\alpha$ makes some correction.

In the T.B. models with $\alpha=45^\circ$, the D term contributions proportional to M_z^2 vanish. If one includes the Yukawa interactions, they produce a 45° rotation in the squark and selectron states i.e. the eigenstates become $\tilde{u}_\pm = (\tilde{u}_L \pm \tilde{u}_R)/\sqrt{2}$ etc. The squark and selectron masses then become [E1,E5,D11,D19]

$$m(\tilde{u}_\pm)^2 = m^2_{\tilde{\nu}} + m_q^2 \pm \beta m_{\tilde{\nu}} m_q$$

$$m(\tilde{e}_\pm)^2 = m(\tilde{d}_\pm)^2;\ m^2_{\tilde{\nu}} = m_g^2 \qquad (7.25)$$

where m_q is the quark mass and β is a model dependent parameter of O(1). Thus there is a small splitting of squark masses between generations produced by the Yukawa interactions proportional to the quark masses giving rise to a natural suppression of flavor changing neutral currents. All the squarks and selectrons are nearly degenerate with mass $\sim m_g = O(M_W)$.

(d) Higgs Bosons

All models contain one pair of Higgs doublets and hence 4 complex or 8 real scalar fields. (The T.B. models contain two additional real scalar fields from the $\hat{U}$ multiplet.) Three of these states are massless Goldstone bosons absorbed by the $W^\pm$ and Z^0 bosons, leaving 5 (or 5+2) massive real modes. The general analysis is somewhat complex, and we summarize here some of the more important features.

(i) T.B. Models

The 7 modes rearrange into one charged state of mass

$$m_{H\pm}^2 = M_W^2 + 2(1+\beta_1^2)m_g^2 \quad , \tag{7.26}$$

one neutral state of mass

$$m_{Ho}^2 = M_z^2 + 2(1+\beta_1^2)m_g^2 \tag{7.27}$$

and four additional neutral modes mixed by the couplings λ' and λ" of Eq. (7.2) [E5,D11]. In Eqs. (7.26), (7.27) β_1 is a model dependent parameter of O(1), and so these Higgs bosons lie above the W and Z bosons [and probably considerably so for $m_g \sim O(M_W)$].

(ii) R.G. Models

Here the couplings λ' and λ" of Eq. (7.2) are zero, and the 5 massive modes arrange themselves into three neutral Higgs mesons H^o, $H^o_{(1,2)}$ and one charged meson $H^{\pm}$ with masses given by [E12]

$$m_{H^o}^{\;2} = \frac{m_g^2}{\cos^2\alpha}\left[1+\frac{\mu^2}{m_1^2} - \frac{1}{2}\cos 2\alpha \frac{M_z^{\;2}}{m_g^{\;2}}\right] \tag{7.28}$$

$$m_{H^o_{(1,2)}}^{\;2} = \frac{1}{2}\left[(M_z^{\;2} + m_{H^o}^{\;2}) \pm \{(M_z^{\;2} + m_{H^o}^{\;2})^2 - 4(\cos 2\alpha)^2 m_{H^o}^{\;2} M_z^2\}^{1/2}\right] \tag{7.29}$$

$$m_{H+}^{\;2} = M_W^{\;2} + m_{H^o}^{\;2} \tag{7.30}$$

(These expressions reduce to the results in [D17] in the limit α → 0.) We note that Eq. (7.28) requires

$$m_g^{\;2} > \frac{1}{2}\cos 2\alpha\, M_z^{\;2} \approx (60\ \text{GeV})^2 \tag{7.31}$$

to prevent the H^o mode from becoming tachyonic. However, if m_g is not large, it is possible for H^o and also $H^o_{(2)}$ to be quite lowlying. Such neutral Higgs bosons if produced would decay into hadron and lepton pairs and hence might be detectable at current accellerators. The remaining two Higgs bosons, H^+ and $H^o_{(1)}$ lie above the W and Z bosons respectively.

VIII. SUPERSYMMETRIC DECAY OF $W^{\pm}$ AND Z^{o} BOSONS

The $W^{\pm}$ and Z^{o} bosons interact with the gauginos and Higgsinos by standard SU(2)XU(1) gauge interactions. Having found the fields representing the physical particles of the theory in Sec. VII, one may eliminate the elementary fields in terms of them and calculate the vertices for the physical decays of the $W^{\pm}$ and Z^{o} particles. There are four interesting supersymmetric decays of these vector bosons.

(i) $W^{\pm} \rightarrow \tilde{W}_-^{\pm} + \tilde{\gamma}$ [E4,E8,E9].

This decay is energetically possible provided

$$\tilde{m}_- + \tilde{m}_\gamma < M_W \tag{8.1}$$

and as we have seen from Eq. (7.16) the lower Wino mass $\tilde{m}_-$ obeys $\tilde{m}_- < M_W$ almost always, and so this decay can occur in almost all models. The interaction governing the decay is

$$L_{W\tilde{W}\tilde{\gamma}} = - e\, \tilde{\lambda}^{\gamma}\gamma^{\mu}[\sin\gamma_+ P_+ - \cos\gamma_- P_-]\tilde{W}_- W_{\mu}{}^{\dagger} + \text{h.c.} \tag{8.2}$$

where $P_{\pm} = (1/2)(1 \pm \gamma_5)$ and $\gamma_{\pm}$ is given in Eq. (7.15)

(ii) $Z^{o} \rightarrow \tilde{W}^{+}_- + \tilde{W}^{-}_-$ [E5,E9,E12]

This mode requires

$$2\tilde{m}_- < M_Z \tag{8.3}$$

and since a light Wino is expected in all models, it is energetically feasible in all models. The vertex interaction governing the decay is

$$L_{Z\tilde{W}\tilde{W}} = - e\, \tilde{W}_-\gamma^{\mu}[A_+P_+ + A_-P_-]\tilde{W}_- Z_{\mu} \tag{8.4}$$

where

$$A_+ = \cot\theta_W \sin^2\gamma_+ + \cot 2\theta_W \cos^2\gamma_+$$

$$A_- = \cot\theta_W \cos^2\gamma_- + \cot 2\theta_W \sin^2\gamma_- \tag{8.5}$$

(iii) $W^{\pm} \rightarrow \tilde{W}^{\pm}_- + \tilde{Z}^{o}_-$ [E9,E12]

This mode requires

$$\tilde{m}_- + \tilde{\mu}_- < M_W \tag{8.6}$$

where the Wino and Zino masses $\tilde{m}_-$, $\tilde{\mu}_-$ are given in Eqs. (7.13) and (7.21a). The mode is feasible only in the T.B. model, for as discussed in Sec. VII only there can the $\tilde{Z}_-$ be light. For $\alpha = 45^{o}$, the decay vertex is given by

$$L_{\widetilde{W}\widetilde{Z}} = \frac{ie}{\sin\theta_W} A\, \bar{\widetilde{Z}}_- \gamma^\mu \widetilde{W}_- W_\mu^{\ +} + \text{h.c.} \qquad (8.7)$$

where

$$A = \cos\theta_W \sin(\beta_+ + \frac{\pi}{4}) O_{12} + \frac{1}{2} \sin\,(\beta_+ - \frac{\pi}{4}) O_{22} \qquad (8.8)$$

and $O_{ik} = N_{ik}/D_k$ is given in Eq. (7.23).

(iv) $Z^o \to \widetilde{Z}_{(3)} + \widetilde{Z}_{(3)}$ [E9,E12]

Here one requires a light Twilight Zino with mass

$$2\tilde{\mu}_3 < M_z \qquad (8.9)$$

and hence this mode occurs in almost all R.G. models (but is energetically forbidden in T.B. models). The decay vertex here is

$$L_{Z\widetilde{Z}_3\widetilde{Z}_3} = -\frac{e}{2\sin 2\theta_W}[\cos 2\alpha\{(O_{23})^2 - (O_{33})^2\}$$

$$-2\sin 2\alpha O_{23} O_{33}]\bar{\widetilde{Z}}_{(3)}\gamma^\mu\gamma_5\widetilde{Z}_3 Z_\mu \qquad (8.10)$$

The above decay interactions depend only on the mixing angle α of Eq. (7.3), the parameter μ of Eq. (7.2) and the photino mass $\tilde{m}_\gamma$. For the R.G. model, α and μ determine the Wino and Twilight masses ($\tilde{m}_-$ and $\tilde{\mu}_3$) while in the T.B. model $\alpha \cong 45^o$ and μ determines $\tilde{m}_-$. Thus for fixed choices of $\tilde{m}_-$, $\tilde{\mu}_3$ and $\tilde{m}_\gamma$ one obtains unique predictions for the decay rates. Characteristic results

are given in Table 1. The remarkable feature is the largeness of the supersymmetric branching ratios, particularly the $Z^o \to \widetilde{W}^+ + \widetilde{W}^-$ which are of "industrial strength" size in all models! In order to see what experimental signals these decays give, it is necessary first, however, to examine the Wino and Zino decay modes.

The $\widetilde{W}_-$ decays proceed through intermediate squark, selectron and W states. The diagrams governing $\widetilde{W}_-$ decays are shown in Fig. 6 with the following decay processes possible [E9,E12]:

$$\widetilde{W}_-^+ \to u_i + \bar{d}_i + \tilde{g}$$

$$\widetilde{W}_-^+ \to u_i + \bar{d}_i + \tilde{\gamma}$$

$$\widetilde{W}_-^+ \to \ell^+ + \nu_1 + \tilde{\gamma} \qquad (8.11)$$

Here $\tilde{g}$ = gluino, ℓ^+ = lepton and u_i and d_i stand for up and down type quark (i is a generation index.) The interactions governing the vertices can be calculated using Eqs. (7.7) and (7.8). Thus the Wino-quark-squark vertex

$$L_{\widetilde{W}q\tilde{q}} = \frac{-ie}{\sqrt{2}\sin\theta_W}[-\sin\gamma_+ \bar{u} P_+ \widetilde{W}_- \tilde{d}_L + \cos\gamma_- \bar{d} P_+ \widetilde{W}_-^c \tilde{u}_L] + \text{h.c.} \qquad (8.12)$$

and the gluino-quark-squark vertex is

$$L_{\tilde{g}q\tilde{q}} = ig_3(\frac{t^r}{2})_{ij}[\bar{u}_i P_+ \lambda_r \tilde{u}_{jL} + \bar{d}_i P_+ \lambda_r \tilde{d}_{jL}]$$

$$+ ig_3(\frac{t^r}{2})_{ij}[\bar{u}_i P_- \lambda_r \tilde{u}_{jR} + \bar{d}_i P_- \lambda_r \tilde{d}_{jL}] + \text{h.c.} \qquad (8.13)$$

where $\lambda_r(x)$ is the Majorana gluino field and t^r the SU(3) matrices. In general there is a significant amount of interference between the W and squark (and W and selectron) poles which must be taken into account, and the gluino modes are strongly reduced due to the gluino mass in the three-body phase space.

A similar set of squark and selectron poles lead to the following Zino decay modes (See Fig. 6):

$$\tilde{Z}_- \to u_i(d_i) + \bar{u}_i(\bar{d}_i) + \tilde{g}$$

$$\tilde{Z}_- \to u_i(d_i) + \bar{u}_i(\bar{d}_i) + \tilde{\gamma}$$

$$\tilde{Z}_- \to \ell^+ + \ell^- + \tilde{\gamma} \qquad (8.14a)$$

In addition, the $W^\pm$ pole diagrams can lead to $\tilde{W}^\pm$ final states, since the $\tilde{W}_-$is lighter than the $\tilde{Z}_-$:

$$\tilde{Z}_- \to u_i(\ell^+) + \bar{d}_i(u_1) + \tilde{W}_-^-$$

$$\tilde{Z} \to \bar{u}_i(\ell^-) + d_i(\bar{u}_1) + \tilde{W}_-^+ \qquad (8.14b)$$

Finally we note that the Twilight zino decay is via the squark and selectron intermediate states:

$$\tilde{Z}_{(3)} \to u_i(d_i) + \bar{u}_i(\bar{d}_i) + \tilde{\gamma}$$

$$\tilde{Z}_{(3)} \to l^+ + l^- + \tilde{\gamma} \qquad (8.15)$$

The decay branching ratios for the $\tilde{W}_-$, $\tilde{Z}_-$ and $\tilde{Z}_3$ are given in Tables 2 and 3 for two values of photino mass $\tilde{m}_\gamma = 2$ and $\tilde{m}_\gamma = 7$ GeV. From Eq. (1.3) one sees that the gluino final states are energetically forbidden for the heavier photino choice, reducing some of the hadronic branching ratios.

Combining Table 1 with Tables 2 and 3 leads to the branching ratios given in Tables 4 and 5 for various final states in the supersymmetric decays of the $W^\pm$ and Z^o bosons. Again we note the largeness of some of the decay modes, particularly in the Z^o decays. There are a number of significant exerimental signals, some of which may be detectable now at the CERN $\bar{p}p$ Collider or at the e^+e^- SLC and LEP machines. These fall into the following general catagories

(a) UFO Events

These are $W^\pm$ and Z^o decays into 1 or 2 jets with unbalanced high $p_\perp$ where "unidentified fermionic objects" (UFOs), i.e. the photinos, take away missing $p_\perp$ with no additional leptons present. Events of this type with one relatively broad jet Fig. 4a (arising from two quarks in a relatively slow $\tilde{W}$ and $\tilde{Z}$ hadronic decay) come from $W \to (\tilde{W} \to \text{jet} + \tilde{\gamma}) + \tilde{\gamma}$, [E4,E8,E9] while events with two broad jets Fig. 4b in opposite hemispheres come from $W \to \tilde{W}+\tilde{Z}$ and $Z \to \tilde{W}+\tilde{W}$ decays where the Winos and Zinos decay hadronically [E9]. One expects that most of these events will have a large missing $p_\perp$. We see from Tables 4 and 5 that UFO events occur with sizable probability even at the CERN $\bar{p}p$ Collider in all models and will be particularly prominant at LEP and SLC.

The two jet UFO events given the stronger of the two signals (particularly for the T.B. models). At the $\bar{p}p$ Collider 2 jet events possess one possible background, i.e. gluino pair production $p+\bar{p} \rightarrow \tilde{g}+\tilde{g}$, though this process becomes negligible at CERN for gluino masses greater than about 70-80 GeV, See e.g. [F5] pg. 252. (In any event the discovery of a gluino or Wino would be equally exciting!)

(b) Lepton-Jet Events With Missing $p_\perp$

These events arise from $W \rightarrow \tilde{W} + \tilde{Z}$ or $Z \rightarrow \tilde{W} + \tilde{W}$ where one "ino" decays hadronically and one leptonically. [E9]. One expects, therefore a lepton in one hemisphere, and a broad jet in the other with missing $p_\perp$ (Fig. 8). We estimate that 30-40% of these events will satisfy the experimental $p_{\perp elec} > 15$ GeV cut, and in the T.B. model roughly at a rate ≈15% of the $W \rightarrow e\nu$ events at CERN. A possible background for this process would be heavy flavor production e.g. top quark pairs [F6]. However, Fig. 8 differs from the t-quark signature in that there will be no b quark decay debris, and there should be additional missing $p_\perp$ from the $\tilde{\gamma}$ arising in the hadronic decay. We note also that the appearance of events such as Fig. 8 at the $\bar{p}p$ Collider would argue in favor of the T.B. models, though lepton-jet events should occur in all models at SLC and LEP from Z^0 decays.

(c) Exotic Leptonic Decays With Missing $p_\perp$

As can be seen from Tables 4 and 5, supersymmetry predicts a number of purely leptonic exotic W and Z decays with small but non-negligible branching ratios [E9]. Thus all models predict a sizable decay rate for $Z \rightarrow$

$\ell^+_1 + \ell^-_2$ + neutrals (where ℓ_1 and ℓ_2 may be in different families). The R.G. models predict $Z \rightarrow \ell^+_1 + \ell^-_1 + \ell^+_2 + \ell^-_2$ + neutrals (from the Twilight decays) while the T.B. models predict $W^+ \rightarrow \ell^+_1 + \ell^+_2 + \ell^-_2$ + neutrals. If exotic decays such as these exist, they could presumably be observed at LEP or SLC.

(d) The low energy effective Lagrangian contains interactions of W and Z with sleptons. Whether or not the corresponding decays of the W and Z occur depends on the slepton mass spectra [E25,E26].

	Branching Ratio (fraction)			
	Tree Breaking		Renormalization Group	
Decay	$\tilde{m}_\gamma$ = 2GeV	$\tilde{m}_\gamma$ = 7GeV	$\tilde{m}_\gamma$ = 2GeV	$\tilde{m}_\gamma$ = 7GeV
$W \rightarrow \tilde{W} + \tilde{\gamma}$	.57	.48	.35	.33
$W \rightarrow \tilde{W} + \tilde{Z}$	1.88	1.63	0	0
$Z^0 \rightarrow \tilde{W}^+ + \tilde{W}^-$	7.14	6.47	4.53	4.45
$Z^0 \rightarrow \tilde{Z}_{(3)} + \tilde{Z}_{(3)}$	0	0	1.31	1.32

Table 1. Branching ratios for the supersymmetric decays of the W and Z bosons (relative to $W \rightarrow e\nu$ and $Z \rightarrow e^+e^-$ respectively) for the tree breaking model and renormalization group model. Values quoted are for Wino mass of 30 GeV, $m_{\tilde{g}} = \sqrt{2} M_W$ and $\tilde{Z}_{(3)}$ mass - 9.5 GeV for two values of the photino mass $\tilde{m}_\gamma$. Results of this table and of the following tables are taken from Ref. [E9].

	Branching Ratio (%)	
	$\tilde{m}_\gamma$ = 2 GeV	$\tilde{m}_\gamma$ = 7 GeV
$\tilde{W} \to l + \nu_l + \tilde{\gamma}$	21.8	26.2
$\tilde{W} \to h + \tilde{\gamma}$	78.2	73.8
$\tilde{Z} \to l + \nu_l + \tilde{W}$	0.4	11.5
$\tilde{Z} \to h + \tilde{W}$	1.2	33.0
$\tilde{Z} \to l + \bar{l} + \tilde{\gamma}$	1.9	16.0
$\tilde{Z} \to h + \tilde{\gamma}$	96.0	36.1

Table 2. Branching ratios for Wino and Zino decays for the tree breaking model for two photino masses. l means e or μ leptons and h means hadrons. The analysis is for $m_g = \sqrt{2}M_W$.

	Branching Ratio (%)	
	$\tilde{m}_\gamma$ = 2 GeV	$\tilde{m}_\gamma$ = 7 GeV
$\tilde{W} \to l + \nu_l + \tilde{\gamma}$	17.6	20.9
$\tilde{W} \to h + \tilde{\gamma}$	82.4	79.1
$\tilde{Z}_{(3)} \to l^+ + l^- + \tilde{\gamma}$	21.6	34.7
$\tilde{Z}_{(3)} \to h + \tilde{\gamma}$	73.8	65.3
$\tilde{Z}_{(3)} \to h + l + \nu_l + \tilde{\gamma}$	4.6	0

Table 3. Branching ratios for Wino and "Twilight Zino" ($Z_{(3)}$) for renormalization group model for two photino masses. l means e or μ leptons and h means hadrons. Wino decays are for m_g = 150 GeV and the $\tilde{Z}_{(3)}$ decays are for $m_g = \sqrt{2}M_W$.

Decay	Branching Ratio (%)	
	$\tilde{m}_\gamma$ = 2 GeV	$\tilde{m}_\gamma$ = 7 GeV
$W \to \tilde{\gamma} + (\tilde{W} \to h + \tilde{\gamma})$	44.6	35.1
$W \to \tilde{\gamma} + (\tilde{W} \to l + \nu_l + \tilde{\gamma})$	12.5	12.5
$W \to \tilde{W} + \tilde{Z} \to (l + \nu_l + \tilde{\gamma}) + (h + \tilde{\gamma})$	41.2	50.6
$W \to (\tilde{W} \to l_1 + \nu_1 + \tilde{\gamma}) + (\tilde{Z} \to l_2 + \bar{l}_2 + \tilde{\gamma})$	0.8	8.2
$W \to (\tilde{W} \to h + \tilde{\gamma}) + (\tilde{Z} \to l + \bar{l} + \tilde{\gamma})$	2.9	19.3
$W \to (\tilde{W} \to h + \tilde{\gamma}) + (\tilde{Z} \to h + \tilde{\gamma})$	142.1	72.8
$Z \to (\tilde{W} \to l + \nu_l + \tilde{\gamma}) + (\tilde{W} \to h + \tilde{\gamma})$	243.7	250.6
$Z \to (\tilde{W} \to l_1 + \nu_1 + \tilde{\gamma}) + (\tilde{W} \to l_2 + \nu_2 + \tilde{\gamma})$	34.0	44.6
$Z \to (\tilde{W} \to h + \tilde{\gamma}) + (\tilde{W} \to h + \tilde{\gamma})$	436.4	352.2

Table 4. Branching ratios for W decays relative to $W \to e + \nu$ and Z decays relative to $Z \to e^+ + e^-$ for the tree breaking model. l, l_1, l_2 stand for e or μ leptons and h for hadrons. The analysis is for $m_g = \sqrt{2}M_W$.

Decay	Branching Ratio (%) $\tilde{m}_\gamma$ = 2 GeV	$\tilde{m}_\gamma$ = 7 GeV
$W \to \gamma\ (\tilde{W} \to h + \tilde{\gamma})$	29.1	26.2
$W \to \gamma + (\tilde{W} \to l + \nu_1 + \tilde{\gamma})$	6.2	6.9
$Z \to (\tilde{W} \to l + \nu_1 + \tilde{\gamma}) + (\tilde{W} \to h + \tilde{\gamma})$	131.0	147.0
$Z \to (\tilde{W} \to l_1 + \nu_1 + \tilde{\gamma}) + (l_2 + \nu_2 + \tilde{\gamma})$	13.9	19.4
$Z \to (\tilde{W} \to h + \tilde{\gamma}) + (\tilde{W} \to h + \tilde{\gamma})$	307.6	278.4
$Z \to (\tilde{Z}_{(3)} \to l_1 + \bar{l}_1 + \tilde{\gamma}) + (\tilde{Z}_{(3)} \to l_2 + \bar{l}_2 + \tilde{\gamma})$	6.1	15.8
$Z \to (\tilde{Z}_{(3)} \to l + \bar{l} + \tilde{\gamma}) + (\tilde{Z}_{(3)} \to h + \tilde{\gamma})$	41.9	59.6
$Z \to (\tilde{Z}_{(3)} \to h + \tilde{\gamma}) + (\tilde{Z}_{(3)} \to h + \tilde{\gamma})$	71.5	56.2
$Z \to (\tilde{Z}_{(3)} \to h + \tilde{\gamma}) + (\tilde{Z}_{(3)} \to h + l + \nu_1 + \tilde{\gamma})$	8.8	0

Table 5. Branching ratios for W decays relative to $W \to e + \nu$ and Z decays relative to $Z \to e^+ + e^-$ for renormalization group model. l, l_1 l_2 stand for e or μ leptons and h for hadrons. Branching ratios through the Wino poles are for m_g = 150 GeV and through the $\tilde{Z}_{(3)}$ poles are for $m_g = \sqrt{2}M_W$.

IX. CONCLUSION

N=1 Supergravity unified models generate a dynamical unification of electro-weak and supergravitational interactions. There are a large number of predictions of such a unification at low energy. The theory predicts an array of new particles, photino, gluino, winos, Zinos, Higgsinos, sleptons, and squarks, with characteristic mass scales governed by the gravitino mass $m_g \sim O(m_W)$. Of these the lighest particles are expected to be the photino, the wino below the W boson and the Zino below the Z boson and hence they represent the best chance of being discovered at current energies at the $\bar{p}p$ collider. The best chance for discovering the selectron and the sneutrino would be at LEP or SLC.

A number of other theoretical consequences of N=1 Supergravity unified models have also been investigated recently. These models suggest possible additional sources of CP violation and could generate contributions to the electric dipole moment of the neutron and the electron which are close to the current experimental upper bounds [E27]. Further, the recent experimental lower limits on the proton decay [F7] require an assessment of the conventional grand unification program [F7], while further theoretical analysis of Supergravity GUT predictions for the strange decay modes is needed. Finally N=1 supergravity models appear to fare better than the ordinary GUT or globally supersymmetric models in allowing for acceptable inflationary scenarios of the early universe [E28].

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NOTATION

Secs. VII and VIII use the Lorentz metric diag $\eta_{\mu\nu} = (-1,+1,+1,+1)$ and standard left handed Weyl spinors (projected by $P_- = (1-\gamma_5)/2, \gamma_5^\dagger = \gamma_5$). The discussion of the supergravity - matter couplings, Sec. II and App. A, are in notation of [C4] and [C7].

APPENDIX A

In this appendix we shall explain in detail the steps needed to construct the Lagrangian of locally supersymmetric grand unified theories [C8].

We consider the coupling of supergravity with the minimal set of auxiliary fields to the gauge vector multiplet and to an arbitrary number of scalar multiplets, (which are representations of the gauge group) and where the full Langrangian must be locally supersymmetric, and locally gauge invariant. For simplicity we assume Eq. (2.21) to hold. The final result can be obtained by different equivalent methods. Our analysis here will be carried in terms of the component fields of the supermultiplets, using the rules of tensor calculus for chiral [C3] and vector multiplets [C2]. The supergravity Lagrangian [C1] is given in Eq. (2.1) where the fields A_μ and u are auxiliary.

This Lagrangian is invarient under the following supersymmetry transformations:

$$\delta_s e^r_\mu = \kappa\bar{\varepsilon}\gamma^r\psi_\mu$$

$$\delta_s\psi_\mu = 2\kappa^{-1}D_\mu[\omega(e,\psi)]\varepsilon + i\gamma_5(\delta^\nu_\mu - \frac{1}{3}\gamma_\mu\gamma^\nu)\varepsilon A_\nu + \frac{1}{3}\gamma_\mu(S - i\gamma_5 P)\varepsilon$$

$$\delta_s S = \frac{1}{2}e^{-1}\bar{\varepsilon}\gamma_\mu R^\mu + i\frac{\kappa}{2}\bar{\varepsilon}\gamma_5\psi_\nu A^\nu - \frac{\kappa}{2}\bar{\varepsilon}(S + i\gamma_5 P)\gamma^\mu\psi_\mu$$

$$\delta_s P = -\frac{i}{2}e^{-1}\bar{\varepsilon}\gamma_5\gamma_\mu R^\mu + \frac{\kappa}{2}\bar{\varepsilon}\psi_\nu A^\nu + i\frac{\kappa}{2}\bar{\varepsilon}\gamma_5(S + i\gamma_5 P)\gamma^\mu\psi_\mu \tag{A.1}$$

$$\delta_s A_\mu = 3\frac{i}{2}e^{-1}\bar{\varepsilon}\gamma_5(\delta^\nu_\mu - \frac{1}{3}\gamma_\mu\gamma^\nu)R_\nu + \kappa\bar{\varepsilon}\gamma^\nu(\psi_\mu A_\nu - \frac{1}{2}\psi_\nu A_\mu)$$

$$- e\frac{\kappa}{4}\varepsilon_{\mu\nu\rho\sigma}\bar{\varepsilon}\gamma_5\gamma^\rho\psi^\sigma A^\nu + i\frac{\kappa}{2}\bar{\varepsilon}\gamma_5(S - i\gamma_5 P)\psi_\mu$$

where $\varepsilon(x)$ is the supergravity parameter. The Lagrangian for the vector multiplet (defined in Eq. (2.10)) in the Wess-Zumino gauge, and with a normalized kinetic energy, when coupled to supergravity is given in Eq. (2.20). The expression appearing in Eq. (2.20) are defined by

$$V = V^\alpha T^\alpha = (V^\alpha_\mu, \lambda^\alpha, D^\alpha) T^\alpha$$

$$[T^\alpha,\ T^\beta] = 2if^{\alpha\beta\gamma}T^\gamma \tag{A.2}$$

$$F_{\mu\nu}{}^\alpha = \partial_\mu V^\alpha_\nu - \partial_\nu V^\alpha_\mu + g_\alpha f^{\alpha\beta\gamma} V^\beta_\mu V^\gamma_\nu$$

$$D_\mu \lambda^\alpha = \partial_\mu \lambda^\alpha + g_\alpha f^{\alpha\beta\gamma} V^\beta_\mu \lambda^\gamma + \frac{1}{2}\omega_{\mu rs}\sigma^{rs}\lambda^\alpha + i\frac{\kappa}{2}A_\mu \gamma_5 \lambda^\alpha ,$$

$$\sigma^{rs} = \frac{1}{4}[\gamma^r, \gamma^s]$$

where g_α is the gauge group coupling constant, and $T^\alpha/2$ are the gauge group generators (The T^α here is normalized, similarly to Pauli-matrices). The Lagrangian of Eq. (2.20) is invariant under the supersymmetry transformations:

$$\delta_s V^\alpha_\mu = \bar{\varepsilon}\gamma_\mu \lambda^\alpha$$

$$\delta_s \lambda^\alpha = -\sigma^{\mu\nu}\varepsilon \hat{F}^\alpha_{\mu\nu} - i\gamma_5 \varepsilon D^\alpha \tag{A.3}$$

$$\delta_s D^\alpha = -i\bar{\varepsilon}\gamma_5 \hat{\not{D}}\lambda^\alpha .$$

The supercovarient quantities $\hat{F}_{\mu\nu}{}^\alpha$ and $\hat{D}_\mu \lambda^\alpha$ that appears in (A.3) are related to $F_{\mu\nu}{}^\alpha$ and $D_\mu \lambda^\alpha$ defined in (A.2) by

$$\hat{F}_{\mu\nu}{}^\alpha = F_{\mu\nu}{}^\alpha - \frac{\kappa}{2}(\bar{\psi}_\mu \gamma_\nu \lambda^\alpha - \bar{\psi}_\nu \gamma_\mu \lambda^\alpha)$$

$$\hat{D}_\mu \lambda^\alpha = D_\mu \lambda^\alpha + \frac{\kappa}{2}(\sigma^{\mu\nu}\hat{F}_{\mu\nu}{}^\alpha + i\gamma_5 D^\alpha)\psi_\mu . \tag{A.4}$$

The left-handed chiral multiplets Σ^a are defined in Eq. (2.9), and under supersymmetry transformations the component fields transform as follows:

$$\delta_s z^a = 2\bar{\varepsilon}_R \chi^a$$

$$\delta_s \chi^a = h^a \varepsilon_L + \hat{\not{D}} z^a \varepsilon_R$$

$$\delta_s h^a = 2\bar{\varepsilon}_R \hat{\not{D}}\chi^a - 2\kappa\bar{\eta}_R \chi^a , \tag{A.5}$$

where

$$\eta_R = \frac{1}{3}(u^* \varepsilon_R - i\not{A}\varepsilon_L)$$

$$\hat{D}_\mu z^a = \partial_\mu z^a - \kappa\bar{\psi}_\mu \chi^a$$

$$\hat{D}_\mu \chi^a = [D_\mu \omega(e,\psi) - i\frac{\kappa}{2}A_\mu]\chi^a - \frac{\kappa}{2}\hat{\not{D}} z^a \psi_{\mu R} - \frac{\kappa}{2}h^a \psi_{\mu L} . \tag{A.6}$$

From the multiplet Σ^a we can construct a multiplet of opposite chirality, denoted by

$$\Sigma_a \equiv (\Sigma^a)^+ = (z_a, \chi_a, h_a) . \tag{A.7}$$

where

$$z_a = z^{a\dagger} = A^a - iB^a \tag{A.8}$$

$$\xi_a = C^{-1}\bar{\chi}^a \; ; \; (\chi^a)^c = \text{right-handed Weyl spinor} \qquad (A.9)$$

$$h_a = h^{a\dagger} = F^a - iG^a \; . \qquad (A.10)$$

In (A . 9), C is the charge conjugation matrix, and in what follows upper and lower indices will be used with left-and right-handed multiplets respectively. To construct a gauge invariant interaction we need the rules of multiplets multiplication, which are

(1) Two multiplets of the same chirality when multiplied form a multiplet of the same chirality:

$$\Sigma_1 \cdot \Sigma_2 = (Z_1,\chi_1,h_1)\cdot(Z_2,\chi_2,h_2) = (Z_1Z_2,\; Z_1\chi_2 + Z_2\chi_1,\; Z_1h_2 + Z_2h_1 - 2\bar{\chi}_1^c\chi_2) \qquad (A.11)$$

This rule, for chiral multiplets with Weyl-spinors, can be obtained directly from superfield multiplication as given by Salam and Strathdee [A2], or indirectly by using the Wess-Zumino rules for multiplets with Majorana spinors [A1]. In the later case we first write the rule for multiplying multiplets with left-handed Majorana spinors (which reads exactly as in (A.11) but with $\bar{\chi}_1^c\chi_2$ replaced by $\bar{\chi}_1\chi_{2L}$), then from each two independent multiplets with Majorana spinors, one multiplet with a Weyl-spinor is formed and rule (A.11) is deduced.

(2) Two multiplets of opposite chiralities, when multipleid symmetrically, give rise to a vector multiplet:

$$\Sigma_1 \times \Sigma_2 \equiv \tfrac{1}{2}(\Sigma_1^\dagger\Sigma_2 + \Sigma_2^\dagger\Sigma_1)$$

$$= (\tfrac{1}{2}Z_1^*Z_2,\, i(Z_1^*\chi_2 - Z_1\chi_2^c),\, -h_1Z_2^*,\, \tfrac{i}{2}(Z_1^*\hat{D}_\mu Z_2 - Z_1\hat{D}_\mu Z_2^* - 2\bar{\chi}_1\gamma_\mu\chi_2,$$

$$\text{[illegible]}$$

$$h_1^*h_2 - \hat{D}_\mu Z_1^*\hat{D}_\mu Z_2 - \bar{\chi}_1\overleftrightarrow{\hat{\not{D}}}\chi_2) + 1 \leftrightarrow 2 \qquad (A.12)$$

Note that the spinor components ξ and λ of the multiplet in (A.12) are both Majorana as is required for vector multiplets. To derive Eq. (A.12) we start with the symmetric product rule for multiplets with Majorana spinors as given by Stelle and West [C2] (this reads exactly as Eq. (A.12) but with χ^c replaced with χ), then as before, from each two independent multiplets with left-handed Majorana spinors, one multiplet with a Weyl spinor is formed. However, in this case the antisymmetric product rule for two multiplets of opposite chiralities, with Majorana spinors, is also needed. To see this let Λ_1, Λ_2, Λ_1', Λ_2' be the left-handed Majorana multiplets, and

$$\Sigma_1 = \Lambda_1 + i\Lambda_2$$

$$\Sigma_2 = \Lambda_1' + i\Lambda_2' \qquad (A.13)$$

then

$$\tfrac{1}{2}(\Sigma_1\Sigma_2^\dagger + \Sigma_2\Sigma_1^\dagger) = (\Lambda_1\Lambda_1' + \Lambda_2\Lambda_2') - i(\Lambda_1\Lambda_2' - \Lambda_1'\Lambda_2) \; .$$

The antisymmetric product rule for multiplets with Majorana spinors reads:

$$\Lambda_1 \wedge \Lambda_2 \equiv -\tfrac{i}{2}(\Lambda_1^\dagger\Lambda_2 - \Lambda_1\Lambda_2^\dagger)$$

$$= (\tfrac{i}{2}Z_1^*Z_2,\, Z_1^*\chi_{2L} + Z_1\chi_{2R},\, iZ_1^*h_2,\, \tfrac{1}{2}(Z_1^*D_\mu Z_2 + Z_1^*D_\mu Z_2 - \bar{\chi}_{1L}\gamma_\mu\chi_{2L}),$$

$$h_2^*\chi_{1L} + \hat{\not{D}}Z_2^*\chi_{1L},\, i(h_1^*h_2 - \hat{D}_\mu Z_1^*\hat{D}_\mu Z_2 - \bar{\chi}_2\hat{\not{D}}\chi_1)) - 1 \leftrightarrow 2 \; . \qquad (A.14)$$

(3) The product of two vector multiplets is a vector multiplet:

$$V_1 \cdot V_2 = (C_1,\xi_1,v_1,V_{\mu 1},\lambda_1,D_1) \cdot (C_2,\xi_2,v_2,V_{\mu 2},\lambda_2,D_2)$$

$$= (\tfrac{1}{2}C_1C_2,\ C_1\xi_2,\ C_1v_2 - \tfrac{1}{2}\bar{\xi}_{1R}\xi_{2L},\ C_1V_{\mu 2} - \tfrac{i}{4}\bar{\xi}_1\gamma_\mu\gamma_5\xi_2\ ,$$

$$C_1\lambda_2 - \tfrac{1}{2}\hat{\not{D}}C_1\xi_2 + \tfrac{1}{2}\hat{v}_1^*\xi_2 + \tfrac{i}{2}\gamma_5\gamma^\mu\xi_2V_{\mu 1}\ ,$$

$$C_1D_2 - \tfrac{1}{2}\hat{D}_\mu C_1\hat{D}^\mu C_2 - \tfrac{1}{2}V_{\mu 1}V_2^\mu + \tfrac{1}{2}v_1^*v_2 - \bar{\xi}_1\lambda_2 - \tfrac{1}{2}\bar{\xi}_2\hat{\not{D}}\xi_1) + 1 \leftrightarrow 2\ , \qquad (A.15)$$

where

$$\hat{v} = K + i\gamma_5 H$$

$$\hat{D}_\mu C = \partial_\mu C + i\tfrac{\kappa}{2}\bar{\psi}_\mu\gamma_5\xi \qquad (A.16)$$

$$\hat{D}_\mu\xi = D_\mu(\omega(e,\psi))\xi - \tfrac{\kappa}{2}(\not{V} + i\gamma_5\hat{\not{D}}C - i\gamma_5\hat{v})\psi_\mu - i\tfrac{\kappa}{2}A_\mu\gamma_5\xi$$

By applying rule (A.15) we can evaluate the component form of the multiplet $\exp(g_\alpha V)$ [C2]:

$$\exp(g_\alpha V) = \exp(g_\alpha C)(1, g_\alpha\xi, g_\alpha\hat{v} - \frac{g_\alpha^2}{2}\bar{\xi}_{1R}\xi_{2L},\ g_\alpha V_\mu - \frac{g_\alpha^2}{4}\bar{\xi}\gamma_5\gamma_\mu\xi,$$

$$g_\alpha\lambda - \frac{g_\alpha^2}{2}\hat{\not{D}}C\xi + \frac{g_\alpha^2}{2}(\hat{v}^* + i\gamma_5\not{V})\xi - \frac{g_\alpha^2}{4}(\bar{\xi}\xi)\xi\ , \qquad (A.17)$$

$$g_\alpha D - \frac{g_\alpha^2}{2}\hat{D}_\mu C\hat{D}^\mu C - \frac{g_\alpha^2}{2}V_\mu V^\mu + \frac{g_\alpha^2}{2}|v|^2 - g_\alpha^2\bar{\xi}\lambda - \tfrac{1}{2}g_\alpha^2\bar{\xi}\hat{\not{D}}\xi$$

$$-\frac{g_\alpha^2}{4}\bar{\xi}(\hat{v}^* + i\gamma_5\not{V})\xi - \frac{g_\alpha^2}{16}(\bar{\xi}\xi)^2)\ .$$

Equation (A.17) simplifies greatly in the Wess-Zumino gauge ($C = \xi = v = 0$):

$$\exp(g_\alpha V) = (1,0,0,g_\alpha V_\mu, g_\alpha\lambda, g_\alpha D - \frac{g_\alpha^2}{2}V_\mu V^\mu)\ . \qquad (A.18)$$

Guided by the vector-scalar coupling of global supersymmetry, we form the gauge invariant interaction

$$\tfrac{1}{4}(\Sigma_{1a}\ \exp\ (g_\alpha V)^a{}_b\Sigma_2^b + 1 \leftrightarrow 2) \qquad (A.19)$$

the above expression is gauge invariant because under group transformations we have:

$$\Sigma^a \to \Omega^a{}_b\Sigma^b$$

$$\Sigma_a \to \Sigma_b\Omega^{\dagger b}{}_a \qquad (A.20)$$

$$\exp(g_\alpha V)^a{}_b \to (\Omega^{-1})^{\dagger a}{}_c\ \exp(g_\alpha V)^c{}_d(\Omega^{-1})^d{}_b$$

where Ω^a_b is a chiral left-handed multiplet whose components are parameters for the group transformations.

The Lagrangian for the gauge invariant and locally supersymmetric interaction of supergravity to the scalar multiplet, can be obtained by applying Eq. (2.13) to the components of the vector multiplet resulting from Eq. (A.19), after setting Σ_2^a to be equal to Σ_1^a. However, because $L_{S \cdot G}$ appears in the last term of Eq. (2.13), a Weyl-scaling will be needed and that changes the kinetic interactions to non-normalized form. To have at least the choice of normalized kinetic energies we generalize Eq. (A.19) to:

$$\frac{1}{2}(\Sigma_{a_1}\ldots\Sigma_{a_m}(A\ \exp(g_\alpha V))^{a_1\ldots a_m}{}_{b_1\ldots b_n}\Sigma^{b_1}\ldots\Sigma^{b_n} + h\cdot c) \quad , \qquad (A.21)$$

where $A^{a_1\ldots a_m}{}_{b_1\ldots b_n}$ are the arbitrary coupling constants that appears in Eq. (2.15). The components of $\Sigma^{a_1}\ldots\Sigma^{a_m}$ can be obtained by repeated application of (A.11):

$$(Z^{a_1}\ldots Z^{a_m}, mZ^{(a_1}\ldots Z^{a_{m-1}}\chi^{a_m)}, mZ^{(a_1}\ldots Z^{a_{m-1}}h^{a_m)} - m(m-1)Z^{(a_1}\ldots Z^{a_{m-2}}\bar{\chi}_{a_{m-1}}\chi^{a_m)}) \qquad (A.22)$$

We form the symmetric product of $\Sigma^{a_1}\ldots\Sigma^{a_m}$ and $\Sigma^{b_1}\ldots\Sigma^{b_n}$, then multiply the resultant vector multiplet with the vector multiplet $(A\ \exp(g_\alpha V))^{a_1\ldots a_m}{}_{b_1\ldots b_n}$. The component form of (A.21) is

$$C = \frac{1}{2}\phi$$

$$\xi = i(\phi,_a\chi^a - \phi,^a\chi_a)$$

$$v = -(\phi,_a h^a - \phi,_{ab}\bar{\chi}_a\chi^b)$$

$$V_\mu = \frac{i}{2}(\phi,_a\mathcal{D}_\mu Z^a - \phi,^a\mathcal{D}_\mu Z_a - 2\phi,^a{}_b\overline{\chi^a}\gamma^\mu\chi^b)$$

$$\lambda = -i\phi,^a{}_b h_a\chi^b + i\phi,^{ab}{}_c(\overline{\chi^a}\chi_b)\chi^c + i\phi,^a{}_b h^b\chi_a$$

$$-i\phi,_{ab}{}^c(\overline{\chi_a}\chi^b)\chi_c - i\phi,^a{}_b\hat{\mathcal{D}}Z_a\chi^b + i\phi,^a{}_b\hat{\mathcal{D}}Z^b\chi_a$$

$$+\frac{g_\alpha}{2}\lambda^\alpha\phi,_a(T^\alpha)^a{}_b Z^b$$

$$D = \phi,^a{}_b h_a h^b - \phi,^{ab}{}_c\overline{\chi^a}\chi_b h^c - \phi,_{ab}{}^c\overline{\chi_a}\chi^b h_c$$

$$+\phi,_{ab}{}^{cd}\overline{\chi_a}\chi^b\cdot\overline{\chi^c}\chi_d - \phi,^a{}_b\hat{\mathcal{D}}_\mu Z_a\hat{\mathcal{D}}^\mu Z^b$$

$$-\phi,^a{}_b\overline{\chi^a}\overset{\leftrightarrow}{\hat{\mathcal{D}}}\chi^b - \phi,^a{}_{bc}\overline{\chi^a}\gamma^\mu\chi^b\hat{\mathcal{D}}_\mu Z^c$$

$$-\phi,_a{}^{bc}\overline{\chi_a}\gamma^\mu\chi_b\hat{\mathcal{D}}_\mu Z_c + \frac{g_\alpha}{2}D^\alpha\phi,_a(T^\alpha)^a{}_b Z^b$$

$$-ig_\alpha\phi,^a{}_b(\bar{\lambda}^\alpha\chi^b(T^\alpha Z)_a - \overline{\chi^a}\lambda^\alpha(T^\alpha Z)^b) \quad . \qquad (A.23)$$

where the function ϕ has been defined in Eq. (2.15) and all script derivatives are the gauge covariant form of the latin ones, e.g.

$$\hat{\mathcal{D}}_\mu Z^a = \hat{D}_\mu Z^a - i\frac{g_\alpha}{2}V_\mu^\alpha(T^\alpha)^a{}_b Z^b \quad . \qquad (A.24)$$

In the above formulas we have distinguished between up and down indices, and the derivatives of ϕ are denoted by:

$$\phi,_a = \frac{\partial\phi}{\partial Z^a} \ , \ \phi,^a = \frac{\partial\phi}{\partial Z_a} \quad . \qquad (A.25)$$

We now apply Eq (2.12) to the multiplet (A.23) to obtain the most general locally supersymmetric gauge invariant Lagrangian (without higher derivatives):

$$e^{-1}L_D = \phi,^a{}_b h_a h^b - \phi,^{ab}{}_c\overline{\chi^a}\chi_b h^c - \phi,_{ab}{}^c\overline{\chi_a}\chi^b h_c$$

$$+\phi,_{ab}{}^{cd}\overline{\chi_a}\chi^b\overline{\chi^c}\chi_d - \phi,^a{}_b\hat{\mathcal{D}}_\mu Z_a\hat{\mathcal{D}}^\mu Z^b$$

$$- \phi,{}^a{}_b \overline{\chi^a}\overleftrightarrow{\hat{\mathcal{D}}}\chi^b - \overline{\chi^a}\gamma^\mu\chi^b(\phi,{}^a{}_{bc}\hat{\mathcal{D}}_\mu z^c - \phi,{}^{ac}{}_b\hat{\mathcal{D}}_\mu z_c)$$

$$- i g_\alpha \phi,{}^a{}_b(\overline{\lambda^\alpha}\chi^b(T^\alpha z)_a - \overline{\chi^a}\lambda^\alpha(T^\alpha z)^b)$$

$$+ \frac{g_\alpha}{2}D^\alpha\phi,{}_a(T^\alpha z)^a + \frac{\kappa}{2}\phi,{}^a{}_b\bar\psi_\mu\gamma_\mu(h_a\chi^b + h^b\chi_a)$$

$$- \frac{\kappa}{2}(\bar\psi_\mu\gamma\mu\chi^c\overline{\chi^a}\chi_b\phi,{}^{ab}{}_c - \overline{\chi^c}\gamma^\mu\psi_\mu\overline{\chi_a}\chi^b\phi,{}_{ab}{}^c)$$

$$- \frac{\kappa}{2}\phi,{}^a{}_b(\bar\psi_\mu\gamma^\mu\gamma^\nu\chi^b\hat{\mathcal{D}}_\nu z_a - \overline{\chi^a}\gamma^\nu\gamma^\mu\psi_\mu\hat{\mathcal{D}}_\nu z^b)$$

$$- i\kappa\frac{g_\alpha}{2}\bar\psi_\mu\gamma_5\gamma^\mu\lambda^\alpha\phi,{}_a(T^\alpha z)^a$$

$$+ \frac{\kappa}{3}[u^*(\phi,{}_a h^a - \phi,{}_{ab}\bar\chi_a\chi^b) + u(\phi,{}^a h_a - \phi,{}^{ab}\overline{\chi^a}\chi_b)]$$

$$+ (i\frac{\kappa}{3}A^\mu + e^{-1}\frac{\kappa^2}{8}\epsilon^{\mu\nu\rho\sigma}\bar\psi_\nu\gamma_\rho\psi_\sigma)(\phi,{}_a\hat{\mathcal{D}}_\mu z^a - \phi,{}^a\hat{\mathcal{D}}_\mu z_a - 2\phi,{}^a{}_b\overline{\chi^a}\gamma_\mu\chi^b)$$

$$+ 4\frac{\kappa}{3}(D_\mu\bar\psi_\nu\sigma^{\mu\nu}\chi^a\phi,{}_a - \phi,{}^a\overline{\chi^a}\sigma^{\mu\nu}D_\mu\psi_\nu)$$

$$- \frac{\kappa^2}{3}\phi(- \frac{R}{2\kappa^2} - \frac{e^{-1}}{2}\epsilon^{\mu\nu\rho\sigma}\bar\psi_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma - \frac{1}{3}|u|^2 + \frac{1}{3}A_\mu A^\mu)$$

$$- \frac{\kappa^3}{8}\epsilon^{\mu\nu\rho\sigma}e^{-1}\bar\psi_\mu\gamma_\nu\psi_\rho(\bar\psi_\sigma\chi^a\phi,{}_a - \overline{\chi^a}\psi_\sigma\phi,{}^a) \qquad (A.26)$$

All derivatives in (A.26) contain torsion pieces and are gauge covariant. It is still possible to add another piece to the Lagrangian that corresponds to the self interactions of the chiral multiplets. The most general form of such interaction is $g(\Sigma)$ where the function g has been defined in Eq. (2.14). The components of this left-handed chiral multiplet are

$$g(\Sigma^a) = (g(z^a)\ ,\ g,{}_a\chi^a,\ g,{}_a h^a - 2g,{}_{ab}\overline{\chi_a}\chi^b) \qquad (A.27)$$

Using Eq. (2.12) on the components of (A.27) we obtain the locally supersymmetric form of this interaction

$$e^{-1}L_{pot} = \frac{1}{2}(g,{}_a h^a - 2g,{}_{ab}\overline{\chi_a}\chi^b + \kappa u g + \kappa g,{}_a\bar\psi_\mu\gamma^\mu\chi^a$$

$$+ \kappa^2 g\bar\psi_\mu\sigma^{\mu\nu}\psi_\nu|_R + h\cdot c) \qquad (A.28)$$

The total Lagrangian is thus

$$L = L_V + L_{pot} + L_D \qquad (A.29)$$

The supergravity Lagrangian is already included in L_D and corresponds to the constant part of the function ϕ.

The auxiliary fields u, A_μ, h^a, and D^α appear in (A.29) and must be eliminated to determine the effective interactions of the physical fields. However, because some contributions of the auxiliary fields are buried in the supercovariant derivatives as in Eqs. (A.2, (A.4), and (A.6), we thus expand all these supercovariant derivatives keeping only their torsionless parts

$$e^{-1}L = \frac{\phi}{6}(R(\omega(e)) + e^{-1}\kappa^2\epsilon^{\mu\nu\rho\sigma}\bar\psi_\mu\gamma_5\gamma_\nu D_\rho(\omega(e))\psi_\sigma + \frac{2}{3}\kappa^2|u|^2 - \frac{2}{3}\kappa^2 A_\mu A^\mu)$$

$$- \phi,{}^a{}_b(\mathcal{D}_\mu z_a\mathcal{D}_\mu z^b + \overline{\chi^a}\overleftrightarrow{\mathcal{D}}(\omega(e))\chi^b - h_a h^b)$$

$$+ \kappa\phi,{}^a{}_b(\mathcal{D}_\nu z^b\overline{\chi^a}\gamma^\mu\gamma^\nu\psi_\mu + \bar\psi_\mu\gamma^\nu\gamma^\mu\chi^b\mathcal{D}_\nu z_a)$$

$$- \overline{\chi^a}\gamma^\mu\chi^b(\phi,{}^a{}_{bc}\mathcal{D}_\mu z^c - \phi,{}_b{}^{ac}\mathcal{D}_\mu z_c)$$

$$-\phi,{}^{ab}{}_{c}\bar{\chi}^{a}\chi_{b}h^{c} - \phi,{}_{ab}{}^{c}\bar{\chi}_{a}\chi^{b}h_{c} + \frac{g_{\alpha}}{2}D^{\alpha}\phi,{}_{a}(T^{\alpha}Z)^{a}$$

$$-ig_{\alpha}\phi,{}^{a}{}_{b}((T^{\alpha}Z)_{a}\bar{\lambda}^{\alpha}\chi^{b} - \overline{\chi^{a}}\lambda^{\alpha}(T^{\alpha}Z)^{b})$$

$$- i\kappa\frac{g_{\alpha}}{4}\bar{\psi}_{\mu}\gamma_{5}\gamma^{\mu}\lambda^{\alpha}\phi,{}_{a}(T^{\alpha}Z)^{a}$$

$$+ \frac{\kappa}{3}(u^{*}(\phi,{}_{a}h^{a} - \phi,{}_{ab}\overline{\chi_{a}}\chi^{b} + \frac{3}{2}g^{*}) + u(\phi,{}^{a}h_{a} - \phi,{}^{ab}\overline{\chi^{a}}\chi_{b} + \frac{3}{2}g))$$

$$+ i\frac{\kappa}{3}A^{\mu}(\phi,{}_{a}\mathcal{D}_{\mu}Z^{a} - \phi,{}^{a}\mathcal{D}_{\mu}Z_{a} - \kappa\phi,{}_{a}\bar{\psi}_{\mu}\chi^{a} + \kappa\phi,{}^{a}\overline{\chi^{a}}\psi_{\mu}$$

$$+ \phi,{}^{a}{}_{b}\overline{\chi^{a}}\gamma_{\mu}\chi^{b} - \frac{3}{4}\bar{\lambda}^{\alpha}\gamma_{\mu}\gamma_{5}\lambda^{\alpha})$$

$$- e^{-1}\frac{\kappa^{2}}{8}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu}\gamma_{\nu}\psi_{\rho}(\phi,{}_{a}\mathcal{D}_{\sigma}Z^{a} - \phi,{}^{a}\mathcal{D}_{\sigma}Z_{a})$$

$$+ 4\frac{\kappa}{3}(\overline{D_{\mu}(\omega(e))\psi_{\nu}}\sigma^{\mu\nu}\chi^{a}\phi,{}_{a} - \phi,{}^{a}\overline{\chi^{a}}\sigma^{\mu\nu}D_{\mu}(\omega(e))\psi_{\nu})$$

$$+ \frac{1}{2}(g,{}_{a}h^{a} - g,{}_{ab}\overline{\chi_{a}}\chi^{b} + \kappa g,{}_{a}\bar{\psi}_{\mu}\gamma^{\mu}\chi^{a} + \kappa^{2}g\bar{\psi}_{\mu}\sigma^{\mu\nu}\psi_{\nu R}$$

$$+ g,{}^{*a}h_{a} - g,{}^{*ab}\overline{\chi_{a}}\chi^{b} + \kappa g,{}^{*a}\overline{\chi^{a}}\gamma^{\mu}\psi_{\mu} + \kappa^{2}g^{*}\bar{\psi}_{\mu}\sigma^{\mu\nu}\psi_{\nu L})$$

$$+ \frac{\kappa^{2}}{6}e^{-1}\phi\partial_{\mu}(e\bar{\psi}_{\nu}\gamma^{\nu}\psi^{\mu})$$

$$- \frac{1}{4}F_{\mu\nu}{}^{\alpha}F^{\mu\nu\alpha} - \frac{1}{2}\bar{\lambda}^{\alpha}\not{D}(\omega(e))\lambda^{\alpha} + \frac{1}{2}D^{\alpha}D^{\alpha} - \frac{\kappa}{2}\bar{\psi}_{\mu}\sigma^{\kappa\lambda}\gamma^{\mu}\lambda^{\alpha}F_{\kappa\lambda}{}^{\alpha}$$

$$+ e^{-1}L^{quartic} , \qquad \text{(A.30)}$$

where we here have grouped all quartic interactions:

$$e^{-1}L^{quartic} = \phi,{}_{ab}{}^{cd}(\overline{\chi_{a}}\chi^{b})(\overline{\chi^{c}}\chi_{d}) - \kappa^{2}\phi,{}^{a}{}_{b}(\overline{\chi^{a}}\psi_{\mu})(\bar{\psi}_{\mu}\chi^{b})$$

$$- \frac{\kappa^{2}}{8}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu}\gamma_{\nu}\psi_{\rho}(\phi,{}^{a}{}_{b}\overline{\chi^{a}}\gamma_{\sigma}\chi^{b} + \frac{1}{2}\bar{\lambda}^{\alpha}\gamma_{5}\gamma_{\sigma}\lambda^{\alpha})$$

$$+ \frac{\kappa^{2}}{4}\bar{\lambda}^{\alpha}\gamma^{\mu}\sigma^{\nu\rho}\psi_{\mu}\bar{\psi}_{\nu}\gamma_{\rho}\lambda^{\alpha}$$

$$+ \frac{\kappa^{4}}{96}\phi((\bar{\psi}_{\mu}\gamma_{\nu}\psi_{\sigma} + 2\bar{\psi}_{\nu}\gamma_{\mu}\psi_{\rho})\bar{\psi}^{\mu}\gamma^{\nu}\psi^{\rho} - 4(\bar{\psi}_{\mu}\gamma^{\nu}\psi_{\nu})^{2})$$

$$- \frac{\kappa^{3}}{6}(\bar{\psi}^{\nu}\gamma^{\rho}\psi_{\rho} - \frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu}\gamma_{\rho}\psi_{\sigma})(\phi,{}^{a}\overline{\chi^{a}}\psi_{\nu})$$

$$- \frac{\kappa^{3}}{6}(\bar{\psi}^{\nu}\gamma^{\rho}\psi_{\rho} + \frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu}\gamma_{\rho}\psi_{\sigma})(\bar{\psi}_{\nu}\chi^{a}\phi,{}_{a}) \qquad \text{(A.31)}$$

In arriving at Eqs. (A.30) and (A.31), we have used the following Fierz identities:

$$\overline{\chi^{a}}\gamma^{\mu}\chi^{b}\bar{\psi}_{\mu}\chi^{c} = \frac{1}{2}\bar{\psi}_{\mu}\gamma^{\mu}\chi_{a}\overline{\chi_{b}}\chi^{c}$$

$$\overline{\chi^{a}}\sigma^{\mu\nu}\sigma^{\rho\sigma}\psi_{\nu}K_{\mu\rho\sigma} = \frac{\kappa^{2}}{8}\overline{\chi^{a}}\psi_{\nu}(\bar{\psi}^{\nu}\gamma^{\rho}\psi_{\rho} - \frac{e^{-1}}{4}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu}\gamma_{\rho}\psi_{\sigma}) , \qquad \text{(A.32)}$$

We now seperate from the Lagrangian all terms that include the auxiliary fields u, A_{μ}, h^{a}, and D^{α}. However, in analogy with the case of one chiral multiplet [C7], we find it more convenient to treat $u - \frac{\kappa}{2}d,{}_{a}h^{a}$ as an independent variable instead of u itself, where the function d is defined in Eq.(2.19). (It is related to the function J that appears in Ref. [C7] by $J = -\frac{\kappa^{2}}{2}d$.) The auxiliary part of the Lagrangian now reads:

$$e^{-1}L^{aux} = \frac{\kappa^2\phi}{9}|u - \frac{\kappa}{2}d,_a h^a|^2 - \frac{\kappa^2}{6}\phi,d\,^a{}_b h_a h^b$$

$$+ \frac{1}{2}[\kappa g(u - \frac{\kappa}{2}d,_a h^a) - \frac{\kappa^2\phi}{9}g(\frac{\kappa^2}{2}d,_a + \frac{g,_a}{g})h^a + h\cdot c]$$

$$+ \frac{\kappa^2\phi}{6}\{[\frac{\kappa}{3}(u - \frac{\kappa}{2}d,_c h^c)(d,^{ab} - \frac{\kappa^2}{6}d,^a d,^b) + h^c(d,_c{}^{ab} - \frac{\kappa^2}{3}d,^a d,^b{}_c)]$$

$$\cdot(\bar{\chi}^a\chi_b) + h\cdot c\} - i\frac{\kappa^3\phi}{18}A^\mu(d,_a\hat{\mathfrak{D}}_\mu Z^a - d,^a\hat{\mathfrak{D}}_\mu Z_a$$

$$+ (d,^a{}_b - \frac{\kappa^2}{6}d,^a d,_b)\overline{\chi^a}\gamma_\mu\chi^b + \frac{9}{2\kappa^2\phi}\bar{\lambda}^\alpha\gamma_\mu\gamma_5\lambda^\alpha) \quad \text{(A.33)}$$

The field equations for $u - \frac{\kappa}{2}d,_a h^a$, h^a, A_μ and D^α give respectively:

$$u - \frac{\kappa}{2}d,_a h^a = -\frac{9}{2\kappa\phi}g^* - \frac{\kappa}{2}(d,_{ab} - \frac{\kappa^2}{6}d,_a d,_b)\overline{\chi_a}\chi^b$$

$$h^a = \frac{3}{\kappa^2\phi}g^*(d^{-1})^a{}_b(\frac{\kappa^2}{2}d,^b + \frac{g,^{*b}}{g^*}) + ((d^{-1})^a{}_b d,^b{}_{ce} - \frac{\kappa^2}{3}\delta^a{}_c d,_e)\overline{\chi_c}\chi^e$$

$$A_\mu = -i\frac{\kappa}{4}[d,_a\hat{\mathfrak{D}}_\mu Z^a - d,^a\hat{\mathfrak{D}}_\mu Z_a + (d,^a{}_b - \frac{\kappa^2}{6}d,^a d,_b)\overline{\chi^a}\gamma^\mu\chi^b + \frac{9}{2\kappa^2\phi}\bar{\lambda}^\alpha\gamma_\mu\gamma_5\lambda^\alpha]$$

$$D^\alpha = -\frac{g_\alpha}{2}\phi,_a(T^\alpha Z)^a . \quad \text{(A.34)}$$

Substituting back from Eq. (A.34) into the auxiliary part of the Lagrangian we obtain:

$$e^{-1}L^{aux} = -\frac{9}{4\phi}|g + \frac{\kappa^2\phi}{9}(d,^{ab} - \frac{\kappa^2}{6}d,^a d,^b)\overline{\chi^a}\chi_b|^2$$

$$+ \frac{6}{\kappa^2\phi}(d^{-1})^a{}_b[\frac{g}{2}(\frac{g,_a}{g} + \frac{\kappa^2}{2}d,_a) + \frac{\kappa^2\phi}{6}(d,^{ce}{}_a - \frac{\kappa^2}{3}d,^c{}_a d,^e)\overline{\chi^c}\chi_e]$$

$$\times [\frac{g^*}{2}(\frac{g^*,^b}{g^*} + \frac{\kappa^2}{2}d,^b) + \frac{\kappa^2\phi}{6}(d,^b{}_{c'e'} - \frac{\kappa^2}{3}d,^b{}_{c'}d,_{e'})\overline{\chi_{c'}}\chi^{e'}]$$

$$- \frac{\kappa^4\phi}{144}|(d,_a\hat{\mathfrak{D}}_\mu Z^a - d,^a\hat{\mathfrak{D}}_\mu Z_a + (d,^a{}_b - \frac{\kappa^2}{6}d,^a d,_b)\overline{\chi^a}\gamma^\mu\chi^b + \frac{3}{2\kappa^2\phi}\bar{\lambda}^\alpha\gamma_\mu\gamma_5\lambda^\alpha)|^2$$

$$- \frac{g_\alpha^2}{8}(\phi,_a(T^\alpha Z)^a)^2 \quad \text{(A.35)}$$

Going back to Eq. (A30), the curvature scalar R appears with a factor $\phi/6$ implying, as has been mentioned earlier, the need for a Weyl-scaling to seperate the physical graviton from the scalar fields. The required scaling is:

$$e_\mu{}^r \to \exp(\frac{\kappa^2 d}{12})e_\mu{}^r \quad \text{(A.36)}$$

Then the connection and the curvature would scale according to:

$$\omega_\mu{}^{rs}(e) \to \omega_\mu{}^{rs}(e) + \frac{\kappa^2}{12}(e_\mu{}^r e^{\nu s} - e_\mu{}^s e^{\nu r})\partial_\nu d$$

$$e\frac{\phi}{6}R(\omega(e)) \to -\frac{e}{2\kappa^2}R(\omega(e)) - e\frac{\kappa^2}{48}(\partial_\mu d)(\partial^\mu d) . \quad \text{(A.37)}$$

and the bosonic part of the Lagrangian becomes as given in Eq. (2.22). Eq. (2.22) can be further simplified and expressed in terms of one function instead of two by defining:

$$\mathcal{G} = -\frac{\kappa^2}{2}d - \log\frac{\kappa^6}{4}|g|^2 \quad \text{(A.38)}$$

In terms of G the effective potential would read

$$e^{-1}V = -\frac{1}{\kappa^4}\exp(-\mathcal{G})((\mathcal{G}^{-1})^a{}_b\mathcal{G},_a\mathcal{G},^b + 3) + \frac{g_\alpha^2}{8\kappa^4}(\mathcal{G},_a(T^\alpha Z)^a)^2 \quad \text{(A.39)}$$

where we have used the fact that the superpotential g is gauge invariant implying for a gauge variation

$$\delta g(Z) = \varepsilon^{\alpha}(g,_{a}(T^{\alpha}Z)^{a}) = 0 \tag{A.40}$$

The fermionic part of the Lagrangian is more complicated. However, the required steps to arrive at the final result are well defined. Firstly, the gravitino field kinetic energy is mixed with those of the spin - 1/2 fields due to the presence of the term

$$\phi,^{a}\overline{\chi^{a}}\sigma^{\mu\nu}D_{\mu}\psi_{\nu} + h \cdot c$$

in the Lagrangian. This implies that the gravitino field has to be redefined. Moreover, the spinors ψ_{μ}, λ^{α}, χ^{a} have to be rescaled to obtain the proper normalizations. The correct transformations are:

$$\chi^{a} = \exp(-\frac{\kappa^{2}d}{24})(\frac{g^{*}}{g})^{1/4}(\chi^{a})^{new}$$

$$\lambda_{L}^{\alpha} = \exp(-\frac{\kappa^{2}d}{8})(\frac{g}{g^{*}})^{1/4}(\lambda_{L}{}^{\alpha})^{new};\ \lambda_{R}^{\alpha} = \exp(-\frac{\kappa^{2}d}{8})(\frac{g^{*}}{g})^{1/4}(\lambda_{R}^{\alpha})^{new}$$

$$\psi_{\mu L} = (\frac{g}{g^{*}})^{1/4}[\exp(\frac{\kappa^{2}d}{24})\psi_{\mu L}{}^{new} + \frac{\kappa}{6}\gamma_{r}(e_{\mu}^{r})^{new}\, d,^{a}(\chi_{a})^{new}]$$

$$\psi_{\mu R} = (\frac{g^{*}}{g})^{1/4}[\exp(\frac{\kappa^{2}d}{24})\psi_{\mu R}{}^{new} + \frac{\kappa}{6}\gamma_{r}(e_{\mu}^{r})^{new}d,_{a}(\chi^{a})^{new}] \tag{A.41}$$

where $(e_{\mu}^{r})^{new}$ is the new vierbein of Eq. (A.36). Substituting Eqs. (A.41) into (A.30) and (A.35) we obtain after a lengthy algebra (and dropping the index "new"):

$$\begin{aligned}
L_F = & -\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu}\gamma_5\gamma_{\nu}D_{\rho}(\omega(e,\psi))\psi_{\sigma} + \frac{e}{\kappa^2}\mathcal{G},^{a}{}_{b}\overline{\chi^{a}}\overleftrightarrow{\mathcal{D}}\chi^{b} - \frac{e}{2}\bar{\lambda}^{\alpha}\not{\mathcal{D}}\lambda^{\alpha} \\
& - e\frac{\kappa}{2}\bar{\psi}_{\mu}\sigma^{\kappa\lambda}\gamma^{\mu}\lambda^{\alpha}F_{\kappa\lambda}{}^{\alpha} + \frac{1}{8}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu}\gamma_{\nu}\psi_{\rho}(\mathcal{G},_{a}\mathcal{D}_{\sigma}Z^{a} - \mathcal{G},^{a}\mathcal{D}_{\sigma}Z_{a}) \\
& - \frac{e}{\kappa}\mathcal{G},^{a}{}_{b}(\mathcal{D}_{\nu}Z^{b}\overline{\chi^{a}}\gamma^{\mu}\gamma^{\nu}\psi_{\mu} + \bar{\psi}_{\mu}\gamma^{\nu}\gamma^{\mu}\chi^{b}\mathcal{D}_{\nu}Z_{a}) \\
& + \frac{e}{\kappa^2}\overline{\chi^{a}}\gamma^{\mu}\chi^{b}((\mathcal{G},^{a}{}_{bc} - \frac{\kappa^2}{4}\mathcal{G},^{a}{}_{b}\mathcal{G},_{c})\mathcal{D}_{\mu}Z^{c} - (\mathcal{G},^{ac}{}_{b} - \frac{\kappa^2}{4}\mathcal{G},^{a}{}_{b}\mathcal{G},^{c})\mathcal{D}_{\mu}Z_{c}) \\
& + \frac{e}{8}\bar{\lambda}^{\alpha}\gamma^{\mu}\gamma_5\lambda^{\alpha}(\mathcal{G}_{a}\mathcal{D}_{\mu}Z^{a} - \mathcal{G}^{a}\mathcal{D}_{\mu}Z_{a}) \\
& + \frac{e}{\kappa^3}\exp(-\frac{\mathcal{G}}{2})[(\mathcal{G}_{ab} - \mathcal{G}_{a}\mathcal{G}_{b} - (\mathcal{G}^{-1})^{c}{}_{d}\mathcal{G}_{c}\mathcal{G}^{d}{}_{ab})\bar{\chi}_{a}\chi^{b} \\
& - \kappa\mathcal{G},_{a}\bar{\psi}_{\mu}\gamma^{\mu}\chi^{a} + \kappa^2\bar{\psi}_{\mu}\sigma^{\mu\nu}\psi_{\nu R} + h \cdot c] \\
& + ie\frac{g_{\alpha}}{4\kappa}(\mathcal{G}_{a}(T^{\alpha}Z)^{a}\bar{\psi}_{\mu}\gamma_5\gamma^{\mu}\lambda^{\alpha} + ei\frac{g_{\alpha}}{\kappa^2}\mathcal{G}^{a}{}_{b}((T^{\alpha}Z)_{a}\bar{\lambda}^{\alpha}\chi^{b} - \overline{\chi^{a}}\lambda^{\alpha}(T^{\alpha}Z)^{b}) \\
& - \frac{e}{\kappa^2}(\mathcal{G}_{ab}{}^{cd} - (\mathcal{G}^{-1})^{e}{}_{f}\mathcal{G},_{ab}{}^{e}\mathcal{G},^{cd}{}_{f} + \frac{1}{2}\mathcal{G},^{c}{}_{a}\mathcal{G}^{d}{}_{b})\overline{\chi_{a}\chi^{b}}\,\overline{\chi^{c}}\chi_{d} \\
& + e\frac{\kappa^2}{4}\bar{\lambda}^{\alpha}\gamma^{\mu}\sigma^{\nu\rho}\psi_{\mu}\bar{\psi}_{\nu}\gamma_{\rho}\lambda^{\alpha} + \frac{e}{8}\mathcal{G},^{a}{}_{b}\overline{\chi^{a}}\gamma^{\mu}\chi^{b}\bar{\lambda}^{\alpha}\gamma_{\mu}\gamma_5\lambda^{\alpha} \\
& + 3e\frac{\kappa^2}{64}\bar{\lambda}^{\alpha}\gamma^{\mu}\gamma_5\lambda^{\alpha}\bar{\lambda}^{\beta}\gamma_{\mu}\gamma_5\lambda^{\beta} + \frac{1}{4}\mathcal{G},^{a}{}_{b}\overline{\chi^{a}}\gamma_{\sigma}\chi^{b}(\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu}\gamma_{\nu}\psi_{\rho} - e\bar{\psi}_{\mu}\gamma_5\gamma^{\sigma}\psi^{\mu})
\end{aligned} \tag{A.42}$$

The Fermionic Lagrangian (A.42) depends only on the function $\mathcal{G}$. In arriving at (A.42) we grouped all terms of the same form together, first expressing them as a function of g and d, and finally simplifying them into a function of $\mathcal{G}$.

The resultant Lagrangian is invariant under supersymmetry transformations only on the smass-shell. To obtain the supersymmetry transformations we substitute the auxiliary fields from Eq. (A.34) into the old transformations and substitute Eq. (A.41). The new transformations are:

$$\delta_s e_\mu{}^r = \kappa\bar{\varepsilon}\gamma^r\psi_\mu$$

$$\delta_s\psi_{\mu L} = 2\kappa^{-1}D_\mu(e,\psi_\nu)\varepsilon_L + \kappa^{-2}\exp(-\tfrac{\mathcal{G}}{2})\gamma_\mu\varepsilon_R + \tfrac{1}{2}(\mathcal{G},_a\bar{\varepsilon}\chi^a - \mathcal{G},^a\overline{\chi^a}\varepsilon)\psi_{\mu L}$$

$$+ \kappa^{-1}(\sigma_{\mu\nu}\varepsilon_L)\mathcal{G},^a{}_b\overline{\chi^a}\gamma^\nu\chi^b - \tfrac{\kappa}{8}(\delta_\mu{}^\nu + \gamma^\nu\gamma_\mu)\varepsilon_L\bar{\lambda}^\alpha\gamma_\nu\gamma_5\gamma^\alpha$$

$$- \frac{\kappa^{-1}}{2}(\mathcal{G},_a\mathcal{D}_\mu z^a - \mathcal{G},^a\mathcal{D}_\mu z_a)\varepsilon_L$$

$$\delta_s V_\mu{}^\alpha = -\bar{\varepsilon}\gamma_\mu\lambda^\alpha$$

$$\delta_s\lambda^\alpha = -\tfrac{1}{2}(\mathcal{G},_a\bar{\varepsilon}\chi^a - \mathcal{G},^a\overline{\chi^a}\varepsilon)\lambda^\alpha - \sigma^{\mu\nu}\varepsilon\hat{F}_{\mu\nu}{}^\alpha - \frac{ig_\alpha}{2\kappa^2}(\gamma_5\varepsilon)(\mathcal{G},_a(T^\alpha z)^a)$$

$$\delta_s z^a = 2\bar{\varepsilon}\chi^a$$

$$\delta_s\chi^a = \gamma^\mu\varepsilon_R\hat{\mathcal{D}}_\mu z^a - \tfrac{1}{2}(\mathcal{G},_b\bar{\varepsilon}\chi^b - \mathcal{G},^b\overline{\chi^b}\varepsilon)\chi^a$$

$$+ (\mathcal{G}^{-1})^a{}_b\mathcal{G},^b{}_{cd}\overline{\chi_c}\chi^d\varepsilon_L - \kappa^{-1}(\mathcal{G}^{-1})^a{}_b\mathcal{G},^b\exp(-\tfrac{\mathcal{G}}{2})\varepsilon_L \qquad (A.43)$$

From the Lagrangian in (2.21) and (A.42) it is possible to read the mass matrices of the physical fields

$$\psi_\mu,\ \lambda^\alpha,\ \chi^a,\ z^a,\ \text{and}\ V_\mu{}^\alpha. \qquad (A.44)$$

The gauge bosons $V_\mu{}^\alpha$ that correspond to broken generators of the gauge group

acquire their masses by the usual Higgs mechanism, while those corresponding to unbroken generators remain massless. The mass formula for the massive gauge bosons are the same as in standard gauge theories.

The mass formula for the complex fields z^a must be given in terms of the real fields A^a and B^a, because their masses will split when supersymmetry is broken. We can expand V in terms of the complex fields:

$$V(z^a,z_b) = \tfrac{1}{2}(V,_{ab})_o z^a z^b + \tfrac{1}{2}(V,^{ab})_o z_a z_b + (V,^a{}_b)_o z_a z^b + \ldots \qquad (A.45)$$

Alternatively, we can write

$$V(z^a,z_b) = \tfrac{1}{2}((M^2{}_{ab})^A A^a A^b + (M^2)^B{}_{ab}B^a B^b) + \ldots$$

Thus:

$$(M^2)^A{}_{ab} = (V,^a{}_b + V,^b{}_a + 2V,_{ab})_o$$

$$(M^2)^B{}_{ab} = (V,^a{}_b + V,^b{}_a - 2V,_{ab})_o \qquad (A.46)$$

It is useful, and considerably simpler, to express the above equations as a function of $\mathcal{G}$ for the case of a flat Kähler manifold:

$$V,_{ab} = \frac{2e}{\kappa^4}\exp(-\mathcal{G})[\tfrac{1}{2}(\mathcal{G},_{ab} - \mathcal{G},_a\mathcal{G},_b) + \frac{1}{\kappa^2}(\mathcal{G},_{abc} - 3\mathcal{G},_{(a}\mathcal{G},_{bc)}$$

$$+ \mathcal{G},_a\mathcal{G},_b\mathcal{G},_c)\mathcal{G}^c] + \frac{g_\alpha^2}{32}z_c z_d(T^\alpha)^c{}_a(T^\alpha)^d{}_b$$

[illegible]

$$-\frac{\kappa^2}{2}\delta^a{}_b + \frac{1}{2}\delta^a{}_b \mathcal{G}_{,c}\mathcal{G}^{,c}] + \frac{g_\alpha^2}{32} z_c z^d (T^\alpha)^e{}_b (T^\alpha)^a{}_d \qquad \text{(A.47)}$$

The mass terms for the fields ψ_μ and χ^a are (for a flat Kähler manifold)

$$L^{mass} = \frac{e}{\kappa^3}\exp(-\frac{\mathcal{G}}{2})[(\mathcal{G}_{,ab} - \mathcal{G}_{,a}\mathcal{G}_{,b})\overline{\chi_a}\chi^b - \kappa\mathcal{G}_{,a}\bar{\psi}_\mu\gamma^\mu\chi^a + \kappa^2\bar{\psi}_\mu\sigma^{\mu\nu}\psi_{\nu R} + h\cdot c]$$

$$- ei\frac{g_\alpha}{8}\kappa(z_a(T^\alpha z)^a)\bar{\psi}_\mu\gamma_5\gamma^\mu\lambda^\alpha - ei\frac{g_\alpha}{2}((T^\alpha z)_a\bar{\lambda}^\alpha\chi^a - \overline{\chi^a}\lambda^\alpha(T^\alpha z)^a) \qquad \text{(A.48)}$$

If supersymmetry is broken, the gravitino field will be described by:

$$\psi'_{\mu R} = \psi_{\mu R} - \frac{\kappa^{-1}}{3}\gamma_\mu(\mathcal{G}_{,a}\chi^a + i\frac{g_\alpha\kappa^3}{4}\exp(\frac{\mathcal{G}}{2})(z_a(T^\alpha z)^a)\lambda^\alpha_L) \qquad \text{(A.49)}$$

After substituting the last equation into (A.48) the mass part of the Lagrangian becomes

$$L^{mass} = \frac{e}{\kappa}\exp(-\frac{\mathcal{G}}{2})(\overline{\psi'_\mu}\sigma^{\mu\nu}\psi'_{\nu L}) - \frac{e}{2}(m_{ab}\overline{\chi_a}\chi^b) - em^\alpha{}_a\bar{\lambda}^\alpha\chi^a$$

$$- \frac{e}{2}m^{\alpha\beta}\bar{\lambda}_L{}^\alpha\lambda_L{}^\beta + h\cdot c \qquad \text{(A.50)}$$

where

$$m_{ab} = 2\kappa^{-3}\exp(-\frac{\mathcal{G}}{2})(\mathcal{G}_{,ab} - \frac{1}{3}\mathcal{G}_{,a}\mathcal{G}_{,b})$$

$$m^\alpha{}_a = i\frac{g_\alpha}{2}z_b(T^\alpha)^b{}_a$$

$$m^{\alpha\beta} = -\exp(-\frac{\mathcal{G}}{2})\frac{g_\alpha^2\kappa^3}{24}(z_a(T^\alpha z)^a)(z_b(T^\beta z)^b) \qquad \text{(A.51)}$$

[illegible]

$$V_{,a} = 0 \qquad \text{(A.52)}$$

and the vanishing of the cosmological constant

$$V = 0$$

which are equivalent to

$$\mathcal{G}_{,ab}\mathcal{G}^{,b} = \frac{\kappa^2}{2}\mathcal{G}_{,a}$$

$$(z_a(T^\alpha z)^a) = 0$$

$$\mathcal{G}_{,a}\mathcal{G}^{,a} = \frac{3}{2}\kappa^2 \qquad \text{(A.53)}$$

one can easily prove that [C9]

$$\text{supertrace } M^2 = \sum_{J=0}^{3/2}(-1)^{2J}(2J+1)m_J^2 = 2(N-1)m_{3/2}^2 \qquad \text{(A.54)}$$

where N is the number of chiral multiplets. This can be seen by noting that the gauge particle contributions are the same as in global supersymmetry while the rest (obtained by letting $g_\alpha = 0$) are given by

$$tr(M^2)^A + tr(M^2)^B - 2\,tr\,m_{ab}^2 - 4m_{3/2}^2 = 4V_{,a}{}^a - \frac{8}{\kappa^6}\exp(-\mathcal{G})(\mathcal{G}_{,ab} - \frac{1}{3}\mathcal{G}_{,a}\mathcal{G}_{,b})$$

$$\cdot(\mathcal{G}^{,ab} - \frac{1}{3}\mathcal{G}^{,a}\mathcal{G}^{,b}) - \frac{4}{\kappa^2}\exp(-\mathcal{G}) \qquad \text{(A.55)}$$

This is equal to, after substituting in Eqs. (A.47) and (A.55),

$$2\chi^{-2}(N - 1)\exp(-\mathcal{G}) \qquad (A.56)$$

Finally, we note that the existence of different sets of auxiliary fields for N = 1 supergravity, poses the question whether the results we obtained remain valid for the other sets.

Kugo and Uehara [C11] have introduced a method based on superconformed tensor calculus [C15] that can accomodate the different sets of auxiliary fields. It was later proved [C12] that all interactions constructed in terms of the other known sets of auxiliary fields are particular examples of the interactions constructed here with the minimal set.

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Figure Captions

Fig. (1): $e^+e^- \to \gamma\tilde{\gamma}\tilde{\gamma}$ process [Fig. (1a)] and competing background $e^+e^- \to \gamma\nu\bar{\nu}$ [Fig. (1b)].

Fig. (2): $e^+e^- \to e^+e^-\tilde{\gamma}\tilde{\gamma}$ process [Fig. (2a)] and competing background $e^+e^- \to e^+e^-\nu\bar{\nu}$.

Fig. (3): Radiative corrections to the gaugino masses arising from Eq. (5.10) [Fig. (3a)] and from Eq. (5.11) [Fig. (3b)].

Fig. (4): Source of significant direct gaugino masses due to the exchange of heavy fields of the GUT sector.

Fig. (5): The ρ parameter as a function of the top quark mass in Supergravity unified theory.

Fig. (6): Decay modes of the Winos and Zinos. In addition to the squark ($\tilde{q}$) intermediate states, the decays can proceed also through sleptons when the final states contain a photino ($\tilde{g}$ = gluino, $\tilde{\ell}$ = lepton).

Fig. (7): Decays with Unidentified Fermionic Objects (UFO), the photinos, with [illegible] jets.

Fig. (8): Lepton-jet decays with [illegible]

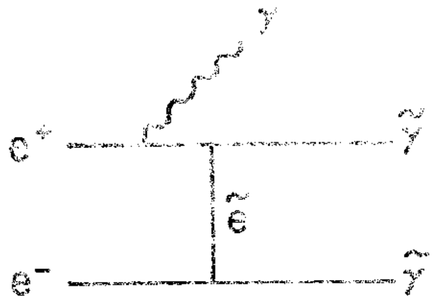


Fig. (1a.)

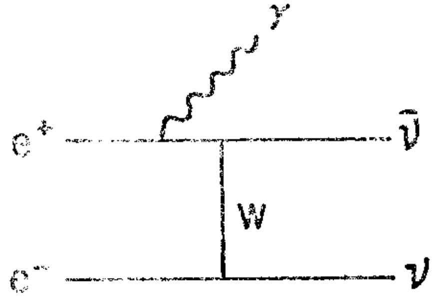


Fig. (1b.)

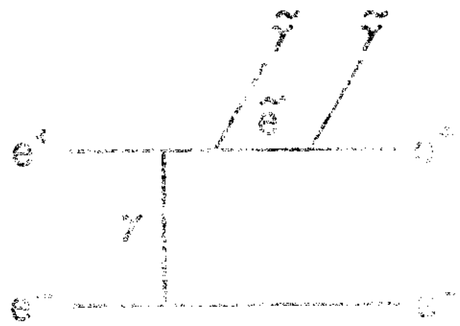
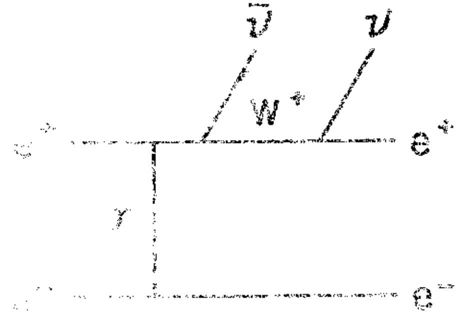


Fig. (2a.)



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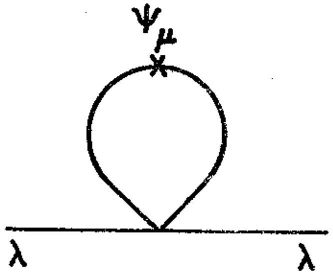


Fig (3a)

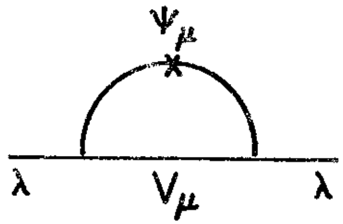


Fig (3b)

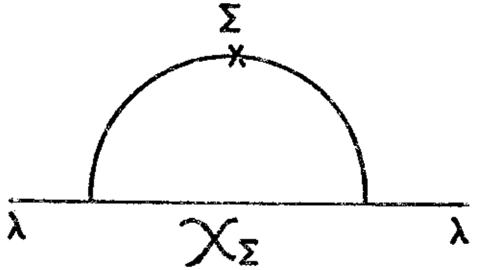


Fig. (4)

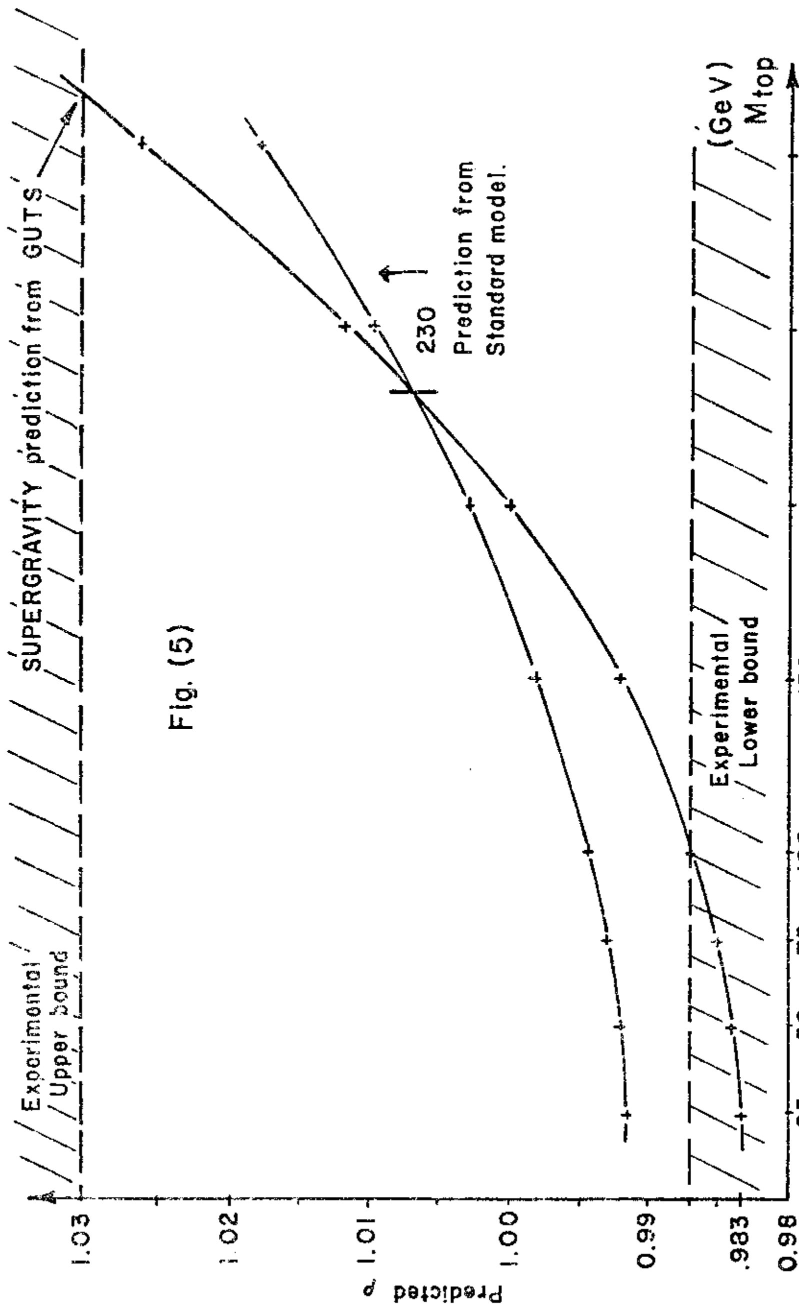
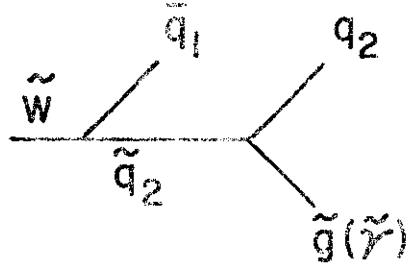
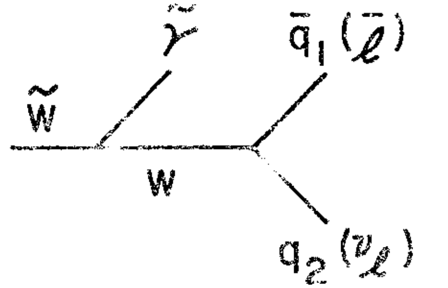


Fig. (5)



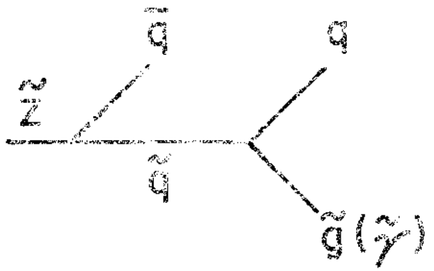
$\tilde{W} \to \bar{q}_1 + q_2 + \tilde{g}\,(\tilde{\gamma})$

(6a)



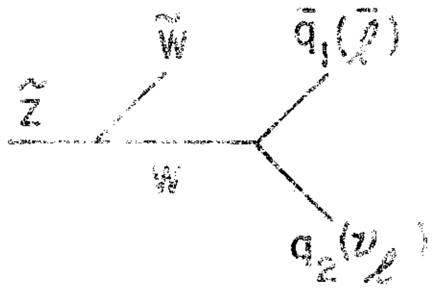
$\tilde{W} \to \tilde{\gamma} + \bar{q}_1(\bar{\ell}) + q_2(\nu_\ell)$

(6b)



$\tilde{Z} \to \bar{q} + q + \tilde{g}\,(\tilde{\gamma})$

(6c)



$\tilde{Z} \to \tilde{W} + \bar{q}_1(\bar{\ell}) + q_2(\nu_\ell)$

(6d)

Fig. (6.)

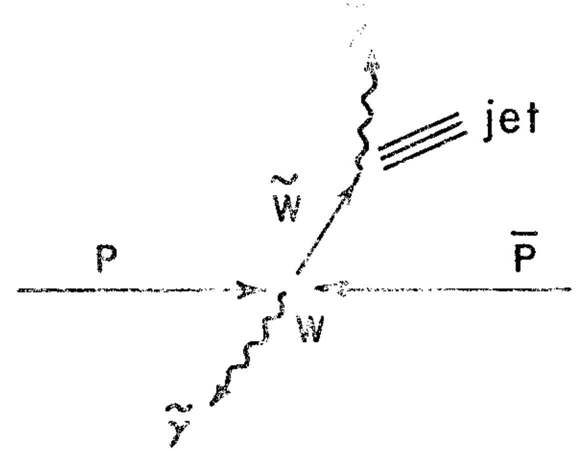


Fig. (7a): One jet UFO event in $W \to \tilde{W}\tilde{\gamma}$

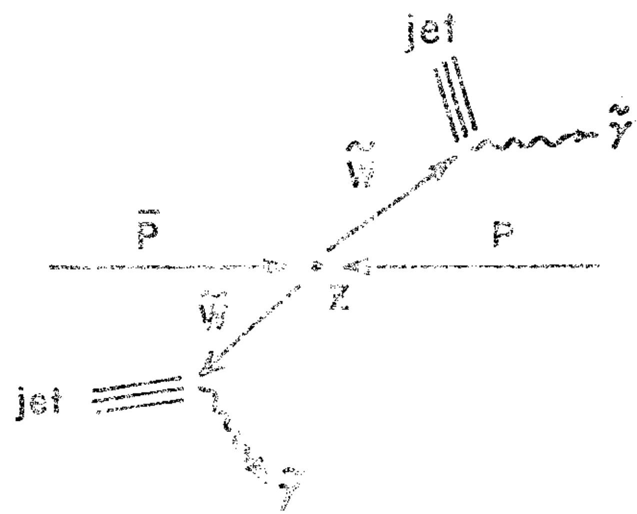


Fig. (7b): Two jet UFO event in $Z \to \tilde{W} + \tilde{W}$

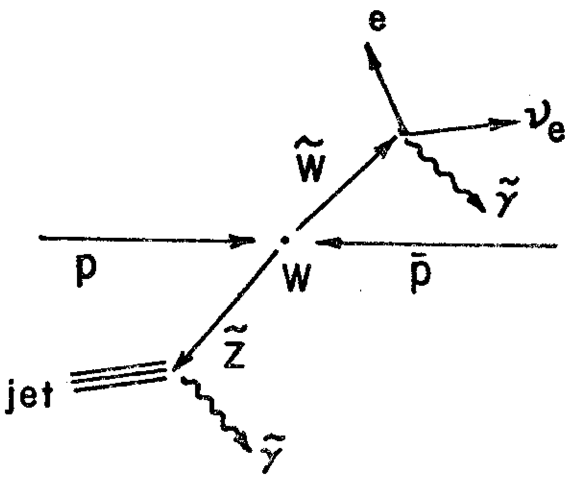


Fig. (8): Lepton-jet event in $W \to \tilde{W} + \tilde{Z}$

Replace $f^{\dagger}(Z^{\dagger})$ by $f^{\dagger}(Z)$ in Eq. (2.18)

In the line after Eq. (2.25), $(d^{-1})_b{}^a$ and $(d)_{,a}{}^b$ should be replaced by $(d^{-1})^a{}_b$ and $(d)_{,}{}^b{}_a$ respectively.

In Eq. (3.9) all G's should be script, D_μ should be script and the argument in the exponential is $-\mathcal{G}/2$. In Eq. (3.10) all G's should be script and $D_\mu Z^a$, $D_\mu Z_a$ should have script D_μ.

In Eq. (3.13), k should be replaced by k^2.

In the last line of page (38), 2 X 10^{-19} gm/cm^3 should read 2 X 10^{-29} gm/cm^3.

The first line of Eq. (7.14) should read:

$$\tilde{W}_+ = i \cos \delta_- \tilde{H}^1 + i \sin \delta_+ \tilde{H}'{}_1{}^c - \sin\gamma_- \lambda_L + \cos \gamma_+ \lambda_R$$

The equation for $\tilde{W}_-$ holds for $\sin 2\alpha \geq \mu\tilde{m}_2/M_W{}^2$. For $\sin 2\alpha < \mu\tilde{m}_2/M_W{}^2$ replace $\tilde{W}_-$ by $\gamma_5 \tilde{W}_-$.

In the bracket of Eq. (7.28) replace "1" by "$1 + \mu^2/m_1{}^2$".

Inside the square root of Eq. (7.29), replace $(M_Z{}^2 + M_{H^o}{}^2)$ by $(M_Z{}^2 + 2M_{H^o}{}^2)^2$ and cos 2α by $(\cos 2\alpha)^2$.

Replace "60 GeV" by "(60 GeV)^2" in Eq. (7.31).

Replace "i" by "-i" in Eq. (8.12).

Add to the r.h.s. of Eq. (8.13) the term $ig_3(t^r/2)_{ij}[\bar{u}_i P_- \lambda_r \tilde{u}_{jR} + \bar{d}_i P_- \lambda_r \tilde{d}_{jL}]$ + h.c.

The matrix M_{ij} of Eq. (7.19) should read $M_{44} = -\mu \sin 2\alpha$ and $M_{54} = \mu'$.

In the first term of Eq. (A.42), Ψ_μ should read $\bar{\Psi}_\mu$.

Eq. (A.43): The right hand side of the last term in $\delta_s \Psi_{\mu L}$ should have an ε_L. The third term of $\delta_s \chi^a$ should have an ε_L.

Notation: Secs. VII and VIII use the Lorentz metric diag $\eta_{\mu\mu} = (-1, +1, +1, +1)$ and standard left handed Weyl spinors (projected by $P_- = (1 - \gamma_5)/2, \gamma_5{}^{\dagger} = \gamma_5$). The discussion of the supergravity - matter couplings, Sec. II and App. A, are in the notation of [C4] and [C7].

ERRATA

Replace $f^{\dagger}(Z^{\dagger})$ by $f^{\dagger}(Z)$ in Eq. ([illegible].18)

In the line after Eq. (2.25), $(d^{-1})_b{}^a$ and $(d)_a{}^b$ should be replaced by $(d^{-1})^a{}_b$ and $(d)^b{}_a$ respectively.

In Eq. (3.9) all G's should be script, D_μ should be script and the argument in the exponential is $-\mathcal{G}/2$. In Eq. (5.10) all G's should be script and $D_\mu Z^a$, $D_\mu Z_a$ should have script D_μ.

In Eq. (5.13), k should be replaced by k^2.

In the last line of page (38), 2×10^{-19} gm/cm^3 should read 2×10^{-29} gm/cm^3.

The first line of Eq. (7.14) should read:

$$\tilde{W}_+ = i \cos \gamma_- \tilde{H}^1 + i \sin \gamma_+ \tilde{H}'_1{}^c - \sin\gamma_- \lambda_L + \cos \gamma_+ \lambda_R$$

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In the bracket of Eq. (7.28) replace "1" by "$1 + \mu^2/m_i{}^2$".

Inside the square root of Eq. (7.29), replace $(M_Z{}^2 + M_{H^0}{}^2)$ by $(M_Z{}^2 + M_{H^0}{}^2)^2$ and $\cos 2\alpha$ by $(\cos 2\alpha)^2$.

Replace "60 GeV" by "(60 GeV)^2" in Eq. (7.31).

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[illegible]

In the last line of Eq. (7.24) $m_{\tilde{u}R}$ should read $m_{\tilde{d}R}$

In the second line after Eq. (8.15) replace "...the choice $m_{\tilde{g}} = \sqrt{2}\, M_W$" by "$\tilde{m}_\gamma = 2$ and $\tilde{m}_\gamma = 7$ GeV".

In captions on Tables 2 and 4 add "The Analysis is for $m_{\tilde{g}} = \sqrt{2}\, M_W$". In the caption on Table 3 add "Wino decays are for $m_{\tilde{g}} = 150$ GeV and $\tilde{Z}(3)$ decays are for $m_{\tilde{g}} = \sqrt{2}\, M_W$". In the caption on Table 5 add "Branching ratios through the Wino pole are for $m_{\tilde{g}} = 150$ GeV and through the $\tilde{Z}(3)$ poles are for $m_{\tilde{g}} = \sqrt{2}\, M_W$".

II. MONOPOLES

THE MAGNETIC MONOPOLE

S. Coleman

(Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA)

Ref: S.C. "The Magnetic Monopole Fifty Years Later"

<u>Plan</u>

① Abelian monopoles from the outside

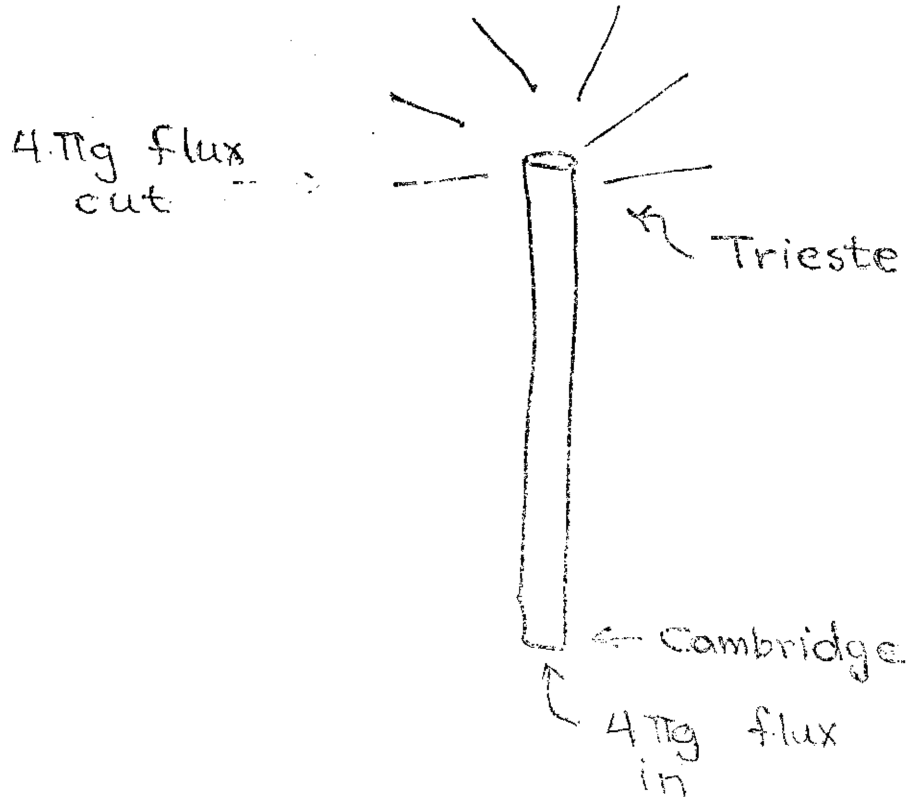
② Nonabelian monopoles from the outside

③ Inside the monopole

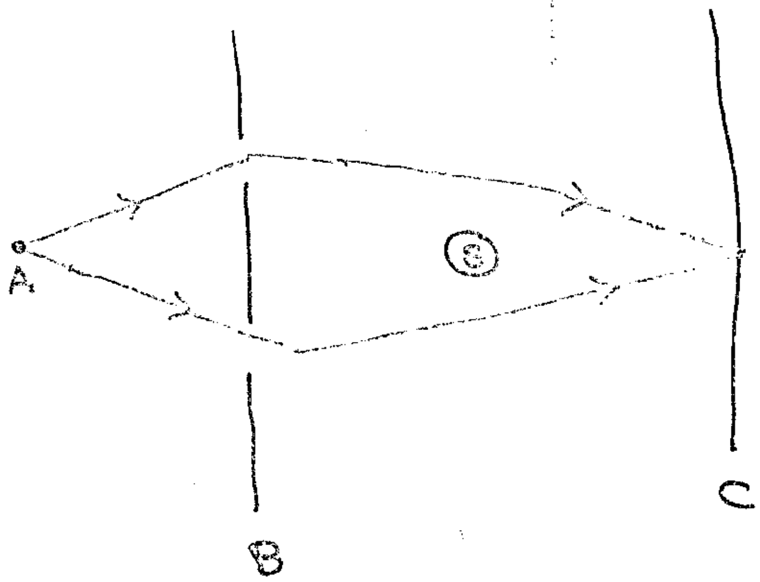
④ Quantum monopoles

The monopole hoax

$$\vec{B} = \frac{g\,\hat{r}}{r^2}$$



Exposing the Hoax by the Bohm-Aharonov Effect



$$\rho = |\psi_1 + e^{ie\Phi}\,\psi_2|^2$$

$$\Phi = 4\pi g$$

∴ solenoid is not detected if

$$\boxed{eg = 0, \pm \tfrac{1}{2}, \pm 1 \ldots} \Leftarrow \text{Dirac's Condition}$$

Gauge Invariance

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}(\vec{\nabla} - ie\vec{A})^2\psi + V\psi$$

$$\psi \to e^{-ie\chi}\,\psi$$

$$\vec{A} \to \vec{A} - \frac{i}{e}e^{ie\chi}\vec{\nabla}e^{-ie\chi}$$

N.B. Only $e^{ie\chi}$ is relevant not χ itself

FOR MONOPOLE

$$\vec{A}\cdot d\vec{x} = g(1-\cos\theta)d\phi$$

CHECK: $A_\phi = g(1-\cos\theta)$

$$F_{\theta\phi} = \partial_\theta A_\phi = g\sin\theta$$

$$F_{\theta\phi}\,d\theta d\phi = \frac{g}{r^2}\,r^2\sin\theta\,d\theta d\phi$$

$\vec{A}$ is singular at $\theta = \pi$ ("Dirac string")

$$\vec{A}\cdot d\vec{x} = g(1-\cos\theta)d\phi \qquad \theta = \pi$$

$$\vec{A}'\cdot d\vec{x} = -g(1+\cos\theta)d\phi \qquad \theta \neq 0$$

Wu-Yang stratagem: Use

$$\left\{\begin{matrix}\vec{A}\\ \vec{A}'\end{matrix}\right\} \text{ for } \left\{\begin{matrix}\theta < \frac{\pi}{2}+\epsilon\\ \theta > \frac{\pi}{2}-\epsilon\end{matrix}\right\}.$$

$\vec{A}$ above red line

$\vec{A}'$ below black line

$$(\vec{A}-\vec{A}')\cdot d\vec{x} = 2g\,d\phi = \vec{\nabla}\chi\cdot d\vec{x}$$

where $\chi = 2\phi g$

$$e^{ie\chi} = e^{i2eg\phi}$$

single-valued if $eg = 0, \pm\frac{1}{2}, \pm 1$ etc.

Fields or particles?

$\hbar$ $\quad e_{field} = e_{part}$

$g\, e_{field} = n/2$ Classical Effect

$g\, e_{part} = n\hbar/2$ Quantum Effect

Yang-Mills Theory Reviewed

$g(x)$: $\quad \phi(x) \to g(x)\,\phi(x) \qquad g \in G$

$$A_\mu = f\, A_\mu^a T^a$$

↑ gauge coupling

$$[T^a, T^b] = C^{abc} T^c$$

$$A_\mu = -A_\mu^\dagger$$

$$D_\mu \phi = (\partial_\mu + A_\mu)\phi$$ "covariant derivative"

g: $\quad D_\mu \phi \to g(x)\, D_\mu \phi$

if

$$A_\mu \to g A_\mu g^{-1} + g\,\partial_\mu g^{-1}$$

Field Strength:

$$[D_\mu, D_\nu]\phi = F_{\mu\nu}\phi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

$g(x)$: $\quad F_{\mu\nu} \rightarrow g F_{\mu\nu} g^{-1}$

$$D_\lambda F_{\mu\nu} = \partial_\lambda F_{\mu\nu} + [A_\lambda, F_{\mu\nu}]$$

Pure (no "matter) Yang-Mills Eqs.

$$\boxed{D_\mu F^{\mu\nu} = 0}$$

comes from

$$\mathcal{L} = \frac{1}{4f^2} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu}$$

Gauging A_0 to zero (a prototype)

$$A_0 \rightarrow g A_0 g^{-1} + g \partial_0 g^{-1} = 0$$

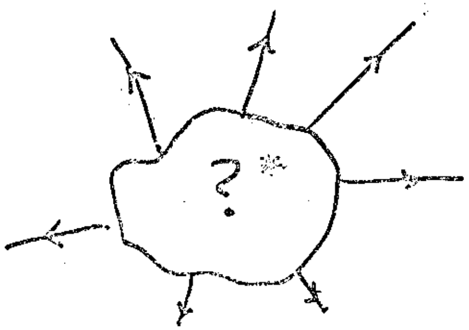
$$\partial_0 g^{-1} = -A_0 g^{-1}$$

$$g^{-1}(\vec{x}, t) = T \exp - \int_0^t dt' A_0(\vec{x}, t')$$

N.B. One is still free to make time-independent gauge transformations

End of Review

Dynamical ([illegible]) Classification of Monopoles



* to be studied in Sec. 3

Outside the black box $\vec{A}$

(1) is time independent

(2) is time reversal invariant

(3) obeys the pure Y. M. Eqs.

What does $\vec{A}$ look like as $r \to \infty$.

$\partial_0 A_\mu = 0$ (time indep.)

$A_0 = 0$ (time-reversal inv.)

$A_r = 0 \quad r > 1$ (gauge choice)

$\vec{A} = \dfrac{\vec{a}(\theta,\phi)}{r} + \cdots$ (asymptotic expansion)

Ignore $\cdots$ (Consistent with Y-M Eq.)

$\vec{A} \cdot d\vec{x} = A_\theta(\theta,\phi)\, d\theta + A_\phi(\theta,\phi)\, d\phi$

$A_\theta = 0$ (gauge choice, but may introduce string singularity at $\theta = \pi$)

Solving the field equations

$$F_{\theta\phi} = \partial_\theta A_\phi$$

$$\partial_\mu \sqrt{g}\, F^{\mu\nu} + [A_\mu, \sqrt{g}\, F^{\mu\nu}] = 0$$

$$\sqrt{g}\, F^{\theta\phi} = \frac{1}{r^2 \sin\theta}\, \partial_\theta A_\phi$$

$$\partial_\theta \frac{1}{\sin\theta} \partial_\theta A_\phi = 0$$

$$A_\phi = \frac{i}{2} Q(\phi)(1-\cos\theta) \qquad [Q = Q^+]$$

$$\partial_\phi \sqrt{g}\, F^{\phi\theta} + [A_\phi, \sqrt{g}\, F^{\phi\theta}] = 0$$

$$\Longrightarrow \partial_\phi Q = 0$$

$$\boxed{\vec{A}\cdot d\vec{x} = \frac{i}{2} Q(1-\cos\theta)\, d\phi}$$ ← G.t.: fn. 14

$$\vec{A}'\cdot d\vec{x} = -\frac{i}{2} Q(1+\cos\theta)\, d\phi$$

$$g = \exp iQ\phi$$

$$e^{i 2\pi Q} = 1$$

or **Q has integer eigenvalues**

↑ GNO Quantization Cond.

E.g. 1: $G = U(1)$

$$\boxed{Q = 0, \pm 1, \pm 2 \ldots}$$

c.f. $Q_{(\text{sec } 2)} \Longleftrightarrow 2eg_{(\text{sec } 1)}$

E.g. 2. $G = SU(n)$

$$Q = \text{diag}\,(q_1, q_2 \ldots q_n) \quad \left(\begin{array}{c}\text{global}\\ \text{gauge}\\ \text{transf.}\end{array}\right)$$

q_r must be integers $\quad (\sum q_r = 0)$

E.g. 3 $G = SU(n)/Z_n$

$$Z_n = \{1, e^{2\pi i/n}, e^{4\pi i/n}, \ldots\}$$

Occurs if all fields transform like adjoint rep:

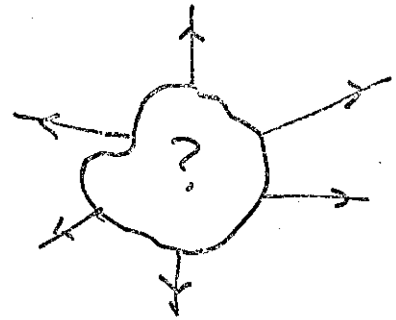
$$g \in SU(n): \quad \phi \to g\phi g^{+}$$

eigenvalues of $Q^{adj} = q_r - q_s$

q_r = integer + const.

$\text{const.} = 0, \frac{1}{n}, \frac{2}{n} \ldots \frac{n-1}{n}$

Topological (Lubkin) Classification of Monopoles



Outside the black box $\vec{A}$ is continuous. We do not assume time-independence, YM Eqs., etc.

Still, for a fixed sphere of radius r at time t, we can still gauge so

$$A_\theta = 0$$

this may still produce a string singularity at $\theta = \pi$,

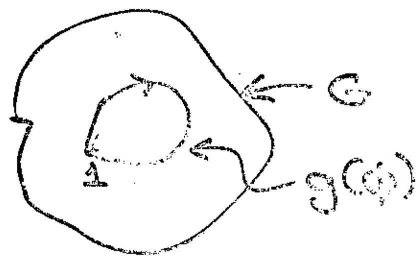
$A_\phi(\theta = \pi, \phi) \neq 0$.

We define $g(\phi)$ by

$$g(0) = 1$$

$$\frac{dg}{d\phi} = A_\phi(\theta=\pi, \phi)\, g$$

If the string is to be unobservable, $g(2\pi) = 1$

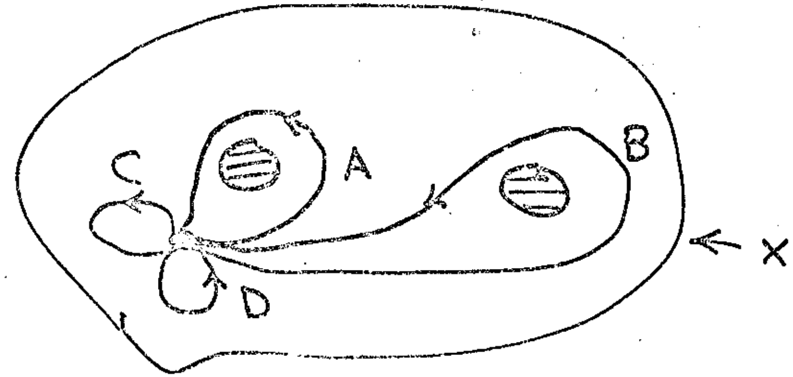


For GNO monopole, $g(\phi) = e^{iQ\phi}$

As we change r or t $g(\phi)$ changes, but continuously, so its homotopy class remains unchanged.

Homotopy?

Homotopy Theory – A short course



Two paths are homotopic if one can continuously be deformed into another. In the drawing, only C and D are homotopic ($\equiv$ in the same homotopy class.)

Multiplication of paths

$\Longrightarrow$ " " homotopy classes

$\Longrightarrow$ (first) homotopy group, $\pi_1(X)$. *

End of course

Separating monopoles $\Longrightarrow$

Some examples

	G	$\pi_1(G)$
1)	$U(1)$	$\mathbb{Z}$
2)	$SU(n)$	1 (simply connected)
3)	$SU(n)/\mathbb{Z}_n$	$\mathbb{Z}_n$

Except for 1) there are many GNO fields in each topological class.

[illegible]

The Big Surprise (Brandt + Neri)

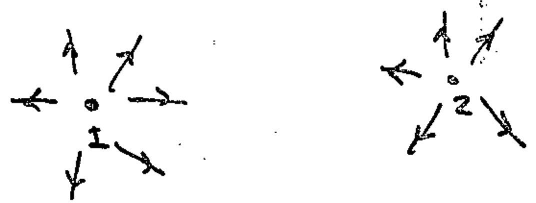
There is one and only one stable GNO field in each topological class.

The only stability is topological stability

↑

The 1st key to monopole theory.

Application: Force between two $SU(n)/Z_n$ monopoles:



Do they attract or repel?

$$d\vec{x}\cdot\vec{A}_1 = \frac{i}{2} Q_1 \,(1-\cos\theta_1)\, d\phi_1$$

$$d\vec{x}\cdot\vec{A}_2 = \frac{i}{2} Q_2 \,(1-\cos\theta_2)\, d\phi_2$$

$\vec{A}_1 + \vec{A}_2$ solves YM Eqs. iff

$$[Q_1, Q_2] = 0$$

$$Q_1 = \mathrm{diag}\,(\lambda_1^{(1)} \cdots \lambda_n^{(1)})$$

$$Q_2 = \mathrm{diag}\,(\lambda_1^{(2)} \cdots \lambda_n^{(2)})$$

Even if we fix these

we can always permute these

Pick one ordering as standard – denote the others as $Q_2^P \Leftrightarrow A_2^P$, P some permutation.

$$\vec{A} = \vec{A}_1 + \vec{A}_2^P$$

as $r \to \infty$

$$\vec{A}\cdot d\vec{x} = \frac{i}{2}(Q_1 + Q_2^P)\,(1-\cos\theta)\, d\phi$$

Only one of these can be stable. Which one?

$$E^{int} \propto \mathrm{Tr}\, Q_1 Q_2^P$$

$$\sum_P E^{int} \propto \mathrm{Tr}\, Q_1 \sum_P Q_2^P = 0$$

(because $\mathrm{Tr}\, Q_2 = 0$)

$\therefore$ Unique stable P = lowest E^{int} $= E^{int} < 0$.

The force is always attractive

SBGFT — A review

ϕ a set of scalar fields

$$\mathcal{L} = \mathcal{L}_{\text{Yang-Mills}} + \tfrac{1}{2} D_\mu\phi \cdot D^\mu\phi - U(\phi) + \cdots$$

↑ fermions, etc.

$U \geq 0$ $\quad U = 0$ for $\phi = \langle\phi\rangle$

$H \subset G$, "the unbroken group" defined by

$h \in H$ iff $h\langle\phi\rangle = \langle\phi\rangle$

Vector mesons associated with H remain massless; others acquire a mass.

An example (Georgi-Glashow Electroweak Model)

$G = SO(3)$ "isospin"

$\vec{\phi}$ "isovector" $\quad \vec{\phi} = (\phi^1, \phi^2, \phi^3)$

$$U = \frac{\lambda}{4}\left(\vec{\phi}\cdot\vec{\phi} - c^2\right)^2$$

$\langle\phi^3\rangle = c \qquad \langle\phi^1\rangle = \langle\phi^2\rangle = 0$

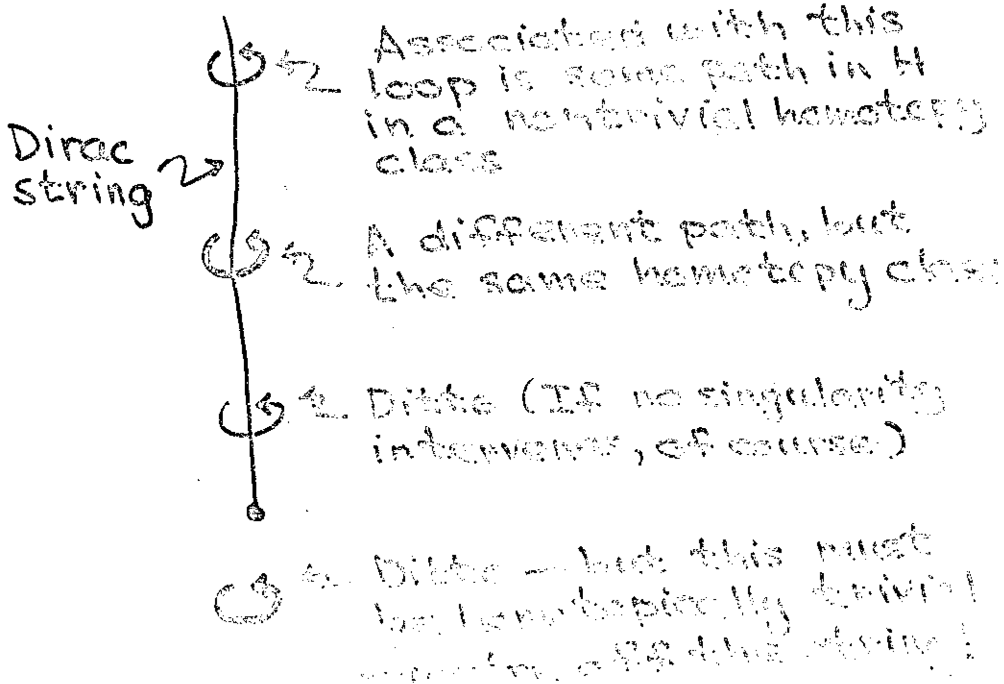
$H = SO(2)$ (rotations about the 3-axis)

only one massless gauge meson "the photon"

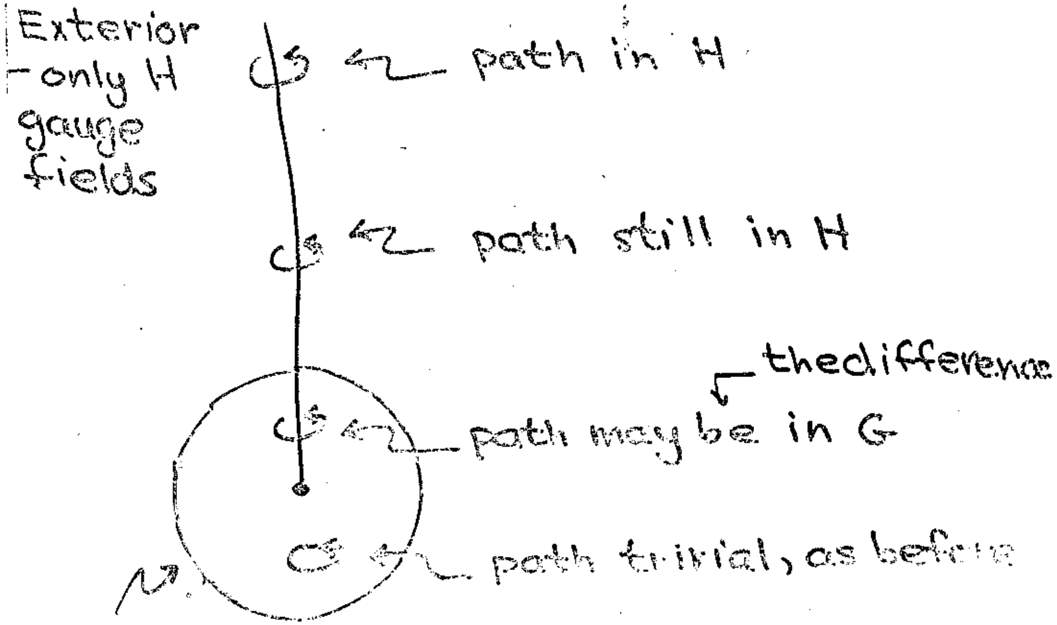
End of Review

Theory of monopoles at large r same as before, except $G_{sec 2} \equiv H_{sec 3}$. In particular, at large r, a stable monopole must be associated with a nontrivial element of $\pi_1(H)$.

Why the monopole would be singular if H were not imbedded in G



Why things are different if H is in G



Monopole core - all G gauge fields can be non zero
Core size $\sim \mu^{-1}$ where μ = heavy gauge meson mass

A path that is homotopically nontrivial in H may be trivial in G.

Example: $G = SO(3)$ $H = SO(2)$
(GG electroweak)

H allows

$$eg = \underline{0}, \pm 1/2, \underline{\pm 1}, \pm 3/2, \underline{\pm 2}, \ldots$$

the underlined entries are OK.
(c.f. 't Hooft / Polyakov)

In general,

$\ker \pi_1(H) \to \pi_1(G)$ is necessary for nonsingularity

← 2nd key to monopole theory

Comments:

(1) It's also sufficient

(2) Nonsingularity $\neq$ time independence

Why monopoles are heavy

Consider for simplicity $H = U(1)$

$$\vec{B} = \frac{g\,\hat{r}}{r^2} \qquad r \gtrsim \mu^{-1}$$

$$E \gtrsim \frac{1}{2} \int_{r > \mu^{-1}} d^3\vec{r}\ |\vec{B}|^2$$

$$\sim g^2 \mu \sim \frac{\mu}{e^2}$$

More massive than the massive gauge mesons by $\frac{1}{e^2}$
[$\frac{1}{g^2}$ in non-Abelian case]

Grand unified monopoles

Georgi-Glashow SU(5) model

$G = SU(5) \qquad H = SU(3)_{color} \times U(1)_{em} / Z_6$

$$g = \left(\begin{array}{c|c} \begin{matrix} 3\times 3 \\ SU(3) \end{matrix} & 0 \\ \hline 0 & 1 \end{array} \right) \Leftarrow \text{embedding of } SU(3)$$

$$Q_{em} = \mathrm{diag}\left(\tfrac{1}{3}, +\tfrac{1}{3}, \tfrac{1}{3}, -1, 0\right)$$

Two possibilities

$$Q_1 = 3 Q_{em}$$

pure electromagnetic monopole

$$Q_2 = Q_{em} + Y_{color}$$

$$Y_{color} = \mathrm{diag}\left(\tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, 0, 0\right)$$

$$Q_2 = \mathrm{diag}(1, 0, 0, -1, 0)$$

chromomagnetic/electromagnetic monopole

Which is lighter?

Estimate the energy outside the core:

$$E \propto \mathrm{Tr}\, Q^2$$

$$Q_1 = \mathrm{diag}(1, 1, 1, -3, 0)$$

$$\mathrm{Tr}\, Q_1^2 = 1+1+1+9 = 12$$

$$Q_2 = \mathrm{diag}(1, 0, 0, -1, 0)$$

$$\mathrm{Tr}\, Q_2^2 = 1+1 = 2 \quad \Leftarrow \text{The winner!}$$

Monopole is a hadron!

What is the classical limit?

Pure Yang-Mills Theory:

$$\mathcal{L} = A^2 + f A^3 + f^2 A^4$$

(all consts., indices, derivatives suppressed)

Similar power counting applies to scalar fields if we say λ (in $\lambda\phi^4$) = const. f^2

$$\mathcal{L} = \cdots + \phi^2 + f\phi^2 A + f^2\phi^2 A^2 + f^2\phi^4.$$

$$fA \equiv A' \qquad f\phi \equiv \phi'$$

$$\mathcal{L}(\phi, A, f) = f^{-2}\,\mathcal{L}(\phi', A', 1)$$

In classical physics f is an irrelevant parameter.

In QM the relevant object is

$$\frac{1}{\hbar}\mathcal{L}(\phi, A, f) = \frac{1}{f^2\hbar}\,\mathcal{L}(\phi', A', 1).$$

	$\hbar \to 0$	fixed f
$\Leftrightarrow$	$f \to 0$	fixed $\hbar$

For simplicity I will

(1) Set $\hbar = 1$

(2) Keep track only of ϕ' (including A' is straightforward)

(3) Drop primes.

$$\mathcal{L} = \left[\tfrac{1}{2}(\partial_0\phi)^2 - \tfrac{1}{2}(\vec{\nabla}\phi)^2 - U(\phi)\right]/f^2$$

Assume has time-independent solution $\phi = \phi_0(\vec{x})$ (like a monopole)

$$\pi = \frac{\partial \mathcal{L}}{\partial(\partial_0\phi)} = \frac{1}{f^2}\,\partial_0\phi$$

$$H = \int d^3\vec{x}\,\left[\pi\,\partial_0\phi - \mathcal{L}\right]$$

$$= \int d^3\vec{x}\,\left[\tfrac{1}{2}f^2\pi^2 - \frac{1}{2f^2}(\vec{\nabla}\phi)^2 - \frac{1}{f^2}U(\phi)\right]$$

$$f^2 H = \int d^3\vec{x}\,\tfrac{1}{2}f^4\pi^2 + V[\phi]$$

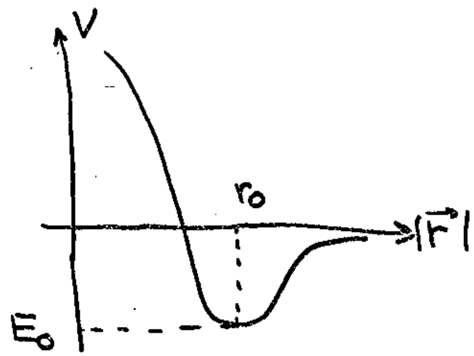
Note: (1) $\phi = \phi_0$ minimum of functional V.

(2) f^2 multiplies H. (trivial)

(3) Small parameter, f^4, multiplies kinetic energy not potential energy. We have [illegible]

The diatomic molecule

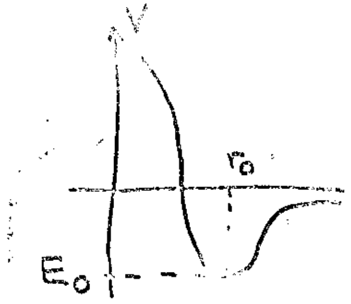
$$H = \frac{\vec{p}^{\,2}}{2M} + V(\vec{r})$$



Analogies

Molecule	Monopole
H	$f^2 H$
$\vec{p}^{\,2}$	$\frac{1}{2}\int d^3\vec{x}\,\pi^2$
$V(\vec{r})$	$V[\phi]$
r_0	ϕ_0
$1/M$	f^4

More degrees of freedom, [illegible]



Order	Eigenstate	Eigenvalue
0	$\vert r_0, \theta, \phi\rangle$	E_0
1	$\vert n, \theta, \phi\rangle$	$+ (n+\frac{1}{2})\sqrt{V''(r_0)/M}$ ("Vibrational levels")
2	$\vert n, \ell, m\rangle$	$+ \ell(\ell+1)/2Mr_0^2 + \cdots$ ("rotational levels")

1) Expansion parameter is $\sqrt{1/M} \longleftrightarrow f^2$

2) "Rotational" spectrum depends on degeneracy of classical solutions under (unbroken) symmetries of theory (c.f. polyatomic molecule).

3) "Vibrational" spectrum depends on number of [illegible] normal modes

Assume for the moment that there is only translational degeneracy

$$\phi_0(\vec{r};\vec{a}) = f(\vec{r}-\vec{a})$$

$$V[\phi_0] \equiv V_0$$

Order	Eigenstate	Eigenvalue
0	$\vert\vec{a}\rangle$	V_0/f^2
1	$\vert\vec{a}, n_1, n_2 \cdots\rangle$	$+ \sum_i (n_i+\frac{1}{2})\omega_i$
2	$\vert\vec{p}, n, \cdots\rangle$	$+ \frac{f^2}{2V_0}\vert\vec{p}\vert^2$

$$|\vec{p}\rangle = \int d^3\vec{a}\; e^{-i\vec{p}\cdot\vec{a}}\, |\vec{a}\rangle/(2\pi)^{3/2}$$

If the classical solution is not invariant under H, there are "translational levels"

(Rotation is to isorotation as spin is to isospin.)

Example: 't Hooft-Polyakov Monopole

Fields in unitarity gauge:

$$W_\mu^\pm \quad A_\mu \quad \phi$$

Action of electromagnetic U(1):

$$\alpha: \begin{Bmatrix} W_\mu^\pm \\ A_\mu \\ \phi \end{Bmatrix} \rightarrow \begin{Bmatrix} e^{\pm i\alpha} W_\mu^\pm \\ A_\mu \\ \phi \end{Bmatrix}$$

Solution invariant iff $W_\mu^\pm(\vec{r}) = 0$ (all $\vec{r}$); this is not so.

$$\alpha: \quad |\theta, \cdots\rangle \rightarrow |\theta+\alpha, \cdots\rangle$$

$$|n, \cdots\rangle = \int \frac{d\theta}{\sqrt{2\pi}} e^{in\theta} |\theta, \cdots\rangle$$

$$\alpha: \quad |n, \cdots\rangle \rightarrow e^{-in\alpha} |n, \cdots\rangle$$

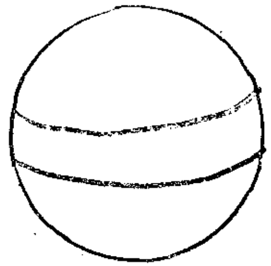
Charge eigenstates!
("Dyons")

$$E = \cdots + a f^2 n^2 + \cdots$$

(a ↑) constant (must be computed)

Do GUTS monopoles have color excitations ("chromodyons")?

They would, if we could define global color rotations. (Abouelsaood; Manohar/Nelson; Balachandran et.al.)



Description A above black line, A' below red line, connected by gauge transformation $g(\theta,\phi)$ in overlap.

In upper region, at some large fixed r, infinitesimal color rotations act on fields at (θ,ϕ) by

$$\lambda_a(\theta,\phi) = h(\theta,\phi)\, \lambda_a(\text{Gell-Mann})\, h(\theta,\phi)^{-1}$$

h continuous $\in SU(3)$ throughout region.

Likewise, in lower region, we define λ'_a, h'.

In overlap

$$\boxed{h(\theta,\phi) = g(\theta,\phi)\, h'(\theta,\phi)}$$

$$g(\theta,\phi) = h(\theta,\phi)\,[h'(\theta,\phi)]^{-1}$$

At $\theta = \pi/2$ (equator)

(1) $h(\frac{\pi}{2},\phi)$ defines a homotopically trivial closed path, because it is obtained by continuous distortion from $h(0,\phi) = \text{const.}$

(2) So does $[h'(\frac{\pi}{2},\phi)]^{-1}$, obtained from $[h'(\pi,\phi)]^{-1} = \text{const.}$

(3) Thus the r.h.s. is homotopically trivial.

(4) But the l.h.s. is nontrivial (there is a monopole).

A CONTRADICTION!

II. TOPOLOGICAL BARYONS AND EFFECTIVE LAGRANGIANS

CURRENT ALGEBRA, BARYONS, AND QUARK CONFINEMENT

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ABSTRACT

It is shown that ordinary baryons can be understood as solitons in current algebra effective Lagrangians. The formation of color flux tubes can also be seen in current algebra, under certain conditions.

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The idea that in some sense the ordinary proton and neutron might be solitons in a nonlinear sigma model has a long history. The first suggestion was made by Skyrme more than twenty years ago.[1] Finkelstein and Rubinstein showed that such objects could in principle be fermions,[2] in a paper that probably represented the first use of what now would be called θ-vacua in quantum field theory. A gauge invariant version was attempted by Faddeev.[3] Some relevant miracles are known to occur in two space-time dimensions;[4] there also exists a different mechanism by which solitons can be fermions.[4]

It is known that in the large N limit of quantum chromodynamics,[5] meson interactions are governed by the tree approximation to an effective local field theory of mesons. Several years ago, it was pointed out[6] that baryons behave as if they were solitons in the effective large N meson field theory. However, it was not clear in exactly what sense the baryons actually <u>are</u> solitons.

The first relevant papers mainly motivated by attempts to understand implications of QCD current algebra were recent papers by Balachandran et. al.[7] and by Boguta.[8]

We will always denote the number of colors as N and the number of light flavors as n. For definiteness we first consider the usual case n=3. Nothing changes for n>3. Some modifications for n<3 are pointed out later. Except where stated otherwise, we discuss standard current algebra with global SU(n)×SU(n) spontaneously broken to diagonal SU(n), presumably as a result of an underlying SU(N) gauge interaction.

Standard current algebra can be described by a field U(x) which -- for each space-time point x -- is a point in the SU(3) manifold. Ignoring quark bare masses, this field is governed by an effective action of the form

$$I = -\frac{F_\pi^2}{16} \int d^4x \; \mathrm{Tr}\, \partial_\mu U \, \partial_\mu U^{-1} + N\Gamma + \text{Higher order terms} \qquad (1)$$

Here Γ is the Wess-Zumino term[9] **which cannot** be written as the integral of a manifestly SU(3)×SU(3) invariant density, and $F_\pi \cong 190$ Mev. In quantum field theory the coefficient of Γ must a priori be an integer,[10] and indeed we will see that the quantization of the soliton excitations of (1) is inconsistent (they obey neither bose nor fermi statistics) unless N is an integer.

Any finite energy configuration U(x,y,z) must approach a constant at spatial infinity. This being so, any such configuration represents an element in the third homotopy group $\pi_3(SU(3))$. Since $\pi_3(SU(3)) \cong Z$, there are soliton excitations, and they obey an additive conservation law. Actually, higher order terms in (1) are needed to stabilize the soliton solutions and prevent them from shrinking to zero size. We will see that such higher order terms (which could be measured in principle by studying meson processes) must be present in the large N limit of QCD and are related to the bag radius. Our remarks will not depend on the details of the higher order terms.

A technical remark is in order. To study solitons, it is convenient to work with a Euclidean space-time M of topology $S^3\times S^1$. Here S^3 represents the spatial variables, and S^1 is a compactified Euclidean time coordinate. A given nonlinear sigma model field U(x) defines a mapping of M into SU(3). We may think of M as the boundary of a five dimensional manifold Q with topology $S^3\times D$, D being a two dimensional disc. Using the fact that $\pi_1(SU(3)) = \pi_4(SU(3)) = 0$, it can be shown that the mapping of M into SU(3) defined by U(x) can be extended to a mapping from Q into SU(3). Then as in ref. (9) the functional Γ is defined by $\Gamma = \int_Q \omega$, where ω is the fifth rank antisymmetric tensor on the SU(3) manifold defined in ref. (9). By analogy with the discussion in ref. (9), Γ is well-defined modulo 2π. (It is essential here that because $\pi_2(SU(3)) = 0$, the five dimensional homology classes in $H_5(SU(3))$ that

can be represented by cycles with topology $S^3\times S^2$ are precisely those that can be represented by cycles with topology S^5. There are closed five-surfaces S in SU(3) such that $\int_S \omega$ is an <u>odd</u> multiple of π, but they do not arise if space-time has topology $S^3\times S^1$ and Q is taken to be $S^3\times D$.)

Now let us discuss the quantum numbers of the current algebra soliton. First, let us calculate its baryon number (which was first demonstrated to be nonzero in reference (7), where, however, different assumptions were made from those we will follow). In previous work[10] it was shown that the baryon number current has an anomalous piece, related to the N Γ term in equation (1). If the baryon number of a quark is 1/N, so that an ordinary baryon made from N quarks has baryon number 1, then the anomalous piece in the baryon number current B_μ was shown to be

$$B_\mu = \frac{\epsilon_{\mu\nu\alpha\beta}}{24\pi^2} \,\mathrm{Tr}(U^{-1}\,\partial_\nu U)\,(U^{-1}\,\partial_\alpha U)\,(U^{-1}\,\partial_\beta U) \qquad (2)$$

So the baryon number of a configuration is

$$B = \int d^3x\; B_0 = \frac{1}{24\pi^2}\int d^3x\; \epsilon^{ijk}\,\mathrm{Tr}(U^{-1}\,\partial_i U)\,(U^{-1}\,\partial_j U)\,(U^{-1}\,\partial_k U) \qquad (3)$$

The right-hand side of equation (24) can be recogized as the properly normalized integral expression for the winding number in $\pi_3(SU(3))$. In a soliton field the right-hand side of (3) equals one, so the soliton has baryon number one; it is a baryon. (In reference (7) the baryon number of the soliton was computed using methods of Goldstone and Wilczek[11]. The result that the soliton has baryon number one would emerge in this framework if the elementary fermions are taken to be quarks.)

Now let us determine whether the soliton is a boson or a fermion. To this end, we compare the amplitude for two processes. In one process, a soliton sits at rest for a long time T. The amplitude is exp-iMT, M being the soliton energy. In the second process, the soliton is adiabatically rotated through a 2π angle in the course of a long time T. The usual term in the Lagrangian $L_0 = -\frac{F_\pi^2}{16}\,\mathrm{Tr}\,\partial_\mu U\,\partial_\mu U^{-1}$ does not distinguish between the two processes, because the only piece in L_0 that contains time derivatives is quadratic in time derivatives, and the integral $\int dt\,\mathrm{Tr}\,\frac{\partial U}{\partial t}\frac{\partial U^{-1}}{\partial t}$ vanishes in the limit of an adiabatic process. However, the anomalous term Γ is linear in time drivatives, and distinguishes between a soliton that sits at rest and a soliton that is adiabatically rotated. For a soliton at rest, $\Gamma=0$. For a soliton that is adiabatically rotated through a 2π angle, a slightly laborious calculation explained at the end of this paper shows that $\Gamma=\pi$. So for a soliton that is adiabatically rotated by a 2π angle, the amplitude is not exp-iMT but exp-iMT $\exp iN\pi = (-1)^N$ exp-iMT.

The factor $(-1)^N$ means that for odd N the soliton is a fermion; for even N it is a boson. This is uncannily reminiscent of the fact that an ordinary baryon contains N quarks and is a boson or a fermion depending on whether N is even or odd.

These results are unchanged if there are more than three light flavors of quarks. How do they hold up if there are only two light flavors? The field U(x) is then an element of SU(2). Because $\pi_3(SU(2))=Z$, there are still solitons. The baryon number current has the same anomalous piece, and the soliton still has baryon number one. But in SU(2) current algebra, there is no Γ term, so how can we see that the soliton can be a fermion?

The answer was given long ago.[2] Although $\pi_4(SU(3)) = 0$, $\pi_4(SU(2)) = Z_2$. With suitably compactified space-time, there are thus two topological classes of maps from space-time to SU(2). In the SU(2) non-linear sigma model, there are hence two "θ-vacua" -- fields that represent the non-trivial class in $\pi_4(SU(2))$ may be weighted with a sign +1 or -1. An explicit field U(x,y,z,t) which goes to 1 at space-time infinity and represents the non-trivial class in $\pi_4(SU(2))$ can (figure (1)) be described as follows (a variant of this description figures in recent work by Goldstone[12]). Start at $t \to -\infty$ with a constant, U=1; moving forward in time, gradually create a soliton-anti-soliton pair and separate them; rotate the soliton through a 2π angle without touching the anti-soliton; bring together the soliton and anti-soliton and annihilate them. Weighting this field with a factor of -1, while a configuration without the 2π rotation of the soliton is homotopically trivial and gets a factor +1, corresponds to quantizing the soliton as a fermion. Thus, internally to SU(2)×SU(2) current algebra, one sees that the soliton can be a fermion. In SU(3)×SU(3) current algebra one finds the stronger result that the soliton <u>must</u> be a fermion if and only if N is odd.

Our results so far are consistent with the idea that quantization of the current algebra soliton describes ordinary nucleons. However, we have not established this. Perhaps there are ordinary baryons and exotic, topologically excited solitonic baryons. However, certain results will now be described which seem to directly show that the ordinary nucleons are the ground state of the soliton.

For simplicity, we will focus now on the case of only twoflavors. Soliton states can be labeled by their spin and isospin quantum numbers -- which we will call J and I, respectively. We will determine semiclassically what values of I and J are expected for solitons. A semiclassical description of current algebra solitons will be accurate quantitatively only in the limit of large N. (Since F_π^2 is proportional to N, N enters the effective Lagrangian (1) as an overall multiplicative factor. Hence, N plays the role usually played by $1/\hbar$.) So we will check the results we find for solitons by comparing to the expected quantum numbers of baryons in the large N limit.

Let us first determine the expected baryon quantum numbers. We make the usual assumption that the multi-quark wave-function is symmetric in space and antisymmetric in color, and hence must have complete symmetry in spin and isospin. The spin-isospin group is $SU(2)\times SU(2) \sim O(4)$. A quark transforms as (1/2, 1/2); this is the vector representation of O(4). We may represent a quark as ϕ_i, where $i=1\ldots4$ is a combined spin-isospin index labeling the O(4) four-vector.

We must form symmetric combinations of N vectors ϕ_i. As is well known, there is a quadratic invariant $\vec{\phi}^2 = \sum_{i=1}^{4} \phi_i^2$. One can also form symmetric traceless tensors of any rank $A^{(p)}_{i_1\ldots i_p} = (\phi_{i_1} \phi_{i_2} \cdots \phi_{i_p}$ - Trace terms); this transforms as (p/2, p/2) under $SU(2)\times SU(2)$. The general symmetric expression that we can make from N quarks is $(\vec{\phi}^2)^k A^{(N-2k)}_{i_1\ldots i_{N-2k}}$, where $0 < k < N/2$. So the values of I and J that are possible are the following:

$$N \text{ even}, \quad I = J = 0,\ 1,\ 2,\ 3,\ \ldots$$

$$N \text{ odd}, \quad I = J = \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2},\ \frac{7}{2},\ \ldots \qquad (4)$$

For instance, in nature we have N=3. The first two terms in the sequence indicated above are the nucleon, of I = J = 1/2, and the delta, of I = J = 3/2. If the number of colors were five or more, we would expect to see more terms in this series. Moreover, simple considerations involving color magnetic forces suggest that, as for N=3, the mass of the baryons in this sequence is always an increasing function of I or J.

Now let us compare to what is expected in the soliton picture. (This question has been treated previously in reference (7).) We do not know the

effective action of which the soliton is a minimum, because we do not know what non-minimal terms must be added to equation (1). We will make the simple assumption that the soliton field has the maximum possible symmetry. The soliton field cannot be invariant under I or J (or any component thereof), but it can be invariant under a diagonal subgroup I+J. This corresponds to an ansatz $U(x) = \exp iF(r)\ \vec{\tau}\cdot\hat{x}$, where $F(r) = 0$ at $r=0$ and $F(r) \to 2\pi$ as $r\to\infty$.

Quantization of such a soliton is very similar to quantization of an isotropic rigid rotor. The Hamiltonian of an isotropic rotor is invariant under an $SU(2)\times SU(2)$ group consisting of the rotations of body fixed and space fixed coordinates, respectively. We will refer to these symmetries as I and J, respectively. A given configuration of the rotor is invariant under a diagonal subgroup of $SU(2)\times SU(2)$. This is just analogous to our solitons, assuming the classical soliton solution is invariant under I+J.

The quantization of the isotropic rigid rotor is well known. If the rotor is quantized as a boson, it has I = J = 0, 1, 2, ... If it is quantized as a fermion, it has I = J = 1/2, 3/2, 5/2, ... The agreement of these results with equation (4) is hardly likely to be fortuitous.

In the case of three or more flavors, it may still be shown that the quantization of collective coordinates gives the expected flavor quantum numbers of baryons. The analysis is more complicated; the Wess-Zumino interaction plays a crucial role.

So far, we have assumed that the color gauge group is SU(N). Now let us discuss what would hapen if the color group were O(N) or Sp(N). (By Sp(N) we will mean the group of N×N unitary matrices of quaternions; thus $Sp(1) \cong SU(2)$.) We will see that also for these gauge groups, the topological properties of the current algebra theory correctly reproduce properties of the underlying gauge theory.

In an O(N) gauge theory, we suppose that we have n multiplets of left-handed (Weyl) spinors in the fundamental N dimensional representation of O(N). There is no distinction between quarks and antiquarks, because this representation is real. (If n is even, the theory is equivalent to a theory of n/2 Dirac multiplets.) The anomaly free flavor symmetry group is SU(n). Simple considerations based on the most attractive channel idea suggest that the flavor symmetry will be spontaneously broken down to O(n), which is the maximal subgroup of SU(n) that permits all fermions to acquire mass. In this case the current algebra theory is based on a field that takes values in the quotient space SU(n)/O(n).

In an Sp(N) gauge theory, we assume the fermion multiplets to be in the fundamental 2N dimensional representation of Sp(N). Since this representation is pseudoreal, there is again no distinction between quarks and antiquarks. In this theory the number of fermion multiplets must be even; otherwise, the Sp(N) gauge theory is inconsistent because of a non-perturbative anomaly[2] involving $\pi_4(Sp(N))$. If there are 2n multiplets, the flavor symmetry is SU(2n). Simple arguments suggest that the SU(2n) flavor group is spontaneously broken to Sp(n), so that the current algebra theory is based on the quotient space SU(2n)/Sp(n). This corresponds to symmetry breaking in the most attractive channel; Sp(n) is the largest unbroken symmetry that lets all quarks get mass.

In O(N), since there is no distinction between quarks and antiquarks, there is also no distinction between baryons and anti-baryons. A baryon can be formed from an antisymmetric combination of N quarks; $B = \epsilon_{i_1 i_2 \ldots i_N} q^{i_1} q^{i_2} \ldots q^{i_N}$. But in O(N), a product of two epsilon symbols can be rewritten as a sum of products of N Kronecker deltas:

$$\epsilon_{i_1 i_2 \ldots i_N}\epsilon_{j_1 j_2 \ldots j_N} = (\delta_{i_1 j_1}\delta_{i_2 j_2}\ldots\delta_{i_N j_N} \pm \text{permutations}).$$

This means that in an O(N) gauge theory, two baryons can annihilate into N mesons.

On the other hand, in an Sp(N) gauge theory there are no baryons at all. The group Sp(N) can be defined as the subgroup of SU(2N) that leaves fixed an antisymmetric second rank tensor γ_{ij}. A meson made from two quarks of the same chirality can be described by the two quark operator $\gamma_{ij} q^i q^j$. In Sp(N) the epsilon symbol can be written as a sum of products of N γ's:

$$\epsilon_{i_1 i_2 \ldots i_{2N}} = (\gamma_{i_1 i_2}\gamma_{i_3 i_4}\ldots\gamma_{i_{2N-1} i_{2N}} \pm \text{permutations}).$$

So in an Sp(N) gauge theory, a single would-be baryon can decay to N mesons.

Now let us discuss the physical phenomena that are related to the topological properties of our current algebra manifolds SU(n)/O(n) and SU(n)/Sp(n). We recall from reference (10) that the existence of SU(n) current algebra with at least three flavors of the Wess-Zumino interaction -- with its a priori quantization law -- is closely related to the fact that $\pi_5(SU(n)) = Z$, $n \geq 3$. The analogue is that

$$\pi_5(SU(n)/O(n)) = Z, \quad n \geq 3$$
$$\pi_5(SU(2n)/Sp(n)) = Z, \quad n \geq 2 \qquad (5)$$

So also the O(N) and Sp(N) gauge theories possess at the current algebra level an interaction like the Wess-Zumino term, provided the number of flavors is large enough. Built into the current algebra theories is the fact that in the underlying theory there is a parameter -- the number of colors -- which a priori must be an integer.

Now we come to the question of the existence of solitons. These are classified by the third homotopy group of the configuration space, and we have

$$\pi_3(SU(n)/O(n)) = Z_2, \quad n \geq 4$$
$$\pi_3(SU(2n)/Sp(n)) = 0, \text{ any } n \qquad (6)$$

Thus, in the case of an O(N) gauge theory with at least four flavors, the current algebra theory admits solitons, but the number of solitons is conserved only modulo two. This agrees with the fact that in the O(N) gauge theory there are baryons which can annihilate in pairs. In current algebra corresponding to Sp(N) gauge theory there are no solitons, just as the Sp(N) gauge theory has no baryons.

For O(N) gauge theories with less than four light flavors we have

$$\pi_3(SU(3)/O(3)) = Z_4$$
$$\pi_3(SU(2)/O(2)) = Z \qquad (7)$$

Thus, the spectrum of current algebra solitons seems richer than the expected spectrum of baryons in the underlying gauge theory. The following remark seems appropriate in this connection. It is only in the multi-color, large N limit that a semiclassical description of current algebra solitons becomes accurate. Actually, large N gauge theories are described by weakly interacting theories of mesons, but it is not only Goldstone bosons that enter; one has an infinite meson spectrum. Corresponding to the rich meson spectrum is an unknown and perhaps topologically complicated configuration space P of the large N theory. Plausibly, baryons can always be realized as solitons in the large N theory, even if all or almost all quark flavors are heavy. Perhaps $\pi_3(P)$ is always Z, Z_2, or 0 for SU(N), O(N), and Sp(N) gauge theories. The Goldstone boson space is only a small subspace of P and would not necessarily reflect the topology of P properly. Our results suggest that as the number of flavors increases, the Goldstone boson space becomes an increasingly good <u>topological</u> approximation to P. In this view, the extra solitons suggested by equation (7) for O(N) gauge theories with two or three flavors become unstable when SU(2)/O(2) or SU(3)/O(3) is embedded in P.

One further physical question will be addressed here. Is color confinement implicit in current algebra?

Do current algebra theories in which the field U labels a point in SU(n), SU(n)/O(n), or SU(2n)/SP(n) admit flux tubes? By a flux tube we mean (figure (2)) a configuration U(x,y,z) which is independent of z and possesses a non-trivial topology in the x-y plane. To ensure that the energy per unit length is finite, U must approach a constant as $x,y \to \infty$. The proper topological classification involves therefore the <u>second</u> homotopy group of the space in which U takes its values. In fact, we have

$$\pi_2(SU(n)) = 0$$
$$\pi_2(SU(n)/O(n)) = Z_2,\ n > 3$$
$$\pi_2(SU(2n)/Sp(n)) = 0 \qquad (8)$$

Thus, current algebra theories corresponding to underlying SU(N) and Sp(N) gauge theories do not admit flux tubes. The theories based on underlying O(N) gauge groups do admit flux tubes, but two such flux tubes can annihilate.

These facts have the following natural interpretation. Our current algebra theories correspond to underlying gauge theories with quarks in the fundamental representation of SU(N), O(N), or Sp(N). SU(N) or Sp(N) gauge theories with dynamical quarks cannot support flux tubes because arbitrary external sources can be screened by sources in the fundamental representation of the group. For O(N) gauge theories it is different. An external source in the spinor representation of O(N) cannot be screened by charges in the fundamental representation. But two spinors make a tensor, which can be screened. So the O(N) gauge theory with dynamical quarks supports only one type of color flux tube -- the response to an external source in the spinor representation of O(N). It is very plausible that this color flux tube should be identified with the excitation that appears in current algebra because $\pi_2(SU(n)/O(n)) = Z_2$.

The following fact supports this identification. The interaction between two sources in the spinor representation of $O(N)$ is, in perturbation theory, N times as big as the interaction between two quarks. Defining the large N limit in such a way that the interaction between two quarks is of order one, the interaction between two spinor charges is therefore of order N. This strongly suggests that the energy per unit length in the flux tube connecting two spinor charges is of order N. This is consistent with our current algebra identification; the whole current algebra effective Lagrangian is of order N (since $F_\pi^2 \sim N$), so the energy per unit length of a current algebra flux tube is certainly of order N.

In conclusion, it still remains for us to establish the contention made earlier that the value of the Wess-Zumino functional Γ for a process consisting of a 2π rotation of a soliton is $\Gamma = \pi$.

First of all, the soliton field can be chosen to be of the form

$$V(x_i) = \left(\begin{array}{c|c} W(x_i) & 0 \\ \hline 0 & 1 \end{array}\right) \tag{9}$$

where the SU(2) matrix W is chosen to be invariant under a combined isospin rotation plus rotation of the spatial coordinate x_i. This being so, a 2π rotation of V in space is equivalent to a 2π rotation of V in isospin. Introducing a periodic time coordinate t which runs from 0 to 2π, the desired field in which a soliton is rotated by a 2π angle can be chosen to be

$$U(x_i, t) = \begin{pmatrix} e^{it/2} & & \\ & e^{-it/2} & \\ & & 1 \end{pmatrix} V(x_i) \begin{pmatrix} e^{-it/2} & & \\ & e^{it/2} & \\ & & 1 \end{pmatrix} \tag{10}$$

Note that $U(x_i, t)$ is periodic in t with period 2π even though the individual exponentials $\exp \pm it/2$ have period 4π. Because of the special form of V, we can equivalently write U in the much more convenient form

$$U(x_i, t) = \begin{pmatrix} 1 & & \\ & e^{-it} & \\ & & e^{it} \end{pmatrix} V(x_i) \begin{pmatrix} 1 & & \\ & e^{it} & \\ & & e^{-it} \end{pmatrix} \tag{11}$$

This field $U(x_i, t)$ describes a soliton that is rotated by a 2π angle as t ranges from 0 to 2π. We wish to evaluate $\Gamma(U)$.

To this end we introduce a fifth parameter ρ $(0 \le \rho \le 1)$ so as to form a five-manifold of which space-time is the boundary; this five-manifold will have the topology of three-space times a disc. A convenient choice is to write

$$\tilde{U}(x_i, t, \rho) = A^{-1}(t, \rho)\, U(x_i, t)\, A(t, \rho) \tag{12}$$

where

$$A(t, \rho) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \rho e^{it} & \sqrt{1-\rho^2} \\ 0 & -\sqrt{1-\rho^2} & \rho e^{-it} \end{pmatrix} \tag{13}$$

Note that at $\rho = 0$, $A(t, \rho)$ is independent of t. So we can think of ρ and t as polar coordinates for the plane -- ρ being the radius and t the usual angular variable. At $\rho = 1$, $\tilde{U}(x_i, t, 1) = U(x_i, t)$ so the product of three-space with the unit circle in the ρ-t plane can be identified with the original space-time. According to equation (14) of reference (11), what we must calculate is

$$\Gamma(U) = \frac{-i}{240\pi^2} \int_0^1 d\rho \int_0^{2\pi} dt \int d^3x\, \epsilon^{ijklm} \operatorname{Tr}\left[\tilde{U}^{-1}\partial_i \tilde{U}\, \tilde{U}^{-1}\partial_j \tilde{U}\, \tilde{U}^{-1}\partial_k \tilde{U}\, \tilde{U}^{-1}\partial_l \tilde{U}\, \tilde{U}^{-1}\partial_m \tilde{U}\right] \tag{14}$$

where i, j, k, l, and m may be ρ, t, x_1, x_2, or x_3. The integral can be done without undue difficulty (the fact that W is invariant under spatial rotations plus isospin is very useful), and one finds $\Gamma(U) = \pi$.

This calculation can also be used to fill in a gap in the discussion of reference (10). In that paper, the following remark was made. Let $A(x,y,z,t)$ be a mapping from space-time into SU(2) that is in the non-trivial homotopy class in $\pi_4(SU(2))$. Embed A in SU(3) in the trivial form $\hat{A} = \left(\begin{array}{cc|c} \multicolumn{2}{c|}{A} & 0 \\ & & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$. Then $\Gamma(\hat{A}) = \pi$. In fact, as we have noted above, the non-trivial homotopy class in $\pi_4(SU(2))$ differs from the trivial class by a 2π rotation of a soliton (which may be one member of a soliton-antisoliton pair). The fact that $\Gamma = \pi$ for a 2π rotation of soliton means that $\Gamma = \pi$ for the non-trivial homotopy class in $\pi_4(SU(2))$.

The following important fact deserves to be demonstrated explicitly. As before, let A be a mapping of space-time into SU(2) and let $\hat{A}$ be its embedding in SU(3). Then $\Gamma(\hat{A})$ depends only on the homotopy class of A in $\pi_4(SU(2))$. In fact, suppose $\hat{A}$ is homotopic to $\hat{A}'$. Let us prove that $\Gamma(\hat{A}) = \Gamma(\hat{A}')$. To compute $\Gamma(A)$ we realize space-time as the boundary of a disc, extend A to be defined over that disc, and evaluate an appropriate integral (figure 2(a)). To evaluate $\Gamma(A')$ we again must extend A' to a disc. This can be done in a very convenient way (figure 2(b)). Since A' is homotopic to A, we first deform A' into A via matrices of the form $\left(\begin{array}{c|c} X & 0 \\ \hline 0 & 1 \end{array}\right)$ -- matrices that are really SU(2) matrices embedded in SU(3) -- and then we extend A over a disc as before. The integral contribution to $\Gamma(A')$ from part I of figure 2(b) vanishes because the fifth rank antisymmetric tensor that enters in defining Γ vanishes when restricted to any SU(2) subgroup of SU(3). The integral in part II of figure 2(b) is the same as the integral in figure 2(a), so $\Gamma(A) = \Gamma(A')$.

The fact that Γ is a homotopy invariant for SU(2) mappings also means that Γ can be used to prove that $\pi_4(SU(2))$ is non-trivial. Since Γ obviously is 0 for the trivial homotopy class in $\pi_4(SU(2))$, while $\Gamma = \pi$ for a process containing a 2π rotation of a soliton, the latter process must represent a non-trivial element in $\pi_4(SU(2))$. What cannot be proved so easily is that this is the only non-trivial element.

I would like to thank A.P. Balachandran and V.P. Nair for interesting me in current algebra solitons.

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Figure Captions

(1) A soliton-antisoliton pair is created from the vacuum; the soliton is rotated by a 2π angle; the pair is then annihilated. This represents the nontrivial homotopy class in $\pi_4(SU(2))$.

(2) A demonstration that Γ is a homotopy invariant for SU(2) mappings.

Table

Some homotopy groups of certain homogeneous spaces

	SU(n)	SU(n)/O(n)	SU(2n)/Sp(n)
π_2	0	Z_2, $n > 3$	0
π_3	Z, all n	Z_2, $n > 4$	0
π_5	Z, $n > 3$	Z, $n > 3$	Z, $n > 3$

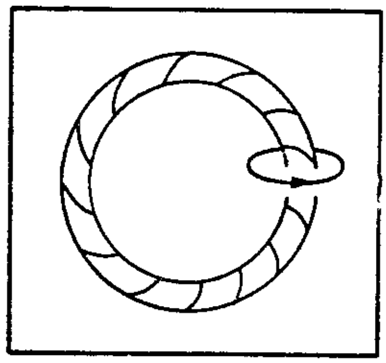

Fig. 1

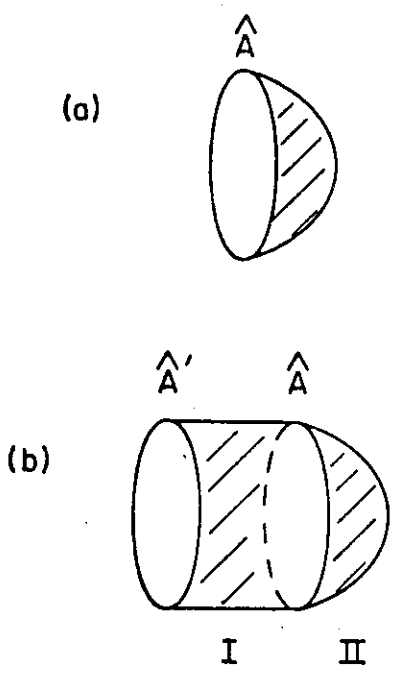

Fig. 2

161

GLOBAL ASPECTS OF CURRENT ALGEBRA

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ABSTRACT

A new mathematical framework for the Wess-Zumino chiral effective action is described. It is shown that this action obeys an a priori quantization law, analogous to Dirac's quantization of magnetic charge. It incorporates in current algebra both perturbative and non-perturbative anomalies.

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The purpose of this paper is to clarify an old but relatively obscure aspect of current algebra -- the Wess-Zumino effective Lagrangian[1] which summarizes the effects of anomalies in current algebra. As we will see, this effective Lagrangian has unexpected analogues in some 2+1 dimensional models discussed recently by Deser, Jackiw, and Templeton[2] and to a recently noted SU(2) anomaly.[3] There also are connections with work of Balachandran, Nair, and Trahern.[4]

For definiteness we will consider a theory with $SU(3)_L \times SU(3)_R$ symmetry spontaneously broken down to the diagonal SU(3). We will ignore explicit symmetry-breaking perturbations, such as quark bare masses. With $SU(3)_L \times SU(3)_R$ broken to diagonal SU(3), the vacuum states of the theory are in one to one correspondence with points in the SU(3) manifold. Correspondingly, the low energy dynamics can be conveniently described by introducing a field $U(x^\alpha)$ that transforms in a so-called non-linear realization of $SU(3)_L \times SU(3)_R$. For each space-time point x^α, $U(x^\alpha)$ is an element of SU(3) -- a 3×3 unitary matrix of determinant one. Under an $SU(3)_L \times SU(3)_R$ transformation by unitary matrices (A,B), U transforms as $U \to AUB^{-1}$.

The effective Lagrangian for U must have $SU(3)_L \times SU(3)_R$ symmetry, and, to describe correctly the low energy limit, it must have the smallest possible number of derivatives. The unique choice with only two derivatives is

$$\mathcal{L} = -\frac{F_\pi^2}{16} \int d^4x \, \mathrm{Tr}\, \partial_\mu U \, \partial_\mu U^{-1} \qquad (1)$$

where experiment indicates $F_\pi \cong 190$ Mev. The perturbative expansion of U is

$$U = 1 + \frac{2i}{F_\pi} \sum_{a=1}^{8} \lambda^a \pi^a + \ldots \qquad (2)$$

where λ^a (normalized so $\mathrm{Tr}\, \lambda^a \lambda^b = 2\delta^{ab}$) are the SU(3) generators and π^a are the Goldstone boson fields.

This effective Lagrangian is known to incorporate all relevant symmetries of QCD. All current algebra theorems governing the extreme low energy limit of Goldstone boson S-matrix elements can be recovered from the tree approximation to it. What is less well known, perhaps, is that (1) possesses an extra discrete symmetry that is not a symmetry of QCD.

The Lagrangian (1) is invariant under $U \to U^T$. In terms of pions this is $\pi^0 \to \pi^0$, $\pi^+ \to \pi^-$; it is ordinary charge conjugation. (1) is also invariant under the naive parity operation $\vec{x} \to -\vec{x}$, $t \to t$, $U \to U$. We will call this P_0. And finally, (1) is invariant under $U \to U^{-1}$. Comparing with equation (2), we see that this latter operation is equivalent to $\pi^a \to -\pi^a$, $a = 1,\ldots,8$. This is the operation that counts modulo two the number of bosons, N_B, so we will call it $(-1)^{N_B}$.

Certainly, $(-1)^{N_B}$ is not a symmetry of QCD. The problem is the following. QCD is parity invariant only if the Goldstone bosons are treated as pseudoscalars. The parity operation in QCD corresponds to $\vec{x} \to -\vec{x}$, $t \to t$, $U \to U^{-1}$. This is $P = P_0\,(-1)^{N_B}$. QCD is invariant under P but not under P_0 or $(-1)^{N_B}$ separately. The simplest process that respects all bona fide symmetries of QCD but violates P_0 and $(-1)^{N_B}$ is $K^+K^- \to \pi^+\pi^0\pi^-$ (note that the ϕ meson decays to both K^+K^- and $\pi^+\pi^0\pi^-$). It is natural to ask whether there is a simple way to add a higher order term to (1) to obtain a Lagrangian that obeys only the appropriate symmetries.

The Euler-Lagrange equation derived from (1) can be written

$$\partial_\mu \left(\frac{F_\pi^2}{8} U^{-1}\, \partial_\mu U\right) = 0 \qquad (3)$$

Let us try to add a suitable extra term to this equation. A Lorentz-invariant term that violates P_0 must contain the Levi-Civita symbol $\epsilon_{\mu\nu\alpha\beta}$. In the

spirit of current algebra, we wish a term with the smallest possible number of derivatives, since, in the low energy limit, the derivatives of U are small. There is a unique P_0-violating term with only four derivatives. We can generalize (3) to

$$\partial_\mu \left(\frac{F_\pi^2}{8} U^{-1}\, \partial_\mu U\right) + \lambda\, \epsilon^{\mu\nu\alpha\beta}\, U^{-1}(\partial_\mu U)U^{-1}(\partial_\nu U)U^{-1}(\partial_\alpha U)U^{-1}(\partial_\beta U) = 0 \quad (4)$$

λ being a constant. Although it violates P_0, (4) can be seen to respect $P = P_0\,(-1)^{N_B}$.

Can equation (4) be derived from a Lagrangian? Here we find trouble. The only pseudoscalar of dimension four would seem to be $\epsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}\,U^{-1}(\partial_\mu U)\cdot U^{-1}(\partial_\nu U)U^{-1}(\partial_\alpha U)U^{-1}(\partial_\beta U)$, but this vanishes, by antisymmetry of $\epsilon^{\mu\nu\alpha\beta}$ and cyclic symmetry of the trace. Nevertheless, as we will see, there is a Lagrangian.

Let us consider a simple problem of the same sort. Consider a particle of mass m constrained to move on an ordinary two dimensional sphere of radius one. The Lagrangian is $\mathcal{L} = \frac{1}{2} m \int dt\, \dot{x}_i^2$ and the equation of motion is $m\ddot{x}_i + m x_i \left(\sum_k \dot{x}_k^2\right) = 0$; the constraint is $\Sigma x_i^2 = 1$. This system respects the symmetries $t \to -t$ and separately $x_i \to -x_i$. If we want an equation that is only invariant under the combined operation $t \to -t$, $x_i \to -x_i$, the simplest choice is

$$m\ddot{x}_i + m x_i \left(\sum_k \dot{x}_k^2\right) = \alpha\, \epsilon_{ijk}\, x_j\, \dot{x}_k \qquad (5)$$

where α is a constant. To derive this equation from a Lagrangian is again troublesome. There is no obvious term whose variation equals the right-hand side (since $\epsilon_{ijk}\, x_i x_j \dot{x}_k = 0$).

However, this problem has a well-known solution. The right-hand side of (5) can be understood as the Lorentz force for an electric charge interacting vector potential A such that $\nabla\times\vec{A} = n\,\vec{x}/|x|^3$; the action for the electron is

$$L = \int \left(\frac{m}{2}\dot{x}_i^2 + e A_i \dot{x}_i\right) dt. \qquad (6)$$

This Lagrangian is problematical because A_i contains a Dirac string and certainly does not respect the symmetries of the problem. To proceed quantum mechanically let us consider the Feynman path integral, $\mathrm{Tr}\, e^{-\beta H} = \int dx_i(t)\, e^{iS}$. In e^{iS} the troublesome term is

$$\exp ie \oint_\gamma A_i\, dx^i \qquad (7)$$

where the integral now goes over the particle orbit γ, a closed orbit if we discuss the simplest object $\mathrm{Tr}\, e^{-\beta H}$.

By Gauss's law we can eliminate the vector potential from (7) in favor of the magnetic field. In fact, the closed orbit γ of figure (1b) is the boundary of a disc D, and by Gauss's law we can write (7) in terms of the magnetic flux through D:

$$\exp ie \oint_\gamma A_i\, dx^i = \exp ie \int_D F_{ij}\, d\Sigma^{ij}. \qquad (8)$$

The precise mathematical statement here is that since $\pi_1(S^2) = 0$, the circle γ in S^2 is the boundary of a disc D (or more exactly, a mapping γ of a circle into S^2 can be extended to a mapping of a disc into S^2).

The right-hand side of (8) is manifestly well defined, unlike the left-hand side, which suffers from a Dirac string. We could try to use the right-hand side of (8) in a Feynman path integral. There is only one problem — D isn't unique. The curve γ also bounds the disc D' (figure (1c)). There is no consistent way to decide whether to choose D or D' (the curve γ could continuously be looped around the sphere or turned inside out). Working with D' we would get

$$\exp\left(-ie \int_{D'} F_{ij}\, d\Sigma^{ij}\right) \qquad (9)$$

where a minus sign appears on the right-hand side of (9) because the curve γ bounds D in a right-handed sense, but bounds D' in a left-handed sense. If we are to introduce the right-hand side of (8) or (9) in a Feynman path integral, we must require that they be equal. This is equivalent to

$$1 = \exp ie \int_{D+D'} F_{ij}\, d\Sigma^{ij} \qquad (10)$$

Since D+D' is the whole two-sphere S^2, and $\int_{S^2} F_{ij}\, d\Sigma^{ij} = 4\pi n$, (10) is obeyed if and only if e is an integer or half-integer. This is Dirac's quantization condition for the product of electric and magnetic charges.

Now let us return to our original problem. We imagine space-time to be a very large four dimensional sphere M. A given non-linear sigma model field U is a mapping of M into the SU(3) manifold (figure (2a)). Since $\pi_4(SU(3)) = 0$, the four sphere in SU(3) defined by U(x) is the boundary of a five dimensional disc Q.

By analogy with the previous problem, let us try to find some object that can be integrated over Q to define an action functional. On the SU(3) manifold there is a unique fifth rank antisymmetric tensor ω_{ijklm} that is invariant under $SU(3)_L \times SU(3)_R$.* Analogous to the right-hand side of

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*Let us first try to define ω at U=1. It can then be extended to the whole SU(3) manifold by an $SU(3)_L \times SU(3)_R$ transformation. At U=1, ω must be invariant under the diagonal subgroup of $SU(3)_L \times SU(3)_R$ that leaves fixed U=1. The tangent space to the SU(3) manifold at U=1 can be identified with the Lie algebra of SU(3). So ω, at U=1, defines a fifth order antisymmetric invariant in the SU(3) Lie algebra. There is only one such invariant. Given five SU(3) generators A, B, C, D and E, the one such invariant is Tr ABCDE − Tr BACDE ± permutations. The $SU(3)_L \times SU(3)_R$ invariant ω so defined has zero curl ($\partial_{[i}\omega_{jklmn]}$ ± permutations = 0), and for this reason (11) is invariant under infinitesimal variations of Q; ω raises only the topological problem discussed in the text.

164

equation (8), we define

$$\Gamma = \int_Q \omega_{ijk\ell m}\, d\Sigma^{ijk\ell m} \tag{11}$$

As before, we hope to include $\exp i\Gamma$ in a Feynman path integral. Again, t problem is that Q is not unique. Our four sphere M is also the boundary of another five disc Q' (figure (2c)). If we let

$$\Gamma' = -\int_{Q'} \omega_{ijk\ell m}\, d\Sigma^{ijk\ell m} \tag{12}$$

(with, again, a minus sign because M bounds Q' with approriate orientation) then we must require $\exp i\Gamma = \exp i\Gamma'$ or equivalently $\int_{Q+Q'} \omega_{ijk\ell m}\, d\Sigma^{ijk\ell m} = 2\pi\cdot$integer. Since Q+Q' is a closed five dimensional sphere, our requirement is

$$\int_S \omega_{ijk\ell m}\, d\Sigma^{ijk\ell m} = 2\pi\cdot\text{integer}$$

for any five sphere S in the SU(3) manifold.

We thus need the topological classification of mappings of the five sphere into SU(3). Since $\pi_5(SU(3)) = Z$, every five sphere in SU(3) is topologically a multiple of a basic five sphere S_o. We normalize ω so that

$$\int_{S_o} \omega_{ijk\ell m}\, d\Sigma^{ijk\ell m} = 2\pi \tag{13}$$

and then (with Γ in equation (11)) we may work with the action

$$I = -\frac{F_\pi^2}{16} \int d^4x\ \mathrm{Tr}\ \partial_\mu U\, \partial_\mu U^{-1} + n\Gamma \tag{14}$$

where n is an arbitrary integer. Γ is, in fact, the Wess-Zumino Lagrangian. Only the a priori quantizationof n is a new result.

The identification of S_o and the proper normalizatin of ω is a subtle mathematical problem. The solution involves a factor of two from the Bott periodicity theorem. Without abstract notation, the result[5] can be stated as

follows. Let y^i, i = 1...5 be coordinates for the disc Q. Then on Q -- where we need it --

$$d\Sigma^{ijk\ell m}\ \omega_{ijk\ell m} = -\frac{i}{240\pi^2}\, d\Sigma^{ijk\ell m}\ [\mathrm{Tr}\ U^{-1}\frac{\partial U}{\partial y^i}\, U^{-1}\frac{\partial U}{\partial y^j}\, U^{-1}\frac{\partial U}{\partial y^k}\ \cdot\ U^{-1}\frac{\partial U}{\partial y^\ell}\, U^{-1}\frac{\partial U}{\partial y^m}]\ . \tag{15}$$

The physical consequences of this can be made more transparent as follows. From equation (2),

$$U^{-1}\,\partial_i U = \frac{2i}{F_\pi}\,\partial_i A + O(A^2), \quad \text{where } A = \Sigma\, \lambda^a \pi^a. \tag{16}$$

So

$$\omega_{ijk\ell m}\, d\Sigma^{ijk\ell m} = \frac{2}{15\pi^2 F_\pi^5}\, d\Sigma^{ijk\ell m}\ \mathrm{Tr}\ \partial_i A\, \partial_j A\, \partial_k A\, \partial_\ell A\, \partial_m A + O(A^6)$$

$$= \frac{2}{15\pi^2 F_\pi^5}\, d\Sigma^{ijk\ell m}\ \partial_i\, (\mathrm{Tr} A\, \partial_j A\, \partial_k A\, \partial_\ell A\, \partial_m A) + O(A^6)$$

So $\int_Q \omega_{ijk\ell m}\, d\Sigma^{ijk\ell m}$ is (to order A^5 and in fact also in higher orders) the integral of a total divergence which can be expressed by Stokes' theorem as an integral over the boundary of Q. By construction, this boundary is precisely space-time. We have, then,

$$n\Gamma = n\,\frac{2}{15\pi^2 F_\pi^5} \int d^4x\ \varepsilon^{\mu\nu\alpha\beta}\ \mathrm{Tr} A\, \partial_\mu A\, \partial_\nu A\, \partial_\alpha A\, \partial_\beta A + \text{higher order} \tag{17}$$

In a hypothetical world of massless kaons and pions, this effective Lagrangian rigorously describes the low energy limit of $K^+K^- \to \pi^+\pi^0\pi^-$.* We reach the remarkable conclusion that in any theory with SU(3) × SU(3) broken to diagonal SU(3), the low energy limit of the amplitude for this reaction must be -- in units given in (17) -- an integer.

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*Our formula should agree for n=1 with formulas of reference (1), as later equations make clear. There appears to be a numerical error on p. 97 of ref. (1) (1/6 instead of 2/15).

What is the value of this integer in QCD? Were n to vanish, the practical interest of our discussion would be greatly reduced. It turns out that if N_c is the number of colors (three in the real world) then $n = N_c$. The simplest way to deduce this is a procedure that is of interest anyway -- coupling to electromagnetism, so as to describe the low energy dynamics of Goldstone bosons and photons.

Let $Q = \begin{pmatrix} 2/3 & & \\ & -1/3 & \\ & & -1/3 \end{pmatrix}$ be the usual electric charge matrix of quarks. The functional Γ is invariant under global charge rotations, $U \to U + i\epsilon[Q,U]$, where ε is a constant. We wish to promote this to a local symmetry, $U \to U + i\epsilon(x)[Q,U]$, where ε(x) is an arbitrary function of x. It is necessary, of course, to introduce the photon field A_μ which transforms as $A_\mu \to A_\mu - (1/e)\partial_\mu \epsilon$; e is the charge of the proton.

Usually a global symmetry can straightforwardly be gauged by replacing derivatives by covariant derivatives, $\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$. In the case at hand, Γ is not given as the integral of a manifestly $SU(3)_L \times SU(3)_R$ invariant expression, so the standard rule of gauging global symmetries of Γ is not available. One can still resort to the trial and error Noether method, widely used in supergravity. Under a local charge rotation, one finds $\Gamma \to \Gamma - \int d^4x\, \partial_\mu \epsilon\, J^\mu$ where

$$J^\mu = \frac{1}{48\pi^2}\epsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}\,\{Q(\partial_\nu U\, U^{-1})(\partial_\alpha U\, U^{-1})(\partial_\beta U\, U^{-1}) + Q\,(U^{-1}\partial_\nu U)(U^{-1}\partial_\alpha U)(U^{-1}\partial_\beta U)\} \qquad (18)$$

is the extra term in the electromagnetic current required -- from Noether's theorem -- due to the addition of Γ to the Lagrangian. The first step in the construction of an invariant Lagrangian is to add the Noether coupling, $\Gamma \to \Gamma' = \Gamma - e\int d^4x\, A_\mu J^\mu(x)$. This expression is still not gauge invariant, because J^μ is not, but by trial and error one finds that by adding an extra term one can form a gauge invariant functional

$$\tilde\Gamma(U, A_\mu) = \Gamma(U) - e\int d^4x\, A_\mu J^\mu + \frac{ie^2}{24\pi^2}\int d^4x\, \epsilon^{\mu\nu\alpha\beta}(\partial_\mu A_\nu) A_\alpha \cdot \mathrm{Tr}\,[Q^2(\partial_\beta U)U^{-1} + Q^2U^{-1}(\partial_\beta U) + QUQU^{-1}(\partial_\beta U)U^{-1}] \qquad (19)$$

The gauge invariant Lagrangian will then be

$$\mathcal{L} = -\frac{F_\pi^2}{16}\int d^4x\, \mathrm{Tr}\, D_\mu U\, D_\mu U^{-1} + n\tilde\Gamma \qquad (20)$$

What value of the integer n will reproduce QCD results?

Here we find a surprise. The last term in (19) has a piece that describes $\pi^0 \to \gamma\gamma$. Expanding U and integrating by parts, (19) has a piece

$$A = \frac{n\, e^2}{48\pi^2 F_\pi}\, \pi^0\, \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \qquad (21)$$

This agrees with the result from QCD triangle diagrams if $n = N_c$, the number of colors. The Noether coupling $eA_\mu J^\mu$ describes, among other things, a $\gamma\pi^+\pi^-\pi^0$ vertex

$$B = -\frac{2}{3}\, ie\, \frac{n}{\pi^2 F_\pi^3}\, \epsilon^{\mu\nu\alpha\beta} A_\mu\, \partial_\nu \pi^+\, \partial_\alpha \pi^-\, \partial_\beta \pi^0 \qquad (22)$$

Again this agrees with calculations[7] based on the QCD VAA anomaly if $n = N_c$. The effective action $N_c\tilde\Gamma$ -- first constructed in another way by Wess and Zumino -- precisely describes all effects of QCD anomalies in low energy processes with photons and Goldstone bosons.

It is interesting to try to gauge subgroups of $SU(3)_L \times SU(3)_R$ other than electromagnetism. One may have in mind, for instance, applications to the standard weak interaction model. In general, one may try to gauge an arbitrary subgroup H of $SU(3)_L \times SU(3)_R$ with generators K^σ, $\sigma = 1 \ldots r$. Each

K^σ is a linear combination of generators T_L^σ and T_R^σ of $SU(3)_L$ and $SU(3)_R$, $K^\sigma = T_L^\sigma + T_R^\sigma$. (Either T_L^σ or T_R^σ may vanish for some values of σ.) For any space-time dependent functions $\epsilon^\sigma(x)$, let $\epsilon_L = \sum_\sigma T_L^\sigma \epsilon^\sigma(x)$, $\epsilon_R = \sum_\sigma T_R^\sigma \epsilon^\sigma(x)$. We want an action with local invariance under $U \to U + i(\epsilon_L(x)U - U\epsilon_R(x))$.

Naturally, it is necessary to introduce gauge fields $A_\mu^\sigma(x)$, transforming as $A_\mu^\sigma(x) \to A_\mu^\sigma(x) - (\frac{1}{e_\sigma}) \partial_\mu \epsilon^\sigma + f^{\sigma\tau\rho} \epsilon^\tau A_\mu^\rho$ where e_σ is the coupling constant corresponding to the generator K^σ, and $f^{\sigma\tau\rho}$ are the structure constants of H. It is useful to define $A_{\mu L} = \sum_\sigma e_\sigma A_\mu^\sigma T_L^\sigma$, $A_\mu^R = \sum_\sigma e_\sigma A_\mu^\sigma T_R^\sigma$.

We have already seen that Γ incorporates the effects of anomalies, so it is not very surprising that a generalizationof Γ that is gauge invariant under H exists only if H is a so-called anomaly-free subgroup of $SU(3)_L \times SU(3)_R$. Specifically, one finds that H can be gauged only if for each σ,

$$\mathrm{Tr}\ (T_L^\sigma)^3 = \mathrm{Tr}\ (T_R^\sigma)^3 \tag{23}$$

which is the usual condition for cancellation of anomalies at the quark level.

If (23) is obeyed, a gauge invariant generalization of Γ can be constructed somewhat tediously by trial and error. It is useful to define $U_{\nu L} = (\partial_\nu U)\, U^{-1}$ and $U_{\nu R} = U^{-1}\, \partial_\nu U$. The gauge invariant functional then turns out to be

$$\tilde{\Gamma}(A_\mu, U) = \Gamma(U) + \frac{1}{48\pi^2} \int d^4x\, \epsilon^{\mu\nu\alpha\beta}\, Z_{\mu\nu\alpha\beta}$$

where

$$\begin{aligned}
Z_{\mu\nu\alpha\beta} = & -\mathrm{Tr}[A_{\mu L}\, U_{\nu L}\, U_{\alpha L}\, U_{\beta L} + (L\to R)] \\
& + i\mathrm{Tr}[[(\partial_\mu A_{\nu L})\, A_{\alpha L} + A_{\mu L}(\partial_\nu A_{\alpha L})]\, U_{\beta L} + (L\to R)] \\
& + i\mathrm{Tr}[(\partial_\mu A_{\nu R})\, U^{-1} A_{\alpha L}\, \partial_\beta U + A_{\mu L}\, U^{-1} (\partial_\nu A_{\alpha R})\, \partial_\beta U] \\
& - \frac{1}{2}\, i\mathrm{Tr}(A_{\mu L}\, U_{\nu L}\, A_{\alpha L}\, U_{\beta L} - (L\to R) \\
& + i\mathrm{Tr}[A_{\mu L}\, U\, A_{\nu R}\, U^{-1}\, U_{\alpha L}\, U_{\beta L} - A_{\mu R}\, U^{-1} A_{\nu L}\, U U_{\alpha R}\, U_{\beta R}] \\
& - \mathrm{Tr}[[(\partial_\mu A_{\nu R})\, A_{\alpha R} + A_{\mu R}\, (\partial_\nu A_{\alpha R})]\, U^{-1} A_{\beta L}\, U \\
& \quad - [(\partial_\mu A_{\nu L})\, A_{\alpha L} + A_{\mu L}\, (\partial_\nu A_{\alpha L})]\, U\, A_{\beta R}\, U^{-1}] \\
& - \mathrm{Tr}[A_{\mu R}\, U^{-1} A_{\nu L}\, U\, A_{\alpha R}\, U_{\beta R} + A_{\mu L}\, U\, A_{\nu R}\, U^{-1} A_{\alpha L}\, U_{\beta L}] \\
& - \mathrm{Tr}[A_{\mu L}\, A_{\nu L}\, U\, (\partial_\alpha A_{\beta R})\, U^{-1} + A_{\mu R}\, A_{\nu R}\, U^{-1} (\partial_\alpha A_{\beta L})\, U] \\
& - i\mathrm{Tr}[A_{\mu R}\, A_{\nu R}\, A_{\alpha R}\, U^{-1} A_{\beta L}\, U - A_{\mu L}\, A_{\nu L}\, A_{\alpha L}\, U\, A_{\beta R}\, U^{-1} \\
& \quad + \frac{1}{2} A_{\mu L}\, A_{\nu L}\, U\, A_{\alpha R}\, A_{\beta R}\, U^{-1} \\
& \quad + \frac{1}{2} A_{\mu R}\, U^{-1} A_{\nu L}\, U\, A_{\alpha R}\, U^{-1} A_{\beta L}\, U]
\end{aligned} \tag{24}$$

If equation (22) for cancellation of anomalies is not obeyed, then the variation of $\tilde{\Gamma}$ under a gauge transformation does not vanish but is

$$\begin{aligned}
\delta\tilde{\Gamma} = & -\frac{1}{24\pi^2} \int d^4x\, \epsilon^{\mu\nu\alpha\beta}\ \mathrm{Tr}\ \epsilon_L\ [(\partial_\mu A_{\nu L})(\partial_\alpha A_{\beta L}) \\
& - \frac{1}{2}\, i\, \partial_\mu\, (A_{\nu L}\, A_{\alpha L}\, A_{\beta L})] - (L\to R)
\end{aligned} \tag{25}$$

in agreement with computations at the quark level[8] of the anomalous variation of the effective action under a gauge transformation.

Thus, Γ incorporates all information usually associated with triangle anomalies, including the restriction on what subgroups H of $SU(3)_L \times SU(3)_R$ can be gauged. However, there is another potential obstruction to the ability to gauge a subgroup of $SU(3)_L \times SU(3)_R$. This is the non-perturbative anomaly[3] associated with $\pi_4(H)$. Is this anomaly, as well, implicit in Γ? In fact, it is.

Let H be an SU(2) subgroup of $SU(3)_L$, chosen so that an SU(2) matrix W is embedded in $SU(3)_L$ as $W = \begin{pmatrix} W & \begin{matrix} 0 \\ 0 \end{matrix} \\ 0 \; 0 & 1 \end{pmatrix}$. This subgroup is free of triangle anomalies, so the functional $\tilde{\Gamma}$ of equation (23) is invariant under infinitesimal local H transformations.

However, is $\tilde{\Gamma}$ invariant under H transformations that cannot be reached continuously? Since $\pi_4\,(SU(2)) = Z_2$, there is one non-trivial homotopy class of SU(2) gauge transformations. Let W be an SU(2) gauge transformation in this non-trivial class. Under W, $\tilde{\Gamma}$ may at most be shifted by a constant, independent of U and A_μ, because $\delta\tilde{\Gamma}/\delta U$ and $\delta\tilde{\Gamma}/\delta A_\mu$ are gauge-covariant local functionals of U and A_μ. Also $\tilde{\Gamma}$ is invariant under W^2, since W^2 is equivalent to the identity in $\pi_4\,(SU(2))$, and we know $\tilde{\Gamma}$ is invariant under topologically trivial gauge transformations. This does not quite mean that $\tilde{\Gamma}$ is invariant under W. Since $\tilde{\Gamma}$ is only defined modulo 2π, the fact that $\tilde{\Gamma}$ is invariant under W^2 leaves two possibilities for how $\tilde{\Gamma}$ behaves under W. It may be invariant, or it may be shifted by π.

To choose between these alternatives, it is enough to consider a special case. For instance, it suffices to evaluate $\Delta = \tilde{\Gamma}\,(U = 1,\ A_\mu = 0) - \tilde{\Gamma}\,(U = W,\ A_\mu = i\,e^{-1}\,(\partial_\mu W)\,W^{-1})$. It is not difficult to see that in this case the complicated terms involving $\epsilon^{\mu\nu\alpha\beta}\,Z_{\mu\nu\alpha\beta}$ vanish, so in fact $\Delta = \Gamma(U{=}1) - \Gamma(U{=}W)$. A detailed calculation shows that

$$\Gamma\,(U = 1) - \Gamma\,(U = W) = \pi \tag{26}$$

This calculation has some other interesting applications and will be described elsewhere.[9]

The Feynman path integral, which contains a factor $\exp\, iN_c\tilde{\Gamma}$, hence picks up under W a factor $\exp\, iN_c\pi = (-1)^{N_c}$. It is gauge invariant if N_c is even, but not if N_c is odd. This agrees with the determination of the SU(2) anomaly at the quark level.[3] For under H, the right-handed quarks are singlets. The left-handed quarks consist of one singlet and one doublet per color, so the number of doublets equals N_c. The argument of ref. 3 shows at the quark level that the effective action transforms under W as $(-1)^{N_c}$.

Finally, let us make the following remark, which apart from its intrinsic interest will be useful elsewhere.[9] Consider $SU(3)_L \times SU(2)_R$ currents defined at the quark level as

$$J^a_{\mu L} = \bar{q}\,\lambda^a\,\gamma_\mu\,\frac{1}{2}\,(1 - \gamma_5)q$$

$$J^a_{\mu R} = \bar{q}\,\lambda^a\,\gamma_\mu\,\frac{1}{2}\,(1 + \gamma_5)q \tag{27}$$

By analogy with equation (17), the proper sigma model description of these currents contains pieces

$$J^{\mu a}_L = \frac{N_c}{48\pi^2}\,\epsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}\,\lambda^a\,U_{\nu L}\,U_{\alpha L}\,U_{\beta L}$$

$$J^{\mu a}_R = \frac{N_c}{48\pi^2}\,\epsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}\,\lambda^a\,U_{\nu R}\,U_{\alpha R}\,U_{\beta R} \tag{28}$$

corresponding (via Noether's theorem) to the addition to the Lagrangian of $N_c\Gamma$. In this discussion, the λ^a should be traceless SU(3) generators. However, let us try to construct an anomalous baryon number current in the same way. We define the baryon number of a quark -- whether left-handed or right-handed -- to be $1/N_c$, so that an ordinary baryon made from N_c quarks has baryon number one. Replacing λ^a by $1/N_c$, but including contributions of both left-handed and right-handed quarks, the anomalous baryon number current would be

$$J^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}\,U^{-1}\partial_\nu U\,U^{-1}\partial_\alpha U\,U^{-1}\partial_\beta U \qquad (29)$$

One way to see that this is the proper, and properly normalized, formula is to consider gauging an arbitrary subgroup not of $SU(3)_L \times SU(3)_R$ but of $SU(3)_L \times SU(3)_R \times U(1)$, $U(1)$ being baryon number. The gauging of $U(1)$ is accomplished by adding a Noether coupling $-eJ^\mu B_\mu$ plus whatever higher order terms may be required by gauge invariance. (B_μ is a $U(1)$ gauge field which may be coupled as well to some $SU(3)_L \times SU(3)_R$ generator.) With J^μ defined in (29), this leads to a generalization of $\tilde{\Gamma}$ that properly reflects anomalous diagrams involving the baryon number current (for instance, it properly incorporates the anomaly in the baryon number - $SU(2)_L$ - $SU(2)_L$ triangle that leads to baryon non-conservation by instantons in the standard weak interaction model).

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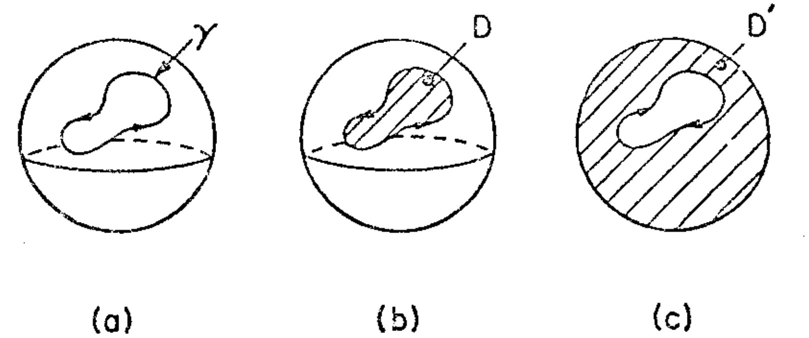
FIGURE CAPTIONS

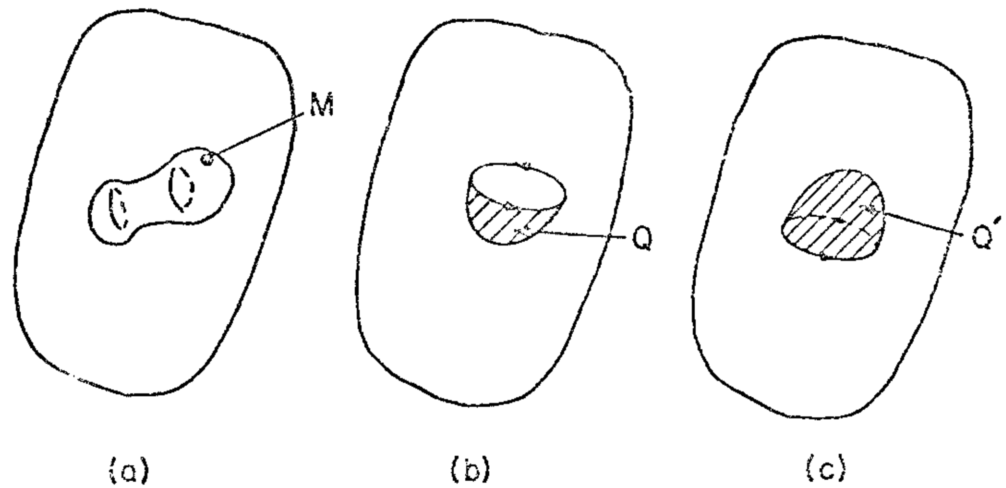
(1) A particle orbit γ on the two-sphere (part (a)) bounds the discs D (part (b)) and D' (part (c)).

(2) Space-time, a four sphere, is mapped into the SU(3) manifold. In part (a), space-time is symbolically denoted as a two sphere. In parts (b) and (c), space-time is reduced to a circle that bounds the discs Q and Q'. The SU(3) manifold is symbolized in these sketches by the interior of the oblong.

I.



II.



170

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STATIC PROPERTIES OF NUCLEONS IN THE SKYRME MODEL

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ABSTRACT

We compute static properties of baryons in an $SU(2) \times SU(2)$ chiral theory (the Skyrme model) whose solitons can be interpreted as the baryons of QCD. Our results are generally within about 30% of experimental values. We also derive some relations that hold generally in soliton models of baryons, and therefore, serve as tests of the 1/N expansion.

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INTRODUCTION

Recent developments have provided partial confirmation of Skyrme's old idea [1] that baryons are solitons in the non-linear sigma model. We know that in the large N limit, QCD becomes becomes equivalent to an effective field theory of mesons [2]. Counting rules suggest [3] that baryons may emerge as solitons in this theory. Although we do not understand in detail the large N theory of mesons, we know that at low energies this theory reduces to a non-linear sigma model of spontaneously broken chiral symmetry. Moreover, the solitons of the non-linear model have precisely the quantum numbers of QCD baryons [4] provided one includes the effects of the Wess-Zumino coupling [5,6].

In this paper we will evaluate the static properties of nucleons such as masses, magnetic moments, and charge radii, in a soliton model. For simplicity we will restrict ourselves to the case of two flavors. One simplification in the SU(2) case is that the Wess-Zumino term vanishes. At a pedestrian level, for $U = 1 + iA + O(A^2)$, the Wess-Zumino term is [5,6]

$$n\Gamma = n \frac{2}{15\pi^2 F_\pi^5} \int d^4x \, \varepsilon^{\mu\nu\alpha\beta} \, \mathrm{Tr}\left[A \, \partial_\mu A \, \partial_\nu A \, \partial_\alpha A \, \partial_\beta A\right] + \text{higher orders} .$$

If $A = a_a \tau_a$, then

$$n\Gamma = n \frac{2}{15\pi^2 F_\pi^5} \int d^4x \, \varepsilon^{\mu\nu\alpha\beta} \, a_a \, \partial_\mu a_b \, \partial_\nu a_c \, \partial_\alpha a_d \, \partial_\beta a_e \, \mathrm{Tr}\left[\tau_a \tau_b \tau_c \tau_d \tau_e\right] .$$

[illegible]

so it needs to be completely antisymmetric in the isospin indices b, c, d and e. But that is impossible because there are only three independent SU(2) generators. More generally, the fifth rank antisymmetric tensor as discussed in [6] vanishes on the SU(2) group manifold. Nonetheless, the anomalous baryon number current, which can be obtained from the $\bar{W}$-Z term or by the method of Goldstone and Wilczek [7], is still present in the two-flavor case.

Since the proper large N effective theory is unknown, we will consider here a crude description in which the large N theory is assumed to be a theory of pions only. In this context, it is necessary to add a non-minimal term to the non-linear sigma model to prevent the solitons from shrinking to zero-size. The simplest reasonable choice is the Skyrme model

$$L = \frac{F_\pi^2}{16}\, Tr\left(\partial_\mu U\, \partial_\mu U^\dagger\right) + \frac{1}{32e^2}\, Tr\left[(\partial_\mu U)U^\dagger\, ,\, (\partial_\nu U)U^\dagger\right]^2 \quad (1)$$

Here U is an SU(2) matrix, transforming as $U \to A\,U\,B^{-1}$ under chiral SU(2) × SU(2); F_π = 186 Mev is the pion decay constant; and the last term, which contains the dimensionless parameter e, was introduced by Skyrme to stabilize the solitons. It is the unique term with four derivatives which leads to a positive Hamiltonian. (It is also the unique term with four derivatives that leads to a Hamiltonian second order in time derivatives.)

Although the Skyrme model is only a crude description, since it omits the other mesons and interactions that are present in the large N limit of QCD, we regard it as a good model for testing the reasonableness of a soliton description of nucleons.

1. [illegible]

From the Lagrangian (1) we find the soliton solution by using the Skyrme ansatz $U_0(x) = \exp(i\,F(r)\,\vec{\tau}\cdot\hat{x})$, where $F(r) = \pi$ at $r = 0$ and $F(r) \to 0$ as $r \to \infty$. If we substitute this ansatz in (1) we get the expression for the soliton mass:

$$M = 4\pi \int_0^\infty r^2 \left\{ \frac{F_\pi^2}{8}\left[\left(\frac{\partial F}{\partial r}\right)^2 + 2\,\frac{\sin^2 F}{r^2}\right] + \frac{1}{2e^2}\,\frac{\sin^2 F}{r^2}\left[\frac{\sin^2 F}{r^2} + 2\left(\frac{\partial F}{\partial r}\right)^2\right]\right\} dr \quad . \qquad (2)$$

The variational equation from (2) is

$$\left(\frac{\tilde{r}^2}{4} + 2\sin^2 F\right)F'' + \frac{\tilde{r}F'}{2} + \sin 2F\; F'^2 - \frac{\sin 2F}{4} - \frac{\sin^2 F\,\sin 2F}{\tilde{r}^2} = 0 \; . \qquad (3)$$

in terms of a dimensionless variable $\tilde{r} = e\,F_\pi\, r$. The behaviour of the numerical solution of equation (3) is shown in Fig. 1.

Now, if $U_0 = \exp[i F(r) \vec{\tau} \cdot \hat{x}]$ is the soliton solution, then $U = A U_0 A^{-1}$, where A is an abritrary constant SU(2) matrix, is a finite energy solution as well. A solution of any given A is not an eigenstate of spin and isospin. We need to treat A as a quantum mechanical variable, as a collective coordinate. The simplest way to do this is to write the Lagrangian and all physical observables in terms of a time dependent A. We substitute $U = A(t) U_0 A^{-1}(t)$ in the Lagrangian, where U_0 is the soliton solution and $A(t)$ is an arbitrary time dependent SU(2) matrix. This procedure will allow us to write a Hamiltonian which we will diagonalize. The eigenstates with the proper spin and isospin will correspond to the nucleon and delta.

So, substituting $U = A(t) U_0 A^{-1}(t)$ in (1), after a lengthy calculation, we get

$$L = - M + \lambda \, \mathrm{Tr}[\partial_0 A \, \partial_0 A^{-1}] \qquad (4)$$

where M is defined in (2) and $\lambda = \frac{4\pi}{6} \frac{1}{e^3 F_\pi} \Lambda$ with

$$\Lambda = \int \tilde{r}^2 \sin^2 F \left[1 + 4 \left(F'^2 + \frac{\sin^2 F}{\tilde{r}^2} \right) \right] d\tilde{r} \qquad . \qquad (5)$$

Numerically we find $\Lambda = 50.9$. The SU(2) matrix A can be written $A = a_0 + i \vec{a} \cdot \vec{\tau}$, with $a_0^2 + \vec{a}^2 = 1$. In terms of

the a's (4) becomes

$$L = - M + 2\lambda \sum_{i=0}^{3} (\dot{a}_i)^2 \qquad .$$

Introducing the conjugate momenta $\pi_i = \frac{\partial L}{\partial \dot{a}_i} = 4 \lambda \dot{a}_i$, we can now write the Hamiltonian

$$H = \pi_i \dot{a}_i - L = 4\lambda \dot{a}_i \dot{a}_i - L = M + 2\lambda \dot{a}_i \dot{a}_i = M + \frac{1}{8\lambda} \sum_i \pi_i^2 \qquad .$$

Performing the usual canonical quantization procedure $\pi_i = - i \frac{\partial}{\partial a_i}$ we get

$$H = M + \frac{1}{8\lambda} \sum_{i=0}^{3} \left(- \frac{\partial^2}{\partial a_i^2} \right) \qquad (6)$$

with the constraint $\sum_{i=0}^{3} a_i^2 = 1$

Because of this constraint, the operator $- \sum_{i=0}^{3} \frac{\partial^2}{\partial a_i^2}$ is to be interpreted as the Laplacian $- \nabla^2$ on the three-sphere. The wave functions (by analogy with usual spherical harmonics) are traceless symmetric polynomials in the a_i's. A typical example is $(a_0 + i a_1)^\ell$, with $- \nabla^2 (a_0 + i a_1)^\ell = \ell(\ell + 2)(a_0 + i a_1)^\ell$. Such a wave function has spin and isospin equal to $\frac{1}{2}\ell$, as one may see by considering the spin and isospin operators

$$I_k = \frac{i}{2}\left(a_0 \frac{\partial}{\partial a_k} - a_k \frac{\partial}{\partial a_0} - \varepsilon_{k\ell m}\, a_\ell \frac{\partial}{\partial a_m}\right)$$

$$J_k = \frac{i}{2}\left(a_k \frac{\partial}{\partial a_0} - a_0 \frac{\partial}{\partial a_k} - \varepsilon_{k\ell m}\, a_\ell \frac{\partial}{\partial a_m}\right) \qquad (7)$$

An important physical point must be addressed here. Since the non-linear sigma model field is $U = A\,U_0\,A^{-1}$, A and $-A$ correspond to the same U. Naively, one might expect to insist that the wave function $\psi(A)$ obeys $\psi(A) = +\psi(-A)$. Actually, as discussed long ago by Finkelstein and Rubinstein [8], there are two consistent ways to quantize the soliton: one may require $\psi(A) = +\psi(-A)$ for all solitons, or one may require $\psi(A) = -\psi(-A)$ for all solitons. The former choice corresponds to quantizing the soliton as a boson. The latter choice corresponds to quantizing it as a fermion. We wish to follow the second road, of course, so our wave functions will be polynomials of <u>odd</u> degree in the a_i's. So, the nucleons, of $I = J = \frac{1}{2}$, correspond to wave functions linear in a_i, while the deltas, of $I = J = \frac{3}{2}$, correspond to cubic functions. Wave functions of fifth order and higher correspond to highly excited states (masses $\geq$ 1730 Mev) which either are lost in the pi nucleon continuum or else are artifacts of the model. The properly normalized wave functions for proton and neutron states

of spin up or spin down along the z axis, and some of the Δ wave functions, are:

$$|p\uparrow\rangle = \frac{1}{\pi}(a_1 + i a_2) \qquad |p\downarrow\rangle = -\frac{i}{\pi}(a_0 - i a_3)$$

$$|n\uparrow\rangle = \frac{i}{\pi}(a_0 + i a_3) \qquad |n\downarrow\rangle = -\frac{1}{\pi}(a_1 - i a_2) \qquad (8)$$

$$\left|\Delta^{++}, s_z = \frac{3}{2}\right\rangle = \frac{\sqrt{2}}{\pi}(a_1 + i a_2)^3$$

$$\left|\Delta^{+}, s_z = \frac{1}{2}\right\rangle = -\frac{\sqrt{2}}{\pi}(a_1 + i a_2)\left(1 - 3\left(a_0^2 + a_3^2\right)\right)$$

Returning to Equation (6), the eigenvalues of the Hamiltonian are $E = M + \frac{1}{8\lambda}\,\ell(\ell+2)$ where $\ell = 2J$. So, the nucleon and delta masses are given by

$$M_N = M + \frac{1}{2\lambda}\,\frac{3}{4}$$

$$M_\Delta = M + \frac{1}{2\lambda}\,\frac{15}{4} \qquad (9)$$

where M, obtained by evaluating (2) numerically, is given by $M = \frac{F_\pi}{e}\,36.5$ and $\lambda = \frac{4\pi}{6}\,\frac{1}{e^3 F_\pi}\,50.9$, as already said. We have found that the best procedure in dealing with this model is to adjust e and F_π to fit the nucleon and delta masses. The results are $e = 5.45$ and $F_\pi = 129$ Mev. Thus, on the basis of the values of the baryon masses, we require (or predict) in this model a value of F_π that is 30% lower than the experimental value of 186 Mev.

2. CURRENTS, CHARGE RADII AND MAGNETIC MOMENTS

In order to compute weak and electromagnetic couplings of baryons, we need first to evaluate the currents in terms of collective coordinates. The Noether current associated with the V-A transformation $\delta U = i\,Q\,U$ is

$$J^{\mu}_{V-A} = i\,\frac{F_\pi^2}{8}\,\mathrm{Tr}[(\partial^\mu U)U^\dagger Q] + \frac{i}{8e^2}\,\mathrm{Tr}\left\{\left[(\partial_\nu U)U^\dagger , Q\right]\left[(\partial^\mu U)U^\dagger , (\partial^\nu U)U^\dagger\right]\right\} \qquad (10)$$

The V+A current is obtained by exchanging U with $U^\dagger$.

The anomalous baryon current is instead [7,6]

$$B^{\mu} = \frac{\varepsilon^{\mu\nu\alpha\beta}}{24\pi^2}\,\mathrm{Tr}[Q(U^\dagger\partial_\nu U)(U^\dagger\partial_\alpha U)(U^\dagger\partial_\beta U)] \qquad (11)$$

where our notation is $\varepsilon_{0123} = -\varepsilon^{0123} = 1$.

If we substitute $U = A(t)\,U_0\,A^{-1}(t)$ in (10), we get rather complicated expressions for the vector and axial currents V and A. The following angular integrals, which are much simpler, are adequate for our purposes:

$$\int d\Omega\, V^{a,0} = \frac{i\,4\pi}{3}\,\Lambda'\,\mathrm{Tr}[(\partial_0 A)A^{-1}\tau_a] \qquad (12)$$

$$\int d\Omega\, \vec{q}\cdot\vec{x}\, V^{a,i} = i\,\frac{\pi}{3}\,\Lambda'\,\mathrm{Tr}(\vec{\tau}\cdot\vec{q}\;\tau_i A^{-1}\tau_a A) \qquad (13)$$

$$\int d\Omega\, A^{a,i} = \frac{\pi}{3}\,D'\,\mathrm{Tr}(\tau_i A^{-1}\tau_a A) \qquad (14)$$

where Λ' and D' are respectively

$$\Lambda' = \sin^2 F\left[F_\pi^2 + \frac{4}{e^2}\left(F'^2 + \frac{\sin^2 F}{r^2}\right)\right] \qquad (15)$$

$$D' = F_\pi^2\left(F' + \frac{\sin 2F}{r}\right) + \frac{4}{e^2}\left(\frac{\sin 2F}{r}F'^2 + 2\,\frac{\sin^2 F}{r^2}F' + \frac{\sin^2 F\,\sin 2F}{r^3}\right) \qquad (16)$$

In the computation of the above formulas from (10) we have neglected terms which are quadratic in time derivatives. In the semiclassical limit the solitons rotate slowly, so terms quadratic in time derivatives are higher order in the semiclassical approximation.

The expression in (7) for the isospin generator I_k can be derived from (12) by integrating over r, and replacing $\dot{a}_i$ by the canonical momentum.

From (11) we derive the baryon current and charge density

$$B^0 = -\frac{1}{2\pi^2}\,\frac{\sin^2 F}{r^2}\,F' \qquad (17)$$

$$B^i = i\,\frac{\varepsilon^{ijk}}{2\pi^2}\,\frac{\sin^2 F}{r}\,F'\,\hat{x}_k\,\mathrm{Tr}\left[(\partial_0 A^{-1})A\,\tau_j\right] \qquad (18)$$

The baryon charge per unit r is therefore

$$\rho_B(r) = 4\pi\, r^2\, B^0(r) = -\frac{2}{\pi} \sin^2 F\, F'$$

and its integral $\int_0^\infty \rho_B(r)\, dr = 1$ gives the baryonic charge.

The isoscalar mean square radius is given by

$$\langle r^2 \rangle_{I=0} = \int_0^\infty r^2\, \rho_B(r)\, dr = \frac{4.47}{e^2 F_\pi^2} = 4.47 \quad (0.28)^2 \text{ fm}^2$$

and we get $\langle r^2 \rangle^{1/2}_{I=0} = 0.59$ fm, while the corresponding experimental value is 0.72 fm.

From (12) and (15) we can compute the isovector charge density per unit r

$$\rho_V(r) = \frac{r^2 \sin^2 F \left[F_\pi^2 + \frac{4}{e^2} \left(F'^2 + \frac{\sin^2 F}{r^2} \right) \right]}{\int_0^\infty r^2 \sin^2 F \left[F_\pi^2 + \frac{4}{e^2} \left(F'^2 + \frac{\sin^2 F}{r^2} \right) \right] dr}$$

and finally derive the proton and neutron charge distributions which are plotted in Figure 2.

The isovector mean square charge radius $\int_0^\infty r^2\, \rho_V(r)\, dr$ is divergent, as expected in the chiral limit [9]. The introduction of quark masses in this model [10] will cure this problem, as it does in nature.

The definitions of isoscalar and isovector magnetic moments are respectively

$$\vec{\mu}_{I=0} = \frac{1}{2} \int \vec{r} \times \vec{B}\, d^3x \qquad (19)$$

and

$$\vec{\mu}_{I=1} = \frac{1}{2} \int \vec{r} \times \vec{V}^3\, d^3x \quad . \qquad (20)$$

Therefore, from (18) the isoscalar magnetic moment density is

$$\rho_M^{I=0}(r) = \frac{r^2\, F' \sin^2 F}{\int_0^\infty r^2\, F' \sin^2 F\, dr}$$

The isoscalar magnetic mean radius is defined by

$$\langle r^2 \rangle_{M,I=0} = \int_0^\infty r^2\, \rho_M^{I=0}(r)\, dr \quad .$$

We get $\langle r^2 \rangle^{1/2}_{M,I=0} = 0.92$ fm, against the experimental value of 0.81 fm.

The simplest way to extract the g factors is to calculate the expectation value of the magnetic moment operators in a proton state of spin up, using the forms given earlier for the wave functions. From (18) and (19) the isoscalar magnetic moment is

$$\left(\mu_{I=0}\right)_i = \frac{1}{2}\int d^3x\, \epsilon_{\ell mi}\, x_\ell < p\uparrow|B_m|p\uparrow>$$

$$= -\frac{1}{2}\frac{i}{2\pi^2}\int d^3x\, \sin^2 F\, F'\, \hat{x}_\ell\, \hat{x}_k\, \epsilon_{\ell mi}\, \epsilon_{mjk} < p\uparrow|Tr[(\partial_0 A^{-1})\, A\, \tau_j]|p\uparrow> \quad .$$

It is easy to check that $< p\uparrow|Tr\left[\left(\partial_0 A^{-1}\right) A\, \tau_j\right]|p\uparrow> =$ $= -\delta_{j3}\frac{i}{2\lambda}$.

It follows that

$$\left(\vec{\mu}_{I=0}\right)_3 = \frac{<\tilde{r}^2>_{I=0}}{\Lambda}\frac{e}{F_\pi}\frac{1}{4\pi} \quad . \tag{21}$$

The g factor is defined by writing $\vec{\mu} = \frac{g}{4M}\vec{\sigma}$. The isoscalar g factor $g_{I=0} = g_p + g_n$ is 1.11 in this model (the experimental value is 1.76, instead).

In order to compute the isovector magnetic moment, we start from (13) and integrate in the radial variable. We get

$$\int d^3x\, \vec{q}\cdot\vec{x}\, V^{3,i} = i\,\frac{\pi}{3}\frac{\Lambda}{F_\pi\, e^3}\, Tr(\vec{\tau}\cdot\vec{q}\, \tau_i\, A^{-1}\, \tau_3\, A)$$

with Λ given in (5).

Now

$$Tr(\vec{\tau}\cdot\vec{q}\,\tau_i\, A^{-1}\,\tau_j\, A) = i\, q_\ell\, \epsilon_{\ell im}\, Tr(\tau_m\, A^{-1}\,\tau_j\, A) \quad .$$

A detailed calculation using the nucleon wave function given

in (8) shows that for any nucleon states N and N'

$$<N'|Tr[\tau_i\, A^{-1}\,\tau_j\, A]|N> = -\frac{2}{3}<N'|\sigma_i\,\tau_j|N> \quad . \tag{22}$$

Therefore

$$<p\uparrow|\int d^3x\, \vec{q}\cdot\vec{x}\, V_i^3|p\uparrow> = -\, q_\ell\, \frac{\pi}{3}\frac{\Lambda}{F_\pi\, e^3}\,\epsilon_{\ell i3}\left(-\frac{2}{3}\right)$$

and

$$<p\uparrow|\int d^3x\, x_\ell\, V_i^3|p\uparrow> = \frac{2\pi}{9}\frac{\Lambda}{F_\pi\, e^3}\,\epsilon_{\ell i3} \quad .$$

In conclusion, from (20) we get

$$\left(\vec{\mu}_{I=1}\right)_3 = \frac{2\pi}{9}\frac{\Lambda}{F_\pi\, e^3} \quad . \tag{23}$$

The isovector g factor $g_{I=1} = g_p - g_n$ turns out to be 6.38 against the experimental value of 9.4. The magnetic moments for the proton and neutron, measured in terms of Bohr magneton , are $\mu_p = \frac{g_p}{2} = 1.87$ and $\mu_n = \frac{g_n}{2} = -1.31$ respectively. The ratio $\left|\frac{\mu_p}{\mu_n}\right|$ turns out to be 1.43 (see Table 1), as opposed to 1.5 in the quark model and 1.46 experimentally.

3. MASS RELATIONS

It is interesting to form certain combinations of experimentally measured quantities from which the parameters of the Skyrme model cancel out. Combining our various formulas, one finds the following formula for the isoscalar g factor in terms of experimentally measured quantities

$$g_{I=0} = \frac{4}{9} \langle r^2 \rangle_{I=0} M_N (M_\Delta - M_N) \qquad (24)$$

This formula is very well satisfied experimentally. The left hand side is 1.76 and the right hand side is 1.66. We also find a formula for the isovector g factor from which the Skyrme model parameters cancel out:

$$g_{I=1} = \frac{2M_N}{M_\Delta - M_N} \qquad (25)$$

This relation is not so well satisfied experimentally, the left hand side being 9.4 and the right hand side 6.38.

Relations (24) and (25) are clearly much more general than the rest of our formulas. For instance, it is easy to see that they continue to hold if an arbitrary isospin conserving potential energy V(U) is included in the model -- the most obvious candidate being a term Tr U to simulate the effects of quark

masses. It is natural to wonder exactly how broad is the range of validity of these formulas.

Consider the soliton before it begins to rotate as a spherically symmetric classical body with an energy density $T_{00}(r)$. (We will treat the soliton as a non-relativistic object and ignore the pressure T_{ij} relative to T_{00}. Actually the proper inclusion of T_{ij} does not modify the formulas.) If such a body begins to rotate with angular frequency $\vec{\omega}$, the velocity at position $\vec{x}$ is $\vec{v}(r) = \vec{\omega} \times \vec{x}$, and the momentum density is $T_{0i}(\vec{x}) = T_{00}(r)\, \varepsilon_{ijk}\, \omega_j x_k$. The angular momentum of the spinning body is

$$\begin{aligned} J_i &= \int d^3x\, \varepsilon_{ijk}\, x_j\, T_{0k}(x) \\ &= \int d^3x\, (\omega_i r^2 - x_i\, \vec{x}\cdot\vec{\omega})\, T_{00} \\ &= \frac{2}{3}\, \omega_i \int d^3x\, T_{00}\, r^2 \end{aligned}$$

We have simply obtained the formula $\vec{J} = I\vec{\omega}$, where the moment of inertia is $I = \frac{2}{3} \int d^3x\, T_{00}\, r^2$. If the body begins to rotate its kinetic energy will be

$$T = \frac{1}{2} \int d^3x\, T_{00}\, \vec{v}^2$$

$$= \frac{1}{2} \int d^3x\, T_{00} (\vec{\omega} \times \vec{x})^2$$

$$= \frac{\vec{\omega}^2}{3} \int d^3x\, T_{00}\, r^2 = \frac{\vec{J}^2}{2I}$$

For the nucleon, $\vec{J}^2 = \frac{3}{4}\hbar^2$; for the Δ, $\vec{J}^2 = \frac{15}{4}\hbar^2$. Interpreting the mass difference between the delta and nucleon as a consequence of the rotational kinetic energy, we find for the moment of inertia $I = (3/2)(M_\Delta - M_N)^{-1}$. The rotational frequency of the nucleon is hence $\vec{\omega} = \vec{J}/I = (2/3)(M_\Delta - M_N)\vec{J}$.

The soliton before it begins to spin has some isoscalar charge density $\rho(r)$, but the isoscalar current density vanishes for a soliton at rest because of spherical symmetry and current conservation (or because of time reversal invariance). A rotating soliton has the current density $\vec{J} = \rho\,\vec{v} = \rho(\vec{\omega} \times \vec{x})$. So the magnetic moment of the rotating soliton is

$$\vec{\mu}_{I=0} = \frac{1}{2} \int d^3x\, \vec{x} \times \vec{J}$$

$$= \frac{1}{2} \int d^3x\, \rho\vec{x} \times (\vec{\omega} \times \vec{x})$$

$$= \frac{1}{3}\, \vec{\omega} \int d^3x\, \rho r^2$$

$$= \frac{\vec{\omega}}{3} \langle r^2 \rangle_{I=0} \qquad (26)$$

Combining this with $\vec{\omega} = (\frac{2}{3})(M_\Delta - M_N)\vec{J}$ and with the definition $\vec{\mu} = \frac{g}{2M}\vec{J}$ of the g factor, we find the result (24) for the isoscalar magnetic moment of the nucleon.

Now, to what extent is this result general? The relations $\vec{J} = I\vec{\omega}$, $T = \vec{J}^2/2I$ are completely general formulas for the angular momentum and kinetic energy of a slowly rotating body. (These formulas hold even when the Hamiltonian -- after elimination of non-propagating degrees of freedom -- is non-local.) The nucleon and delta are slowly rotating bodies in the large N limit, with I of order N and $\vec{\omega}$ of order 1/N. The formula $\vec{\omega} = \frac{2}{3}(M_\Delta - M_N)\vec{J}$ is a rigorous formula for the rotational frequency of the nucleon or delta in the large N limit or in any semi-classical soliton description.

Unfortunately, the formula $\vec{J} = \rho(\vec{\omega} \times \vec{x})$ is not a completely general formula for the current density induced in a static object when it begins to rotate. This formula holds for a macroscopic body, but whether it holds for a microscopic body such as a soliton depends on how the current and charge densities are constructed from the elementary fields. Likewise the formula (26) is not a completely general formula for the magnetic moment of a rotating sphere. In general there may be a non-locality in the relation between the charge density and the induced current; this non-locality spoils the relation $\vec{\mu}_{I=0} = \frac{\vec{\omega}}{3} \langle r^2 \rangle_{I=0}$.

If the baryon current is given by Skyrme's formula (11) then we will have (25), and the successful relation (24) will hold regardless of the choice of the chiral model Lagrangian. However, a realistic description of nature requires additions to the Skyrme current. For instance, the J = 1, I = 0 ω meson is observed to couple to the isoscalar current. This suggests the addition to the current of an extra term $\Delta B_\mu = a\, \partial_\nu \omega_{\mu\nu}$, where $\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$. With this addition to the current the relation (26) no longer holds for a rotating soliton, and with it eq. (24) is lost.

Thus, the successful formula (24) depends on the definition of the baryon current but not on the choice of the Lagrangian. It can likewise be shown that eq. (25) holds as long as the Lagrangian only involves spinless fields and their first derivatives, but can be modified by including higher derivatives or fields of higher spin.

4. AXIAL COUPLING AND GOLDBERGER-TREIMAN RELATION

To evaluate the axial coupling g_A we calculate the integral $\int d^3x\, A_i^a(x)$ in a soliton state. The relation of this integral with the axial coupling is slightly subtle. The standard definition of axial current matrix element is

$$\langle N'(p_2)|A_\mu^a(0)|N(p_1)\rangle = \bar{u}(p_2)\, \tau^a (\gamma_\mu \gamma_5 g_A(q^2) + q_\mu \gamma_5 h_A(q^2))\, u(p_1) \tag{27}$$

Current conservation implies $2m\, g_A(q^2) + q^2 h_A(q^2) = 0$. In the nonrelativistic limit, for the spatial components of the current, (27) becomes

$$\langle N'(p_2)|A_i^a(0)|N(p_1)\rangle = g_A(q^2)\left(\delta_{ij} - \frac{q_i q_j}{|q|^2}\right)\langle N'|\sigma_j \tau^a|N\rangle \tag{28}$$

The $\frac{1}{|q|^2}$ singularity in (28) reflects, of course, the pion pole. The $\vec{q} \to 0$ limit of (28) is ambiguous. Taking the limit in a symmetric way - replacing $q_i q_j$ by $\frac{1}{3}\delta_{ij}|q|^2$ - the right hand side of (28) becomes $\frac{2}{3} g_A \langle N'|\sigma_i \tau^a|N\rangle$ in the limit $q \to 0$; here $g_A = g_A(0)$ is the usual axial coupling constant.

Corresponding to this subtlety, the integral $\int d^3x\, A_i^a(x)$ in a soliton state is not absolutely convergent. Performing first the angular integral and then the radial integral corresponds to the symmetric limit just described. With this prescription for the integral we find

$$\int d^3x\, A_i^a(x) = \frac{\pi}{3e^2}\, D\, \mathrm{Tr}[\tau_i A^{-1} \tau_a A] \tag{29}$$

where

$$D = \int_0^\infty d\tilde{r}\, \tilde{r}^2 \left[\left(F' + \frac{\sin 2F}{\tilde{r}}\right) + 4\left(\frac{\sin 2F}{\tilde{r}}(F')^2 + \frac{2\sin^2 F\, F'}{\tilde{r}^2}\right.\right.$$

$$\left.\left. + \frac{\sin^2 F \sin 2F}{\tilde{r}^3}\right)\right]$$

Numerically we find $D = -17.2$. As we have discussed before (22) $Tr[\tau_i A^{-1}\tau_a A]$, evaluated in a nucleon state, equals $-\frac{2}{3}\langle\sigma_i\tau_a\rangle$. Setting (29) equal to $\frac{2}{3}g_A$ (corresponding to the symmetric $\vec{q}\to 0$ limit of (28)) we get

$$g_A = \frac{3}{2}\left(-\frac{2}{3}\right)\frac{\pi}{3e^2}\, D = 0.61 \qquad (30)$$

which unfortunately is not in good agreement with the experimental value $g_A = 1.23$. Although the Adler-Weisberger sum rule, which is a consequence of chiral symmetry, is surely obeyed in the Skyrme model, we do not know how it works out.

There is another useful way to compute g_A , which links it to the long distance behaviour of the soliton solution $F(r)$, and turns out to be particularly useful for proving the Goldberger-Treiman relation.

The requirement of current conservation $\partial_\mu A^\mu = 0$ reduces to $\partial_i A^i = 0$ in the static approximation. Therefore the volume integral of the axial current can be computed as a surface integral by using the divergence theorem, as follows:

$$\int d^3x\, A_i^a = \int d^3x\, \partial_j(x_i A^a{}_j) = \int_S x_i A_j^a \hat{x}_j\, dS \qquad (31)$$

The definition of axial current from (10) is

$$A_i^a = \frac{iF_\pi^2}{8} Tr\left[\left((\partial_i U_0)U_0^+ + U_0^+\partial_i U_0\right)A^{-1}\tau^a A\right] + \text{higher derivatives} \qquad (32)$$

where $U_0 = \cos F + i \sin F\, \vec{\tau}\cdot\hat{x}$ is the soliton solution. At large distances $F(r)$ goes like $\frac{B}{r^2}$ where B can be extracted from the computer solution and is $B = \frac{B'}{e^2F_\pi^2}$ with $B' = 8.6$. Therefore at large distances

$$U_0 = 1 + i\frac{B}{r^2}\vec{\tau}\cdot\hat{x}$$

and

$$\partial^i U_0 = -i\frac{B}{r^3}\left(\tau_i - 3\vec{\tau}\cdot\hat{x}\,\hat{x}^i\right)$$

It follows from (32) that the current to be used in formula (31) is

$$A_i^a = \frac{F_\pi^2}{4}\frac{B}{r^3}\left[\left(\tau_i - 3\vec{\tau}\cdot\hat{x}\,\hat{x}_i\right)A^{-1}\tau_a A\right] + \ldots \qquad (33)$$

Therefore from (31) we obtain

$$\int d^3x\, A_i^a = -F_\pi^2 B \frac{2\pi}{3} \operatorname{Tr}\left[\tau_i A^{-1}\tau^a A\right]$$

From (22) and the definition of g_A we get therefore

$$g_A = \frac{3}{2} F_\pi^2 B \frac{2\pi}{3} \frac{2}{3} = 2B' \frac{\pi}{3e^2} = 0.61 \qquad (34)$$

as before. Equations (30) and (34) imply a relation between D and B'. Indeed, by using (3) one can integrate D with the result $D = -2B'$.

Finally, let us check the Goldberger-Treiman relation in this model. The old fashioned Lagrangian for pions π coupled to nucleons ψ is

$$L = \frac{1}{2}\left(\partial_\mu \pi^a\right)^2 + i g_{\pi NN}\, \pi^a \bar{\psi}\gamma_5 \tau^a \psi$$

The nonrelativistic reduction of the coupling term is $\frac{g_{\pi NN}}{2M_N}\, \partial_i \pi^a \bar{\psi}\sigma_i \tau^a \psi$. From this form one can find the large distance behaviour of the expectation value of the pion field in a nucleon state

$$\langle \pi^a(x) \rangle = -\frac{g_{\pi NN}}{8\pi M_N} \frac{x_i}{r^3} \langle \sigma_i \tau^a \rangle \quad . \qquad (35)$$

On the other hand, we can find the expectation value of the pion field at great distances from a soliton by studying the asymptotic behaviour of the soliton solution. The small fluctuations of U around its vacuum expectation value are related to the pion field by

$$U = 1 + 2i \frac{\vec{\tau}\cdot\vec{\pi}}{F_\pi} + \dots$$

With $U = AU_0A^{-1}$ and $U_0 = 1 + i \frac{B}{r^2} \vec{\tau}\cdot\hat{x} \dots$, we find the large distance behaviour of the pion field:

$$\pi^a = \frac{BF_\pi}{4} \frac{x_i}{r^3} \operatorname{Tr}\left[\tau_i A^{-1}\tau^a A\right]$$

By using (22) and (34)

$$\langle \pi^a \rangle = -B \frac{F_\pi}{6} \frac{x_i}{r^3} \langle \sigma_i \tau^a \rangle = -\frac{g_A}{F_\pi} \frac{1}{4\pi} \frac{x_i}{r^3} \langle \sigma_i \tau^a \rangle \qquad (36)$$

So comparing (35) and (36) we finally get the Goldberger-Treiman relation

$$g_A = \frac{F_\pi\, g_{\pi NN}}{2M_N}$$

The predicted value of $g_{\pi NN}$ is 8.9 compared with the experimental value of 13.5.

5. DECAYS OF THE Δ

In this section, we will calculate the amplitudes for the decay processes $\Delta \to N\pi$ and $\Delta \to N\gamma$. The decay $\Delta \to N\gamma$ is related by a simple quark model argument [11] to the nucleon magnetic moment. A similar quark model argument [12] relates the amplitude $\Delta \to N\pi$ to the pion-nucleon coupling. For a review of the quark model relations, see [13]. We will see that the 1/N expansion makes predictions for Δ decays analogous to the predictions of the quark model. These predictions are model-independent in the sense that they hold for any soliton model of baryons and serve as quantitative tests of the $\frac{1}{N}$ expansion. The Skyrme model will not enter in this section except in the concluding paragraph.

In the large N limit the Δ and the nucleon are nearly degenerate, so the decays $\Delta \to N\pi$ and $\Delta \to N\gamma$ involve soft pions and photons. Also, the nucleon and the Δ are described by the same classical soliton solution with different but known wave functions for the collective coordinates (8). Hence the coupling of the soft pion or photon in Δ decay can be computed in terms of the static coupling of pions or photons to nucleons.

In view of chiral symmetry, the pion couplings to baryons can be expressed as derivative couplings. For soft pions, the coupling will involve mainly the first derivative of the pion field $\partial_i \pi^a$, multiplied by some operator $\mathcal{O}_i^{\ a}$ acting on the

collective coordinates. In $\mathcal{O}_i^{\ a}$ time derivatives of A can be neglected (since the nucleon rotates slowly in the large N limit) so $\mathcal{O}_i^{\ a}$ must be a function of A only. The only function of A that transforms properly under spin and isospin ($\mathcal{O}_i^{\ a}$ must have $I = J = 1$) is $\mathrm{Tr}[\tau_i A^{-1} \tau_a A]$. So in the large N limit, irrespective of other details, the coupling of soft pions to baryons is of the form

$$L_\pi = \delta\, \partial_i \pi^a \,\mathrm{Tr}[\tau_i A^{-1} \tau_a A] \tag{37}$$

for some δ.

The pion-nucleon coupling is related to δ by evaluating the matrix element of (37) between initial and final nucleon states. We have already done this, in effect, in computing $g_{\pi NN}$ in the Skyrme model, and the relation is $\delta = \frac{3}{4} \frac{g_{\pi NN}}{M_N}$, M_N being the nucleon mass. On the other hand, we can describe the hadronic decay of the Δ by taking the matrix element of (37) between an initial Δ and a final nucleon.

Let us define a coupling $g_{\pi N\Delta}$ as follows (it is called $M_{\uparrow\uparrow}$ in [12]). For a decay $\Delta^{++}\left(s_z = \frac{3}{2}\right) \to p\left(s_z = \frac{1}{2}\right) + \pi^+$, we define the amplitude to be $g_{\pi N\Delta}\left(k_x + ik_y\right)\frac{1}{2M_N}$, where $\vec{k}$ is the center of mass momentum of the pion. Evaluating the matrix element of (37), we find $g_{\pi N\Delta} = \frac{3}{2}\, g_{\pi NN}$. The quark model relation of [12] is instead $g_{\pi N\Delta} = \frac{6}{5}\, g_{\pi NN}$. The relation

$g_{\pi N\Delta} = \frac{3}{2}\, g_{\pi NN}$, which follows from the 1/N expansion without other assumptions, is in excellent agreement with experiment. With the experimental value $g_{\pi NN} = 13.5$, it gives a val 125 Mev for the width of the Δ; the experimental value is about 120 Mev.

A similar analysis can be made for the electromagnetic decay of the Δ. The decay $\Delta \to N\gamma$ violates isospin, so it involves only the isovector part of the electromagnetic current. The isovector coupling of the magnetic field $\vec{B}$ to baryons must be of the form $\vec{B}\cdot\vec{\mu}$, where $\vec{\mu}$ is an operator acting on the collective coordinates of baryons. $\vec{\mu}$, the isovector magnetic moment operator of baryons, must be the third component of an isovector. Neglecting time derivatives, the only possibility is $\mu_i = \alpha\, \mathrm{Tr}\left[\tau_i A^{-1} \tau_3 A\right]$ where α is some constant. So the magnetic coupling to baryons is

$$L_{Mag} = \vec{B}\cdot\vec{\mu} = \alpha B_i\, \mathrm{Tr}\left[\tau_i A^{-1} \tau_3 A\right] \qquad (38)$$

A relation of this form holds in any soliton description of baryons; only the value of α is model dependent. The value of α determines the isovector part of the nucleon magnetic moment. The relation is obtained by calculating the matrix element of (38) between initial and final nucleon states; the calculation is essentially the one we have already performed in deriving

eq. (23). Writing the proton and neutron magnetic moments as $\vec{\mu}_p = \mu_p \vec{\sigma}$, $\vec{\mu}_n = \mu_n \vec{\sigma}$, the relation is $\alpha = \frac{3}{4}(\mu_p - \mu_n)$.

We can now calculate the amplitude for $\Delta \to N\gamma$ by evaluating the matrix element of (38) between initial Δ and final nucleon. Let us define a transition moment $\mu_{N\Delta}$ by the formula $\mu_{N\Delta} = \langle p, s_z = \frac{1}{2} | \mu_z | \Delta^+, s_z = \frac{1}{2} \rangle$ where $\mu_z = \frac{3}{4}(\mu_p - \mu_n)\, \mathrm{Tr}\left[\tau_3 A^{-1} \tau_3 A\right]$ is the z component of the baryon magnetic moment operator. Using wave functions in (8) we find $\mu_{N\Delta} = (\mu_p - \mu_n)/\sqrt{2}$. This agrees very well with the experimental value $\mu_{N\Delta} = (.70 \pm .01)(\mu_p - \mu_n)$. The quark model (11) gives $\mu_{N\Delta} = \frac{2}{5}\sqrt{2}\, (\mu_p - \mu_n) = .57(\mu_p - \mu_n)$ (this relation is often written $\mu_{N\Delta} = \frac{2}{3}\sqrt{2}\, \mu_p$; we are using here the quark model prediction $\mu_n = -\frac{2}{3}\mu_p$).

The model independent tests of the 1/N expansion $g_{\pi N\Delta} = \frac{3}{2}\, g_{\pi NN}$ and $\mu_{N\Delta} = (\mu_p - \mu_n)/\sqrt{2}$ work very well (perhaps fortuitously so) if one takes $g_{\pi NN}$ and $\mu_p - \mu_n$ from experiment. The Skyrme model, however, is less successful. Since the Skyrme model values of $g_{\pi NN}$, μ_p, and μ_n are all about 30% too small, the predictions for $\mu_{N\Delta}$ and $g_{\pi N\Delta}$ are too low (see table I) by a similar margin.

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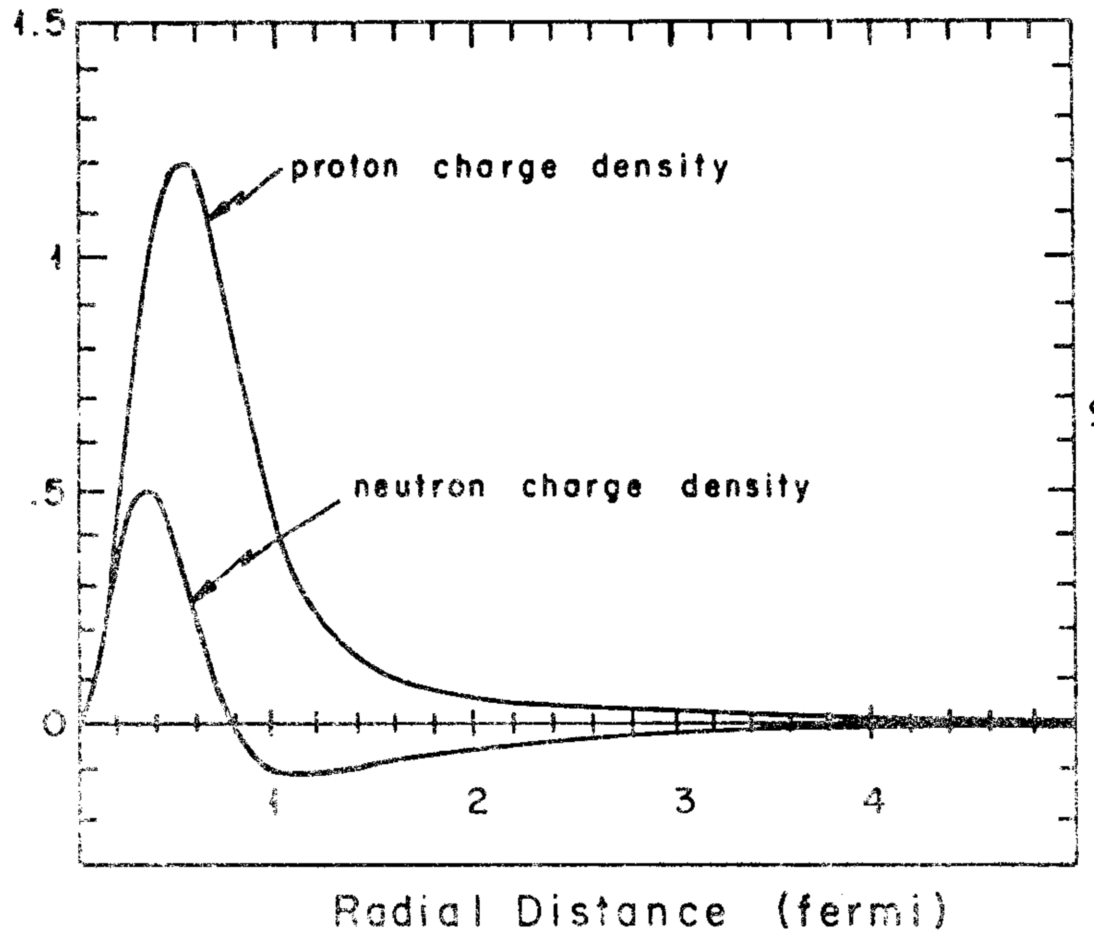
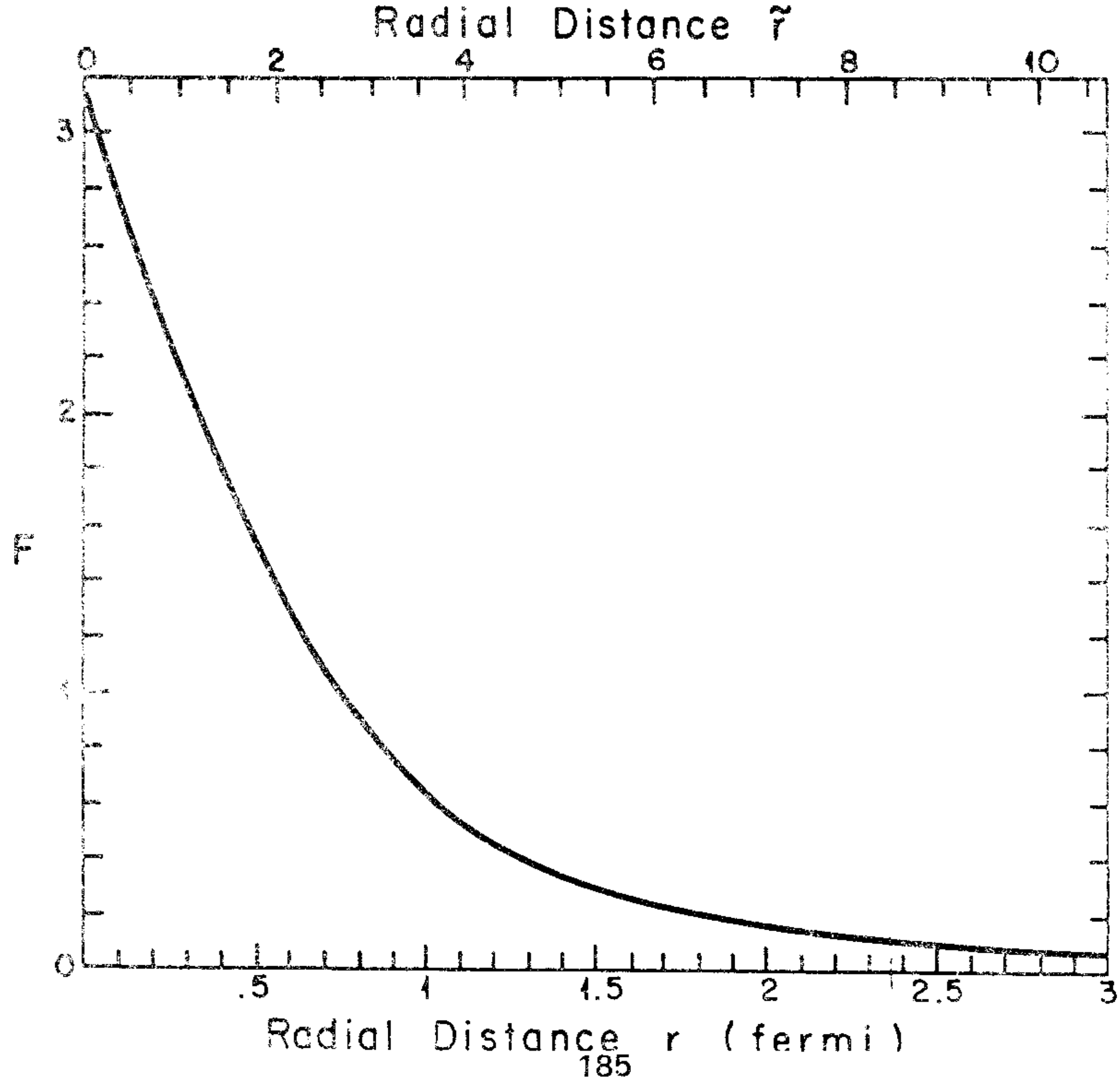
FIGURE CAPTIONS

Fig. 1: Plot of F, the numerical solution of eq. (3). F appears in the Skyrme ansatz $U_0(x) = \exp[iF(r)\vec{\tau}\cdot\Omega]$. The radial distance is measured in fermi, and also in the dimensionless variable $\tilde{r} = eF_\pi r$.

Fig. 2: Plot of the proton and neutron charge densities. These charge densities are given as functions of the radial distance r, and include a factor of $4\pi r^2$.


proton charge density
neutron charge density
1.5
1
.5
0
1
2
3
4
Radial Distance (fermi)

31


Radial Distance $\tilde{r}$
0
2
4
6
8
10
F
3
2
0
.5
1
1.5
2
2.5
3
Radial Distance r (fermi)

32

TABLE 1

Quantity	Prediction	Experiment
M_N	input	939 Mev
M_Δ	input	1232 Mev
F_π	129 Mev	186 Mev
$\langle r^2\rangle^{1/2}_{I=0}$	.59 fm	.72 fm
$\langle r^2\rangle^{1/2}_{M,I=0}$	.92 fm	.81 fm
μ_p	1.87	2.79
μ_n	-1.31	-1.91
$\left\|\frac{\mu_p}{\mu_n}\right\|$	1.43	1.46
g_A	0.61	1.23
$g_{\pi NN}$	8.9	13.5
$g_{\pi N\Delta}$	13.2	20.3
$\mu_{N\Delta}$	2.3	3.3

IV. HIGHER DIMENSIONAL THEORIES

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KALUZA-KLEIN APPROACH TO SUPERGRAVITY

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References:

Supergravity: [1]

N = 8 theories: [2, 3]

N = 1 phenomenology: [4]

Compactification of d = 11 supergravity on S^7: [5 → 24]

Other Kaluza-Klein refs: [25 → 35]

Other relevant refs: [35 → 42]

KALUZA KLEIN APPROACH TO SUPERGRAVITY

M. J. Duff.

I. INTRODUCTION

Why Kaluza-Klein?
Why Supergravity?

II THE d=11 THEORY

Spontaneous compactification
Holonomy groups and broken supersymmetries: Killing spinors
Known solutions

III THE SEVEN-SPHERE AND SPONTANEOUS SYMMETRY BREAKING

Round S^7, N=8 theory, massless & massive states
Squashing S^7, the Higgs effect, "Space Invaders"
Parallelizability of S^7, more Higgs
Breaking of N=8 to N=1 and to N=0

IV REALISTIC THEORY?

The cosmological constant and fermion condensates
$SU(3) \times SU(2) \times U(1)$
Bound states.

REFS: Supergravity [1→4], Seven-sphere compactification [5→24], other Kaluza-Klein [25→35], other [35→].

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Consider
$$S = -\frac{1}{2\pi K_5^2}\int d^4x\,dy\sqrt{-g}\,R$$

$\Rightarrow \quad R_{MN} = 0$

consistent with but not imply $\langle g_{MN}\rangle = M^4 \times S^1$

Write
$$g_{MN} = \phi^{-1/3}\begin{bmatrix} g_{\mu\nu} + \kappa^2\phi A_\mu A_\nu & \kappa\phi A_\mu \\ \kappa\phi A_\nu & \phi \end{bmatrix}$$

$g_{MN} = \det g_{\mu\nu}$

with $\phi = \exp\sqrt{3}\,\kappa\sigma \qquad \kappa^2 = 16\pi G = m\,K_5^2$

$$\Rightarrow \quad S = \int d^4x\sqrt{-g}\left[-\frac{1}{\kappa^2}R - \frac{1}{4}e^{\sqrt{3}\kappa\sigma}F_{\mu\nu}F^{\mu\nu} + \ldots\right]$$

where $\langle g_{\mu\nu}\rangle = \eta_{\mu\nu} \quad \langle A_\mu\rangle = 0 \quad \langle\sigma\rangle = 0$

Expand about ground state:
$$h_{\mu\nu}(x,y) = \sum_n h^n_{\mu\nu}(x)\,e^{inmy}$$
$$A_\mu(x,y) = \sum_n A^n_\mu(x)\,e^{inmy}$$
$$\sigma(x,y) = \sum_n \sigma^n(x)\,e^{inmy}$$

$\Rightarrow$ tower of charged massive states $q_n = n\,q$ $\qquad e = \kappa m$

Symmetries
$$\delta g_{MN}(x,y) = -\nabla_M\xi_N - \nabla_N\xi_M$$
$$\xi_M = \xi_M(x,y) = \sum_n \xi^{(n)}_M(x)\,e^{inmy}$$
$$\delta\hat{g}_{\mu\nu} = \delta(\phi^{-1/4}g_{\mu\nu} + \kappa^2\phi^{3/4}A_\mu A_\nu)$$
$$= -\hat{\nabla}_\mu\xi_\nu - \hat{\nabla}_\nu\xi_\mu$$

$$\Rightarrow \quad \begin{aligned} \delta\mathring{g}_{\mu\nu} &= -\nabla_\mu\mathring{\xi}_\nu - \nabla_\nu\mathring{\xi}_\mu \\ \delta\mathring{A}_\mu &= -\xi^\nu\nabla_\nu\mathring{A}_\mu - \mathring{A}_\nu\nabla^\nu\mathring{\xi}_\mu - \nabla_\mu\mathring{\xi}_5 \\ \delta\mathring{\sigma} &= -\xi^\mu\nabla_\mu\mathring{\sigma} \end{aligned}$$

i.e. by keeping only massless states, $d = 5$ gen covariance
$\Rightarrow$ $d=5$ gen covariance and $U(1)$ gauge transformation.

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{\kappa^2}R - \frac{1}{4}e^{\sqrt{3}\kappa\sigma}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\,\partial_\nu\sigma\right]$$

also global symmetry $A_\mu \to \lambda A_\mu \qquad \phi \to \lambda^{-2}\phi$

$\lambda =$ constant

EXAMPLE: ~~generalize~~ to 4 + K

$$S = -\frac{1}{(2\pi)^K \kappa_K^2}\int d^{4+K}\sqrt{-g}\,R \qquad \Rightarrow \qquad M^4 \times \underbrace{S^1 \times S^1}_{K\ \text{time}}$$

$$g_{MN} = \Delta^{-1/4}\begin{bmatrix} g_{\mu\nu} + \kappa^2 g_{mn} A_\mu^m A_\nu^n & \kappa\, g_{nm} A_\mu^m \\ \kappa\, g A_\nu^n g_{mn} & g_{mn} \end{bmatrix}$$

Δ = det g_{nm}

Symmetry of massless states $d = 4$ gen covariance
$U(1) \times U(1) \ldots K$ times
$GL(K, R)$ $\quad A_\mu^n \to \lambda^{-1\,n}{}_m A_\mu^m$
$g_{nm} \to \lambda_n^p \lambda_m^q g_{pq}$

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{\kappa^2}R - \frac{1}{4} g_{nm} F_{\mu\nu}^n F^{m\mu\nu} - \frac{1}{2}\partial_\mu g_{nm}\partial^\mu g^{nm}\right.$$

$q_n^{(p)} = n\kappa\, m^{(p)}$

COMMENTS (1) ~~Ground~~ state has both spacetime & extra dimensions flat. Consistent with 4+K equations of motion but 1) compactification not implied 2) $d=4$ for spacetime is chosen by hand. Stable? Ref WITTEN
(2) $R_{MN} = 0 \Rightarrow$ Einstein + Maxwell + Klein Gordon $\Box\sigma = \sqrt{3}\kappa e^{\sqrt{3}\kappa\sigma} F_{\mu\nu}F_{\mu\nu}$
(3) $\Lambda = 0$ both in $d = 4 + K$ and $d = 4$ — can't put $\sigma = 0$
(4) signature $-+++$ important $-$ve energies

3) no potential for scalars (no Higgs)

4) gauge group abelian

5) 1 spin 2
K spin 1
$\frac{K(K+1)}{2}$ spin 0

total degrees of freedom $= 2 + 2K + \frac{K(K+1)}{2} = (4+K)(1+K)/2$

same as in 4 + K. Not true in general.

6) scalars are singlets under gauge group.

REINSTATE MASSIVE MODES

cf SALAM & STRATHDEE
DUFF & TOMS (2)
CHODOS & DETWEILER
ROTH & RUBIN

Symmetry of $\mathcal{L}$ $d = 4+K$ gen cov
" " $\langle g_{MN}\rangle$ $d = 4$ Poincaré $\times U(1) \times$

1) Symmetries of $d = 4$ theory now given by ∞ dimensional noncompact group. Salam and Strathdee $\Rightarrow$ SO(1, ~~2~~) in 4+1 theory

$\delta x^\mu = 0 \qquad \delta y = \omega^1(x)\cos my + \omega^2(x)\sin my + \omega^3(x)$

$Q_1 = -\cos my \frac{\partial}{\partial y} \qquad Q_2 = -\sin my \frac{\partial}{\partial y} \qquad Q_3 = -\frac{\partial}{\partial y}$

$[Q_1, Q_2] = -Q_3 \qquad [Q_2, Q_3] = Q_1 \qquad [Q_3, Q_1] = Q_2$

2) Power counting now 4+K, not 4. See by

$$\sum_n \int d^4x \quad \sim \int d^{4+K}y$$

Keep massive modes ⇒ quantize in $d = 4+K$
discard " " " " $d = 4$

N.B. $D = (d-2)L + 2$ degree of divergence L loops

no compelling reason for $d = 4$.

3) Philosophy: No operational distinction between $d = 4+K$ theory and $d = 4$ theory with special spectrum of massive states. N

4) Careful about gauges when quantizing.

5) Finite in $d = 5$ at 1-loop, $d = 4$ explanation

$$\sum_n m_n^4 \sqrt{g} + m_n^2 \sqrt{g} R + m_n^0 \sqrt{g} R^2$$

$$2m^4 \zeta_R(-2)\sqrt{g} + 2m^2 \zeta_R(\cdot)\sqrt{g}R + (1 + 2\zeta_R(0))\sqrt{g}R^2$$

$$\quad = 0 \qquad\qquad = 0 \qquad\qquad = 0$$

6) $q_n \sim ne$ $\quad e = Km$ $\quad \dfrac{q_n}{m_n} = K$ universal!
$m_n = nm$

[Spin 1 equivalence principle]

REFERENCES II

LECTURE 2

See Trieste Lectures

Spontaneous Compactification — Horvath et al, Duff & van N., Freund & Rubin.

Take pure gravity

$$R_{MN} = 0$$

consistent with $R_{\mu\nu} = 0$ $\quad R_{mn} = 0$

extra dimensions must be Ricci flat ⇒ $U(1) \times \dots U(1)$ at most. For interesting non-abelian groups need $R_{mn} \neq 0$

(e.g. $R_{mn} \propto g_{mn}$ all coset spaces and some cosets G/H e.g. $SO(K+1)/SO(K)$ ...

Would like $M = M_1 \times M_2$

M_1 $\quad R_{\mu\nu} = \Lambda_1 g_{\mu\nu}$ $\quad \Lambda_1 \leq 0$

(compact) M_2 $\quad R_{mn} = \Lambda_2 g_{mn}$ $\quad \Lambda_2 > 0$

$\Lambda_2 > 0$ because Einstein & $\Lambda_2 > 0$ ⇒ compact (& discrete spectrum) and prefer $\Lambda_1 \leq 0$ (eventually $= 0$) ...

$\langle g_{\mu\nu} \rangle$ = AdS permissible field theory (classically stable ...

compatible with supersymmetry. So pure gravity with bare

Λ no good

$$G_{MN} + \Lambda g_{MN} = 0$$

$$g^{MN} R_{MN} - \frac{d}{2} R + d\Lambda = 0$$

$$\frac{2-d}{2} R = -d\Lambda \qquad R_{MN} = \frac{2\Lambda}{d-2} g_{MN}$$

$$\Rightarrow \qquad R_{\mu\nu} = c_1 g_{\mu\nu}$$
$$R_{mn} = c_1 g_{mn}$$

can't have AdS × compact space nor M^4 × compact space.

⇒ Need gravity + matter [Cremmer et al]

could have gravity + Yang-Mills but would defeat the object of the exercise. Better suggestion made by Freund & Rubin. Try antisymmetric tensor (motivated of course by supergravity)

[illegible]

… κ_{11}^2

+++ ·)

hel [illegible]

d dimensional Lagrangian

$$\mathcal{L} = \frac{1}{4}\sqrt{g}R - \frac{1}{48}\sqrt{g}\, F_{MNPQ} F^{MNPQ}$$

↙ could choose other ro[u]nd

where $F_{MNPQ} = 4\,\partial_{[M} A_{NPQ]}$

i.e. invariant under abelian gauge transformations

$$\delta A_{NPQ} = \partial_{[N} \Lambda_{PQ]} \qquad \Lambda_{[PQ]}(x) \text{ arbitrary}$$

Field equations:

$$R_{MN} - \frac{1}{2} g_{MN} R = \frac{1}{3}\left[F_{MPQR} F_N{}^{PQR} - \frac{1}{8} g_{MN} F_{PQRS} F^{PQRS} \right]$$

$$\nabla_M F^{MPQR} = 0$$

Look solutions of the product form $M = M_1 \times M_2$

(better if one could prove instability of other solutions)

(but leave dim of spacetime unspecified)

$$\langle g_{\mu\nu} \rangle = \mathring{g}_{\mu\nu}(x) \qquad \langle F_{\mu\nu\rho\sigma} \rangle = \mathring{F}_{\mu\nu\rho\sigma}(x)$$

$$\langle g_{mn} \rangle = \mathring{g}_{mn}(y) \qquad \langle F_{mnpq} \rangle = \mathring{F}_{mnpq}(y)$$

But $\langle g_{\mu n} \rangle = 0 \qquad \langle F_{\mu\nu\rho q} \rangle = \langle F_{\mu\nu p q} \rangle$

$$= \langle F_{\mu n p q} \rangle = 0$$

since require Lorentz invariant ground state

Freund & Rubin set

$$F_{\mu\nu\rho\sigma} = f\ \epsilon_{\mu\nu\rho\sigma}$$
$$F_{mnpq} = 0$$

/ spacetime $d = 4$! [illegible] (c.f. Duff & van N.). [illegible] no real understanding [illegible]

Substitute into Einstein equations $\Rightarrow$ $M = M_1 \times M_2$

(-+++) $\qquad R_{\mu\nu} = \frac{10-2d}{d-2} f^2 g_{\mu\nu} \qquad c_1 < 0$

(++...+) $\qquad R_{mn} = \frac{6}{d-2} f^2 g_{mn} \qquad c_2 > 0$

Two cases

a) $f = 0$ max symm M^{11}. ($M^4 \times T^K$ [illegible] but also have $d=4$ [illegible])

b) $f \neq 0$ max symm $AdS(d=4) \times S^7$ [S^K fewer symmetries than T^K]

Compactification is automatic in case (b).

Hence "SPONTANEOUS" compactification because compactification is consequence of equations of motion. Just as in "spontaneous" symmetry breaking. [illegible] symmetry?

True vacuum not determined at the classical level. [illegible] not necessary to assume transition from unbroken [illegible] (M^{11}) to broken one ($AdS \times S^7$). May never have been M^{11}.

COMMENTS

a) Could have chosen other rank and obtained different spacetime dimension. Eg in $d=4$ rank one $A_M \Rightarrow$ $F_{MN} = \epsilon_{MN}$ QED compactifies down to $d=2$!

[Raises question of "true vacuum". At this classical level each solution can claim to be the ground state because energies cannot be compared $\Rightarrow$ quantum theory. Does it pick one and stay there?]

b) COSMOLOGICAL constant: Λ for spacetime related to curvature of extra dimensions.
Could put $\Lambda = 0$ by having Λ bare

$$R_{\mu\nu} = 0 \qquad \Lambda_0 = (d-5) f^2$$

$$R_{mn} = 2 f^2 g_{mn} \qquad (\text{N.B. } d \text{ independent})$$

Λ_0 defined by

$$G_{MN} + \Lambda_0 g_{MN} = \frac{1}{3}\left[F^2_{MN} - \frac{1}{12} g_{MN} F^2\right]$$

BUT i) ad hoc
ii) fine tuning spoiled by known effects
iii) (as we shall see) breaks SUSY explicitly rather than [illegible]
iv) perhaps fermions will supply effective Λ_0 in quantum theory.

Need both A_{MNP} and Λ_0 to get Minkowski space.

Ansatze for Massless States   Duff & Pope (this paper)

Suppose we have a ground state $\mathring{g}_{MN}$ and $\mathring{A}_{MNP..}$, write

$$g_{MN} = \mathring{g}_{MN} + h_{MN}$$

$$\Phi_{MNP..} = \mathring{A}_{MNP..} + \phi_{MNP..}$$

decompose each fluctuation

$$\phi^{mn\ldots}_{\mu\nu\ldots}(x,y) = \phi^{\alpha}_{\mu\nu}(x)\, Y^{mn}_{\alpha}(y)$$

where $M^2 Y^{mn}_{\alpha} = 0$

and M^2 is mass operator determined by expanding field equations to 1st order in fluctuations. Similarly fermions

$$M\, Y = 0$$

and index α runs over number of such zero modes.

ple:

$$\mathcal{L} = \frac{i}{2} \det e_M{}^A\, \bar{\Psi}\, \Gamma^M D_M \Psi \qquad \bar{\Psi} = \Psi^\dagger \Gamma_0$$

$$\{\Gamma_A, \Gamma_B\} = -2\eta_{AB} \qquad \Gamma_A^\dagger = \pm \Gamma_A \quad \begin{matrix} + & \text{time} \\ - & \text{space} \end{matrix}$$

[Take $\bar{\Psi} C = \Psi$ Majorana]

$$\Gamma^M D_M \Psi = 0$$

But $\Gamma_A = (\gamma_\mu \mathbb{1},\ \gamma_5 \Gamma_a)$ $\qquad \{\gamma_\alpha, \gamma_\beta\} = -2\eta_{\alpha\beta}$, $\{\Gamma_a \Gamma_b\} = -2\delta_{ab}$

$$(\gamma^\mu D_\mu + \gamma_5 \Gamma^m D_m)\, \Psi(x,y) = 0$$

kinetic term — mass term (N.B γ_5 mass term typical of K.K.).

Ansatz is $\Psi^{\alpha A} = \psi^{\alpha I}(x)\, \eta^{AI}(y)$

where I runs over solutions of

$$\Gamma^m D_m \eta = 0$$

(N.B since Ψ & ψ anti commuting, η is commuting)

Effectively we are Fourier decomposing

$$\Psi(x,y) = \sum_n \psi_n(x)\, Y_n(y)$$

and keeping lowest ($n=0$) mode.

BUT Lichnerowicz theorem $(\Gamma^m D_m)^2 = -\Box + \frac{R}{4}$

positive if $R \geq 0$ (Problem).

What about g_{MN} itself?

$$g_{MN} = \begin{bmatrix} g_{\mu\nu} + g_{mn} V_\mu{}^m V_\nu{}^n & V_\mu{}^m g_{mn} \\ g_{mn} V_\nu{}^n & g_{mn} \end{bmatrix}$$

Then require to know y dependence of

$g_{\mu\nu}(x,y)$	spin 2	rank 2 Tensor
$V_\mu{}^m(x,y)$	spin 1	vector
$g_{mn}(x,y)$	spin 0	scalar

under $\delta g_{MN} = -\nabla_{(M}\xi_{N)}$ with $\xi_M = (\xi_\mu(x), 0)$

Answer is model-dependent for scalars. Suppose we ignore (temporarily) the scalars (i.e. this is [illegible]) then ansatz is

$$g_{\mu\nu}(x,y) = g_{\mu\nu}(x)$$

$$V_\mu{}^m(x,y) = A_\mu{}^\alpha(x) K_\alpha{}^m(y)$$

$$g_{mn}(x,y) = g_{mn}(y) = \text{metric on } M_2$$

i.e. and $K_{\alpha m}$ are Killing vectors on M_2

$$\nabla_{(m} K_{n)} = 0$$

Number of massless vector bosons = N^o of Killing vectors.

<u>Explanation</u>

Consider gen coord transformation $\delta x^M = \xi^M$

$$\delta g_{MN} = -\nabla_M \xi_N - \nabla_N \xi_M$$

of special kind

$$\xi_m(x,y) = \varepsilon_\alpha(x) K_m{}^\alpha(y)$$

where $K_m{}^\alpha$ are the Killing vectors, such that

$$[K_\alpha, K_\beta] = f_{\alpha\beta}{}^\gamma K_\gamma \qquad K_\alpha = K_\alpha{}^m \partial_m$$

$f_{\alpha\beta}{}^\gamma$ structure constants of G. Then can show

$$\delta g_{\mu\nu}(x) = 0$$

$$\delta A_\mu{}^\alpha(x) = D_\mu \varepsilon^\alpha = \partial_\mu \varepsilon^\alpha(x) - f_{\gamma\beta}{}^\alpha A_\mu{}^\beta \varepsilon^\gamma$$

A_μ is the Yang-Mills gauge field appropriate

Similarly $g_{\mu\nu}(x)$ transforms like metric tensor $\delta g_{\mu\nu}(x)$

$\Rightarrow$ graviton and hence massless.

N B In presence of scalars, may be a Higgs mechanism giving masses to some of the vector bosons, and so ansatz not guaranteed to yield massless states.

Will return to problem of scalar ansatz when we settle on a specific model, namely $d = 11$ Supergravity.

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The ideas we have discussed may in principle be applied to any theory of gravity + matter in $d > 4$ but it is particularly compelling to apply them to supergravity for the following reasons.

SUSY ⇒ $\nexists$ [illegible] Nahm Nucl Phys B135 (
Maximum d is $d = 11$. Components of Majorana spinors

$$2^{\alpha - 1} \qquad \alpha = \frac{d}{2} \quad d \text{ even}$$
$$\alpha = \frac{d-1}{2} \quad d \text{ odd}$$

$d = 11 \Rightarrow \alpha = 5 \Rightarrow 2^5 = 32$ components

ψ^{α} $SO(1,10) \to SO(1,3) \times SO(7)$

$32 = 4 \times 8 \Rightarrow 8$ supersymmetries

helicity $-2 \xrightarrow{1} -3/2 \xrightarrow{2} -1 \xrightarrow{3} -1/2 \xrightarrow{4} 0 \xrightarrow{5} 1/2 \xrightarrow{6} 1 \xrightarrow{7} 3/2 \xrightarrow{8}$

⇒ max spin 2.

If $d > 11 \Rightarrow N > 8 \Rightarrow s_{max} > 2 \Rightarrow$ [illegible]

[illegible] $d = 11$ [illegible] Nahm [illegible] ?

⇒ unique particle content $e_M{}^A$ ψ_M A_{MNP} (F_4)
(or A_{MNPQRS} ⇒ F_7)

but only A_{MNP} exists and no cosmological constant.
Nicolai, Townsend
& Van N. Lett Nuovo Cim 30 (1980) 315.

$\mathcal{L}$ is (Cremmer & Julia Nucl Phys. B159 (79) 141). PAGE 9

N.B. Shall not use words like spin 2 spin 3/2 for $e_M{}^A$ ψ_M because "spin" is $SO(1,3)$ concept.

complicated but might be "theory of the world". [illegible] philosophical point. 1 superfield

C is antisym. [illegible] $[\bar\psi \chi = \ldots$

[N.B.

$C\Gamma_1$	S
$C\Gamma_2$	S
$C\Gamma_3$	A
$C\Gamma_4$	A
$C\Gamma_5$	S

]

2) Extreme KK philosophy $\Rightarrow$ symmetries PAGE 10

N.B discrete symmetries

3) Lichnerowicz

a) include gauge fields
b) torsion
c) squash
d) SUPERGRAVITY : only consistent spin 3/2 theory

4) SPONTANEOUS COMPACTIFICATION (PTO)

set $\langle \psi_M \rangle = 0$

Ansatz $\Rightarrow$ $\langle F_{\mu\nu\rho\sigma} \rangle = \alpha m \, \epsilon_{\mu\nu\rho\sigma}$

$R_{\mu\nu} =$

5) SPONTANEOUS SYMMETRY BREAKING

Magic properties of S^7 permit geometric origin of gauge SUSY P, C, CP breaking.

$\langle F_{\mu\nu\rho\sigma} \rangle = f \, \epsilon_{\mu\nu\rho\sigma}$ $\qquad \mathring{F}_{mnpq} \neq 0$

① $$\nabla_m \mathring{F}^{mnpq} = +\frac{f}{12} \, \epsilon^{rstumnpq} \mathring{F}_{rstu}$$ $\qquad d^*F = 2f\,F$, $\quad (\Delta - 4f^2)F = 0$

$\mathring{F}_4^2 = \mathring{F}_{\mu\nu\rho\sigma} \mathring{F}^{\mu\nu\rho\sigma} \leqslant 0$ $\qquad \mathring{F}_7^2 = \mathring{F}_{mnpq} \mathring{F}^{mnpq} \geqslant 0$

$= -24 f^2$

② $$\mathring{R}_{\mu\nu} = \frac{1}{3}\left[F_{\mu\alpha\beta\gamma} F_\nu{}^{\alpha\beta\gamma} - \frac{1}{12} g_{\mu\nu} (F_4^2 + F_7^2) \right]$$

③ $$R_{mn} = \frac{1}{3}\left[F_{mabc} F_n{}^{abc} - \frac{1}{12} g_{mn} (F_4^2 + F_7^2) \right]$$

N.B. $$R^\mu{}_\mu = \frac{1}{36}\left[8F_4^2 - 4F_7^2 \right] \leqslant 0$$ $\qquad \Lambda = 0$ iff $f =$

$$R^m{}_m = \frac{1}{36}\left[-7F_4^2 + 5F_7^2 \right] \geqslant 0$$

~

~~TEST FOR~~

A priori, any solution of these equations can claim to be the ground state, but "true" vacuum should presumably be distinguished by its symmetries.

Could try "maximal symmetry", but instead look to supersymmetry.

TEST FOR SUPERSYMMETRY

Have put $\langle \psi_M \rangle = 0$: unbroken supersymmetry ⇒ spinors ϵ for which $\langle \delta \psi_M \rangle = 0$.

i) For simplicity put $F_{mnpq} = 0$, $F_{\mu\nu\rho\sigma} = 3m\,\epsilon_{\mu\nu\rho\sigma}$ ⇒ $R_{\mu\nu} = \ldots$, $R_{\mu\nu} = 6 \ldots$

using $\Gamma_M = (\gamma_\mu \otimes 1, \gamma_5 \otimes \Gamma_a)$ find

$$\bar{D}_\mu = D_\mu + m\,\gamma_\mu \gamma_5$$

$$\bar{D}_m = D_m - \frac{m}{2} \Gamma_m$$

Unbroken supersymmetry requires, with $\epsilon^{\alpha i}(x,y) = \epsilon^{\alpha I}(x)\, \eta^{iI}(y)$

KILLING SPINORS $\quad \bar{D}_m \eta^I(y) = 0 \qquad I =$ number of solutions

$$[\bar{D}_m, \bar{D}_n]\eta = C_{mn}{}^{ab}\Gamma_{ab}\eta - \frac{1}{4} R_{mn}{}^{ab}\Gamma_a\Gamma_b\eta + \frac{m^2}{4}[\Gamma_m, \Gamma_n]\eta$$

~~C is the Weyl tensor~~

$$[\bar{D}_\mu, \bar{D}_\nu] = -\frac{1}{4} R_{\mu\nu}{}^{\alpha\beta}\gamma_\alpha\gamma_\beta - m^2[\gamma_\mu, \gamma_\nu]$$

Matrix has 8×8 ⇒ all eigenvalues zero

$$R_{mnpq} = m^2 (\mathring{g}_{mp}\mathring{g}_{nq} - \mathring{g}_{mq}\mathring{g}_{np})$$

⇒ unbroken maximal supersymmetry ⇒

$d=4$ AdS $\times S^7$ $\qquad S^7 = \frac{SO(8)}{SO(7)}$

Let us now adopt this solution, tentatively, as the ground state & examine the consequences.

N.B. $SO(8) \not\supset SU(3) \times SU(2) \times U(1)$ ⇒ hidden symmetries

contrast with WITTEN.

$d=7$ isometry group $\supset SU(3) \times SU(2) \times U(1)$. but did not [illegible] solutions.

e.g. $S^5 \times S^2 \sim SO(4) \times SO(2)$ $\qquad N = 0$
$\quad CP^2 \times S^3 \sim SU(3) \times SU(2) \times SO(2)$ $\qquad N = 0$

but related couplings. But CP^2 is bad.

Exp. why such non-maximally symm ground-states?

LECTURE 4

SUMMARY FROM LECTURE 3 : Start with $d=11$ supergravity
Unique. No matter! SLIDE

Bosonic field equations

I $\quad R_{MN} - \frac{1}{2} g_{MN} R = \frac{1}{3}\left[F_{MPQR} F_N{}^{PQR} - \frac{1}{8} g_{MN} F_{PQRS} F^{PQRS} \right]$

II $\quad \nabla_M F^{MPQR} = -\frac{1}{576} \varepsilon^{M_1 \cdots M_8 PQR} F_{M_1 \cdots M_4} F_{M_5 \cdots M_8}$

Look for solution with

$$F_{\mu\nu\rho\sigma} = \pm 3m\, \varepsilon_{\mu\nu\rho\sigma} \qquad F_{mnpq} = 0$$

$$\Rightarrow \qquad R_{\mu\nu} = -12 m^2 g_{\mu\nu}$$

$$R_{mn} = 6 m^2 g_{mn}$$

Covariant derivative $\bar{D}_M$ appearing in

$$\delta \psi_M = \bar{D}_M \varepsilon = D_M \varepsilon - \frac{i}{144}\left(\Gamma^{NPQR}{}_M + 8\, \Gamma^{PQR} \delta^N_M \right) \hat{F}_{NPQR}$$

splits

$$\bar{D}_\mu = D_\mu + m\, \gamma_\mu \gamma_5$$

$$\bar{D}_m = D_m - \frac{m}{2} \Gamma_m \qquad \text{using } \Gamma_A = (\gamma_\alpha \mathbb{1},\ \gamma_5 \Gamma_a)$$

KNOWN SOLUTIONS WITH S^7 TOPOLOGY

	Holonomy	[illegible]	Supersymmetry	Gauge Symmetry	Ref
round	$\mathbb{1}$	✓	$N=8$	$SO(8)$	5, 6
round + "[illegible]"	$SO(7)$	✓	$N=0$	$SO(7)$	7
[illegible]t squashed	G_2	✓	$N=1$	$SO(5)\times SU(2)$	12
[illegible]t squashed	G_2	✓	$N=0$	$SO(5)\times SU(2)$	18
[illegible]t squashed "Torsion"	$SO(7)$	✓	$N=0$	$SO(5)\times SU(2)$	12 (16, 8)

KNOWN SOLUTIONS WITH OTHER TOPOLOGY

	Holonomy		Supersymmetry	Gauge Symmetry	Ref
T^7	$\mathbb{1}$	✓	$N=8$	$[U(1)]^7 \times \mathbb{R}^{21}$	2
$3\times T^3$	$SU(2)$	✓	$N=4$	$[U(1)]^3 \times \mathbb{R}^{25}$	32
$[illegible] \times S^2$	$SO(7)$	✓	$N=0$	$SU(4) \times SU(2) \times \mathbb{R}$	6
$[illegible] \times S^3$	$SO(7)$	✓	$N=0$	$SO(5)\times SU(2) \times SU(2)$	
$[illegible]\times S^2 \times S^3$ ([illegible])	$SO(7)$	✓ / ×	$N=0$	$\mathbb{R}^2 \times SU(2)\times SO(2)\times SO(2)\times SU(2)$ / $\mathbb{R}^2 \times U(1) \times SU(2) \times SU(2) \times SU(2)$	
[illegible]	$SO(7)$	×	$N=0$	$\mathbb{R} \times SU(3)\times SU(2)\times SU(2)$	

Requirement of unbroken supersymmetry

$$\varepsilon(x,y) = \varepsilon^{I}(x)\,\eta^{I}(y)$$

where I runs over number of Killing spinors

$$\bar{D}_m \eta^{I} = 0$$

$$\Rightarrow \quad [\bar{D}_m, \bar{D}_n]\,\eta^{I} = -\frac{1}{4} C_{mn}{}^{ab}\,\Gamma_{ab}\,\eta^{I} = 0$$

Maximum number 8 achieved when $C_{mn}{}^{ab} = 0$

$\Rightarrow$ seven-sphere with standard metric $\quad S^7 = \dfrac{SO(8)}{SO(7)}$.

Adopt this (tentatively as our ground state). Want to determine effective $d=4$ theory. Fourier expand

$e_\mu{}^A(x,y) \quad \psi_\mu(x,y) \quad A_{MNP}(x,y)$ into harmonics on S^7

$\Rightarrow$ finite number of $m=0$ states + ∞ tower of $m \neq 0$ states

all in reps of $SO(8)$ and all with spins ≤ 2.

$m=2$ ———

$m=1$ ——— $\quad$ 8 64 216 224 336

$m=\ $ [illegible] ——— $\quad$ 1 8 [illegible] 56 70

Heavy states $\gg 10^{19}$ GeV concentrate on massless states

BUT Caution: ~~states which~~ S^7 was but one of many possible solutions, and with $N=8$ supersymmetry is (hopefully) not the true vacuum. States which are massive in vacuum A may be massless in vacuum B and vice-versa. So do not discard massive states — they may be needed later!

QUESTIONS ?

MASSLESS ANSATZ Proceed in steps: ① determine spins and quantum numbers of massless states i.e. obtain $d=4$ Lagrangian in linearized approximation ② Determine full Lagrangian (should be done properly but can "cheat" by [illegible] the non-linear $\mathcal{L}$ from the linear one by supersymmetry).

We have $\Lambda = -12 m^2$ m^{-1} radius of S^7

deW/N have $4\pi G \Lambda = -3e^2$

combine ⇒ $e^2 = 16\pi G m^2$

c.f. Weinberg $g^2 = 32\pi G m^2$ (convention?)

e defined by

$$F_{\mu\nu}{}^{IJ} = \partial_\mu A_\nu{}^{IJ} - \partial_\nu A_\mu{}^{IJ} - e A_\mu{}^{IK} A_\nu{}^{KJ} - e A_\nu{}^{IK} A_\mu{}^{KJ} \ldots$$

$e_\mu{}^a$	1	1	e_μ
$\psi_\mu{}^I$	8_s	8_c	ψ_μ
$A_\mu{}^{[IJ]}$	28	28	$A_\mu{}^{I}$
$\chi^{[IJK]}$	56_s	56_c	$\chi^{[I'}$
$S^{[IJKL]+}$	35_v	35_v	$S^{I'J'}$
$P^{[IJKL]-}$	35_c	35_s	$P^{(I'}$

N.B. Special property of SO(8) is triality: 3 inequivalent 8 dimensional representation

index: 1–

8_v	[0010]	vector	i
8_s	[0001]	spinor	I
8_c	[1000]	conjugate spinor	I'

Only one 28

N.B. $\Gamma_{ij}{}^{IJ}$ $\Gamma_a{}^{IJ}$ $\Gamma_{as}{}^{IJ} = +\Gamma_a{}^{IJ}$ $\Gamma_{as}{}^{I'J'} = -\Gamma_a{}^{I'J'}$

3 35's index

35_v	[0020]	(ij)	$[IJKL]_+$	$[I'J'K'L']_-$
35_s	[0002]	$[ijkl]_-$	(IJ)	$[I'J'K'L']_+$
35_c	[1000]	$[ijkl]_+$	$[IJKL]_-$	$(I'J')$

Mass ... in Tower of massive states M_n ... [...
$\bar{D}_m \Psi^{(n)} = m \ldots$

[cannot switch off gauge coupling to get $e = 0$ theory because $G M_n$ ... . No Cremmer-Julia E_7 !]

e.g. $8_v \times (1, 8_s, 28, 56_s, 35_v, 35_c)$

$= (8_v, 8_c + 56_c, 8_v + 56_v + 160_v, 8_c + 56_c + 160_c + 224_{vc},$
$8_v + 112_v + 160_v, 56_v + 224_{cv})$

Higgs
$\Rightarrow (8_v, 8_c + \boxed{56_c}, 56_v + 160_v, 160_c + 224_{vc}, 112_v, 224$

$M_2 = 2m$ ———

$M_1 = m$ ——— PTO for level 2

8, 64, 216, 384, 336

$M_0 = 0$ ———

1, 8, 28, 56, 70

N.B Supersymmetry parameter $\varepsilon(x, y) = \sum_n \varepsilon_n(x) Y_n(y)$

$\Gamma^p \bar{D}_p Y_n = n m Y_n$

lowest mode 8 fold degenerate $\psi_\mu(x,y) = \psi_\mu^I(x) Y^I(y)$

8 $m=0$ gravitinos $N=8$ unbroken s.s.

But also ∞ number of broken supersymmetries $\Rightarrow \infty$ number of massive gravitinos.

Similar remarks apply to general covariance parameter $\xi_\mu(x,y)$ gauge parameter $\xi_m(x,y)$ & Lorentz parameters $\omega^{\alpha\beta}(x,y)$ and SO(7) parameters $\omega^{ab}(x,y)$.

[Include massive states : symmetry of $\mathcal{L}$ is $d=11$ SS, gen cov, SO(1,10) etc BUT symmetry of vacuum is only $SO(3,2) \times SO(8)$ & $N=8$ ss $\Rightarrow OSp(4|8)$. cf Salam & Strathdee

WHERE DO WE GO FROM HERE ?

PHYSICS

1) Want to break $N=8$ to $N=1$ and then to $N=0$

2) Want to break SO(8) [and SO(7)].

3) Want to eliminate Λ (in true vacuum)

$\Rightarrow$ Spontaneous Symmetry Breaking $\langle S \rangle \neq 0$ or $\langle P \rangle \neq 0$

[illegible]

1) S^7 is $k=\pm 1$ Yang Mills instanton over S^4

2) S^7 admits not one but two Einstein metrics
(true for all S^{4n+3} $n \geq 1$ S^7 S^{11} . . .)

3) Only compact manifolds with absolute parallelism, i.e. [illegible]

there exist torsion $S_{\mu\nu}{}^{\rho}$ ∋

$$R_{\mu\nu\rho\sigma}\left(\mathring{\Gamma}_{\mu\nu}{}^{\rho} + S_{\mu\nu}{}^{\rho}\right) = 0 \qquad \text{parallelizable}$$

$$S_{\mu\nu\rho} = S_{[\mu\nu\rho]} \qquad \text{absolute}$$

$$\Downarrow$$

$$\Rightarrow \qquad \ddot{y}^m + \Gamma^m{}_{np}\,\dot{y}^n\dot{y}^p = 0 \qquad \text{same as with } \mathring{\Gamma}$$

[Cartan & Schouten Proc Akad Wet Amsterdam 29 (1926) [illegible]

Answer a) group manifolds $S_{mnp} = \pm \frac{1}{2} f_{mnp}$

m^{-1} radius of S^7 b) S^7 $\nabla^m S_{mnpq} = \pm \frac{m}{6} \epsilon_{npq}$ [illegible]

$S_{mnpq}\, S^{mnpq'} \propto$ [illegible]

(same as A_{mnp} equation in supergravity)

[illegible] i) Consider a $d=7$ metric

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + \lambda^2 g_{mn}(y)\left(dy^m - K^m{}_\alpha A_\mu{}^\alpha dx^\mu\right)\left(dy^n - K^n{}_\beta A_\nu{}^\beta d\right.$$

where
$g_{\mu\nu}(x) = S^4$ $\mu,\nu = 1\,2\,3\,4$
$g_{mn}(y) = S^3$ $m,n = 1\,2\,3$
$A_\mu{}^\alpha(x) = SU(2)$ instanton $\alpha = 1\,2\,3$
with $k = \pm 1$

$$\text{i.e.} \quad \begin{bmatrix} g_{\mu\nu} + A_\mu{}^\alpha K_\alpha{}^m A_\nu{}^\beta K_\beta{}^n g_{mn} & \lambda^2 g_{mn} K^m_\beta A_\nu^\beta \\ \lambda^2 g_{mn} K^n{}_\alpha A^\alpha_\nu & \lambda^2 g_{mn} \end{bmatrix}$$

$$k = \frac{1}{8\pi^2}\,\mathrm{Tr}\int F \wedge F \qquad k = \pm 1$$

This metric has S^7 topology

$\lambda^2 = 1$ isometry group $SO(8)$

$\lambda^2 \neq 1$ $SO(5) \times SU(2)$ [$Sp(2) \times Sp(1)$

Amazingly, $R_{mn} = \begin{bmatrix} \alpha & & & & & & \\ & \alpha & & & & & \\ & & \alpha & & & & \\ & & & \beta & & & \\ & & & & \beta & & \\ & & & & & \beta & \\ & & & & & & \beta \end{bmatrix}$ $\alpha = 3 - \frac{3\lambda^2}{2}$

$\beta = \lambda^2 + \frac{1}{2\lambda^2}$

[illegible] Squashed.

Both solve $R_{mn} = 6m^2 g_{mn}$ but with different Λ

for space-time if we hold fixed conserved charge.

$$Q = \int_{S^7} {}^*F \qquad {}^*F \text{ is 7-form}$$

$$\Lambda_S = \left(\frac{3}{5}\right)^{7/2} \Lambda_R$$

so squashed S^7 is another candidate ground-state

INTERPRETATION

$$\overset{\circ}{g}_{mn}(y) = \langle g_{mn}(x,y) \rangle$$

but $\quad g_{mn}(x,y) = \sum_i S^i(x) Y^i_{mn}(y) \qquad i$ SU(8) index

so $\quad = \overset{\circ}{g}_{mn}(y) + h_{mn}(x,y)$

So if $\langle g_{mn}(x,y) \rangle \neq \overset{\circ}{g}_{mn}(y)$

$\Rightarrow$ some $\langle S^i(x) \rangle \neq 0$

$\Rightarrow$ HIGGS mechanism.

SUPERSYMMETRY

Even more amazingly

$$[\bar{D}_m, \bar{D}_n]\eta = -\frac{1}{4} C_{mn}{}^{ab} \Gamma_{ab} \eta = 0$$

has one solution satisfying

$$\bar{D}_m = \left(\bar{D}_m \mp \frac{m}{2}\Gamma_m\right)\eta \qquad R = \pm 1$$

N.B [for $F_{\mu\nu\rho\sigma} = +m\,\varepsilon_{\mu\nu\rho\sigma}$, supersymmetry corresponds to

$$\left(D_m - \frac{m}{2}\Gamma_m\right)\eta = 0.$$

$\Rightarrow$ $N=1$ supersymmetry $\qquad R = +1$

$N=0$ supersymmetry $\qquad R = -1$

[vice-versa for $F_{\mu\nu\rho\sigma}$]

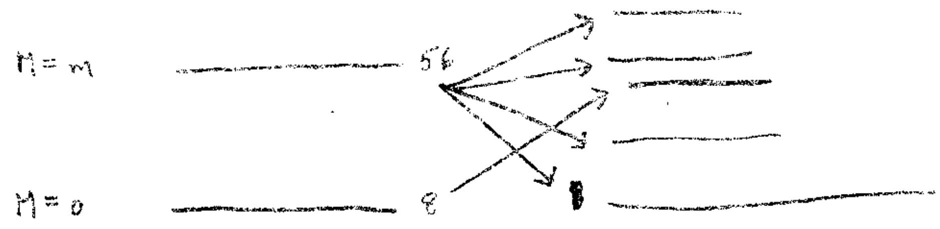
2 different $SO(5) \times SU(2)$ subgroups of $SO(8)$

J		
2	(1,1)	(1,1)
3/2	(4,2)	(5,1) + (1,3)
1	(10,1) + (5,3) + (1,3)	(10,1) + (5,3) + (1,3)
1/2	(16,2) + (4,4) + (4,2)	(10,1) + (10,3) + (5,3) + (1,1)
0^+	(10,3) + (5,1)	(10,3) + (5,1)
0^-	(14,3) + (5,3) + (1,5) + (1,1)	(10,3) + (5,1)

But wait a minute, in N=1 case all 8 ~~massless~~ gravitinos will get a mass (4,2), how can we have unbroken supersymmetry?

SPACE INVADERS SCENARIO

S^7 does have singlet (10,1) + (10,3) + (5,3) + (1,1)



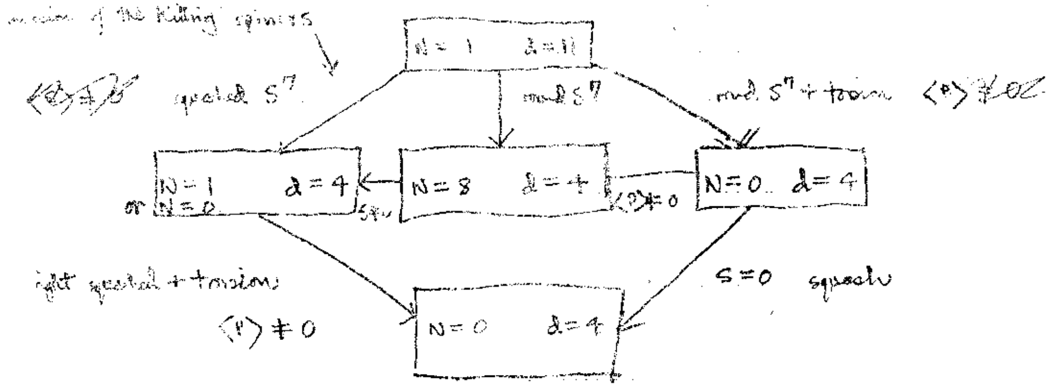
LECTURE 6

A REALISTIC THEORY

We have seen how N = 8 breaks to N = 1 at the Planck scale spontaneously at the tree level. There are 3 outstanding problems

1) Does N = 1 break to N = 0 at M = Mp or $M_p \ll M < M_W$

The first is easy to achieve by a) right-squashed S^7, no torsion b) round S^7 with torsion c) round S^7 no torsion.



But current theories predict the latter.

2) How to get SU(3) × SU(2) × U(1) when not ⊂ SO(8)?

3) How to get Λ = 0 N.B. no amount of $g_{\mu\nu}$ g_{mn} $F_{\mu\nu\rho\sigma}$ F_{mnpq} works.

$\Rightarrow \quad \langle g_{\mu\nu} \rangle = g_{\mu\nu}(x) \qquad \langle g_{mn} \rangle = g_{mn}(y)$

$\langle F_{\mu\nu\rho\sigma} \rangle = 2m\,\varepsilon_{\mu\nu\rho\sigma} \qquad \langle F_{mnpq} \rangle = F_{mnpq}(y)$

$$\Rightarrow \quad R_{\mu\nu} = \frac{1}{3}\left[F_{\mu\alpha\beta\gamma} F_\nu{}^{\alpha\beta\gamma} - \frac{1}{12} g_{\mu\nu}\left(F_4^2 + F_7^2\right) \right]$$

$$R_{mn} = \frac{1}{3}\left[F_{mabc} F_n{}^{abc} - \frac{1}{12} g_{mn}\left(F_4^2 + F_7^2\right) \right]$$

$$F_4^2 = F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma} \leqslant 0. \qquad F_7^2 = F_{mnpq} F^{mnpq} \geqslant 0$$

$$\nabla_m F^{mnpq} = \frac{m}{6} \varepsilon^{npqrstu} F_{rstu}$$

$$R^\mu{}_\mu = \frac{1}{36}\left[8F_4^2 - 4F_7^2 \right] \leqslant 0$$

$$R^m{}_m = \frac{1}{36}\left[-7F_4^2 + 5F_7^2 \right] \geqslant 0$$

BOSONS ALONE CAN NEVER GIVE $\Lambda = 0$.

BUT Lorentz invariance permits

$$S_{mnp}(y) \equiv \frac{i}{2} \langle \bar{\psi}_n \Gamma_s \psi_r \rangle \neq 0$$

N.B $d = 11$

$\omega_M{}^{AB}(e) + K_M{}^{AB} \qquad K$ contorsion

~~K_{ABC}~~ ... K

$$\left[\; T_{MN}{}^A = e_M{}^B K_{NB}{}^A - e_N{}^B K_{MB}{}^A = D_M e_N{}^A - D_N e_M{}^A \;\right].$$

Permits a field configuration

$$R_{\mu\nu}(e) = 0$$

$$R_{mn}(\hat{\omega}) = 0$$

$$\hat{\omega}_{mrs} = \dot{\omega}_{mrs} + S_{mrs} - S_{mrs} + S_{smn}$$

N.B Special properties of S^7 allow non-trivial solutions

$R_{mn}(\omega) = 6m^2 g_{mn}$. still S^7

Either A) Round S^7, if $\hat{\omega}$ is ∥ Torsion

$$R_{mnpq}(\hat{\omega}) = 0 \quad \Rightarrow \quad R_{mp}(\hat{\omega}) = 0$$

B) Right Squashed S^7, if $\quad S_{mrs} = \pm \frac{m}{4} \bar{\eta}\, \Gamma_{abc}\, \eta$

$$R_{mnpq}(\hat{\omega}) = C_{mnpq}(\omega) \qquad R_{mn}(\hat{\omega}) = 0$$

So IF quantum theory produced fermion condensate

$$\langle \bar{\Psi}_a \hat{\Gamma}_b \Psi_c \rangle \sim c \; \bar{\eta} \, \Gamma_{abc} \, \eta$$

then ⇒ spontaneous compactification with squashed S^7

and $\Lambda = 0$, provided c chosen correctly. Moreover

radius of S^7 and hence Y-M couplings would

be calculable if c calculable cf. Weinberg

N.B. Spin ½ only because

$$\Psi_a = \sum_n \chi^n(x) \, Y^n_a(y)$$

↑ spin ½

$$\langle \bar{\Psi}_a \Gamma_b \Psi_c \rangle = \sum_{nm} \langle \bar{\chi}^n(x) \gamma_5 \chi^m(x) \rangle \, \underline{\bar{Y}^n_a \Gamma_b Y^m_c}$$

Puzzle: Why totally antisymmetric?

(Perhaps not!)

Puzzle: Why should $\Lambda = 0$ in true vacuum

[illegible] U(1), [illegible] ?

HOW TO GET SO(3) × SU(2) × U(1)

1) KK UNIFICATION

Two bosonic gravitational symmetries in $d = 11$

A) General covariance $x_M \to x_M'(x)$

B) Local Lorentz invariance $\delta e_M{}^A = \alpha^A{}_B \, e_M{}^B$

$\delta \omega_M{}^{AB} = D_M \alpha^{AB}$

(NOT combined in a simple group)

A) Isometry group $SO(8) \subset d = 11$ G.R.

Obvious symmetry with 28 elementary gauge bosons.

$B_\mu{}^{IJ}(x)$ coming from $g_{\mu m}(x,y) = A_\mu{}^{IJ}(x) \, K^m{}_{IJ}(y)$

B) $SO(7)$ local $\subset d = 11$ Lorentz $SO(1,10) = SO(1,3) \times SO(7)$

Hidden symmetry with 21 composite gauge bosons

coming from $\omega_\mu{}^{ab}(x,y)$.

N.B. B) (Maybe enlargeable to SO(8), or even SU(8)).

[illegible]

Possible unification is

Electroweak $SU(2) \times U(1) \subset SO(5) \subset d = 11$ G.R.

Strong $SU(3) \subset SO(7) \subset d = 11$ Lorentz

$\Rightarrow$ NOT simple group, NOT GUTS!

~

BUT VEV of $e_M{}^A$ breaks $d = 11$ GR and $d = 11$ Lorentz

so that vacuum has only $SO(3,2) \times$ internal $SO(8)$ spacetime symmetry.

Faced with the old problem that $SO(8) \not\supset SU(3) \times SU(2) \times U(1)$.

$\Rightarrow$ Not all $28 + 21$ gauge bosons are massless; only 28 are massless.

SUPERUNIFICATION FROM 11 DIMENSION

(with B. Nilsson & C. Pope)

(1) N = 8 SUPERSYMMETRY BREAKS SPONTANEOUSLY TO N = 1, AT THE TREE LEVEL; AT SCALE ~ M_{PLANCK}

& SO(8) BREAKS TO SO(5) × SU(2).

BUT N = 8 THEORY IS OBTAINED BY "SPONTANEOUS COMPACTIFICATION" OF N = 1 SUPERGRAVITY IN d = 11 DIMENSION: ON THE SEVEN-SPHERE $S^7 = SO(8)/SO(7)$.

AWADA, DUFF & POPE Phys. Rev. Lett **50**, 294 (1983

N.B. MASSLESS GRAVITINO IN N = 1 PHASE COMES FROM MASSIVE N = 8 MULTIPLET, AS DO HIGGS SCALARS (300 OF SO(8)).

② REALISTIC $SU(3) \times SU(2) \times U(1)$ THEORY ?

(N.B. $SO(8) \not\supset SU(3) \times SU(2) \times U(1)$)

MAYBE, IF (LIKE E.G.M.Z) POSTULATE SOME GAUGE BOSONS ARE COMPOSITES.

BUT (UNLIKE E.G.M.Z)

1) $N=8 \rightarrow N=1$ NOT ASSUMED BUT HAPPENS!

2) NEED MASSLESS $m=0$ BOUND STATES IN $N=1$ PHASE ONLY & NOT $N=8$: NO "FUNNY" PARTICLES

3) UNWANTED HIGH SPIN STATES CAN GET MASSIVE & DISAPPEAR.

4) NO NON-COMPACT (E_7) PROBLEMS

N.B. NO GUTS KALUZA-KLEIN PICTURE

ELECTROWEAK	$SU(2) \times U(1)$	$\subset$ $d=11$ GENERAL COORDINATE GROUP
STRONG	$SO(8)$	$\subset$ $d=11$ LOCAL LORENTZ GROUP

KALUZA-KLEIN SUPERGRAVITY

c.f. Witten
Nucl. Phys B186, 412

A) BASIC K-K IDEA :

			+ ∞ MASSIVE
GRAVITY IN $d>4$ ⇒	MASSLESS GRAVITY IN $d=4$	(spin 2)	2
	+ YANG MILLS	(spin 1)	1
	+ σ-MODEL	(spin 0)	0

• SPACETIME SYMMETRIES IN $d>4$ ⇒ SPACETIME AND INTERNAL SYMMETRIES IN $d=4$

B) WHY $d=11$ SUPERGRAVITY?

1. SUSY ⇒ MAXIMUM $d=$ ELEVEN!
⇒ UNIQUE INTERACTION OF g_{MN} ψ_M F_{MNPQ}
44 128 84

2. EXTREME K-K PHILOSOPHY: NO INTERNAL SYMMETRIES (EVEN CPT REPLACED BY $P_7 P_3 T$)

3. LICHNEROWICZ THEOREM: MASSLESS SPIN ½ IN $d=4$ REQUIRES SPIN $\frac{3}{2}$ IN $d>4$ ⇒ SUPERGRAVITY

4. SPONTANEOUS COMPACTIFICATION NOT ONLY WORKS (S^7) BUT PREDICTS $d=4$ ($11-7=4$) FOR SPACETIME.

.3 5. GEOMETRIC ORIGIN OF HIGGS & SUPERHIGGS EFFECT.

AUTOMATIC SPONTANEOUS BREAKING OF GAUGE SUSY C, P, CP SYMMETRIES

SPONTANEOUS COMPACTIFICATION

LOOK FOR GROUND STATE SOLUTIONS OF $d=11$ EQUATIONS ($\Psi_M = 0$) $M,N = 1 \dots 11$

$$R_{MN} - \frac{1}{2} g_{MN} R = \frac{1}{3}\left[F_{MPQR} F_N{}^{PQR} - \frac{1}{8} g_{MN} F^2 \right]$$

$$\nabla_M F^{MNPQ} = -\frac{1}{576}\, \varepsilon^{M_1 \dots M_8 NPQ} F_{M_1 \dots M_4} F_{M_5 \dots M_8}$$

N.B $d=4$!

FREUND-RUBIN CHOICE

$F_{\mu\nu\rho\sigma} = 3m\, \varepsilon_{\mu\nu\rho\sigma}$ $\mu,\nu = 1 \dots 4$

$F_{mnpq} = 0$ $m,n = 5 \dots 11$

others $= 0$

$\Rightarrow$ $R_{\mu\nu} = -12 m^2 g_{\mu\nu}$ $R_{mn} = 6 m^2 g_{mn}$

$d=4$ SPACETIME (x^μ) $\times$ $d=7$ COMPACT SPACE (y^m)

CRITERION FOR UNBROKEN SUPERSYMMETRY

$\delta \Psi_M = \bar{D}_M \varepsilon = 0$.

$\Gamma_A = (\gamma_\alpha \otimes \mathbb{1}, \gamma_5 \otimes \Gamma_a)$ $\varepsilon(x,y) = \varepsilon(x)\eta(y)$

N = no. of KILLING SPINORS on $d=7$ space

i.e solutions of

$$\bar{D}_m \eta = \left(D_m - \frac{m}{2} \Gamma_m \right) \eta = 0$$

THE SEVEN SPHERE AND N=8

$$\bar{D}_m \eta = 0 \quad \Rightarrow \quad [\bar{D}_m, \bar{D}_n]\eta = -\frac{1}{4} C_{mn}{}^{ab} \Gamma_{ab} \eta = 0$$

($C_{mn}{}^{ab}$: WEYL TENSOR; Γ_{ab}: SO(7) GENERATORS)

LINEAR COMBINATION OF Γ_{ab} GENERATE HOLONOMY GROUP.

$\mathcal{H} \subset SO(7)$. N = no. of unbroken supersymmetries = no. of spinors left invariant by $\mathcal{H}$

MAXIMALLY SYMMETRIC SOLUTION OF $R_{mn} = 6m^2 g_{mn}$ IS

$S^7 = SO(8)/SO(7)$, $C_{mn}{}^{ab} = 0$, $\mathcal{H} = \mathbb{1}$, $N = N_{max} = 8$

radius m^{-1}

KALUZA-KLEIN YIELDS EFFECTIVE $d=4$ THEORY WITH $N=8$, LOCAL SO(8) WITH $e^2 = 16\pi G m^2$ (WEINBE

$\Lambda = -12 m^2$ (MUCH TOO BIG) AND SPECTRUM :

~ order M_P^2

	2	3/2	1	1/2	0^+	0^-
$M=0$	1	8_s	28	56_s	35_v	35_c
BUT ALSO $M \neq 0$ e.g. $M=m$	8_v	$8_c + 56_c$	$56_v + 160_v$	$160_c + 224_{vc}$	112_v	224_{cv}
+ ∞ tower osp(4|8)	⋮	⋮	⋮	⋮	⋮	⋮

UNIQUE PROPERTIES OF S^7 (OCTONIONS)

1. S^7 ADMITS TWO EINSTEIN ($R_{mn} = 6m^2 g_{mn}$) METRICS

$$e^0 = d\mu \qquad e^i = \frac{1}{2}\sin\mu\,\omega_i \ \text{etc} \qquad e^{4} = \frac{\lambda}{2}(\nu_i + \cos\mu\,\omega_i)$$

here $\nu_i = \sigma_i + \Sigma_i \qquad \omega_i = \sigma_i - \Sigma_i \qquad \lambda = \text{constant}$

SU(2) 1-forms: $d\sigma_1 = -\sigma_2\wedge\sigma_3 \qquad d\Sigma_1 = -\Sigma_1\wedge\Sigma_3$ etc

$\lambda^2 = 1 \Rightarrow$ ROUND $S^7 = SO(8)/SO(7)$

$\lambda^2 = 1/5 \Rightarrow$ SQUASHED $S^7 = \dfrac{Sp(2)\times Sp(1)}{Sp(1)\times Sp(1)}$ $\qquad Sp(2) \simeq SO($ $\quad Sp(1) \simeq SU($

(works only for S^{4n+3} $n \geq 1$)

2. ONLY COMPACT MANIFOLDS ADMITTING "ABSOLUTE PARALLELISM" ARE (CARTAN 1926)

A) GROUP SPACES ($S_{mnp} = \pm\frac{1}{2} f_{mnp}$)

B) SEVEN SPHERE ($\nabla^m S_{mnpq} = \pm\frac{m}{6}\,\varepsilon_{npqrstu}\,S^{rstu}$

$S_{mnpq} = \partial_{[m} S_{npq]}$) ← d = 11 [illegible] (ENGLERT

i.e. exists $S_{mnp} = S_{[mnp]} \ni R_{mnpq}(\hat{\Gamma} + S) = 0$

3. SO(8) HAS "TRIALITY": 3 INEQUIVALENT 8's

VECTOR 8_v SPINOR 8_s CONJUGATE SPINOR 8_c

SQUASHING S^7 = HIGGS + SUPERHIGGS

FOURIER EXPANDING $g_{mn}(x,y)$ IN HARMONICS ON S^7 YIELDS MASSLESS SCALARS (35_v) PLUS TOWER OF MASSIVE SCALARS, $S(x)$.

IF GROUND STATE IS

$\langle g_{mn}(x,y)\rangle$ = round $S^7 \qquad \Rightarrow$ ALL $\langle S(x)\rangle = 0$

BUT IF GROUND STATE IS

$\langle g_{mn}(x,y)\rangle$ = squashed $S^7 \qquad \Rightarrow$ SOME $\langle S(x)\rangle \neq 0$

DIFFERENCE $h_{mn} = \text{diag}(0,0,0,0,1,1,1) \qquad \nabla_{(a}h_{bc)} = 0$

(KILLING TENSOR) $\Rightarrow$ 1, 35_v or 300. CONSTANT TRACE EXCLUDES 35_v. FIND 1 = DILATATIONS 300 = SQUASHING

NON-ZERO VEV FOR 300 BREAKS SO8 TO SO(5) × SU(

AMAZINGLY: SQUASHED S^7 HAS $\mathcal{H} = G_2 \Rightarrow N = 1$

N = 8 BROKEN TO N = 1

[SUBTLETY: N=1 RIGHT, N=0 LEFT

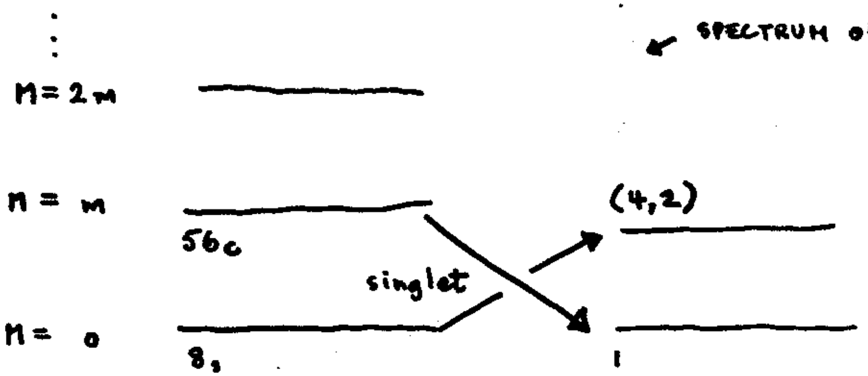
PUZZLE: MASSLESS $N=8$ SECTOR $(1, 8_s, 28, 56_s, 35_v, 35_c)$ ALONE CANNOT FORM $N=1$ MULTIPLET

UNDER $SO(8) \rightarrow SO(5) \times SU(2)$

E.G. 8_s GRAVITINOS $\rightarrow (4,2) \rightarrow$ ALL 8 MASSIVE

RESOLUTION: SPACE INVADERS SCENARIO:

10^{19} GeV STATES ZOOM DOWN FROM PLANCKIAN SKY TO BECOME MASSLESS!



$\lambda^2 = 1 \rightarrow$ SQUASH $\rightarrow \lambda^2 = 1/5$

$56_c \rightarrow (10,1) + (10,3) + (5,3) + (1,1)$

↑ singlet

KALUZA KLEIN SUPERUNIFICATION

TWO BOSONIC GRAVITATIONAL SYMMETRIES IN $d=11$: GENERAL COVARIANCE: $x_M \rightarrow x'_M(x)$, AND LOCAL LORENTZ $SO(1,10)$ INVARIANCE:

$$\delta e_M{}^A = \alpha^A{}_B(x)\, e_M{}^B \qquad \delta \omega_M{}^{AB} = \nabla_M \alpha^{AB}(x).$$

↑ 'ELFBEIN' ↑ SPIN CONNECTION

NOT COMBINED IN SIMPLE GROUP!

ISOMETRY GROUP $SO(8) \subset d=11$ G.R.

$\Rightarrow$ OBVIOUS SYMMETRY WITH ELEMENTARY GAUGE BOSONS

BUT ALSO

$SO(3,1) \times SO(7) \subset SO(1,10)$

$d=4$ LORENTZ ⇓

"HIDDEN" SYMMETRY WITH COMPOSITE GAUGE BOSONS

SUGGESTS:

ELECTROWEAK $SU2 \times U(1) \subset SO(8) \subset d=11$ G.R.

STRONG $SU(3) \subset SO(7) \subset d=11$ LORENTZ

$\Rightarrow$ NOT GRAND UNIFICATION

BOUND STATES

E.G.M.Z based on Cremmer-Julia $E_7 \times SU(8)$

N=8 THEORY. SU(8) MULTIPLETS LIKE

-3/2	-1	-1/2	0	1/2	1	3/2	2	5/2
$\bar{8}$	63	216	420	504	$\overline{378}$	$\overline{168}$	$\overline{36}$	$\bar{8}$
	1	8	28	56	70	$\overline{56}$	28	

BUT m=0 SPIN 1 NOT IN ADJOINT

(ALSO m=0 " 3/2 " " VECTOR

m=0 " 2 " " SINGLET

m=0 " 5/2 " ABSENT)

ALSO, CANNOT GIVE MASSES TO UNWANTED λ'S IF

$SO(3) \times SO(3) \times U(1) \subset SU(8)$

ALSO, E_7 NON COMPACT

ALSO, NO MECHANISM FOR $N=8 \to N=1$

THESE PROBLEMS ABSENT IN $SO(8) \times SO(7)$ PICTURE

(COULD POSTULATE BOUND STATES FORM ONLY IN N=1 CASE AND NOT N=8).

MAJOR UNSOLVED PROBLEM : QUARK/LEPTON REPRESENTATIONS.

(N.B C, P, CP BREAKING BUILT IN TO d=11 THEORY)

RHAPS

COSMOLOGICAL CONSTANT PROBLEM: TORSION

(with G. CASTELLANI)

1 COMPACTIFICATION OF d=11 SUPERGRAVITY WITH $F_{\mu\nu\rho\sigma} = 3m\,\epsilon_{\mu\nu\rho\sigma}$ CAN NEVER YIELD $\Lambda=0$ USING BOSON FIELDS ALONE.

2 CAN GET $\Lambda=0$ IF TORSION IS SUCH AS TO "PARALLELIZE" EXTRA DIMENSIONS (MAGIC PROPERTY OF S^7) BUT REQUIRES NON-ZERO VEV FOR FERMIONIC BILINEARS.

$$R_{\mu\nu}(\omega) = 0 \qquad R_{mn}(\omega + S) = 0$$

$$S_{abc} = \underbrace{\langle \bar{\psi}_a \Gamma_b \psi_c \rangle}_{\text{spin } 1/2 \text{ condensate}} \quad \text{ANTISYMMETRIC}$$

$\Lambda=0$ PROBLEM

BREAKING OF N=1 TO N=0

BOUND STATES

} FERMION CONDENSATES ?

N = 8 SO(8) PHASE

$M=2m$	35_v	$56_s + 224_{vs}$	$28 + 350 + 567_v$	$8_s + 160_s$ $672_{vc} + 840_s$	$\boxed{1}$ 294 $\boxed{300}$	$\boxed{35_s}$ 840
$M=m$	8_v	$8_c + \boxed{56_c}$	$56_v + 160_v$	$160_c + 224_{vc}$	112_v	22[illegible]
$M=0$	1	8_s	28	56_s	35_v	3[illegible]
spin	2	$3/2$	1	$1/2$	0^+	0^-

PHYSICAL MOTIVATION

PHYSICS : (1) want to break $N=8$ to $N=1$ (and $N=0$)

(2) want to break $SO(8)$

(3) want to eliminate Λ in "true" vacuum.

MATHS :

(1) S^7 ADMITS TWO EINSTEIN METRICS (true only for S^{4n+3} $n \geq 1$)

(2) ABSOLUTE PARALLISM

—

SQUASHING = HIGGS

$$\langle g_{mn}(x,y)\rangle \neq \mathring{g}_{mn}(y) \Rightarrow \langle h_{mn}(x,y)\rangle \neq 0$$

$$\langle S\rangle \neq 0 \qquad h_{mn}(x,y) = \sum_N S^N(x)\, Y^N_{mn}(y)$$

TORSION

$$\langle A_{mnp}(x,y)\rangle \neq 0 \qquad A_{mnp}(x,y) = \sum_N P^N(x)\, Y^N_{mnp}(y)$$

$$\nabla_\ell h_{mn} = 0 \Rightarrow 1 \;\; 35_v \text{ or } 300 \Rightarrow 1 \text{ and } 300$$

$$\nabla_\ell A_{mnp} = \partial_\ell A_{mnp} \Rightarrow 35_s \text{ or } 35_v \Rightarrow 35_s \text{ (round } S^7)$$

$SO(8) \to [SO(5) \times SU(2)]_s$ SQUASHED

$300 \to (14,5) + (35,3) + (35,1) + (5,5) + (14,1)$

$(10,3) + (1,5) + (5,3) + \boxed{(1,1)}$

$SO(8) \to SO(7)$ ROUND + TORSION

$35_s \to 27 + 7 + \boxed{1}$

$SO(8) \to [SO(5) \times SU(2)]_c$ R. SQUASH + TORSION

$35_c \to (14,1) + (5,3) + (1,5) + \boxed{(1,1)}$

$\Rightarrow$ TORSION EXTREMA OF DEWIT - NICOLAI (...

$\Rightarrow$ SQUASHING NOT : $N=8 \to N=1$ requires extra dimensions N.B $E = \frac{1}{4}\Sigma Q^{+1}_\alpha Q^1_\alpha$

$= \frac{1}{4} \sum_\alpha Q^{+2}_\alpha Q^2_\alpha$ etc

LOOK AT $N=1$

$1 \to (1,1)$

$8_s \to (4,2)$

$28 \to (10,1) + (5,3) + (1,3)$

$56_s \to (16,2) + (4,4) + (4,2)$

$35_v \to (10,3) + (5,1)$

$35_c \to (14,1) + (5,3) + (1,5) + (1,1)$

PUZZLE : ALL 8 GRAVITINOS ACQUIRED A MASS.

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1. EMBEDDING IN $\mathbb{R}^8$
   - Isometry Group, Isotropy Group
   - Curvature tensor
   - Killing vectors

2. KILLING SPINORS
   - Definition
   - Number of Killing Spinors
   - Relation to supersymmetry
   - Representation of Killing vectors

3. TORSION
   - Parallelizability of S^7
   - Cartan - Schouten torsion
   - Representation in terms of Killing spinors
   - Invariance group

4. SPECTRA OF OPERATORS ON S^7
   - SCALAR LAPLACIAN
   - DIRAC OPERATOR
   - "LICHNEROWICZ THEOREM"

EMBEDDING IN $\mathbb{R}^8$:

FLAT EUCLIDEAN $\mathbb{R}^8$: COORDINATES X^A $A=1\cdots 8$

Metric $\quad ds^2 = dx^A dx^A$

Hypersurface $\quad x^A x^A = m^{-2} \quad - \quad S^7$, radius m^{-1}

$\Rightarrow$ Induced metric on S^7 has curvature given by

$$R_{abcd} = m^2 (g_{ac} g_{bd} - g_{ad} g_{bc}) \qquad a = 1 \cdots 7$$

$$\Rightarrow R_{ab} = 6 m^2 g_{ab} \qquad \text{EINSTEIN SPACE.}$$

$$\Rightarrow R = 42 m^2$$

ISOMETRY GROUP, G : Write $X = \begin{pmatrix} x^1 \\ \vdots \\ x^8 \end{pmatrix}$

$\therefore$ $\mathbb{R}^8$ metric is $\quad ds^2 = dX^T dX$

Hypersurface : $\quad X^T X = m^{-2}$

Let S be an element of $SO(8)$, then if

$$X' = S X,$$

$$ds'^2 = ds^2 \quad , \quad X'^T X' = X^T X$$

$\therefore$ Metric on S^7 invariant under $SO(8)$

ISOTROPY SUBGROUP, H :

H : Group which leaves a point in S^7 fixed

S^7 homogeneous, so all points equivalent.

E.g. $\quad x^A = (0, 0, \cdots 0, m^{-1})$

Subgp. of G which preserves this is $SO(7)$:

$$\begin{pmatrix} SO(7) & \\ & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ m^{-1} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ m^{-1} \end{pmatrix}$$

$\therefore \quad S^7 \cong G/H = SO(8)/SO(7) \qquad$ Coset space

KILLING VECTORS : $\quad \mathcal{L}_{K_a} g_{bc} = 0$

$$\Leftrightarrow \nabla_a K_b + \nabla_b K_a = 0$$

In a space with isometry group G, Killing vectors in the adjoint rep. of G.

$\therefore$ For round S^7, $\exists$ 28 KVs in adj. rep. of $SO(8)$

Explicitly,

$$K^{AB} = K^{[AB]} = x^A \frac{\partial}{\partial x^B} - x^B \frac{\partial}{\partial x^A} \qquad A = 1, \cdots 8$$

$$[K^{AB}, K^{CD}] = C^{ABCD}{}_{EF} K^{EF}$$

$\uparrow$ Structure constants of $SO(8)$

KILLING SPINORS

DIRAC MATRICES Γ_a (8x8 matrices) $\quad a = 1 \cdots 7$

$$\{ \Gamma_a , \Gamma_b \} = -2 g_{ab} \qquad \text{ANTI-HERMITEAN}$$

$$\Gamma_{a \cdots b} = \Gamma_{[a} \cdots \Gamma_{b]}$$

SPIN CONNECTION $\quad D_m = \partial_m - \frac{1}{4} \omega_m{}^{ab} \Gamma_{ab}$

↑ Connection 1-form

KILLING SPINOR η satisfies

$$\bar{D}_m \eta = 0 , \quad \text{where} \quad \bar{D}_m = D_m - \frac{m}{2} \Gamma_m$$

HOW MANY SOLUTIONS ?

$$\bar{D}_m \eta = 0 \Rightarrow [\bar{D}_m , \bar{D}_n] \eta = 0 \qquad \text{Integrability Condition}$$

i.e. $\quad [D_m , D_n] \eta + \frac{m^2}{4} \Gamma_{mn} \eta = 0$

$$\Rightarrow \quad -\frac{1}{4} R_{mn}{}^{ab} \Gamma_{ab} \eta + \frac{m^2}{4} \Gamma_{mn} \eta = 0$$

But $\quad R_{mnpq} = m^2 (g_{mp} g_{nq} - g_{mq} g_{np})$

$$\Rightarrow \quad [\bar{D}_m , \bar{D}_n] \eta = 0$$

→ 8 SOLUTIONS , $\bar{D}_m \eta^I = 0$, $I = 1, \cdots 8$

I - SO(8) Spinor index

KILLING SPINORS ⟷ SUPERSYMMETRIES IN KALUZA-KLEIN SUPERGRAVITY

Recall that in $d=11$ ground state, $\langle \Psi_M \rangle = 0$

Vacuum supersymmetric $\Rightarrow \langle \delta \Psi_M \rangle = 0$

$$\delta \Psi_M = D_M(\hat{\omega}) \varepsilon - \frac{i}{144} (\hat{\Gamma}^{NPQR}{}_M + 8 \hat{\Gamma}^{PQR} \delta^N_M) \hat{F}_{NPQR} \, \varepsilon$$

∴ If $\quad F_{\mu\nu\rho\sigma} = 3m \, \varepsilon_{\mu\nu\rho\sigma}$, $\qquad$ (Freund / Rubin)

$$\delta \Psi_m = D_m \varepsilon - \frac{m}{2} \Gamma_m \varepsilon = \bar{D}_m \varepsilon$$

∴ Writing $\quad \varepsilon(x,y) = \varepsilon^I(x) \, \eta^I(y)$,

$$\langle \delta \Psi_m \rangle = 0 \iff \bar{D}_m \eta^I = 0$$

∴ Kaluza-Klein $d=11$ supergravity with M_7 = round S^7 has $N = 8$ supersymmetry.

KILLING VECTORS FROM KILLING SPINORS :

Can choose η^I to be Majorana $(\eta^\dagger = \eta^T C)$,

and normalized so that $\bar{\eta}^I \eta^J = \delta^{IJ}$.

Consider $\bar{\eta}^I \Gamma_m \eta^J = -\bar{\eta}^J \Gamma_m \eta^I$ (Majorana)

$\therefore$ 28 of $SO(8)$

$$\nabla_m (\bar{\eta}^I \Gamma_n \eta^J) = (\overline{D_m \eta^I}) \Gamma_n \eta^J + \bar{\eta}^I \Gamma_n D_m \eta^J$$

$$= -\frac{m}{2} \left(\bar{\eta}^I \Gamma_m \Gamma_n \eta^J - \bar{\eta}^I \Gamma_n \Gamma_m \eta^J \right)$$

$$= -m \, \bar{\eta}^I \Gamma_{mn} \eta^J$$

$\Rightarrow \bar{\eta}^I \Gamma_n \eta^J$ satisfies Killing's eqn.

<u>NB</u> Previously had $K^{AB} = x^A \frac{\partial}{\partial x^B} - x^B \frac{\partial}{\partial x^A} = K^{[AB]}$

A, B $SO(8)$ Vector indices

Here $K_m^{IJ} = \bar{\eta}^I \Gamma_m \eta^J$ I, J $SO(8)$ Spinor indices

Triality of $SO(8)$: $\exists$ 3 inequivalent 8-dim reps.:

Vector	$A, B, \cdots$	8_v
Spinor	$I, J, \cdots$	8_s
Conjugate Spinor	$I', J', \cdots$	8_c

($\exists$ an 8_c of $\eta^{I'}$, satisfying $\tilde{D}_m \eta^{I'} = (\partial_m + \frac{m}{2} \Gamma_m) \eta^{I'} = 0$)

But $\exists$ only one 28 of $SO(8)$ ...

TORSION

<u>Parallelizability</u>: n-dimensional manifold M //izable if

$\exists$ n linearly independent <u>globally</u> <u>defined</u> vector fields

$\Leftrightarrow$ $\exists$ a connection $\bar{\Gamma}^\mu{}_{\alpha\beta}$ (not symmetric in α, β)

such that $\text{Riem}(\bar{\Gamma}^\mu_{\alpha\beta}) = 0$

<u>Absolute Parallelizability</u>: $\exists$ totally antisymmetric torsion tensor $S_{\mu\nu\rho} = S_{[\mu\nu\rho]}$ such that

$$\bar{\Gamma}^\mu{}_{\alpha\beta} = \Gamma^\mu{}_{\alpha\beta} + S^\mu{}_{\alpha\beta}$$

($\Gamma^\mu{}_{\alpha\beta}$: Riemannian Connection)

has

$$\text{Riem}(\bar{\Gamma}^\mu{}_{\alpha\beta}) = 0$$

$S_{\mu\nu\rho} = S_{[\mu\nu\rho]} \Rightarrow$ Geodesics same in $\bar{\Gamma}$, and Γ.

$\exists$ 2 classes of absolutely //izable manifolds :

1) Group manifolds

2) Seven Sphere

Generically,

$$0 = \bar{R}^\mu{}_{\nu\rho\sigma} = R^\mu{}_{\nu\rho\sigma} + 2 \nabla_{[\rho} S^\mu{}_{\sigma]\nu} - 2 S_{\alpha[\rho}{}^\mu S_{\sigma]\nu}{}^\alpha$$

$$\nabla_{[a} S_{bcd]} = \nabla_a S_{bcd} = \pm \frac{m}{6} \varepsilon_{abcdefg} S_{efg} \qquad (1)$$

$$S_{acd} S_{bcd} = 6 m^2 g_{ab} \qquad (2)$$

$$(1) \Rightarrow \quad dS = \pm 4m * S$$

$$\Rightarrow \quad \Delta S = (d + d^*)^2 S = 16 m^2 S \qquad (3)$$

(Δ = Hodge de Rham Laplacian on 3-forms)

Can show that on S^7, (3) has 70 solutions

35 satisfy (1) with $+\frac{m}{6}$

35 " " " $-\frac{m}{6}$

Can represent these in terms of η^I, or $\eta^{I'}$:

$$\bar{\eta}^I \Gamma_{abc} \eta^J = S^{IJ}_{abc} \quad \text{satisfy (1) with } +\frac{m}{6}$$

$$\bar{\eta}^{I'} \Gamma_{abc} \eta^{J'} = S'^{I'J'}_{abc} \quad " \quad " \quad " \; -\frac{m}{6}$$

(Majorana $\Rightarrow$ $S^{IJ} = S^{JI}$, $S^{IJ}\delta_{IJ} = 0$; $S'^{I'J'} = S'^{J'I'}$, $S'^{I'J'}\delta_{I'J'} = 0$)

S'_{abc} can be used to obtain a new KK supergravity soln, S^7 with "torsion", with [illegible] - Englert

NB S' [illegible] SO(8) [illegible]

1. SCALAR LAPLACIAN

x^A - coordinates on $\mathbb{R}^8$ $\quad A = 1 \dots 8$

Let $f_n(x) = T_{A_1 \cdots A_n} x^{A_1} \cdots x^{A_n}$

$T_{A_1 \cdots A_n}$ constant SO(8) irreducible tensors – i.e. symmetric, tracefree

$\therefore \partial_A \partial_A f_n(x) = 0$ i.e. f_n Harmonic

Write $\mathbb{R}^8$ metric $ds^2 = dx^A dx^A = dr^2 + r^2 h_{mn} dy^m dy^n$, ($h_{mn}$: S^7 metric)

where $r^2 = x^A x^A$, and y^m are coordinates on S^7

$$\partial_A \partial_A f_n = \frac{1}{r^7} \frac{\partial}{\partial r}\left(r^7 \frac{\partial f_n}{\partial r} \right) + \frac{1}{r^2} \Box f_n$$

($\Box$: S^7 Laplacian on unit sphere)

$$\Rightarrow \quad -\Box \phi_n = n(n+6)\phi_n, \quad \text{where}$$

$$\phi_n = \frac{1}{r^n} f_n$$

$\therefore$ On sphere of radius m^{-1},

$$-\Box \phi_n = n(n+6) m^2 \phi_n$$

degeneracy $\quad d_n = \frac{2(n+3)(n+5)!}{6!\, n!}$

DIRAC OPERATOR   Project Harmonic spinors from $\mathbb{R}^8$

$d = 8$ Dirac matrices $\hat{\Gamma}_A$: $\{\hat{\Gamma}_A, \hat{\Gamma}_B\} = -2g_{AB}$

($\hat{\Gamma}_A$: 16×16 matrices)

$$\hat{\Gamma}_a = \begin{pmatrix} 0 & i\Gamma_a \\ -i\Gamma_a & 0 \end{pmatrix}, \quad \hat{\Gamma}_8 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

$a = 1 \cdots 7$

$$\hat{\Gamma}_9 = \frac{1}{8!} \varepsilon^{AB\cdots C} \underbrace{\Gamma_A \Gamma_B \cdots \Gamma_C}_{8} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

DIRAC OPERATOR ON $\mathbb{R}^8$: $\hat{P} = \hat{\Gamma}^A \partial_A$

" " " unit S^7 : $P = \Gamma^a \mathcal{D}_a$

$$\Rightarrow \hat{P} = \begin{pmatrix} 0 & i\left(\frac{\partial}{\partial r} + \frac{7}{2r} + \frac{1}{r} P\right) \\ i\left(\frac{\partial}{\partial r} + \frac{7}{2r} - \frac{1}{r} P\right) & 0 \end{pmatrix}$$

Let $\Psi_n(x) = \Psi_{A_1 \cdots A_n} x^{A_1} \cdots x^{A_n}$

where $\Psi_{A_1 \cdots A_n}$ are constant $SO(8)$ irreducible spinor tensors — symmetric on $A_1 \cdots A_n$ and

$$\hat{\Gamma}^{A_1} \Psi_{A_1 \cdots A_n} = 0$$

Furthermore, $\Psi_n = \begin{pmatrix} \Psi_n^+ \\ \Psi_n^- \end{pmatrix}$,

where $\hat{\Gamma}_9 \begin{pmatrix} \Psi_n^+ \\ 0 \end{pmatrix} = \begin{pmatrix} \Psi_n^+ \\ 0 \end{pmatrix}$, $\hat{\Gamma}_9 \begin{pmatrix} 0 \\ \Psi_n^- \end{pmatrix} = -\begin{pmatrix} 0 \\ \Psi_n^- \end{pmatrix}$

Then setting $\psi_n^{\pm} = \frac{1}{r^n} \Psi_n^{\pm}$,

$$P \psi_n^{\pm} = \pm \left(n + \tfrac{7}{2}\right) \psi_n^{\pm} \quad \text{on unit } S^7$$

$\therefore$ On round S^7 of radius m^{-1}, Dirac operator has eigenvalues

$$\pm \left(n + \tfrac{7}{2}\right) m$$

$+ \;\hat{=}\;$ +ve $\hat{\Gamma}_9$ eigenstates
$- \;\hat{=}\;$ −ve " "

degeneracy $d = \dfrac{8\,(n+6)!}{6!\; n!}$

+ve spectrum $\hat{=}$ $I', J', \cdots$ $SO(8)$ indices

−ve spectrum $\hat{=}$ $I, J, \cdots$ $SO(8)$ indices

Note that $D_m \eta^I = \frac{m}{2} \Gamma_m \eta^I$,

so $\underline{P \eta^I = -\frac{7m}{2} \eta^I}$

sim. $\underline{P \eta^{I'} = +\frac{7m}{2} \eta^{I'}}$

So η^I, $\eta^{I'}$ are the lowest −ve, +ve eigenvalues modes of the Dirac operator

LICHNEROWICZ THEOREM FOR $\bar{D}_m$, $\bar{D}'_m$

CONSIDER AN ARBITRARY $d=7$ SPACE WITH $R = 42m^2$

CLEARLY $\quad \bar{D}_m \psi = 0 \Rightarrow P\psi = -\frac{7m}{2}\psi$

or $\quad \bar{D}'_m \psi = 0 \Rightarrow P\psi = +\frac{7m}{2}\psi$

CONVERSE IS ALSO TRUE. PROVE FOR $-\frac{7m}{2}$:

$$\int |\bar{D}_m \psi|^2 = \int |D_m \psi|^2 + m \int \bar{\psi} P \psi + \frac{7m^2}{4} \int |\psi|^2$$

Now
$$\int \bar{\psi} P^2 \psi = -\int \bar{\psi} \Box \psi + \frac{R}{4} \int \bar{\psi}\psi$$
$$= \int |D_m \psi|^2 + \frac{21m^2}{2} \int |\psi|^2$$

$\therefore$ If $\quad P\psi = \kappa \psi$,

$$\int |\bar{D}_m \psi|^2 = \left[\left(\kappa + \frac{m}{2}\right)^2 - 9m^2 \right] \int |\psi|^2$$

$\therefore$ if κ $-$ve, $\quad \kappa \leq -\frac{7m}{2}$

Equality $\Rightarrow \int |\bar{D}_m \psi|^2 = 0 \Rightarrow \bar{D}_m \psi = 0$

(For κ +ve, repeat with $\bar{D}'_m$.)

$\therefore$ If $R = 42m^2$, $\quad |\kappa| \geq \frac{7m}{2}$

If $\kappa = -\frac{7m}{2}$, $\quad \bar{D}_m \psi = 0$

If $\kappa = +\frac{7m}{2}$, $\quad \bar{D}'_m \psi = 0$

[illegible] BETWEEN $\pm\frac{7m}{2}$

SQUASHED SEVEN SPHERE

1. DESCRIPTION OF SQUASHED SPHERE

   Quaternionic Projective Plane, $P_2(H)$

   Squashed S^7 as distance sphere in $P_2(H)$

   Isometry group and isotropy group

   Left and right squashings

2. KILLING SPINORS

   Number of Killing spinors

   Holonomy group

   Breaking of $SO(8) \rightarrow Sp(2) \times Sp(1)$

   Invasion of the Killing Spinors

3. SPECTRA OF OPERATORS

   Scalar Laplacian

   Dirac operator

4. TORSION

   Ricci flattening torsion

   Kaluza Klein supergravity solution

   Invariance group

Quaternionic Projective Plane, $P_2(H)$

FLAT H^3, coordinates Q_1^{α}, Q_2, Q_3 (12 real dimensions)

metric on H^3 : $ds^2 = d\bar{Q}_\alpha \, dQ_\alpha$

restrict to unit S^{11} : $\bar{Q}_\alpha Q_\alpha = 1$

Introduce homogeneous coordinates

$$q_1 = Q_1 Q_3^{-1} \quad , \quad q_2 = Q_2 Q_3^{-1}$$

$\therefore$ In terms of q_i, Q_3 $(i = 1, 2)$,

S^{11} metric is

$$ds^2 = \left| \bar{q}_i \, dq_i \, \bar{Q}_3 Q_3 + dQ_3 Q_3^{-1} \right|^2 + \frac{1}{1+\bar{q}_k q_k} d\bar{q}_i \, dq_i - \frac{1}{(1+\bar{q}_k q_k)^2} \bar{q}_i \, dq_i \, d\bar{q}_j \, q_j$$

S^{11} is S^3 bundle over $P_2(H)$

$\therefore$ $P_2(H)$ obtained from S^{11} metric by projecting orthogonally to the S^3 fibers.

Now in $P_2(H)$, $(Q_1, Q_2, Q_3) \equiv (\lambda Q_1, \lambda Q_2, \lambda Q_3)$, $\lambda \in H$

$\therefore$ Project S^{11} metric orthogonally to the orbits $Q_\alpha \to U Q_\alpha$, $U \in Sp(1)$

$\therefore$ $P_2(H)$ metric is

$$\boxed{ds^2 = \frac{1}{1+\bar{q}_k q_k} d\bar{q}_i \, dq_i - \frac{1}{(1+\bar{q}_k q_k)^2} \bar{q}_i \, dq_i \, d\bar{q}_j \, q_j}$$

$i = 1, 2$. 8 real dimensions

Introduce a real parametrization of $P_2(H)$:

$$q_1 = \tan\chi \cos\tfrac{\mu}{2} \, U \quad , \quad q_2 = \tan\chi \sin\tfrac{\mu}{2} \, V$$

U, V unit quaternions : $\bar{U}U = \bar{V}V = 1$

Introduce 2 sets of real left-invariant 1-forms, σ_i, Σ_i :

by

$$2U^{-1}dU = i\sigma_1 + j\sigma_2 + k\sigma_3$$
$$2V^{-1}dV = i\Sigma_1 + j\Sigma_2 + k\Sigma_3$$

$$d\sigma_i = -\tfrac{1}{2}\varepsilon_{ijk} \, \sigma_j \wedge \sigma_k \quad , \quad d\Sigma_i = -\tfrac{1}{2}\varepsilon_{ijk} \, \Sigma_j \wedge \Sigma_k$$

$$\left(\begin{aligned} \sigma_1 &= \cos\psi \, d\theta + \sin\psi \sin\theta \, d\phi \\ \sigma_2 &= -\sin\psi \, d\theta + \cos\psi \sin\theta \, d\phi \\ \sigma_3 &= d\psi + \cos\theta \, d\phi \\ \Sigma_1 &= \cdots \end{aligned} \right)$$

Substitute into $P_2(H)$ metric $\Rightarrow$

$$ds^2 = d\chi^2 + \tfrac{1}{4}\sin^2\chi \left[d\mu^2 + \tfrac{1}{4}\sin^2\mu \, \omega_i^2 + \tfrac{1}{4}\cos^2\chi \left(\nu_i + \cos\mu \, \omega_i\right)^2 \right]$$

$i = 1, 2, 3$

where

$$\nu_i = \sigma_i + \Sigma_i$$
$$\omega_i = \sigma_i - \Sigma_i$$

Distance sphere : Pick a point in $P_2(H)$ (any point, since all points equivalent). Look at level surface defined by all geodesics of length r emanating from the point. This defines a 7-dimensional hypersurface — the DISTANCE SPHERE

Choose the point $\chi = 0$. (i.e. $q_1 = q_2 = 0$)

Distance Sphere is the level surface $r = \tan\chi$ = constant.

$\therefore$ Induced metric on the distance sphere (up to constant scaling factor) is

$$ds^2 = d\mu^2 + \tfrac{1}{4}\sin^2\mu\, \omega_i^2 + \tfrac{1}{4}\lambda^2 (\nu_i + \cos\mu\, \omega_i)^2$$

SQUASHED S^7 METRIC λ = Squashing Parameter

Notation: Indices $a, b, \ldots$ now run $0, \ldots 6$

$a = (0, i, \hat{i})$ $i = 1, 2, 3$; $\hat{i} = 4, 5, 6 = \hat{1}, \hat{2}, \hat{3}$

Introduce the orthonormal basis

$$e^0 = d\mu \qquad e^i = \tfrac{1}{2}\sin\mu\, \omega_i \qquad e^{\hat{i}} = \tfrac{1}{2}\lambda(\nu_i + \cos\mu\, \omega_i)$$

Connection 1-form $\omega_{ab} = \omega_{[ab]}$, $de^a = -\omega_{ab} \wedge e^b$

Curvature 2-form $\Theta_{ab} = d\omega_{ab} + \omega_{ac} \wedge \omega_{cb}$

$$\Theta_{0i} = (1 - \tfrac{3}{4}\lambda^2)\, e^0 \wedge e^i + \tfrac{1}{4}(1-\lambda^2)\, \varepsilon_{ijk}\, e^{\hat{j}} \wedge e^{\hat{k}}$$

$$\Theta_{0\hat{i}} = \frac{\lambda^2}{4}\, e^0 \wedge e^{\hat{i}} - \tfrac{1}{4}(1-\lambda^2)\, \varepsilon_{ijk}\, e^j \wedge e^{\hat{k}}$$

$$\Theta_{ij} = (1 - \tfrac{3}{4}\lambda^2)\, e^i \wedge e^j + \tfrac{1}{2}(1-\lambda^2)\, e^{\hat{i}} \wedge e^{\hat{j}}$$

$$\Theta_{\hat{i}\hat{j}} = \frac{1}{4\lambda^2}\, e^{\hat{i}} \wedge e^{\hat{j}} + \tfrac{1}{2}(1-\lambda^2)\left[\varepsilon_{ijk}\, e^0 \wedge e^k + e^i \wedge e^j\right]$$

$$\Theta_{i\hat{j}} = \frac{\lambda^2}{4}\, e^i \wedge e^{\hat{j}} - \tfrac{1}{4}(1-\lambda^2)\, e^j \wedge e^{\hat{i}} + \tfrac{1}{6}(1-\lambda^2)\left[\delta_{ij}\, e^k \wedge e^{\hat{k}} - \varepsilon_{ijk}\, e^0 \wedge e^{\hat{k}}\right]$$

$$\Theta_{ab} = \tfrac{1}{2} R_{abcd}\, e^c \wedge e^d$$

$\therefore$ Ricci tensor $R_{ab} = \mathrm{diag}(\alpha, \alpha, \alpha, \alpha, \beta, \beta, \beta)$,

$$\alpha = 3 - \tfrac{3}{2}\lambda^2, \qquad \beta = \lambda^2 + \frac{1}{2\lambda^2}$$

Einstein condition: $R_{ab} = c\,\delta_{ab}$

i.e. $\alpha = \beta$

$\Rightarrow$

a) $\lambda^2 = 1$ ROUND S^7

or b) $\lambda^2 = \frac{1}{5}$ SQUASHED EINSTEIN S^7

ISOMETRY GROUP OF SQUASHED SPHERE:

Return to quaternionic metric on $P_2(H)$. Let $w = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$

$$\therefore P_2(H): \quad ds^2 = (1 + w^\dagger w)^{-1} dw^\dagger dw - (1 + w^\dagger w)^{-2} w^\dagger dw\, dw^\dagger w$$

Distance Sphere: $w^\dagger w$ = constant

Metric and constraint preserved by

i) $w \to A w$, $A \in Sp(2)$

or ii) $\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \to \begin{pmatrix} q_1 a \\ q_2 a \end{pmatrix}$, $a \in Sp(1)$

$\therefore$ Isometry group $G = Sp(2) \times Sp(1)$

(Locally, $Sp(2) \cong SO(5)$, $Sp(1) \cong SU(2)$)

(When $\lambda^2 = 1$, $\exists$ additional symmetry, and $G = SO(8)$)

Isotropy Subgroup, H

Pick a point, e.g. $w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Subgroup of $Sp(2) \times Sp(1)$ which preserves this is left-multiplication by $Sp(2)$ matrices

$$\begin{pmatrix} b & 0 \\ 0 & c \end{pmatrix}$$

and simultaneous right-multiplication by b^{-1}, where $b, c \in Sp(1)$

$$\therefore \quad H = Sp(1) \times Sp(1)$$

Squashed S^7 is homogeneous, and G acts transitively (in fact $Sp(2)$ alone acts transitively)

$\therefore$ Can be described by the coset space G/H

$$= \frac{Sp(2) \times Sp(1)}{Sp(1) \times Sp(1)}$$

[Let $Sp(1) \times Sp(1)$ subgroup of $Sp(2)$ be $Sp(1)_A \times Sp(1)_B$

Then $$S^7 = \frac{Sp(2) \times Sp(1)_C}{Sp(1)_A \times Sp(1)_{B+C}}$$

where $Sp(1)_{B+C}$ = diagonal subgroup of $Sp(1)_B \times Sp(1)_C$]

Squashed Spheres are Handed

Recall that in the squashed S^7 metric,

$$\omega_i = \sigma_i - \Sigma_i$$
$$\nu_i = \sigma_i + \Sigma_i$$

and σ_i, Σ_i satisfy

$$d\sigma_i = -\tfrac{1}{2} \varepsilon_{ijk}\, \sigma_j \wedge \sigma_k \quad , \quad d\Sigma_i = -\tfrac{1}{2} \varepsilon_{ijk}\, \Sigma_j \wedge \Sigma_k$$

σ_i, Σ_i <u>left</u>-invariant 1-forms.

Can instead have <u>right</u>-invariant 1-forms σ_i', Σ_i'

$$d\sigma_i' = \tfrac{1}{2} \varepsilon_{ijk}\, \sigma_j' \wedge \sigma_k' \quad , \quad d\Sigma_i' = \tfrac{1}{2} \varepsilon_{ijk}\, \Sigma_j' \wedge \Sigma_k'$$

(defined by $2\, dU\, U^{-1} = i\sigma_1' + j\sigma_2' + k\sigma_3'$, etc)

$\Rightarrow$ Θ's given by previous expressions, but with $\varepsilon_{ijk} \to -\varepsilon_{ijk}$

Crucial when considering first-order self-adjoint operators (e.g. Dirac operator)

Spectrum of +ve and −ve eigenvalues is different

Also Killing Spinors

Original squashed S^7 — <u>left</u>-squashed

S^7 with σ_i', Σ_i' — <u>right</u>-squashed

Killing Spinors

How many η satisfying $\bar{D}_m \eta \equiv D_m \eta - \frac{m}{2} \Gamma_m \eta = 0$?

or ξ " $\bar{D}'_m \xi \equiv D_m \xi + \frac{m}{2} \Gamma_m \xi = 0$?

Integrability condition:

$$[\bar{D}_a, \bar{D}_b] = [\bar{D}'_a, \bar{D}'_b] = -\frac{1}{4} R_{abcd} \Gamma_{cd} + \frac{m^2}{2} \Gamma_{ab}$$

$$= -\frac{1}{4} C_{abcd} \Gamma_{cd} \qquad \text{if } R_{ab} = 6m^2 g_{ab}$$

↑ Weyl Tensor

Substitute Weyl tensor of $\lambda^2 = \frac{1}{5}$ sphere : ⇒

14 linearly independent Γ_{ab} combinations

$$C_{0iab} \Gamma_{ab} = \frac{4}{5} [\Gamma_{0i} + \frac{1}{2} \varepsilon_{ijk} \Gamma_{\hat{j}\hat{k}}]$$

$$C_{ijab} \Gamma_{ab} = \frac{4}{5} [\Gamma_{ij} + \Gamma_{\hat{i}\hat{j}}]$$

$$C_{\hat{i}\hat{j}ab} \Gamma_{ab} = \frac{4}{5} [-\Gamma_{\hat{i}\hat{j}} - \frac{1}{2} \Gamma_{j\hat{i}} + \frac{1}{2} \delta_{ij} \Gamma_{k\hat{k}} - \frac{1}{2} \varepsilon_{ijk} \Gamma_{0\hat{k}}]$$

Represent $\Gamma_0 = \gamma_0 \otimes 1$, $\Gamma_i = \gamma_i \otimes 1$, $\Gamma_{\hat{i}} = i \gamma_5 \otimes \tau_i$

Find that only solution of $C_{abcd} \Gamma_{cd} \psi = 0$ is proportional to $\begin{pmatrix} 0 \\ i \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

Integrability condition is necessary <u>but</u> <u>not</u> <u>sufficient</u> condn for existence of solns. Find that

$\eta = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ i \\ 1 \\ \vdots \\ 0 \end{pmatrix}$ solves $\bar{D}_a \eta = 0$ but no soln of $\bar{D}'_a \xi = 0$

(LEFT-SQUASHED SPHERE)

$C_{ab} = C_{abcd} \Gamma_{cd}$ singles out 14 linearly indep. combinations of the 21 Γ_{ab}'s which generate $SO(7)$

∴ C_{ab} generates a 14 dimensional subgroup of $SO(7)$

This subgroup is G_2

But $[\bar{D}_a, \bar{D}_b]$ generates the <u>HOLONOMY</u> <u>GROUP</u> of the connection $\bar{D}_a$.

∴ In $\lambda^2 = \frac{1}{5}$ squashed sphere, holonomy groups of $\bar{D}_a$, and $\bar{D}'_a$ are both G_2.

G_2 is the stability group of a spinor in $d=7$ dimensions.

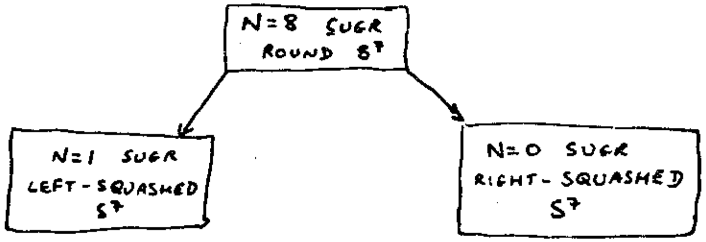
∴ Would expect at most one solution of $\bar{D}_a \psi = 0$ or $\bar{D}'_a \psi = 0$

And in fact $\bar{D}_a \psi = 0$ has 1 solution, $\bar{D}'_a \psi = 0$ has no solution } LEFT SQUASHED SPHERE

However, for <u>RIGHT</u>-<u>SQUASHED</u> sphere,

$\bar{D}_a \psi = 0$ has no solution

$\bar{D}'_a \psi = 0$ has [illegible] solution



But $8_{\psi_m} \rightarrow (4,2)$ LEFT-SQUASHED

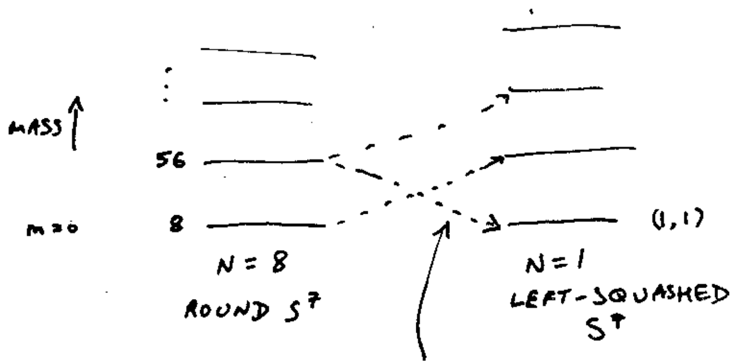
$\phantom{But\ 8_{\psi_m}} \rightarrow (5,1)+(1,3)$ RIGHT-SQUASHED

$SO(8) \qquad Sp(2)\times Sp(1)$

Qu: WHEN $N=8$ BREAKS TO $N=1$, ALL 8 MASSLESS GRAVITINI BECOME MASSIVE.

WHERE DOES THE 1 MASSLESS GRAVITINO COME FROM ?

Ans: OUT OF THE SKY !



INVASION OF THE KILLING SPINOR

ON LEFT-SQUASHED S^7, $\exists$ ONE KILLING SPINOR η :

$$\bar{D}_m \eta \equiv D_m \eta - \frac{m}{2}\Gamma_m \eta = 0$$

WANT TO TRACE THIS BACK TO THE ROUND S^7.

DO THIS BY LOOKING FOR THE SPINOR ψ, SATISFYING

$$P\psi = \kappa\psi \qquad (P = \Gamma^m D_m)$$

FOR ALL VALUES OF SQUASHING PARAMETER λ, SUCH THAT

$$\psi = \eta \qquad \left(\text{and so } \kappa = -\frac{7m}{2}\right)$$

ON THE $\lambda^2 = \frac{1}{5}$ SPHERE.

THE SOLUTION IS $\psi = \eta$ FOR ALL λ.

$\Rightarrow$ ON ROUND SPHERE $(\lambda^2 = 1)$,

$$P\psi = -\frac{9m}{2}\psi$$

($\hat{=}$ 1'ST MASSIVE LEVEL OF GRAVITINOS IN THE ROUND SPHERE THEORY)

Spectra of Operators

1. SCALAR LAPLACIAN

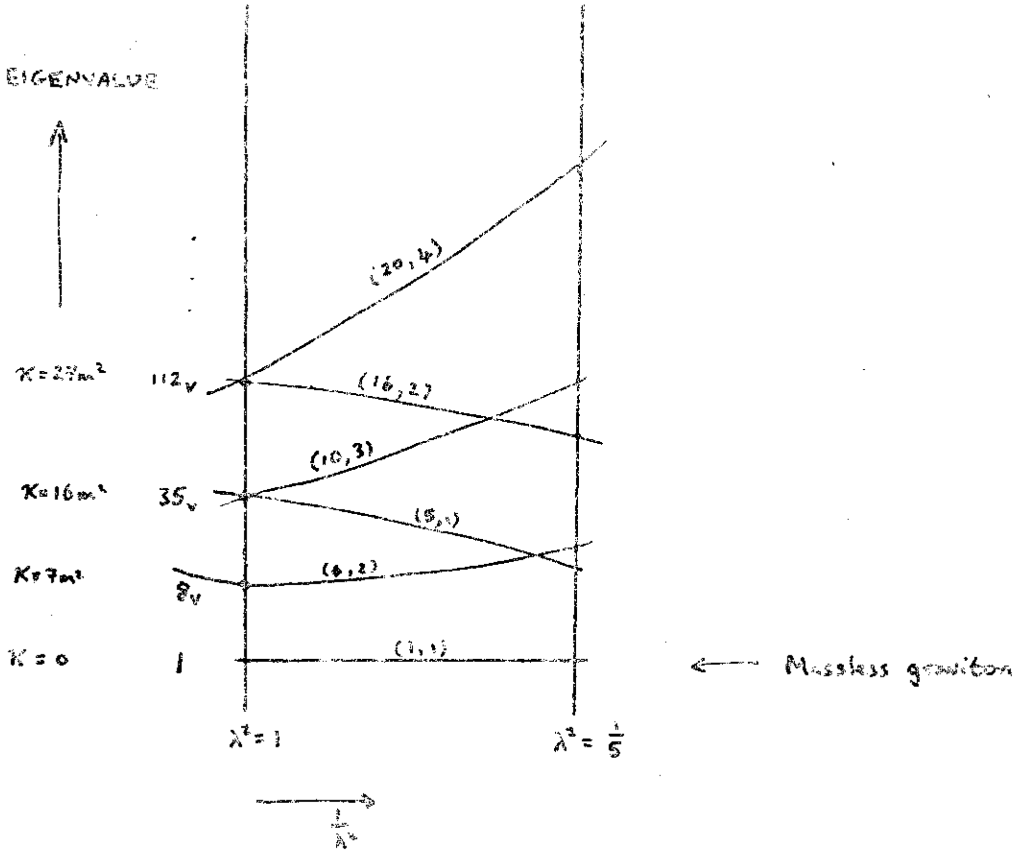
Can show that

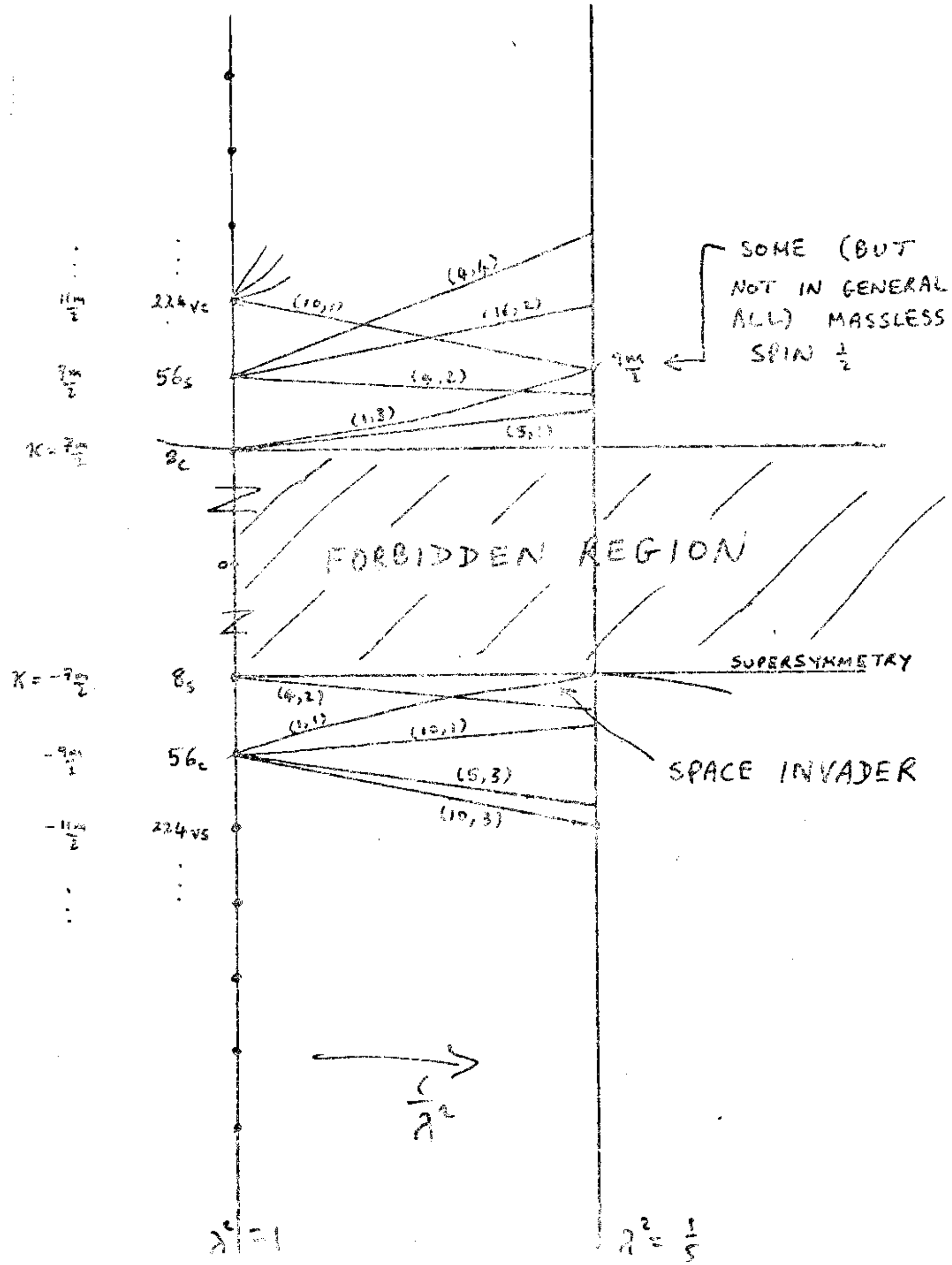
$$-\Box = -\Box^0 + \frac{1-\lambda^2}{\lambda^2} K_i^2$$

↑ Squashed S^7 Laplacian

↑ Round S^7 Laplacian

↑ K_i generates $Sp(1)$ in $Sp(2) \times Sp(1)$





Criterion for massless spin $\frac{1}{2}$ physical fields :

$$\Psi_m(x,y) = \sum \chi(x)\,\eta_m(y)$$

summation is over η_m's which satisfy

1) $\Gamma^n D_n \eta_m - m\Gamma_m \chi - \frac{3m}{2}\eta_m = 0$

and 2) $D_n \eta_n = \frac{m}{2}\chi$

and 3) $\Gamma^n D_n \chi = \frac{9m}{2}\chi$

where $\chi = \Gamma^n \eta_n$

If $\chi \neq 0$, χ is a $+\frac{9m}{2}$ mode of Dirac

Conversely, such η_m can be expressed as

$$\eta_m = (D_m + \tfrac{3m}{2}\Gamma_m)\psi ,$$

where ψ is a $\frac{9m}{2}$ mode of Dirac.

a) ROUND S^7 All η_m's can be constructed in this way.

$\therefore$ 56_s of $SO(8)$ of massless spin $\frac{1}{2}$

b) LEFT-SQUASHED S^7 SOLUTION

$(10,1) + (1,3)$ of η_m's can be constructed in this way — The $N=1$ supersymmetric partners of the $Sp(2)\times Sp(1)$ gauge bosons.

ARE THERE ANY MORE? OPEN QUESTION

TORSION

Recall : 1. S^7 parallelizable (Topological property)

2. Round S^7 Absolutely //izable (Metric property)

SQUASHED EINSTEIN S^7 $(\lambda^2 = \frac{1}{5})$ admits a torsion

$S_{abc} = S_{[abc]}$

such that $\bar{\Gamma}^a{}_{bc} = \Gamma^a{}_{bc} + S^a{}_{bc}$ has

$\bar{R}_{ab} = 0$ (But not $\bar{R}_{abcd} = 0$)

S_{abc} satisfies

$$\nabla_a S_{bcd} = \nabla_{[a} S_{bcd]} = \pm \tfrac{m}{2}\,\varepsilon_{abcdefg} S_{efg}$$

$$S_{acd} S_{bcd} = 6m^2 g_{ab}$$

LEFT-SQUASHED : $S_{abc} \propto \bar{\eta}\,\Gamma_{abc}\,\eta$ $\qquad \bar{D}_a \eta = 0$

RIGHT-SQUASHED : $S_{abc} \propto \bar{\xi}\,\Gamma_{abc}\,\xi$ $\qquad \bar{D}'_a \xi = 0$

BUT To obtain new KK Supergravity solution, with $F_{abcd} \propto \nabla_a S_{bcd}$, field equations imply

$$\nabla_a S_{bcd} = -\tfrac{m}{6}\,\varepsilon_{abcdefg}\, a_{efg}$$

$\therefore$ Can only find supergravity solution with squashing and "torsion" for the RIGHT-SQUASHED S^7.

(So $N=0$ already before "torsion" added)

ξ is a singlet under $Sp(2)\times Sp(1)$

$\therefore$ RIGHT-SQUASHED S^7 + "TORSION" Has $Sp(2)\times Sp(1)$ invariance

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SUPERSTRINGS

THE SUPERSYMMETRIC STRING OF SCHWARZ AND GREEN IS A VARIANT OF THE OLD FERMIONIC STRING THEORY OF RAMOND-NEVEU-SCHWARZ.

CONSISTENT ONLY IN TEN DIMENSIONS, IT APPARENTLY MUST BE INTERPRETED IN THE SENSE OF A KALUZA-KLEIN THEORY ... FOUR MINKOWSKIAN DIMENSIONS AND SIX COMPACT ONES.

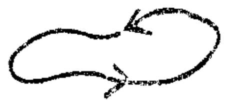
NAIVELY, THE TEN DIMENSIONAL SUPERSTRING THEORY REDUCES AT LOW ENERGIES TO TEN DIMENSIONAL SUPERGRAVITY ... BUT UNLIKE THAT THEORY, IT IS APPARENTLY

(1½)

WHY DOES A STRING THEORY DESCRIBE GRAVITY? IT DESCRIBES A RICH SPECTRUM OF PARTICLES

...

STRING OSCILLATIONS



INCLUDING OSCILLATIONS OF THE EXTRA DIMENSIONS ....

THE LIGHTEST STATES OF THIS SPECTRUM ARE MASSLESS PARTICLES OF SPINS

$$0, \tfrac{1}{2}, \ldots, 2$$

WITH JUST THE LONG WAVELENGTH COUPLINGS OF SUPERGRAVITY ....

IN FACT THE SUPERSTRING THEORY WOULD APPEAR AT THE MOMENT TO BE THE ONE REAL CONTENDER AS A RENORMALIZABLE, PHYSICALLY SENSIBLE QUANTUM THEORY OF GRAVITY.

PHENOMENOLOGY INCLUDES WHATEVER IS POSSIBLE IN

$N=8$ SUPERGRAVITY (4 DIM)

OR IN

$D=10$ SUPERGRAVITY

BUT MANY OTHER POSSIBILITIES AS WELL.

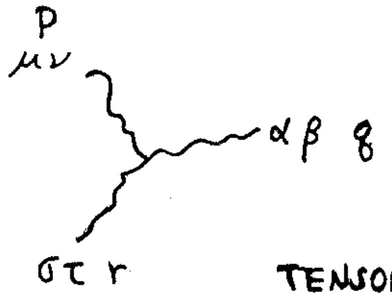
A SCORECARD

	$N=8$ SUPERGRAVITY	10 DIM SUPERGRAVITY	10 DIM SUPERSTRING
FINITE OR RENORMALIZABLE?	PROBABLY NOT	NO	PROBABLY
UNIQUE	FAR FROM IT	ALMOST AND MAYBE	ALMOST AND MAYBE
PHENOMENOLOGICAL PROSPECTS	POOR	PROBLEMATIC	PROBLEMATIC
AESTHETICS	O.K. EXCEPT FOR NONUNIQUENESS	BEAUTIFUL	UNSATISFACTORY IN PRESENT FORMULATION

WHAT IS REALLY UNSATISFACTORY ABOUT THE STRING THEORY AT THE MOMENT IS THAT IT ISN'T YET A THEORY ... IT IS A (NOT ENTIRELY COMPLETE) SET OF FEYNMAN RULES FOR THREE STRING, FOUR STRING VERTICES, ETC.

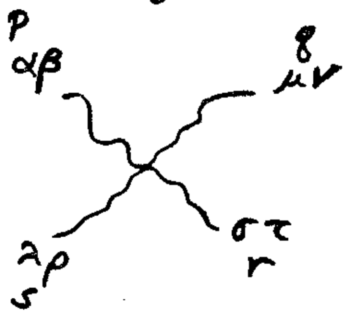
SUPPOSE THAT GENERAL RELATIVITY HAD NEVER BEEN INVENTED AND SOMEONE WAS TRYING TO CONSTRUCT A LORENTZ INVARIANT THEORY OF A MASSLESS SPIN TWO PARTICLE.

AFTER CONSTRUCTING THE FREE FIELD THEORY, ONE TRIES TO FIND AN ACCEPTABLE THREE BODY VERTEX....



A PRIORI IT WOULD BE A TENSOR $T_{\alpha\beta\gamma\delta\sigma\tau}(p\,q\,r)$

BUT THERE IS A UNIQUE ACCEPTABLE CHOICE. EVEN MORE COMPLEX, IF ONE HAS NEVER HEARD OF RIEMANNIAN GEOMETRY, IS THE FOUR BODY VERTEX



THE EXISTENCE OF A PHYSICALLY ACCEPTABLE 4 BODY VERTEX IS A BIZARRE MIRACLE .... UNTIL ONE DISCOVERS RIEMANNIAN GEOMETRY AND WRITES

$$\sqrt{g}\,R$$

THE STRING THEORY IS IN SUCH A STATE. THERE SEEM TO BE CONSISTENT STRING INTERACTIONS

AND A FINITE, PHYSICALLY SENSIBLE QUANTUM THEORY OF GRAVITY ...
(NO GHOSTS, NO TACHYONS ...)
BUT THE CONSISTENCY IS A MIRACLE, THE VERTICES ARE LABORIOUSLY CONSTRUCTED AND PROVED CONSISTENT.

WE DON'T HAVE AN ANALOGUE OF RIEMANNIAN GEOMETRY UNDERLYING STRING THEORY ... AND WE CAN'T SUM THE VERTICES IN A FORM ANALOGOUS TO

$$\sqrt{g}\, R.$$

THIS IS A CRUCIAL PROBLEM ON THE ESTHETIC PLANE. IF THERE ISN'T AN ANALOG OF RIEMANNIAN GEOMETRY AND $\sqrt{g}R$ IN STRING THEORY, I DOUBT STRING THEORY IS ATTRACTIVE AS A FUNDAMENTAL THEORY OF NATURE.

ALSO ON THE PRACTICAL LEVEL, IT IS A CRUCIAL PROBLEM

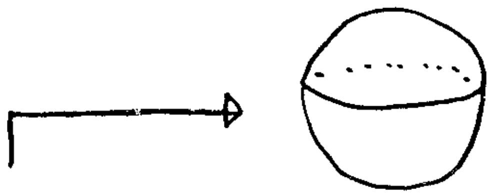
- * I DON'T THINK IT IS REASONABLE TO DESCRIBE THE EARTH GOING AROUND THE SUN BY EXCHANGE OF 10^{80} OR 10^{90} STRINGS. WE SHOULD WORK WITH A NONTRIVIAL SOLUTION OF SOMETHING.

- * EVEN WORSE ... TO DO ANY REASONABLE PHENOMENOLOGY IN STRING THEORY WILL REQUIRE A KALUZA-KLEIN APPROACH ...

AND PROBABLY NOT WITH FLAT EXTRA DIMENSIONS (A CASE ALREADY CONSIDERED BY SCHWARZ AND GREEN).

TO DISCUSS NON-TRIVIAL COMPACTIFICATION ONE NEEDS THE STRING ANALOGUE OF RIEMANNIAN GEOMETRY.

i.e. what equations to solve?
what is a "non-singular" solution?



(THIS IS A SINGULAR SPACE IF YOU ONLY KNOW ABOUT FUNCTIONS $g_{\mu\nu}(x,y)$ AND DON'T KNOW ABOUT COORDINATE TRANSFORMATIONS AND RIEMANNIAN GEOMETRY.)

HOW DO MASSLESS PARTICLES AND SUPERGRAVITY ARISE IN STRING THEORY? I'LL CONSIDER THE SLIGHTLY SIMPLER CASE OF OPEN STRINGS... LEADING TO MASSLESS PARTICLES OF SPINS $(\frac{1}{2}, 1)$ AND SUPER YANG-MILLS THEORY.

THE ORDINARY $(D=26)$ BOSONIC STRING THEORY DESCRIBES THE MOTION OF A STRING — PARAMETRIZED BY (σ, τ) — IN A SPACETIME OF COORDINATES X^μ.

$(0 \le \sigma \le \pi)$

IN SOME GAUGE

$$\mathcal{L} = \int \frac{1}{2}\left(\partial_\alpha X^\mu \, \partial_\alpha X_\mu\right) d\sigma \, d\tau$$

$$\alpha = \sigma, \tau$$

THE X^μ — ALTHOUGH SPACETIME COORDINATES — APPEAR AS FREE SCALAR FIELDS IN A TWO DIM. $\sigma\tau$ WORLD.

IN THE RAMOND-NEVEU-SCHWARZ FERMIONIC STRING THEORY, THE STRING ALSO CARRIES ANTI-COMMUTING DEGREES OF FREEDOM $\Psi^\mu(\sigma,\tau)$.

THE ψ^μ FIELD IS RATHER ODD BECAUSE IT ANTICOMMUTES AND IS A SPINOR UNDER TRANSFORMATIONS OF (σ,τ); HOWEVER UNDER LORENTZ ROTATIONS OF X^μ THE ψ^μ TRANSFORM AS A <u>VECTOR</u>.

HENCE ψ^μ MAPS BOSONS INTO BOSONS AND FERMIONS INTO FERMIONS.

$$\psi^\mu |B\rangle = |B'\rangle \qquad \Delta J = 0, 1$$
$$\psi^\mu |F\rangle = |F'\rangle$$

IN FACT AS WE'LL SEE THE MODEL CAN BE CONSTRUCTED SO THAT ALL STATES ARE BOSONS OR SO ALL ARE FERMIONS

THE LAGRANGIAN IS

$$\mathcal{L} = \int d\sigma\, d\tau \Big[\tfrac{1}{2} (\partial_\alpha X^\mu)(\partial_\alpha X_\mu) + \tfrac{1}{2}\, i\, \bar{\Psi}_\mu \gamma^\alpha \frac{\partial}{\partial \sigma^\alpha} \psi^\mu \Big]$$

IT HAS A VERY PECULIAR SYMMETRY

$$\delta X^\mu = i \bar{\epsilon} \psi^\mu$$
$$\delta \psi^\mu = \not{\partial} X^\mu \epsilon$$

WHERE ϵ_α = TWO COMPONENT ANTICOMMUTING SPINOR IN $\sigma\tau$ SPACE ... BUT A LORENTZ SCALAR.

THE CORRESPONDING CONSERVED CHARGE Q_α COMMUTES WITH ANGULAR MOMENTUM

parity

$$Q_\alpha | J, J_z; \eta \rangle = | J, J_z; \tilde{\eta} \rangle$$

THIS PECULIAR SYMMETRY WAS THE GENESIS OF SUPERSYMMETRY (AT LEAST IN THE WEST) .... WESS AND ZUMINO

ERASED THE μ INDEX FROM X^μ, INTERPRETED X^μ AS A SCALAR FIELD ϕ, AND GENERALIZED THE STRING PARAMETERS (σ, τ) TO FOUR SPACE TIME COORDINATES. THIS IS WHAT SUPERSYMMETRY DEVELOPED FROM.

THE LAGRANGIAN FROM THE LAST TRANSPARENCY DOES NOT HAVE SPACE-TIME SUPERSYMMETRY. I.E. IT DOES NOT HAVE BOSE-FERMI SYMMETRY. BUT A SLIGHT VARIANT OF IT DOES.

FIRST OF ALL, I'VE WRITTEN THE LAGRANGIAN

$$\mathcal{L} = \frac{1}{2} \int d\sigma \, d\tau \left(\partial_\alpha X^\mu \partial_\alpha X_\mu + \bar{\psi}_\mu i \not{\partial} \psi^\mu \right)$$

IN A PARTICULAR GAUGE. NO NEED TO WRITE THE (COMPLICATED) GAUGE INVARIANT FORM. BUT WE MUST IMPOSE THE CONSTRAINT EQUATIONS (ANALOGOUS TO GAUSS' LAW) THAT ARE CONJUGATE TO THE GAUGE CONDITIONS. THE CONSTRAINTS ARE THE VANISHING OF THE SYMMETRY GENERATORS (REPARAMETRRATION OF STRING AND "SUPERSYMMETRY") OR OF THE STRING ENERGY-MOMTM TENSOR AND SUPERCURRENT:

$$0 = \partial_\alpha X^\mu \partial_\beta X_\mu - \frac{1}{2} \eta_{\alpha\beta} (\partial_\sigma X^\mu)^2$$

$$0 = (\partial_\alpha X^\mu) \gamma^\alpha \gamma^\delta \psi_\mu$$

THE CONSTRAINTS CAN BE USED TO ELIMINATE TWO COMPONENTS OF X^μ (SAY X^0, X^9) AND OF THE CORRESPONDING TWO COMPONENTS OF ψ^μ (ψ^0, ψ^9). WE ARE THEN LEFT WITH EIGHT FREE BOSE AND FERMI FIELDS.

$$\mathcal{L} = \frac{1}{2} \sum_{i=1}^{8} \int d\sigma d\tau \left((\partial_\alpha X^i)^2 + \bar{\psi}^i \gamma^\alpha \partial_\alpha \psi^i \right)$$

AS I'VE MENTIONED, THIS LAGRANGIAN CAN BE QUANTIZED SO THAT THE STATES ARE ALL BOSONS OR ALL FERMIONS.

REMEMBER (OPEN STRINGS) $0 \le \sigma \le \pi$ THE FIELD ψ HAS A FOURIER EXPANSION

$$\psi^i(\sigma) = \sum \psi_n^i \, e^{in\sigma}$$

where one CAN CHOOSE $n =$ integer or $n =$ half integer

$$\psi^i(\sigma) = \sum e^{in\sigma} \, \psi_n^i$$

$n =$ integer ... fermions

$n =$ half integer ... bosons

WHY? CANONICAL QUANTIZATION

$$\{ \psi^i_{-n}, \psi^j_m \} = \delta^{ij} \delta_{n,m}$$

ONE MAY REGARD ψ^j_m $m > 0$ as "ANNIHILATION OPERATORS" AND ψ^j_{-m} $m > 0$ as "CREATION OPERATORS"

FOR BOSONS -- $n \in \mathbb{Z} + \frac{1}{2}$

THE HILBERT SPACE IS AS FOLLOWS:

- UNIQUE GROUND STATE OF STRING $|\Omega\rangle$ $J=0$, BOSON.

EXCITATIONS

$$\psi^{i_1}_{-m_1} \psi^{i_2}_{-m_2} \cdots \psi^{i_k}_{-m_k} |\Omega\rangle$$

ALL BOSONS SINCE ψ^i IS A VECTOR.

X^μ

$X^i_{i=1 \ldots 8}$

· HOW COULD IT BE OTHERWISE?

IF n is <u>INTEGER</u> $\psi_1 \quad \psi_2$

THE ψ^i_n FOR $n>0$, $n<0$ $\quad \psi_{-1} \quad \psi_{-2}$

ARE ANNIHILATION, CREATION OPERATORS

BUT ψ^i_0 IS LEFT OVER, SELF CONJUGATE

THE ψ^i_0 OBEY

$$\{\psi^i_0 \; \psi^j_0\} = 2\delta^{ij} \qquad i,j = 1 \cdots 8$$

AN 8 DIMENSIONAL CLIFFORD ALGEBRA

··· GAMMA MATRICES OF $O(8)$

IRREDUCIBLE REPRESENTATION IS UNIQUE, <u>SIXTEEN DIMENSIONAL</u>

THE <u>SPINOR</u> OF $O(8)$ ··· SO GROUND STATE IS A <u>SPINOR</u> $|\Omega^\alpha\rangle$ $\alpha = 1 \cdots 16$

AND EXCITATIONS

$$\psi^{i_1}_{-m_1}\psi^{i_2}_{-m_2} \cdots \psi^{i_k}_{-m_k}|\Omega^\alpha\rangle$$

ARE ALSO FERMIONS.

· LOW-LYING STATES:

$n \in Z + \frac{1}{2}$ BOSON	$n \in Z$ FERMION
$\lvert\Omega\rangle$ 1 state $J=0$	$\lvert\Omega^\alpha\rangle$ 16 states
$\psi^i_1\lvert\Omega\rangle = \lvert\Omega^i\rangle$ 8 states $J=1$	$m=0$
$m=0$	

CAN'T HAVE S[illegible] SUPERSYMMETRY SINCE COUNTING OF STATES IS DIFFERENT.

ACTUALLY FIRST BOSE [illegible]TATION $\psi^i_1|\Omega\rangle$ IS <u>MASSLESS</u>; IT IS A VECTOR BUT A MASSIVE VECTOR NEEDS NINE COMPONENTS. GROUND STATE $|\Omega\rangle$ OF BOSE SPECTRUM IS THUS A <u>TACHYON</u>.

HOWEVER, GLIOZZI, SCHERK, AND OLIVE (1976) MADE THE FOLLOWING OBSERVATION:

HALF THE BOSONS ARE IN CORRESPONDENCE WITH HALF THE FERMIONS

KEEP ONLY "EVEN G" BOSONS ← tachyon $J=0$

$$\underbrace{\psi^{i_1}_{m_1}\,\psi^{i_2}_{m_2}\cdots\psi^{i_k}_{m_k}}_{\text{odd } k}\,|\Omega\rangle$$

THIS ELIMINATES TACHYONS
GROUND STATE OF STRING IS NOT A MASSLESS VECTOR.

FOR FERMIONS DEFINE $\underbrace{\tilde{\Psi} = \psi^1_0\,\psi^2_0\cdots\psi^8_0}_{\text{ANALOGUE OF } \gamma_5}$

AND KEEP ONLY STATES OF $\tilde{\Psi} = +1$
GROUND STATE OF FERMI SPECTRUM HAS NOW 8 COMPONENTS ... MUST BE $M=0$ SINCE $M\neq 0$ FERMION IN $D=10$ NEEDS 16 COMPONENTS.

SO LOWEST STATE

BOSE	FERMI
$m=0$ vector	$m=0$ spin ½, definite chirality

THIS IS THE $(\frac{1}{2}, 1)$ MULTIPLET OF $D=10$ SUPERSYMMETRY.

GLIOZZI, SCHERK, OLIVE SHOWED – BY COUNTING – THAT ALSO THE EXCITATIONS MIGHT BE SUPERSYMMETRIC; AND FROM THE RECENT WORK (SCHWARZ AND GREEN) WE KNOW THAT RESTRICTED TO HALF THE BOSONS AND HALF THE FERMIONS THE INTERACTIONS TOO ARE SUPERSYMMETRIC.

OUR LAGRANGIAN WAS OBTAINED (BY FIXING A GAUGE) FROM ONE THAT WAS MANIFESTLY LORENTZ INVARIANT, BUT HAD NO 10 DIMENSIONAL SUPERSYMMETRY.

IT CAN BE CONVERTED TO A FORM THAT [illegible] DIMENSIONAL SUPERSYMMETRY [illegible] [illegible] [illegible] [illegible] [illegible] $(\partial\phi)^2$ [illegible] [illegible] [illegible] WHAT [illegible] THE [illegible] [illegible] THE FERMIONS

ψ^i, [illegible] 8 BY INTRODUCING FOUR REAL SCALARS ϕ^i [illegible] 4

$\frac{1}{\sqrt{2}}\epsilon_{\mu\nu}\partial_\nu\phi_1 = [illegible]$

$\frac{1}{\sqrt{2}}\epsilon_{\mu\nu}\partial_\nu\phi_2 = [illegible]$

$\frac{1}{\sqrt{2}}$ [illegible] $= [illegible]$

$\frac{1}{\sqrt{2}}$ [illegible] $= [illegible]$

SHUFFLE THE SCALARS

$$\sigma_1 = \tfrac{1}{2}(\phi_1 + \phi_2 + \phi_3 + \phi_4)$$

$$\sigma_2 = \tfrac{1}{2}(\phi_1 + \phi_2 - \phi_3 - \phi_4)$$

$$\sigma_3 = \tfrac{1}{2}(\phi_1 - \phi_2 + \phi_3 - \phi_4)$$

$$\sigma_4 = \tfrac{1}{2}(\phi_1 - \phi_2 - \phi_3 + \phi_4)$$

[illegible] [illegible] [illegible] [illegible] [illegible] NEW [illegible] [illegible] [illegible]

$\frac{1}{\sqrt{2}}\epsilon_{\mu\nu}\partial_\nu\sigma_1 = [illegible]$

$\frac{1}{\sqrt{2}}\epsilon_{\mu\nu}\partial_\nu\sigma_2 = [illegible]$

$\frac{1}{\sqrt{2}}\epsilon_{\mu\nu}\partial_\nu\sigma_3 = [illegible]$

$\frac{1}{\sqrt{2}}\epsilon_{\mu\nu}\partial_\nu\sigma_4 = [illegible]$

[illegible]

THE LAGRANGIAN IS NOW

$$\bar{\mathcal{L}} = \frac{1}{2}\int d\sigma dz \left((\partial_\alpha X^i)^2 + \bar{\lambda}^\alpha i \not\partial \lambda_\alpha \right)$$

BUT IT IS **NOT QUITE** EQUIVALENT TO THE OLD ONE.

BOSONIZATION OF FERMIONS IS EXACT ON THE OPEN LINE. HERE, ON $0 \le \sigma \le \pi$ IT IS NOT QUITE EXACT. $\bar{\mathcal{L}}$ DIFFERS FROM $\mathcal{L}$ EXACTLY AS DESIRED:

(1) IT AUTOMATICALLY DESCRIBES BOTH FERMIONS AND BOSONS, UNLIKE $\mathcal{L}$ WHICH CAN BE QUANTIZED WITH ONLY FERMIONS OR ONLY BOSONS. AFTER ALL λ^α HAS $J = \frac{1}{2}$

(2) QUANTIZATION OF $\bar{\mathcal{L}}$ GIVES **HALF THE BOSE SECTOR** OF $\mathcal{L}$ AND **HALF THE FERMI SECTOR**.

(3) $\bar{\mathcal{L}}$ IS SUPERSYMMETRIC, THE CONSERVED CURRENTS BEING

$$S_\alpha^\epsilon = \gamma_\alpha \lambda^\epsilon \qquad (8)$$

$$\tilde{S}_\alpha^\epsilon = (\gamma^\beta \partial_\beta X^i \gamma_i \lambda)^\epsilon \qquad (8)$$

$\bar{\mathcal{L}}$ IS NOT MANIFESTLY LORENTZ INVARIANT AND THIS IS NOT OBVIOUS SINCE $\bar{\mathcal{L}}$ IS NOT REALLY EQUIVALENT TO $\mathcal{L}$. $\bar{\mathcal{L}}$ (to our knowledge) cannot be obtained by gauge fixing in a manifestly Lorentz Invariant Lagrangian.

$\bar{\mathcal{L}}$ CAN BE LABORIOUSLY SHOWN TO BE LORENTZ INVARIANT (FOR INSTANCE S^ϵ AND $\tilde{S}^\epsilon$ ABOVE COMBINE INTO ONE LORENTZ MULTIPLET) AND INTERACTIONS CAN BE LABORIOUSLY CONSTRUCTED.

OUTSTANDING PROBLEM:

FIND THE ANALOGUE OF GENERAL COVARIANCE AND $\sqrt{g}\,R$.

V. COSMOLOGY AND ASTROPHYSICS

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Cosmology

cosmic red-shift
microwave background $\Rightarrow$ expansion

As $t \searrow$, $\rho, T \nearrow$

$t \lesssim 1\,s \qquad T \gtrsim 1\,MeV$

$\Rightarrow$ particle processes dominate

Particle physics

unified gauge theories
$\Rightarrow$ phase transitions

QCD: quark – hadron transition
$T_c \sim 100$ MeV

electroweak: Weinberg – Salam trans.
$T_c \sim 100$ GeV

? GUTs: $T_c \gtrsim 10^{15}$ GeV
(or others in range
10^2 GeV $\to 10^{19}$ GeV)

2 problems

1. Gauge theories
$\Rightarrow$ phase transitions.

2. Phase transitions
$\Rightarrow$ cosmological implications

What could have survived?

a. topologically stable defects
monopoles, strings, domain walls

b. predictions of cosmological parameters
density, baryon number,
fluctuation spectrum

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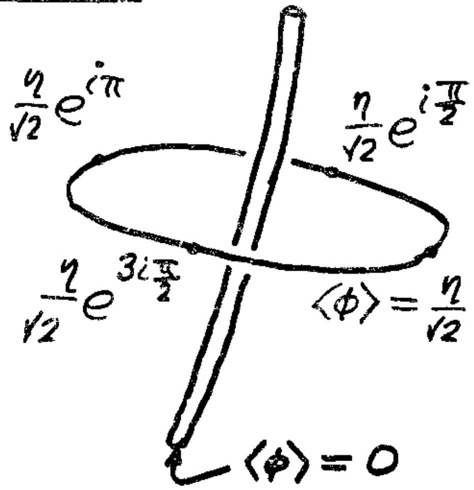
1. Do we know how to calculate the nature of the phase transitions in gauge theories?

Topological Defects

eg String



cf:

fluxtubes in superconductors

vortex lines in superfluids

stable because manifold of vacuum states (labelled by α) has non-trivial loop

Gauge group $= G$

Unbroken subgroup $= H$

Manifold of vacuum states $= G/H$.

Require non-trivial $\pi_1(G/H)$

NB: if G is simply connected $\pi_1(G/H) \simeq \pi_0(H)$

i.e. strings if H is disconnected

Spontaneously Broken Gauge Theories

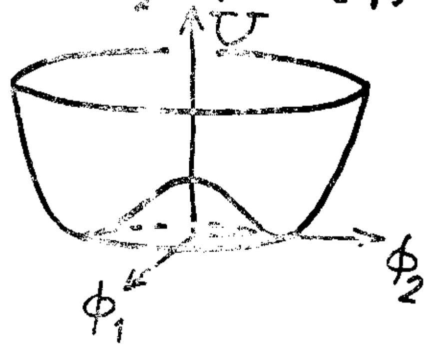
$$\mathcal{L} = \frac{1}{2} D_\mu\phi \cdot D^\mu\phi - \frac{1}{4} F_{\mu\nu}\cdot F^{\mu\nu} - U(\phi)$$

invariant under G

e.g.

$$\mathcal{L} = D_\mu\phi^* D^\mu\phi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\lambda^2\left(\phi^*\phi - \frac{1}{2}\eta^2\right)^2$$

$$D_\mu\phi = \partial_\mu\phi + ieA_\mu\phi$$



$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\eta\, e^{i\alpha}$$

different $\alpha \Rightarrow$ different (equivalent) vacuum states

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

e.g. $\alpha = 0$: $\phi_1 = \eta + \phi_1'$

ϕ_1': Higgs $\quad m^2 = \lambda^2\eta^2$

$A_\mu + \frac{1}{e}\partial_\mu\phi_2$: $\quad m^2 = e^2\eta^2$

But:

gauge invariance ?

Defects	require non-trivial
domain walls	$\pi_0(G/H)$
strings	$\pi_1(G/H) \cong \pi_0(H)$
monopoles	$\pi_2(G/H) \cong \pi_1(H)$

Domain walls:
broken discrete symmetry
unacceptable in visible universe today

Monopoles:
unavoidable if G semisimple & H contains U(1) factor
$\Rightarrow$ monopole problem

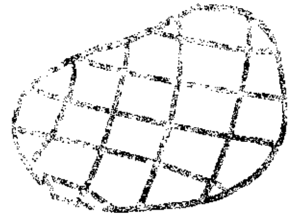
Strings: optional

e.g. Spin(10)
$\downarrow$ 126
$SU(5) \times Z_2$
$\downarrow$ 45
$\sim SU(3) \times SU(2) \times U(1) \times Z_2$
$\downarrow$ 10
$\sim SU(3) \times U(1) \times Z_2$

315

Composite structures

1. Walls bounded by strings



e.g. Spin(10)
$\downarrow$ 54
"Spin(6) × Spin(4)" = H_1
$\downarrow$ 126
$\sim SU(3) \times SU(2) \times U(1)$
$\downarrow$ 10
$\sim SU(3) \times U(1)$

$\pi_1(H_1) = Z_2$: monopoles
$\pi_0(H_2) = Z_2'$: strings

Walls generated at 2nd transition (Z_2' broken)

2. Strings terminated by monopoles

$G \to H_1 \to H_2$

$\pi_1(H_1) = Z_2$: monopoles
broken at 2nd transition
$\Rightarrow$ strings

Effective Action

$$Z[j] = e^{iW[j]} = \langle 0, \text{out} | 0, \text{in} \rangle_j$$

$$= \int [d\phi]\, e^{iS[\phi] + i\phi\cdot j}$$

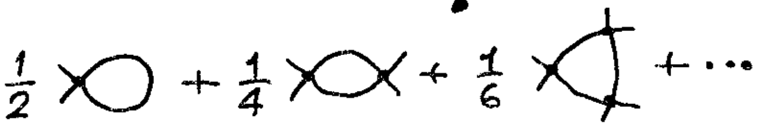
$$\phi\cdot j = \int d^4x\, \phi(x)\, j(x)$$

$$\frac{\delta W}{\delta j(x)} = \langle \hat{\phi}(x) \rangle \equiv \bar{\phi}(x)$$

Def: $\Gamma[\bar{\phi}] = W[j] - \bar{\phi}\cdot j$

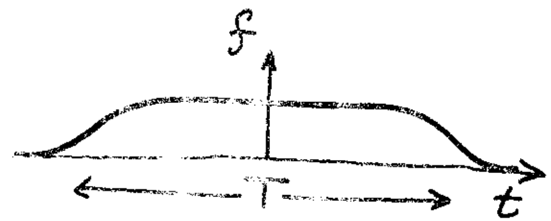
$$\frac{\delta \Gamma}{\delta \bar{\phi}(x)} = -j(x)$$

$$\Gamma[\bar{\phi}] = S[\bar{\phi}] + \hbar\,(\text{1-loop corr.}) + \cdots$$



Slowly varying j:

$j(x) = f(t)\, j(\underline{x})$



Let

$$\hat{H}_j = \hat{H} + \int d^3\underline{x}\, j(\underline{x})\, \phi(\underline{x})$$

$$\hat{H}_j |n\rangle_j = E_n[j] |n\rangle_j$$

$$e^{iW[j]} = \langle 0, \text{out} | 0, \text{in} \rangle_j$$

$$\simeq \sum_n \langle 0 | \ldots | n \rangle_j\, e^{-iE_n[j]T}\, {}_j\langle n | \ldots | 0 \rangle$$

$$\Rightarrow \lim_{T\to\infty} \frac{W[j]}{T} = -E_0[j]$$

Assume $j(\underline{x}) = g(\underline{x})\, j$

$g(\underline{x}) \simeq 1$ in Ω

$\simeq 0$ outside Ω

If ground state (equilibrium state) is homogeneous, $\frac{E_0[j]}{\Omega} \xrightarrow[\Omega\to\infty]{} \varepsilon(j)$

$\Rightarrow$ As $\Omega T \to \infty$,

$$W[j] \simeq -\Omega T\, \varepsilon(j)$$

<u>Effective potential $V(\bar\phi)$</u>

Similarly $\Gamma[\bar\phi] = -\Omega T\, V(\bar\phi)$

for slowly varying $\bar\phi$

$$V(\bar\phi) = U(\bar\phi) + \hbar(\text{1-loop corrs}) + \cdots$$

diagrams with $j=0$ external ϕ lines

Note: $V(\bar\phi) = \varepsilon(j) + \bar\phi j$

$$\frac{\partial \varepsilon}{\partial j} = -\bar\phi \qquad \frac{\partial V}{\partial \bar\phi} = j$$

Cf. $F(V,T) = G(p,T) - Vp$

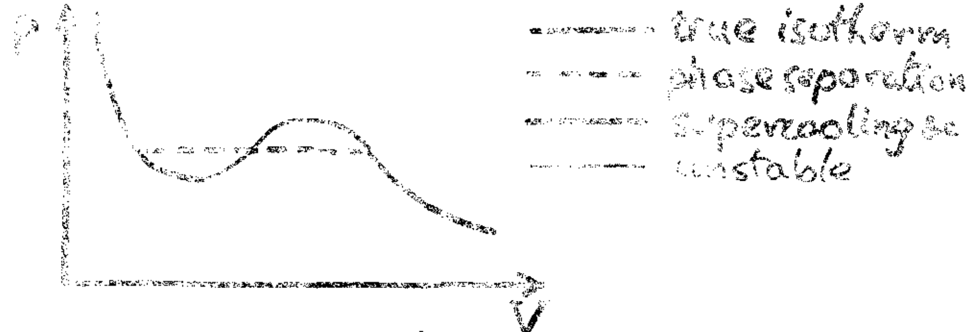
$$\frac{\partial G}{\partial p} = V \qquad \frac{\partial F}{\partial V} = -p$$

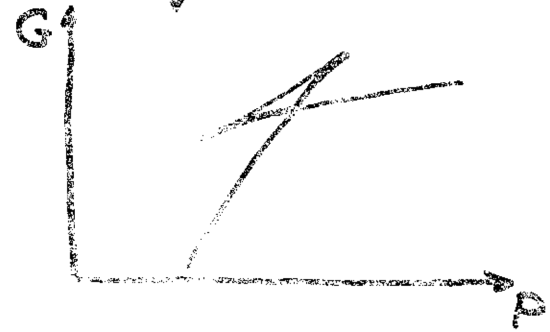
At finite T, $\varepsilon(j)$ <u>is</u> the free energy of equilibrium state with given j

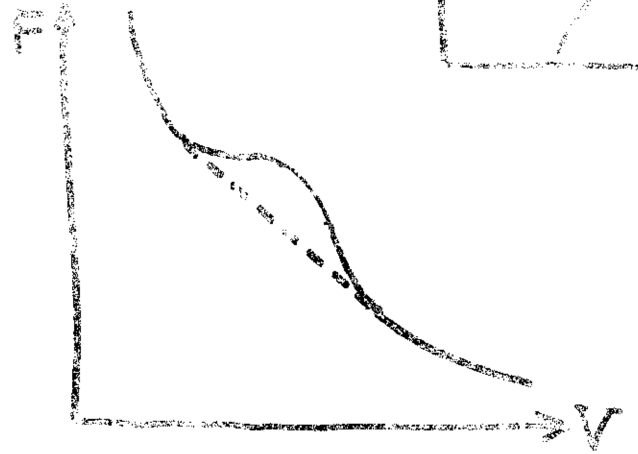
Compressibility $-\frac{1}{V}\frac{\partial V}{\partial p} = -\frac{1}{V}\frac{\partial^2 G}{\partial p^2} \geq 0$

$$-\frac{\partial p}{\partial V} = \frac{\partial^2 F}{\partial V^2} \geq 0 \qquad F \text{ is convex}$$

Maxwell construction







Lattice theory: can prove

$$-\frac{\partial^2 \varepsilon}{\partial j^2} = \langle \phi^2 \rangle - \langle \phi \rangle^2 \geq 0$$

$$\frac{\partial \bar{\phi}}{\partial j} = -\frac{\partial^2 \varepsilon}{\partial j^2} \qquad \frac{\partial j}{\partial \bar{\phi}} = \frac{\partial^2 V}{\partial \bar{\phi}^2}$$

$$\Rightarrow \quad \frac{\partial^2 V}{\partial \bar{\phi}^2} \geq 0$$

<u>V is convex</u>

318 (presumably true also in continuum theory)

Also, effective mass is

$$m^2(\bar{\phi}) = \frac{\partial^2 V}{\partial \bar{\phi}^2}$$

convexity $\Longleftrightarrow$ no tachyons

Finite-T Effective Potential

$$V(\bar{\phi}) = U(\bar{\phi}) + V^{(1)}_{T=0}(\bar{\phi}) + V^{(1)}_{Temp}(\bar{\phi}) + \cdots$$

$$V^{(1)}_{T=0} = \sum (\pm) \; \frac{m^4(\bar{\phi})}{64\pi^2} \ln \frac{m^2(\bar{\phi})}{\mu^2}$$

($\pm$: bosons / fermions) + polynomial

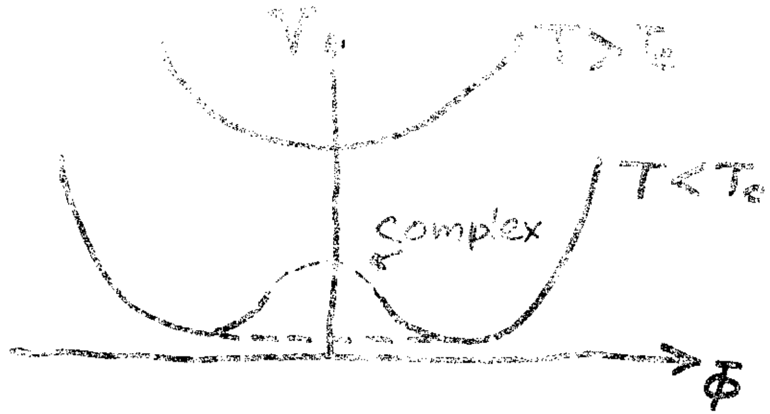
polynomial (degree 4) fixed by renormalization conditions on parameters in U

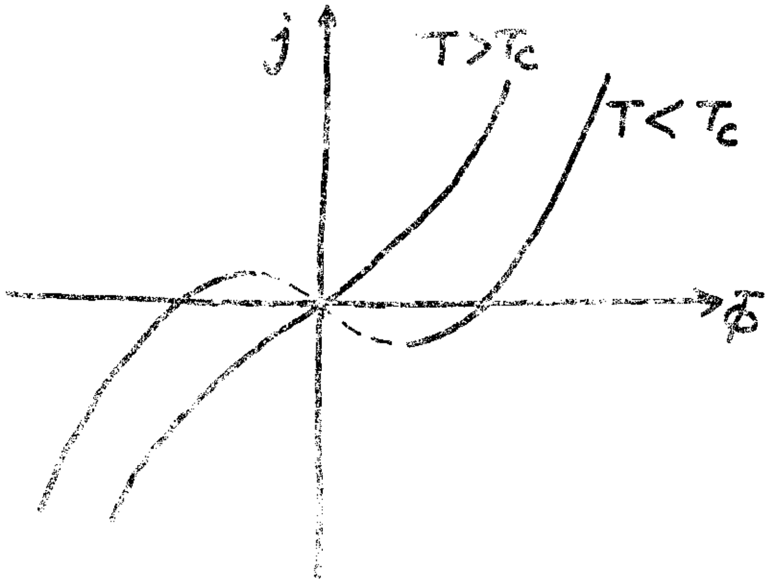
$$V^{(1)}_{Temp} = -\frac{\pi^2}{90} \mathcal{N} T^4 + \frac{1}{24} \mathcal{M}^2 T^2 + \mathcal{O}(T)$$

$\mathcal{N}$ = total no. of helicity states of light particles ($m_i \ll T$) [bosons + $\frac{7}{8}\times$ fermions]

$\mathcal{M}^2 = \sum_i m^2(\bar{\phi})$ [bosons + $\frac{1}{2}\times$fermions]

Note: coeff. of $\bar{\phi}^2 < 0$ at $T=0$, becomes >0 at $T > T_c$.

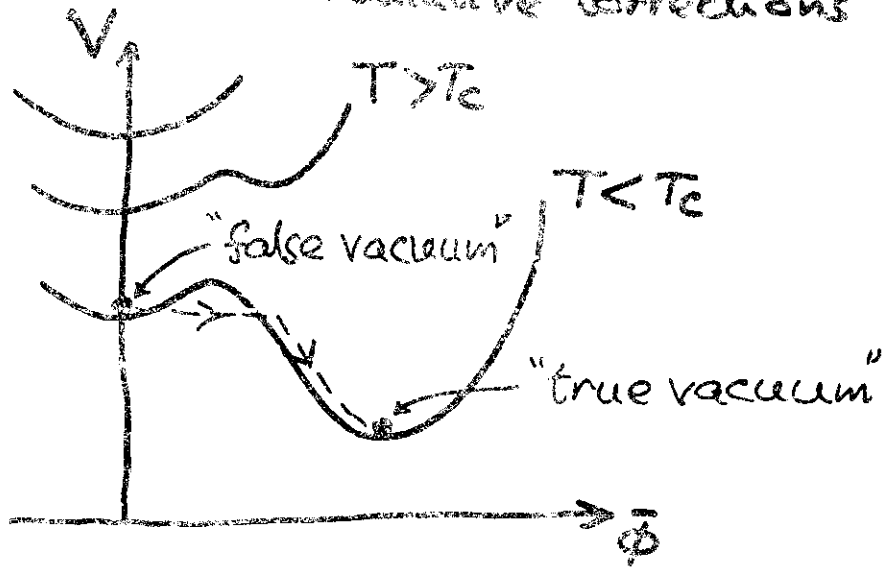




Note: if $\bar{\Phi}$ is more than 1-dimensional, different ground states are not separated by a barrier, so transition is 2nd order.

First Order Phase Transitions

Because of: cubic terms in V or radiative corrections



barrier ⇒ supercooling

⇒ transition by quantum tunnelling or thermal fluctuations, then "roll down"

But: is this use of $V(\bar{\Phi})$ justified?

15

Elitzur's Theorem

Elitzur '75
DeAngelis, de Falco + Guerra '78

Spontaneous breaking of local gauge symmetry is impossible.

$$\lim_{j\to 0} \lim_{\Omega\to\infty} \langle \phi \rangle_{\Omega,j} = 0$$

Proved in gauge-invariant lattice theory — no gauge fixing.

320

But:
what does $V(\bar\phi)$ mean?

Börner + Seiler:
$V(\bar\phi)$ is wrong thing to calculate,
$\bar\Phi$ is not a good order parameter

N.b.: similar problem in superconductivity

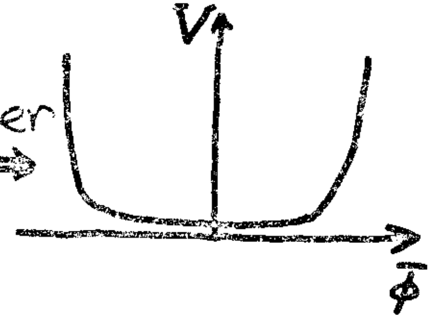
On lattice: phase transition does occur
[Osterwalder + Seiler '78 ⇒ mass gap in lattice Higgs model]
but doesn't show up in $V(\bar\phi)$

16

Failure of Loop Expansion

Tree level ($V = U$)
⇒ symmetry breaking for any space-time dimension d

Not true
$d=0$: Bender + Cooper
⇒



steepest descent,
2 competing saddle points

n.b.: soliton connecting 2 minima of U plays an important role

cf: lattice calculations
– condensation of monopoles

Marques + Ventura:
approach based on condensation of domain walls.

[illegible]

Hawking - Moss

At inflationary phase to [illegible]

geometry [illegible] de Sitter

⇒ Euclidean version S^4

(finite volume [illegible])

⇒ even with proper fixing

$$\lim \langle \phi \rangle = 0$$

use $\langle \phi^2 \rangle$,

2-point source function [illegible]

$S[\phi] \to S[\phi] + \frac{1}{2} \int [illegible]$

$\rho(x, y) = \langle \phi(x) \phi(y) \rangle = [illegible]$

$\Rightarrow \Gamma[\varphi] = W - \frac{1}{2} [illegible]$

⇒ calculate ρ in Hartree [illegible]

$\Rightarrow \langle \phi^2 \rangle \Rightarrow m^2$

Gauge symmetry [illegible] phase transition

Conclusion

Flat space:

Calculate $V(\phi)$ but beware

of nonperturbative effects

due to [illegible]

De Sitter [illegible]

$V(\phi)$ not [illegible]

Calculate $\Gamma[\varphi]$

[illegible] ce)

[illegible] mean?

[illegible] alternative

[illegible]

invariant [illegible]

1

Cosmological Problems

1. Flatness $\rho \sim \rho_{crit}$
2. Baryon number $\frac{n_B}{n_\gamma} \sim 10^{-10}$
3. Horizon problem
4. Smoothness – size of fluctuations
5. Monopole problem
6. Cosmological constant $\Lambda \sim 0$.

New Inflationary Universe

proposed solution to $1 \to 5$

322

2

Baryon Number

$$\left.\begin{array}{l} n_B \sim .1\ m^{-3} \\ n_\gamma \sim 400\ cm^{-3} \end{array}\right\} \frac{n_B}{n_\gamma} \sim 10^{-10}$$

Generation of B requires:

B violation

CP violation

Thermal non-equilibrium

e.g.

$$\frac{X \to \bar{q}\bar{q}}{X \to q\ell} \neq \frac{\bar{X} \to qq}{\bar{X} \to \bar{q}\bar{\ell}}$$

$\tau_X > \frac{R}{\dot{R}}$ when X go out of equilibrium: $\langle n\,\sigma_{prod}\,v\rangle \sim \frac{\dot{R}}{R}$ $(T \sim m_X)$

But: uncertain parameters

p decay?

323

Flatness Problem.

$$H^2 = \left(\frac{\dot R}{R}\right)^2 = \frac{8\pi}{3 m_{Pl}^2}\rho - \frac{K}{R^2}$$

assuming $\Lambda = 0$.

$$m_{Pl} = G^{-1/2} \sim 10^{19}\,\mathrm{GeV}$$

$$\rho \gtrless \rho_{crit} = \frac{3 m_{Pl}^2}{8\pi} H^2 \sim 10^{-29}\,\mathrm{g\,cm^{-3}}$$

$$\Longleftrightarrow \quad K \gtrless 0 \quad \Longleftrightarrow \quad \text{universe} \begin{Bmatrix}\text{closed} \\ \text{open}\end{Bmatrix}$$

Now $\frac{|\rho - \rho_{cr}|}{\rho} \lesssim 10$

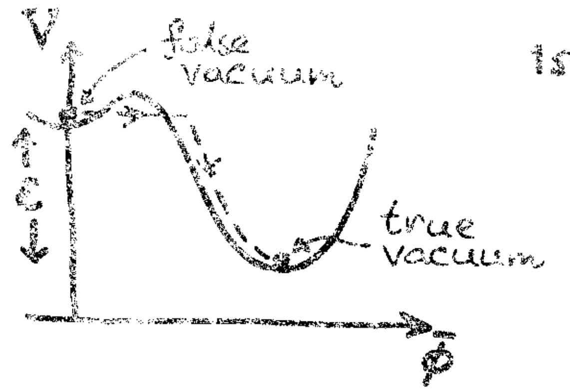
$\rho \propto R^{-4}$ or R^{-3}, $\quad |\rho - \rho_{cr}| \propto R^{-2}$

$\Rightarrow$ at T_{Pl}, $\quad \frac{|\rho - \rho_{cr}|}{\rho} \lesssim 10^{-58}$

Why so small?

Inflationary Universe

Guth



1st order phase transition

$\Rightarrow$ supercooling

$\Rightarrow$ energy density dominated by

$$\varepsilon = V_{\phi=0} - V_{min} \qquad V_{min} \simeq 0 \;(\Lambda = 0)$$

$$\Rightarrow \left(\frac{\dot R}{R}\right)^2 = \frac{8\pi}{3 m_{Pl}^2}\varepsilon = H^2 \;(\text{const.}) \qquad H \sim \frac{T_c^2}{m_{Pl}}$$

$$\Rightarrow R \propto e^{Ht} \quad (\text{inflation})$$

Note: $T_{\mu\nu} = \varepsilon g_{\mu\nu} \Rightarrow p = -\varepsilon$.

Cure of flatness problem:

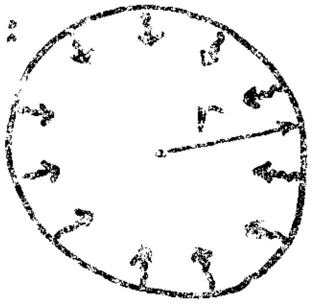
$\rho = \varepsilon$ (const), $\quad \rho - \rho_{cr} \propto R^{-2}$

$\frac{|\rho - \rho_{cr}|}{\rho} \propto e^{-2Ht} \qquad Ht \gtrsim 60$?65

[within

Horizon Problem

Last scattering surface:



at time of last scattering

$$\begin{cases} r \sim 10^7 \text{ l.yr} \\ t \sim 10^5 \text{ yr} \end{cases}$$

no prior contact between opposite points: why same T?

Cure: during inflation r increases by factor $\gtrsim e^{60}$

true horizon $\gg$ apparent horizon.

[inflation from $t = -\infty$ corresponds to de Sitter space; which has <u>no</u> particle horizon]

Smoothness Problem

Assume: initial state at t_{Pl} is a thermal equilibrium state at $T_{Pl} \sim 10^{19}$ GeV

(perhaps because of quantum gravity)

Then: on a galactic scale,

$$\frac{\delta\rho}{\rho} \sim \frac{1}{\sqrt{N}} \sim 10^{-40}$$

Grows like t or R^2 until it comes within horizon, i.e. by factor $\sim 10^{54}$

$$\Rightarrow \frac{\delta\rho}{\rho} \gg 1.$$

But: why should initial state be thermal equilibrium one?

Cure: Inflation:

entire visible universe now comes from very small initial region

But: Size of perturbations in inflationary universe?

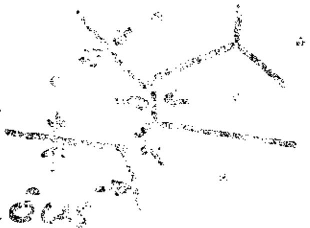
Tunnelling through barrier
⇒ bubble of new phase
(true vacuum)

Probability of bubble nucleation per unit space-time volume $p \sim e^{-S}$
(S = action for "bounce" solution)

If $p \gtrsim H^4$ $\left(H \sim \frac{T_c^2}{m_{Pl}} \sim 10^{10}\,\text{GeV}\right)$

many bubbles form

energy
⇒ KE of bubble walls



⇒ very inhomogeneous universe

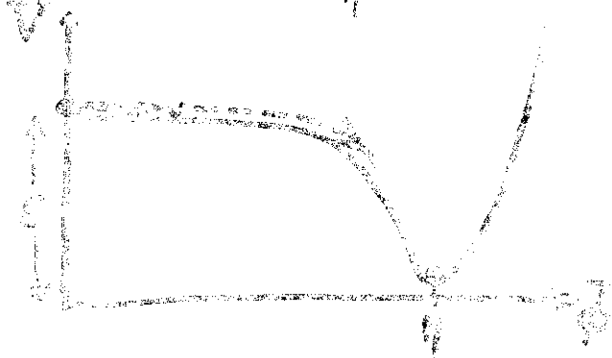
If $p \ll H^4$

bubbles never coalesce
large regions either [illegible] old phase
[illegible] (closed) [illegible] universe

Linde
Albrecht + Steinhardt
Hawking

Coleman-Weinberg mechanism

$$V = \lambda\left(\phi^4 \ln\frac{\phi^2}{\eta^2} - \frac{1}{2}(\phi^4 - \eta^4)\right)$$



very flat near $\phi = 0$

slow "roll-down", time scale
$\sim (V'')^{-1/2} \gg H^{-1}$

$$H^2 \sim \frac{\varepsilon}{m_{Pl}^2}$$

equivalently $\frac{V''}{V} \ll \frac{1}{m_{Pl}^2}$

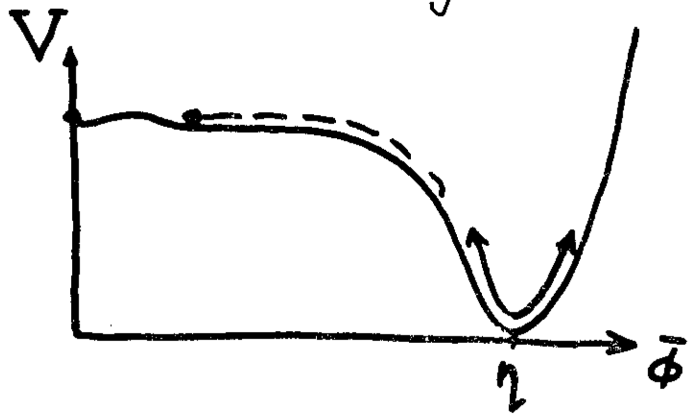
⇒ inflation continues after the transition

But: V'' not too small - still require $p \ll H^4$

Inflation after transition

⇒ 1. single bubble can expand to be ≫ present visible universe

2. energy not deposited immediately in bubble walls



Energy ⇒ oscillations in $\bar{\phi}$
equiv. to coherent state of Higgs bosons
⇒ decay → thermalises energy

Note: $V''(\eta)$ must be large, $\gg H^2$ to reheat universe

Monopole Problem

cured by NIU

No. of monopoles ~ 1 per bubble
→ diluted by inflation

monopoles could be made at later transition

Too many?
- perhaps not in 2-stage transition (Moss)

Strings –

also could be made at a later transition

Baryon + Lepton Number

New Inflationary Universe

$\Rightarrow \quad B = 0$

$\quad\quad L = 0$

Nonzero B must be created after inflation

– reheat universe to $T \sim m_X$

$\Rightarrow$ X's (or Higgs)

$\Rightarrow$ decays $\rightarrow$ B asymmetry

Contributions also from decay of ϕ oscillations

strings } produced at later
monopoles } transitions

Prediction: $L \approx 0$

$n_L \quad n_B \quad (\ldots)$

Fine Tuning

1. $\Lambda = 0 \Rightarrow V(\phi_{min}) = 0$

Choice of constant in V

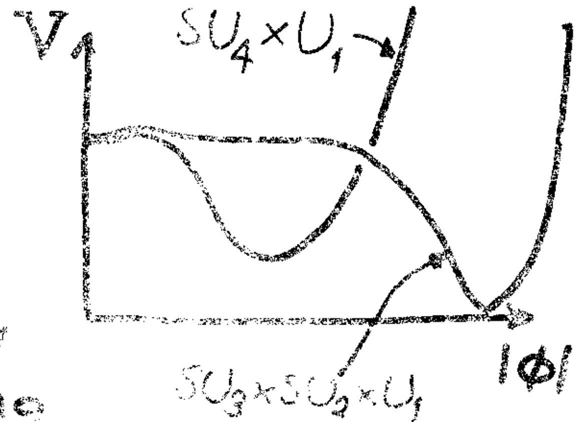
2. $V''(0) \ll H^2$

$V''(0) = m_0^2 + \xi R + cg^2T^2 + 3h^2\langle\phi^2\rangle$

$\uparrow$ $\xi \approx 1/6$

Problem with SU(5):

Breit, Gupta + Zaks



Inflationary path is along a ridge

Universe $\rightarrow SU_4 \times U_1$ phase

$\Rightarrow$ 1st order transition to $SU_3 \times SU_2 \times U_1$

$\rightarrow$ very inhomogeneous universe

Supersymmetric Inflation

Very flat potential natural in SUSY model:

$$V^{(1)}_{T=0} = \frac{1}{64\pi^2} \sum (\pm) m^4 \left(\ln \frac{m^2}{\mu^2} - \frac{1}{2} \right)$$

$\uparrow$ (bosons / fermions) + poly. terms

Exact SUSY $\Rightarrow$ coeff. = 0

Broken SUSY $\Rightarrow$ reduced by factor m_S^2/m_X^2

Superpotential $W(\phi_i)$

of superfields ϕ_i

$\Rightarrow$ effective potential $V(\phi_i)$ of corresponding scalar fields ϕ_i:

$$V(\phi_i) = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2$$

Also

$$V_{Temp}(\phi_i) = \frac{T^2}{8} \sum_{i,j} \left| \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \right|^2$$

O'Raifeartaigh SUSY breaking

Steinhardt, Albrecht et al

$$W = \lambda_1 X A^2 + \lambda_2 Y (A^2 - M^2) \qquad (M = \text{const})$$

$$\Rightarrow V = |\lambda_1 A^2|^2 + |\lambda_2 (A^2 - M^2)|^2 + |2\lambda_1 A X + 2\lambda_2 A Y|^2$$

Min. at $A = \frac{\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} M, \quad \lambda_1 X + \lambda_2 Y = 0$

X, Y undetermined at tree level

Degeneracy broken by

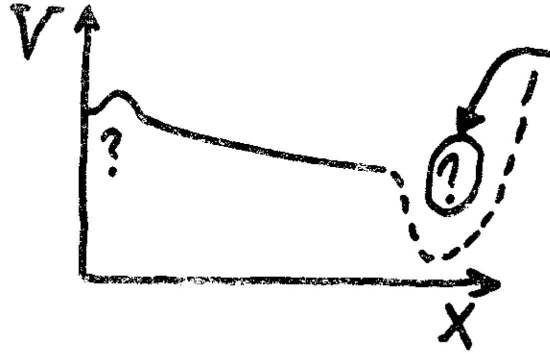
$$V^{(1)}_{T=0} \sim \alpha M^4 \ln \frac{X^2}{\mu^2}, \quad \text{large } X$$

Gauge theory: $\alpha < 0$

$\Rightarrow$ minimum at large X

reverse hierarchy — Witten

or geometric hierarchy — Dimopoulos + Raby



Problem: how to get large V'' at min.

Fill Monopoles Other Remedies

Fine tuning constraints easier to satisfy in SUSY model if T_c is close to m_{pl}

Inflationary transition at $T_c \sim m_{pl}$

[illegible]

[illegible] fine tuning [illegible] (perhaps) fluctuations

But can't solve monopole problem

near m_{pl}, quantum gravity important $\Rightarrow$ non-renormalizability $\Rightarrow$ non-calculable effects

Inflationary Universe

Starobinsky
Guth+Pi
Hawking

$$\ddot{\phi} + 3\frac{\dot{R}}{R}\dot{\phi} = -\frac{\partial V}{\partial \phi} + \frac{1}{R^2}\nabla^2\phi$$

$$ds^2 = dt^2 - R^2(t)\,dx^2 \qquad R(t) \sim e^{Ht}$$

treat $\frac{\partial V}{\partial \phi}$ negligible

$$[\Delta\phi(x,t)]^2 = \frac{H^2}{(2\pi)^3}\int [illegible]$$

$$\simeq \frac{1}{16\pi^2}\left\{ H^2 [illegible] \right\}$$

Inflation ends when $\phi \to \eta$

$\Rightarrow Ht \sim \frac{\#}{g^2} \sim 40$ [Depends on size of $V''(0)$]

(should be $\gtrsim 60$: requires fine tuning)

large t: $\Delta\phi(x,t) \simeq -\dot{\bar{\phi}}(t)\,\delta\tau(x)$

ie $\phi(x,t) \simeq \bar{\phi}(t - \delta\tau(x))$

$$\Rightarrow \left(\frac{\Delta\rho}{\rho}\right)_{hor} \sim \left(\ln\frac{H}{|k|}\right)^{3/2}$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{hor} \sim \left(\ln \frac{H}{|\underline{k}|}\right)^{3/2}$$

nearly scale-invariant

as required by Zel'dovich theory of galaxy formation.

But, for galactic scale $\sim 10^6$ l.yr

$$\frac{\Delta\rho}{\rho} \sim 50 \quad \text{(should be } 10^{-4}\text{)} \; !$$

SUSY model:

may be better because of fermi-bose cancellations in V

Primordial inflation:
tree-level calculation suggests reasanable value for $\frac{\Delta\rho}{\rho}$

But: quantum gravity
$\Rightarrow$ loop corrections uncertain

Conclusions

New Inflationary Universe

solves
- flatness problem
- horizon problem
- monopole problem*
- smoothness problem (?)

predicts
- $\rho \simeq \rho_{cr}$
- $n_L \sim n_B \ll n_\gamma$
- $\sim$ scale-independent $\frac{\Delta\rho}{\rho}$

But: requires fine tuning
($\lambda \to 0$, coeff. of φ^2)
fluctuations too big

Possible cure: **SUSY**

But: reheating difficult or

*monopole problem unsolved
+ theory non-renormalizable

IC/83/131

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

STATUS OF THE RUBAKOV-CALLAN EFFECT *

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Trieste, Italy.

ABSTRACT

In this review we try to bring together the various approaches that are being developed to analyze what happens when fermions scatter off monopoles in grand unified theories. The material is divided according to the following table of contents.

MIRAMARE - TRIESTE
August 1983

* Extended write-up of talks given at the EPS Conference on High Energy Physics, Brighton, July 1983, the "Summer Workshop in Particle Physics", [illegible], July 1983 and [illegible], August 1983

TABLE OF CONTENTS

"WHAT IS THE RUBAKOV CALLAN EFFECT"?

In 1981 Rubakov [1] pointed out a remarkable effect that would take place if an SU(5) 't hooft-Polyakov monopole passed through nuclear matter, namely it would induce protons to decay at rates comparable to strong interactions. This effect was independently discovered a little later by Callan [2], who has since tried to develop a complete description of the above catalysis reaction in the framework of quantum field theory. We shall briefly recall the essential details in Rubakov's and Callan's work and then go on to discuss what has been done since by various people. It is crucial to fully understand this effect and to be able to calculate cross-sections and branching ratios because of the following observable consequences: i) a monopole passing through a giant proton decay detector would lead to a chain of proton decays, one every 10 to 100 cm, depending on the density of the detector and the magnitude of the cross-section $\sigma_{Rubakov}$. ii) Astrophysical bodies like neutron stars become monopole detectors [3] and tight limits on the product of the monopole flux and $\sigma_{Rubakov}$ are said to come from a study of X-ray emission from such stars. In order to understand the origin of the effect it is perhaps useful to recall the problem pointed out by Kazama, Yang and Goldhaber [4], when one considers a charged fermion scattering of a point Dirac monopole, namely the Hamiltonian of the system is not self-adjoint and physics is not defined at $r = 0$. In a simple way one can pinpoint the origin of the problem by noting that the total angular momentum is given by

$$\vec{J} = \vec{L} + \vec{s} + eg\,\vec{r}/r \quad , \tag{1.1}$$

where L is the orbital angular momentum, $\vec{s}$ is the spin of the fermion and the last piece comes from the charge field interaction. Like the fermion spin it also gives rise to a half unit of angular momentum by virtue of the Dirac condition $eg = 1/2$. For s-wave fermions the spin orientation can be chosen to exactly cancel the latter piece, giving rise to an angular momentum zero state. However as the fermion passes the core and $\vec{r} \to -\vec{r}$, the angular momentum will not be conserved unless $\vec{s} \to -\vec{s}$ (helicity flip) or $e \to -e$ (charge exchange), which would require some special boundary condition at $r = 0$. On the other hand, a 't Hooft-Polyakov monopole is non-singular at $r = 0$, so a careful study of fermions in this system should tell us exactly what happens to s-wave fermions as they scatter off the monopole core.

In brief what comes out of such a study is the following. If an s-wave fermion reaches the core of a GUT monopole, it will pop out again with a change in identity, e.g. $d \to e$, $u_1 \to \bar{u}_2$, etc. The monopole is a state of indefinite fermion number and it distorts the fermionic vacuum around it due to such processes occurring virtually in the surrounding vacuum, i.e. leptons and quarks pair up locally outside the core and such correlations fall off slowly like a power $(r^{-1})^d$ as $r \to \infty$. One consequence of this phenomenon in the case of monopoles in theories like SU(5) is that baryon-number violating processes can occur outside the core of the monopole and manifestations of this phenomenon include (at the quark level)

$$U + M \to M + e^+ + \bar{u} + \bar{d}$$

or at the hadronic level

$$p + M \to M + e^+ + \pi^0 + \pi^0$$

i.e. a proton decay can be induced by a monopole passing through it. The cross-section for this process is believed to be compatible to that which saturates the s-wave unitarity bound, i.e. $\sigma \sim \pi/E^2$.

333

II. BRIEF RESUME OF THE SU(2) MONOPOLE FERMION SYSTEM

Before entering into details of the analysis started by Rubakov and Callan, let us briefly review some of the basic kinematics and notions relevant to the problem.

The basic system one studies is the familiar SU(2) Georgi-Glashow chiral model

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} (D_\mu \phi^a)^+ (D^\mu \phi^a) + V(\phi^+\phi)$$
$$+ \sum_i \bar{\psi}^{(i)}_L \gamma . D \psi^{(i)}_L \quad + \text{possible Yukawa couplings} \qquad (2.1)$$

(ψ_L is a 2-component left-handed Weyl spinor).

Vacuum structure (in the spherically symmetric non-singular gauge) - The Higgs field around the monopole is supposed to take on the configuration

$$\phi^a(x) \longrightarrow \frac{x^a}{r} \langle \phi \rangle , \quad r \to \infty \qquad (2.2)$$

and the gauge symmetry breaks according to $SU(2) \to U(1)_{em}$, where the U(1) is the set of rotations which leaves ϕ^a invariant. The 't Hooft-Polyakov monopole solution corresponding to (2.2) is given by

$$A^a_0 = 0 \quad , \quad A^a_i = \frac{1}{e} \epsilon_{aij} \frac{x^j}{r^2} \quad , \quad r \to \infty$$

Electromagnetism and the monopole charge are, respectively, given by

$$F^{em}_{\mu\nu} = \frac{\phi^a}{|\phi|} F^a_{\mu\nu} = \frac{1}{e r^2} \epsilon_{\mu\nu a} \hat{n}_a \qquad (2.3)$$

$$B^{em}_i = \epsilon_{ijk} F^{em}_{jk} = \hat{n}_i / e r^2 \qquad (2.4)$$

and

$$g = \frac{1}{4\pi} \int d^3x \, \vec{\nabla} . \vec{B}^{em} = \frac{1}{4\pi} \int_{R \to \infty} dS \, \hat{n} . \vec{B}^{em}$$
$$= \frac{1}{e} \qquad \text{(i.e. the Dirac condition)} \qquad (2.5)$$

The Dirac equation (non-singular equation)

$$[\gamma . D + m_f]_{ij} \psi_{L_j}(x) = 0 \qquad (2.6)$$

where $[D_\mu]_{ij} = \delta_{ij} \partial_\mu - i \epsilon_{a\mu k} \frac{x^k}{r^2} T^a_{ij}$ and we can write $\psi_{L,i} = \begin{bmatrix} a^+ \\ a^- \end{bmatrix}_L$, where $a^\pm$ refer, respectively, to the $Q = \pm 1$ eigenstate with respect to the U(1) charge operator outside the monopole core. [Note $Q = 1/2\, e$ and $Qg = 1/2$.]

Properties of this system

1. The total angular momentum is given by $\vec{J} = \vec{L} + \frac{1}{2} \vec{\sigma} + \frac{1}{2} \vec{\tau}$ where [illegible] = 0, i.e. monopole turns isospin into spin and it enters on an identical footing. For s-waves (in the $m_f = 0$ limit) and zero total angular momentum scattering states ($J = L = 0$) we have the special decomposition due to Jackiw [illegible]

$$\psi^{J=0}_{L\,\alpha,i} = \frac{1}{\sqrt{8\pi}\,r}\left(\epsilon_{\alpha i}\,h(r) + (\hat{n}^a\tau^a)_{\alpha\beta}\,\epsilon_{\beta i}\,g(r)\right) \qquad (2.7)$$

We can define the two-component spinor $f_\kappa = \begin{bmatrix} h \\ g \end{bmatrix}$ and obtain the two-dimensional radial Dirac equation $\hat{\not{D}}\, f = 0$ with boundary condition $g(r) = 0$ at $r = 0$

$$\hat{\gamma}_\mu = (\sigma_3, \sigma_1) \quad \text{and} \quad \hat{D}_\mu = \partial_\mu + a_\mu \,, \qquad (2.8)$$

where $a_\mu = (a_0, a_r)$ refer to the U(1) quantum fluctuations of the monopole field (the static field as $r \to \infty$ does not enter since its contribution cancels the angular momentum term).

2. <u>Exact solution in static monopole field</u> (see Marciano and Muzinich for the scattering solution for $m_f \neq 0$ in the full SU(5) context [6]), proceeds by pure charge exchange, namely

$$\psi^+_L\Big|^{in} \longrightarrow \psi^-_L\Big|^{out} \qquad \text{i.e. } a^+ \to a^-$$

and not by helicity flip

$$\text{i.e. } \psi^+_L\Big|^{in} \not\to \psi^+_R\Big|^{out} \,,$$

where

$$\psi^+_L = \begin{pmatrix} a^+ \\ 0 \end{pmatrix}, \quad \psi^-_L = \begin{pmatrix} 0 \\ a^- \end{pmatrix} .$$

This conclusion holds for $m_f \neq 0$, provided $m_f/E \ll 1$.

<u>Question of zero modes</u>

Although for $m_f = 0$ the above Dirac equation has non-normalizable zero modes, it has been recently pointed out by Walsh, Weiz and Wu [7] that if we add a mass term in SU(5) model $m_f = \phi_5(r)$, where ϕ_5 is the Higgs field which breaks $SU(2) \times U(1) \to U(1)_{em}$, then there are no corresponding normalizable zero modes.

However the existence of normalizable zero modes in a static monopole field has no direct bearing on the Rubakov-Callan effect.

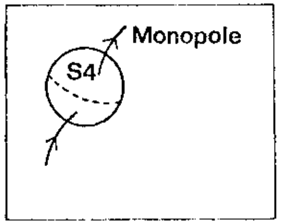
<u>Reason</u> - A monopole is not exactly like an instanton for which

$$\text{index } \not{D} = N \text{ (the winding number of the field)} \qquad (2.9)$$

and

$$\langle N+1\,|\,\psi_1(x)\,\psi_2(x)\,|\,N\rangle \neq 0 \qquad (2.10)$$

(i.e. there exists a non-vanishing probability for a fermionic pair $\psi_1\psi_2$ to appear at some point x_μ if the winding number of the field changes from N to N+1 as we go from $t = -\infty$ to $t = +\infty$) . One cannot use the Gauss theorem to turn the local statement of the anomaly $\partial^\mu J^5_\mu = cF\tilde{F}$ into the index theorem because the monopole sweeps out a world line, which punctures any 4 volume. Furthermore, the one-particle description is inadequate because it entails the excitation of the dyon degree of freedom which is forbidden energetically. One therefore needs a full QFT treatment of the problem (Fig.1).



<u>Fig.1</u>

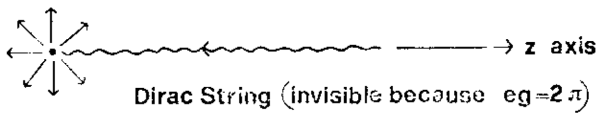
<u>Monopole-fermion system in the singular unitarity gauge</u> - For completeness let us briefly describe the monopole-fermion system in the unitarity gauge in which the U(1) electromagnetism points along the third component of the SU(2) generators, i.e.

$$F^{em}_{\mu\nu} = \partial_\mu A^3_\nu - \partial_\nu A^3_\mu \qquad (2.11)$$

$$A^3_i = \frac{1-\cos\theta}{r\cos\theta}\,\hat{r}_i \;; \quad A^\pm_i = 0 \quad ; \; r \to \infty \qquad (2.12)$$

$$B^{mon}_i = \frac{\hat{r}_i}{e r^2} - \frac{2\pi}{e}\,\delta_{i3}\,\delta(x)\,\delta(y) \qquad (2.13)$$

where the last term in Eq.(2.13) represents the Dirac string which has been chosen to point along the positive z axis (Fig.2).



<u>Fig.2</u>

The Hamiltonian (Kazama, Yang and Goldhaber [4])

$$H = \underbrace{\vec{\alpha}\cdot\vec{\nabla} + \beta m_f}_{H_0} \underbrace{- \frac{\rho}{r}}_{\text{dyon}} \underbrace{- \kappa e \frac{\vec{\sigma}\cdot\vec{r}}{2 m_f r^3}}_{\text{fermion anomalous magnetic moment}} \qquad (2.14)$$

The problem pointed out in Ref.4 in H_0 is not self-adjoint and the physics at $r = 0$ is not defined. This can be remedied using ρ and κ as regulators. For example $\kappa \neq 0$ makes H self-adjoint and the scattering problem well defined. The conserved total angular momentum is $J = L + \frac{1}{2}\sigma$, i.e. $[H, L + \frac{1}{2}\sigma] = 0$, which implies a different partial wave expansion than that used for the spherically symmetric non-singular gauge, for which $[H, L + \frac{1}{2}\sigma + \frac{1}{2}\tau] = 0$. One uses

$$\psi_\alpha = \frac{1}{\sqrt{4\pi} r} \begin{bmatrix} f(r) & \eta \\ g(r) & \eta \end{bmatrix} \qquad \text{4-component spinor}$$

$$\eta_\alpha = \begin{bmatrix} Y_{1/2,\, m - \frac{1}{2}}(\theta,\varphi) \\ Y_{1/2,\, m + \frac{1}{2}}(\theta,\varphi) \end{bmatrix} , \qquad (2.15)$$

where $Y_{m,m'}(\theta,\varphi)$ refer to monopole harmonics introduced by Yang and collaborators. The boundary condition at $r = 0$ in the limit $\rho, \kappa \to 0$ depends on how the limits $\rho, \kappa \to 0$ (for a detailed discussion see Wu [8]). In particular, one can define a class of different solutions characterized by an angle ω, where boundary condition at $r = 0$ is characterized by

$$\lim_{r \to 0} \arg \begin{pmatrix} g(r) \\ f(r) \end{pmatrix} = \omega . \qquad (2.16)$$

Different ω corresponds to a different fermion spectrum. This monopole phase angle presumably has an interesting physical interpretation.

Indeed Callan and Das [9] have recently pointed out that this angle is closely related to the θ angle of instanton physics. Furthermore, when one has more than one fermion and a more complex monopole group, then the boundary condition at $r = 0$ can be represented by a unitary transformation $\psi^{in}(0) = U \psi^{out}(0)$. In the case of a single Dirac fermion, the boundary condition in s-wave fermions is given by $\psi_L(0) = e^{i\omega} \psi_R(0)$.

Full SU(5) context – The SU(5) gauge theory breaks down to $SU(3)_C \otimes SU(2)_L \otimes U(1)$ at a mass scale $M_X \sim 10^{15}$ GeV, by a superheavy Higgs field in the adjoint representation 24. The vacuum expectation of this Higgs field ϕ_{ab} represented as a 5 x 5 traceless matrix is given by

$$\phi_{ab} = \langle\phi\rangle \begin{bmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{bmatrix} \begin{matrix} d_1 \\ d_2 \\ d_3 \\ e \\ \nu \end{matrix} .$$

There is then a subsequent breakdown of $SU(2)_L \otimes U(1)$ to $U(1)_{em}$ at the 100 GeV scale. The monopole is quantized with regard to an U(1) subgroup made up of diagonal generators of $SU(3) \otimes U(1)_{em}$, which can be traced to the spontaneous breakdown of a SU(2) lepto-quark subgroup of SU(5), which breaks to U(1), i.e,

$$SU(2) \xrightarrow{M_X} U_Q(1) ,$$

$$Q = Q_{em} + I_{3c} + \frac{1}{\sqrt{3}} Y_c .$$

The Dirac condition reads $g\,Q = 1/2$. Let us choose the monopole to sit in $(\bar{d}_3, e^-)$ space, so that

$$Q = \begin{bmatrix} 0 & & & & \\ & 0 & & & \\ & & 1 & & \\ & & & -1 & \\ & & & & 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1/3 & & & & \\ & 1/3 & & & \\ & & 1/3 & & \\ & & & -1 & \\ & & & & 0 \end{bmatrix}}_{Q_{em}} + \underbrace{\begin{bmatrix} -1/3 & & & & \\ & -1/3 & & & \\ & & 2/3 & & \\ & & & 0 & \\ & & & & 0 \end{bmatrix}}_{\frac{1}{3} Y_c}$$

Then with regard to the monopole SU(2) subgroup of SU(5) we have the following left-handed fermion doublets.

$$\begin{matrix} Q = +1 \\ Q = -1 \end{matrix} \begin{bmatrix} \bar{d}_3 \\ e^- \end{bmatrix}_L ; \begin{bmatrix} e^+ \\ d_3 \end{bmatrix}_L ; \begin{bmatrix} u_1 \\ \bar{u}_2 \end{bmatrix}_L ; \begin{bmatrix} u_2 \\ -\bar{u}_1 \end{bmatrix}_L .$$

In the scattering problem we have the following transitions at $r = 0$:

$$\bar{d}_{3L} \to e^-_L$$
$$e^+_L \to d_{3L}$$
$$u_{1L} \to \bar{u}_{2L}$$
$$u_{2L} \to -\bar{u}_{1L} ,$$

i.e. the top line represents an incoming s-wave, while the bottom line represents an outgoing s-wave. The situation is reversed for $L \to R$, i.e. $e^-_R \to d_{3R}$ etc.

III. THE RUBAKOV VACUUM PAIRING ARGUMENT

There are two aspects of the argument:

i) the existence of anomaly driven by the monopole field;

ii) the existence of baryon number, etc. violating boundary conditions at the core for s-wave fermions.

Note both points are essential to understand Rubakov's analysis.

Significance of the anomaly - Consider chiral charge $Q_5 = \int \psi^+\gamma_0\gamma_5\psi d^3\vec{r} = n_L - n_R$ in SU(2)model. However the monopole gives rise to the Adler, Bell and Jackiw anomaly (here we absorb the coupling constant into the definition of the vector potential)

$$\frac{dQ_5}{dt} = \frac{1}{16\pi^2}\int \vec{E}\cdot\vec{B}\, d^3\vec{r} + 2m\int \psi^+\gamma_0\gamma_5\psi\, d^3\vec{r} \tag{3.1}$$

where the second term is a mass effect. By virtue of the spherical symmetry of the monopole field the anomaly only couples s-wave fermions. Further, using Eq.(2.4) ($g = 4\pi$)

$$\frac{1}{16\pi^2}\int \vec{E}\cdot\vec{B}\, d^3\vec{r} = \int_0^R dr\, E_r \tag{3.2}$$

However by the Gauss equation $\frac{1}{e^2}\partial E/\partial r = Q\psi^+\psi = Q\,[\rho_+(r) + \rho_-(r)]/r^2$ for s-wave fermions, where $\rho_\pm(r)$ is the radial charge density for helicity ($\pm$) fermions, respectively. Hence the first term is of order $\alpha\int dr\,(\rho_+ + \rho_-)/r$, while the mass term is of order $2m\int dr\,(\rho_+ - \rho_-)$. This means if we examine what happens to fermions outside the monopole core but inside a radius $R \ll \alpha m^{-1}$, then the anomaly tells us that $n_L - n_R$ must change. What does it mean that $n_L - n_R$ has to change, since the U(1) interactions outside the core are vectorial? The answer appears to be that: a) there must be some adjustment in the fermion vacuum around the monopole, b) this must persist as long as $R^{-1} \gtrsim m/\alpha$. However the precise nature of the fermion pairing structure depends on the boundary conditions at $r = r_0$, the number of SU(2) fermion doublets and the way fermions propagate in the monopole quantum field for $r > r_0$.

Calculation of the vacuum pairing - One starts by considering what is involved in calculating the correlation functions:

$$\langle \psi\bar\psi \cdots \rangle = \int \psi\bar\psi\cdots e^{-\int\mathcal{L}}\,[dA][d\psi][d\bar\psi] \tag{3.3}$$

where

$$\mathcal{L} = -\tfrac{1}{4}F^2 + i\bar\psi \not{D}\psi - m\bar\psi\psi$$

If we restrict ourselves to J = 0 fields and m = 0, then using Eq.(2.7)

$$\psi_L = \frac{1}{\sqrt{8\pi}\, r}\left(g + \frac{\vec{r}\cdot\vec{\sigma}}{r}h\right)\varepsilon \tag{3.4}$$

and $4\pi r^2\bar\psi\not{D}\psi = \bar{f}\,\hat\gamma\cdot D\, f$, where $f = \begin{pmatrix} g \\ h \end{pmatrix}$ and $\hat{D} = (D_0, D_r)$. Outside the core $\hat{D}_\mu$ no longer contains the static monopole field. If we note $\hat{D}$ only depends on the time and radial components of the electromagnetic field and the fact that the transverse components can be trivially integrated out, then one sees that all the dynamics of s-wave fermions is contained in a two-dimensional Abelian theory on the half line $0 \leq r \leq \infty$, with the boundary condition $h(r) = 0$ at $r = 0$, because of the singular term $\vec{r}\cdot\vec{\sigma}/r$ in Eq.(3.4). We can write

$$\langle \psi(r)\,\psi'(r')\cdots\rangle = \frac{1}{\sqrt{8\pi}}\frac{1}{r}\,\frac{1}{\sqrt{8\pi}}\frac{1}{r'}\cdots\langle f(r)\, f'(r')\cdots\rangle \tag{3.5}$$

where

$$\langle f f'\cdots\rangle = \int f f'\cdots e^{-\int\mathcal{L}}\,[da_\mu][df][d\bar f]$$

with

$$\int\mathcal{L} = \int_{-\infty}^{\infty} dt\int_0^{\infty} dr\left[\frac{4\pi r^2}{e^2} f_{\mu\nu}f^{\mu\nu} + \bar{f}\,\hat\gamma\cdot D\, f\right] \tag{3.6}$$

and $\hat\gamma\cdot\hat{D} = \sigma_3\hat{D}_0 + \sigma_1\hat{D}_r$; $\hat\gamma_5 = i\sigma_2 = \varepsilon$, $\hat{D}_\mu = \partial_\mu + a_\mu$.

Following Schwinger, this field theory can be explicitly solved. One starts with a chiral rotation

$$ f = e^{i\hat{\gamma}_5 \alpha(r) + \beta(r)} f^{\circ}(r) $$

(3.7)

which is chosen so as to cancel the U(1) potential in $\hat{D}_\mu$, i.e.,

$$ \bar{f} \hat{\not{D}} f \rightarrow \bar{f}^{\circ} [\hat{\not{D}} + \sigma_\mu (\partial^\mu \beta + \varepsilon_{\mu\nu} \partial_\nu \alpha)] f^{\circ} \qquad (3.8) $$

where we choose α so that

$$ a_\mu + \partial_\mu \beta + \varepsilon_{\mu\nu} \partial_\nu \alpha = 0 \qquad (3.9) $$

Note that β corresponds to a pure gauge rotation. From Eq.(3.9) one notes that the field strength is given by

$$ E^{U(1)} = \Box \alpha \qquad (3.10) $$

337

However on changing the variables according to Eq.(3.7) we must include the Jacobian

$$ [da_\mu][df][d\bar{f}] = J \, [d\alpha][df^{\circ}][d\bar{f}^{\circ}] $$

Using Fujikawa's method[10], this leads to the two-dimensional anomaly factor

$$ \text{Log}\, J = N_D \int \alpha(x) E(x) d^2x \qquad \text{(for } N_D \text{ doublets)} \qquad (3.11) $$

which is nothing but the full four-dimensional anomaly [illegible], since by virtue of its spherical symmetry one can [illegible]

$$ \text{Log}\, J = N_D \int \alpha(x) \vec{E} \cdot \vec{B} \, 4\pi r^2 dr\, dt \qquad (3.12) $$

i.e. the monopole transforms the four-dimensional [illegible] two-dimensional anomaly for the radial field theory. Thus we [illegible] transformation of the theory:

$$ \int \mathcal{L} = \int dt\, dr \left[\frac{4\pi r^2}{e^2} (\Box \alpha)^2 + N_D ([illegible]) \right] $$

(3.13)

[illegible] fermion Green's function can be estimated using

$$ \langle f(1) f(2) \cdots \rangle_M = \int [d\alpha] \, e^{S[\alpha]} \prod_i e^{\sigma_2 \alpha(i)} \times \langle f^{\circ}(1) f^{\circ}(2) \cdots \rangle_{free} \qquad (3.14) $$

Now one notes that (for $N_D = 2$)

$$ \langle e^{2\sigma_2 \alpha(x)} e^{2\sigma_2 \alpha(y)} \rangle_\alpha = e^{-4K(x,y) + 2K(x,x) + 2K(y,y)} $$

Where $x = (t,r)$, $y = (t',r')$ and

$$ \left[\frac{8\pi^2}{e^2} \Box r^2 \Box - \frac{2}{\pi} \Box \right] K(x,y) = \delta^{(2)}(x-y) $$

(3.15)

subjected to boundary condition at r = 0. As noted by Rubakov[1] and Callan[2], this system can be explicitly solved and the asymptotic form of K(x,y) is known. For N_D doublets

$$ \langle e^{N_D \sigma_2 \alpha(x)} e^{N_D \sigma_2 \alpha(y)} \rangle_{|t-t'| \to \infty} \longrightarrow e^{N_D \text{Log} \frac{|t-t'|^2}{rr'}} \qquad (3.16) $$

Now consider the model with the two doublets

$$ \psi_L^{(1)} = \begin{bmatrix} a \\ \bar{a} \end{bmatrix}_L \quad ; \quad \psi_L^{(2)} = \begin{bmatrix} b \\ \bar{b} \end{bmatrix}_L $$

and consider the pairing operator

$$ F(x) = \varepsilon_{\alpha\beta} \varepsilon_{ij} \psi^{(1)}_{\alpha i}(x) \psi^{(2)}_{\beta j}(x) $$

$$ = \varepsilon_{\alpha\beta} [a_{L\alpha} \bar{b}_{L\beta} + \bar{a}_{L\alpha} b_{L\beta}] \qquad (3.17) $$

[illegible] represents the charge conjugation operation in this [illegible] system. We can explicitly evaluate the limit

$$\underset{T\to\infty}{Lt} \langle M | F(T,r) F^{+}(0,r') | M \rangle$$

$$\to \frac{1}{r^2}\frac{1}{r'^2} e^{2\log \frac{T^2}{rr'}} \underbrace{Tr\{S_F^0(x,y)\, S_F^0(y,x)\}}_{\sim Tr\{(\frac{\sigma_3 T}{T^2})^2\}}$$

$$\to \frac{1}{r^3}\frac{1}{r'^3} \tag{3.18}$$

Hence by the cluster decomposition (or by observing that this limit singles out the lowest intermediate state, namely the monopole), one obtains

$$\langle M | F(r) | M \rangle \sim \frac{1}{r^3}$$

i.e. (3.19)

$$\langle M | (a\bar{b} + \bar{a}b) | M \rangle \sim r^{-3}$$

For $N_D = 4,6,\ldots$ doublets, one obtains the generalization

$$\langle M | F^{(1)}(r) \cdots F^{(N_D/2)}(r) | M \rangle \sim (r^{-3})^{N_D/2} \tag{3.20}$$

One can try to give a graphical interpretation of this result as indicated in Fig.3(a) for $N_D = 2$ or Fig.3(b) for $N_D = 4,6,\ldots$ Fig.3(a) is meant to show the vacuum pairing at some point r outside the core of a monopole due to a virtual process in which a fermion a_L falls into the core only to reappear as b_L. However this picture is not quite correct since the scattering solution for a_L takes $a_L \to \bar{a}_L$ at the core. Instead one should think of the neutral system $\frac{1}{\sqrt{2}}(a_L + \bar{b}_L)$ involving half fermions falling into the core and popping out as the half fermion number system $\frac{1}{\sqrt{2}}(\bar{a}_L + b_L)$. Fig.3(b) represents the corresponding generalization for $N_D = 4,6,\ldots$ (The role of half fermion number becomes mathematically clear in Callan's bosonization and soliton treatment in which the phenomenon of fermion number factorization is becoming well understood.)

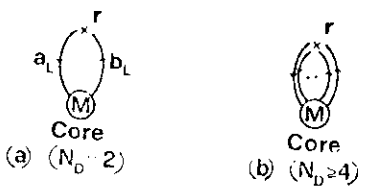


Fig.3

SU(5) "condensates" - Consider the case of only the one generation

$$\begin{bmatrix} \bar{d}_3 \\ e^- \end{bmatrix}_L ; \quad \begin{bmatrix} e^+ \\ d_3 \end{bmatrix}_L ; \quad \begin{bmatrix} u_1 \\ \bar{u}_2 \end{bmatrix}_L ; \quad \begin{bmatrix} u_2 \\ -\bar{u}_1 \end{bmatrix}_L .$$

Then Rubakov's analysis yields the following vacuum pairing:

$$\langle (\bar{u}_{2L} u_{2R} + \bar{u}_{1L} u_{1R})(\bar{d}_{3L} d_{3R} + e^+_L e^-_R) \rangle_M \sim \frac{1}{r^6}$$

$$\langle (\bar{u}_2 \bar{d}_3 - u_1 e^-)(\bar{u}_1 e^+ - u_2 d_3) \rangle_M \sim \frac{1}{r^6}$$

$$\langle (\bar{u}_{1L} \bar{d}_{3R} + u_{2L} e^-_R)(\bar{u}_{2L} e^+_R + u_{1L} d_{3R}) \rangle_M \sim \frac{1}{r^6} \tag{3.21}$$

In particular, one has the $\Delta B \neq 0$ "condensate" (i.e. vacuum pairing around the monopole)

$$\langle \bar{u}_{1L}\, \bar{u}_{2L}\, \bar{d}_{3R}\, e^+_R \rangle_M \sim \frac{1}{r^6} \quad . \tag{3.22}$$

We note that all these local correlations among quarks and leptons are completely neutral as regards the Q charge (i.e. they always involve two up and two down charges). What is remarkable is that they are also colour neutral and electrically neutral. Further, provided we average over the monopole's orientation with respect to colour hypercharge, they are also colour singlets. (We should remember the only forces that have been taken into account so far are the long range $U_Q(1)$ interaction, left over after the breakdown of the monopole SU(2) subgroup of SU(5).)

Physical picture of the catalysis reaction - From the above analysis one has the following picture. The vacuum around the monopole is polarized by virtual processes including baryon number violating ones ($\Delta B \neq 0$) because quark and leptons in the Fermi vacuum around the core can fall into the monopole and pop out with a different identity $\bar{d} \to e^-$, $u_1 \to \bar{u}_2$ etc. An incoming proton can encounter one of these processes and decay without one of its fermions actually having, in a one-particle sense, to reach the core. In the full quantum field context the catalysis reaction can occur without exciting the Coulomb energies of the colour and electric charges on or around the monopole core. This reaction is shown in Fig.4.

Other examples of reactions which could occur if the energy permitted would include $p_L \to p_R + e^+e^-$, this is shown in Fig.5. Hence one must also take into account the branching ratio into the different channels, many of which will not involve a violation of baryon number. In the naive picture, the branching ratio for $\Delta B = 0$ and $\Delta B \neq 0$ processes would be equal if the fermions were strictly massless.

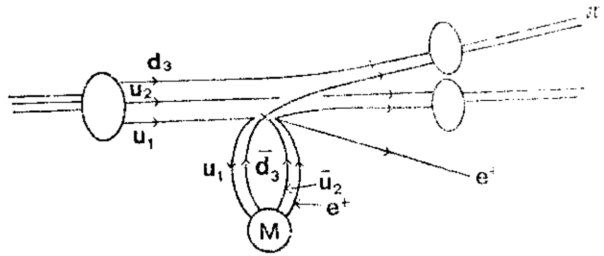


Fig.4

$p \to e^+ + \pi + \pi$ (catalyzed proton decay)

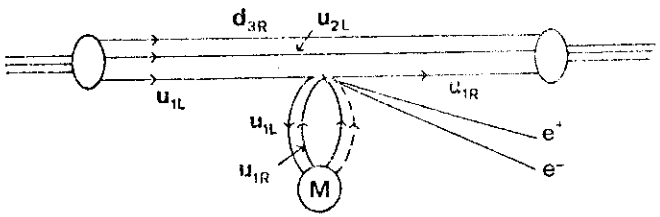


Fig.5

$p_L \to p_R + e^+e^-$ (helicity flip)

Is the Rubakov analysis complete? - There are in fact many open questions:

1. What if $m_f \neq 0$?
2. What about the effect of other interactions? viz. the horizontal SU(2) colour strong interactions, which cause

$$\begin{bmatrix} u_1 \\ \bar{u}_2 \end{bmatrix} \longleftrightarrow \begin{bmatrix} u_2 \\ -\bar{u}_1 \end{bmatrix}$$

3. Can we calculate cross-sections and branching ratios?
4. What happens in other GUT theories, do anomalous magnetic moments, heavy generations or electroweak effects play a role and so on.

To answer some of these questions, other approaches are being developed to which we now turn.

As regards mass effects, the anomaly argument suggests that the pairing parameters will have the following behaviour at $r \to \infty$

$$\langle M|F(r)|M\rangle \sim \frac{1}{r^3}\, e^{-m_f/\alpha\, r} \qquad (3.23)$$

I shall argue that fermion masses only spoil explicit solvability not the conclusions. They also provide the clue to the "freezing out" at low energies of the heavy flavours.

IV. CALLAN'S BOSONIZATION APPROACH

In Callan's approach to the problem the SU(2) horizontal colour interactions are replaced by $U_{I_{3c}}(1) \otimes U_{Y_c}(1)$ so that the two-dimensional radial field theory

$$\mathcal{L}_F = \bar{f}.\hat{\not{\partial}} f + m_0 \bar{f} f \qquad (4.1)$$

can be bosonized using the work of Coleman [11] and Mandelstam [12]. Here one can write the fermion fields in a bosonized form:

$$f(x) = :\exp\left[i\sqrt{\pi}\left(\varphi(x) - \int_{-\infty}^{x} dz_\mu \epsilon_{\mu\nu} \partial_\nu \varphi(z)\right)\right] \qquad (4.2)$$

for which one has the following identities:

$$\bar{f}\,\not\partial\, f = \frac{1}{2}(\partial_\mu \varphi)^2$$

$$\bar{f}\,\hat{\gamma}_\mu f = \epsilon_{\mu\nu}\partial_\nu \varphi$$

$$m_0 \bar{f} f = \mu m_0 : \cos 2\sqrt{\pi}\varphi :_\mu \tag{4.3}$$

A Coulomb or Abelian interaction can be included by using the Gauss law

$$\partial_r E = \frac{1}{8\pi r}\sum_f e_f \bar{f}\gamma^0 f \tag{4.4}$$

If we collect the relevant fermion degrees of freedom into conjugate pairs

$$\varphi_{u_1} = (u_1, \bar{u}_1) \quad ; \quad \varphi_{u_2} = (u_2, \bar{u}_2)$$

$$\varphi_{d_3} = (d_3, \bar{d}_3) \quad ; \quad \varphi_e = (e^+, e^-)$$

then the bosonized Lagrangian of the system can be written in the form

$$L = \int_0^\infty dr \left[\mathcal{L}_K + \mathcal{L}_M + \mathcal{L}_C \right] \tag{4.5}$$

where

$$\mathcal{L}_K = \sum_{i=u_1,u_2,d_3,e} \frac{1}{2}\left[\partial_\mu \varphi_i\right]^2$$

$$\mathcal{L}_M = \sum_i \mu_i^2 \cos 2\sqrt{\pi}\varphi_i \quad ; \quad \mu_i^2 = m_i \mu$$

$$\mathcal{L}_C = \frac{Q^2(r)}{r^2} + \frac{Y_c^2(r)}{r^2} + \frac{I_{3c}^2(r)}{r^2}$$

and

$$Q(r) = \frac{1}{\sqrt{\pi}}\left[\frac{2}{3}\varphi_{u_1} + \frac{2}{3}\varphi_{u_2} - \frac{1}{3}\varphi_{d_3} - \varphi_e\right]$$

$$Y_c(r) = \frac{1}{\sqrt{\pi}}\left[-\sqrt{\frac{2}{3}}\varphi_{d_3} + \frac{1}{\sqrt{6}}(\varphi_{u_1} + \varphi_{u_2})\right]$$

$$I_{3c}(r) = \frac{1}{\sqrt{\pi}}\left[\frac{1}{2}\varphi_{u_1} - \frac{1}{2}\varphi_{u_2}\right]$$

Boundary conditions at $r = 0$

1) $\phi_{e^+}(0) = \phi_{\bar{d}_3}(0)$, $\phi_{u_1}(0) = \phi_{u_2}(0)$ } SU(5) $\Delta B \neq 0$, $\Delta(B-L) \neq 0$

2) $Q(0) = Y_c(0) = I_{3c}(0) = 0$, i.e. vanishing Coulomb charges

Ground state: $L_K = 0$, $L_M = L_M\,(\min)$

$$\Rightarrow \varphi_i = \pm N_i\sqrt{\pi} \quad \text{and} \quad L_c = 0$$

$$\Rightarrow \varphi_{u_1} = \varphi_{u_2} = N\sqrt{\pi} \quad ; \quad \varphi_{e^+} = \varphi_{\bar{d}_3} = -N\sqrt{\pi} \tag{4.6}$$

$\therefore$ Each N labels an equivalent degenerate ground state, which can be losely thougnt of as being associated with a bare monopole with N times the $u_1u_2d_3e^-$ system attached to it. Tunnelling from the state N to N+1 represents a proton and electron being excited off the monopole core, i.e. we have the following kink picture (Fig.6) corresponding to transition $\gamma M \rightarrow M + p + e^-$

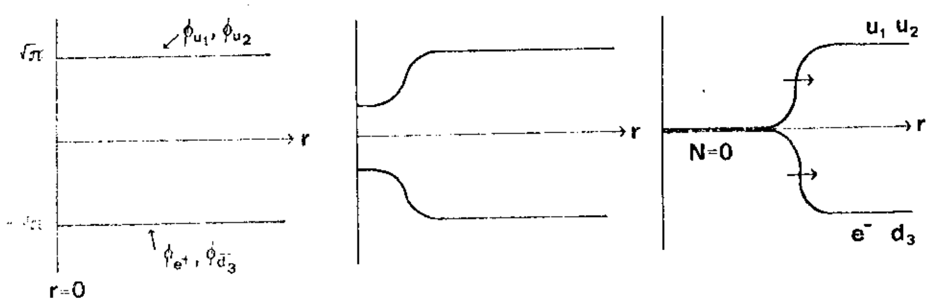


Fig.6

Kink picture of the transition $M(N=1) \rightarrow M(N=0) + u_1 + u_2 + d_3 + e^-$

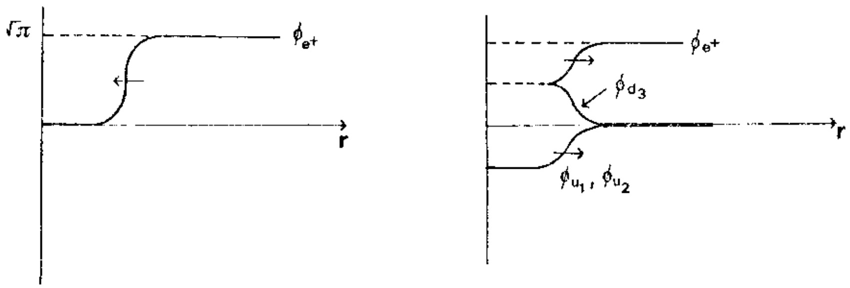
One important feature of Callan's kink picture is the way the kinks annihilate or get created at the core without exciting Coulomb energies.

Charge or fermion number factorization in fermion-monopole scattering

Recently Callan [11] has pointed out that 1/2 fermion solitons emerge in scattering processes of the form:

$$e^+_L + M \to M + \frac{1}{2}e^+_R + \frac{1}{2}u^+_{1R} + \frac{1}{2}u^+_{2R} + \frac{1}{2}d3_L$$

In the kink picture this proceeds as indicated in Figs.7(a) and (b), where the half soliton structure reflects results from the effect of the boundary conditions at r = 0



(a) incoming soliton (b) outgoing ½ solitons

Fig.7

Kink picture representing the process $e^+_L + M \to M + \frac{1}{2}e^+_R + \frac{1}{2}u_{1R} + \frac{1}{2}u_{2R} + \frac{1}{2}d_3$

The interpretation of this phenomenon which represents the transition $\Delta B = \Delta L = 1/2$, $\Delta(B-L) = 0$ is that the outgoing state is a superposition of the form

$$|out\rangle = \frac{1}{\sqrt{2}}|e^+_R\rangle + \frac{1}{\sqrt{2}}|u_{1R}\,u_{2R}\,d_{3L}\rangle \qquad (4.7)$$

and for which the relevant fermion number expectation values are half integers. This suggests that the true state is a coherent state involving an indefinite number of fermions, i.e.

$$|out\rangle = \sum_i \left(a_i|e^+_R, i\rangle + b_i|u_{1R}, u_{2R}, d_{3L}, i\rangle\right) \qquad (4.8)$$

where

$$\sum_i \left(|a_i|^2 + |b_i|^2\right) = 1/2$$

Finally, Callan notes in Ref.12 that both the processes

$$e^+_L + M \to M + e^+_R + \cdots \qquad (\Delta B = 0)$$

and

$$e^+_L + M \to M + P + \cdots \qquad (\Delta B \neq 0)$$

are possible. Further, one expects

$$\sigma(e^+M \to MX\ (\Delta B = 0)) \approx \frac{1}{2}\,\frac{\pi}{E^2}$$
$$\sigma(e^+M \to MX\ (\Delta B \neq 0)) \approx \frac{1}{2}\,\frac{\pi}{E^2} \qquad (4.9)$$

where the value $\sigma \sim \pi/E^2$ is the total s-wave cross-section from unitarity.

V. OBSERVATION ON THE EFFECT OF QUANTUM CORRECTIONS TO THE BOZONIZATION TREATMENT BY VERASORO

In Callan's treatment one can define the following transformation of the fields $\psi_i = M_{ij}\,\phi_j$ so that:

$$\mathcal{L}_c = \frac{2e^2}{r^2}\varphi_2^2 + \frac{g^2}{3r^2}\left[\varphi_2 + \sqrt{2}\,\varphi_4\right]^2 + \frac{g^2}{2r^2}\varphi_4^2 \qquad (5.1)$$

Near the core one might expect that $\psi_i \sim 0$, $i = 2,3,4$ and the action to be approximated by

$$L = \int dt \int_0^\infty dr \left[\frac{1}{2}(\partial_\mu \varphi_1)^2 + \sum_i \mu_i^2 \cos\sqrt{\pi}\,\varphi_i\right] \qquad (5.2)$$

However Virasoro pointed out [12] on studying the effect of the quantum fluctuations of the ψ_i fields, one obtains an effective mass term of the form

$$\mathcal{L}_m = c \sum_i m_i \frac{1}{r^{3/4}} \cos\sqrt{\pi}\,\psi_i \qquad (5.3)$$

This acts like a repulsive barrier for kink creation and annihilation near the core. To see this note that if $L_M = \mu^2 \cos\sqrt{\pi}\,\psi$ and we write $\psi_{kink} = \tanh\mu(r-R)$, then the kink momentum $P_r \approx \nabla_r \psi_{kink} \sim \mu e^{-\mu(r-R)}$. Hence very roughly the k.E energy associated with the kink is $\sim \mu^{-1}$. If $\mu \sim 1/r$ then the K.E $\sim 1/R$. For the above case in fact K.E $\sim 1/R^{1/2}$. If we try to picture what happens at low energies, then one notices that there are two competing scales. Firstly, there is the confinement radius which is further squeezed by the above effect as shown in Fig.8, secondly, however, the kinks themselves have a finite size which one would naturally associate with the fermion Compton wavelength λ_f. Hence at low energies the application of the soliton picture may be problematic.

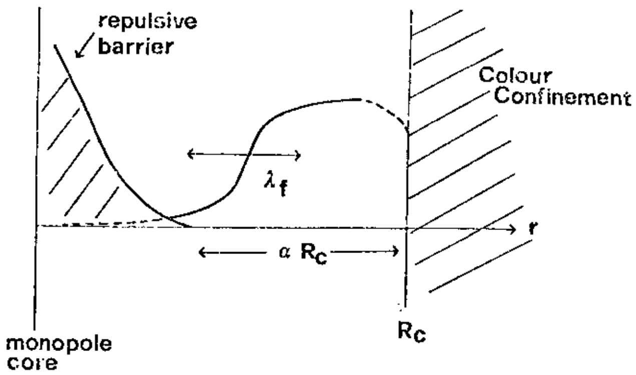


Fig.8

The distortion of the kink picture at low energies

VI. TREATMENT OF THE NON-ABELIAN SU(2) COLOUR INTERACTIONS IN THE FERMION-MONOPOLE SCATTERING PROBLEM

I would like to outline a new approach which I have been developing in collaboration with Werner Nahm and Valerie Rubakov for calculating s-wave fermion Green's functions in the fermion-monopole scattering problem, which takes into account in some approximations the non-Abelian colour strong interaction.

Consider an arbitrary 2n s-wave fermion correlation function and note, following the argument in the Rubakov analysis, that it can be written in the form

$$\langle f_{\alpha_1}(x_1)\cdots f_{\alpha_2}(x_n)\, f^+_{\beta_1}(y_1)\cdots f^+_{\beta_n}(y_n)\rangle_{\text{all fields}}$$
$$= \int [d\alpha_Q][d\alpha_c]\, e^{S_Q + S_c} \times \text{Products of } U(1) \text{ factors } \left(\prod_i e^{i\sigma_2\alpha_Q(i)} \prod_j e^{i\sigma_2\alpha_c(j)}\right)$$
$$\times \langle \hat f_{\alpha_1}\cdots \hat f_{\alpha_n}\, \hat f^+_{\beta_1}\cdots \hat f^+_{\beta_n}\rangle_{\text{horizontal SU(2) colour interactions}} \qquad (6.1)$$

The separation of Abelian U(1) factors, even in the presence of non-Abelian interactions can be demonstrated, as before, using the Fujikawa argument based on the change of variables

$$f \to e^{i\sigma_2\alpha}\,\hat f \,.$$

In the case of the original Rubakov and Callan analysis which was based only on the $U_Q(1)$ forces, $\hat f = f^{free}$. In order to deal with the SU(2) colour let us define $\hat f = (\chi_{in}, \chi_{out})$ where

$$\chi_{in} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_L \; ; \quad \chi_{out} = \begin{bmatrix} \bar u_2 \\ \bar u_1 \end{bmatrix}_L$$

Boundary condition at $r = 0$ $\chi_{in}(0) = \chi_{out}(0)$ with a charge conjugation property $\bar{\chi}_L = \chi_R^c$. The action for the radial QCD is

$$Z(\eta,\eta^+) = \int [da_\mu][df][d\bar{f}]\, e^{\int dt\, dr \left[-\frac{1}{4} f^a_{\mu\nu} f^{a\mu\nu} + \bar{f}[\hat{D\!\!\!/} + m] f + \bar{\eta} f + \bar{f}\eta\right]} \tag{6.2}$$

where $x_\mu = (t,r)$; $D_\mu = [\partial_\mu - g\, a^a T^a]_{ij}$. If we choose $a^a_0 = 0$ gauge then

$$f^a_{\mu\nu} f^{a\mu\nu} = \sum_{a=1}^{3} (4\pi r\, \partial_t a^a_r)^2 \tag{6.3}$$

This can be justified only up to Gaussian quantum fluctuations. [We assume everything else can be treated as renormalization corrections.] Note $a_0 = 0$ is similar to the $a_\pm = 0$ gauges used in QCD_2 in the large N_c treatments 14). After integrating out the radial colour fields

$$Z(\eta,\bar{\eta}) = \int [df][d\bar{f}]\, e^{\int d^2x\, [\bar{\eta} f + \bar{f}\eta]} \times$$

$$\exp\left[\int d^2x\, [\bar{f}_i \hat{\gamma}\cdot\partial f_i + \int d^2x\, d^2y \sum_{a,b} (ig)^2 J^a_\mu(x) \Delta^{-1}_{\mu\nu} J^b_\nu(y)]\right] \tag{6.4}$$

where $$J^a_\mu(x) = \bar{f}_i(x) \hat{\gamma}_\mu \tau^a_{ij} f_j(x)$$

In (6.4) we notice the non-local current-current interaction, which is typical of a non-Abelian theory. This suggests we try to develop a bilocal or string theory along the lines done for ordinary QCD_2 (see, e.g. Ref.14). One writes the functional integral in the form

$$Z = \int [df][d\bar{f}]\, e^{[(S_F^{0^{-1}}, f\bar{f}) + (f_i\bar{f}_j,\, K_{ij,\ell m} f_\ell \bar{f}_m)]} \times Z_{source}$$

where

$$[K_{ij,\ell m}]_{\alpha\beta,\gamma\delta} = (\hat{\gamma})_{\delta\beta} (\hat{\gamma})_{\gamma\alpha}\, \tau^a_{mi} \tau^a_{j\ell}\, K$$

$$K = g^2 (4\pi r^2 \partial_t^2)^{-1}$$

and

$$(A, KB) = \int d^2x\, d^2y\, A(x,y) (KB)_{x,y}$$
$$= \int d^2x\, d^2y\, d^2x'\, d^2y'\, A(x,y) K(x,y|x',y') B(x',y') \tag{6.5}$$

Now one makes use of the following relations:

1. $$f_i \bar{f}_j = \frac{1}{2}\left[f_\ell \bar{f}_\ell \delta_{ij} + Tr\{f_\ell \tau^a_{\ell m} \bar{f}_m\}\right] \tau^a_{ij}$$

2. $$K_{ij,\ell m} = \left[\delta_{mi}\delta_{j\ell} - \frac{1}{N_c}\delta_{mj}\delta_{i\ell}\right] K$$

3. $$\int [d\Sigma(x,y)]\, e^{(\Sigma, K^{-1}\Sigma) + (\Sigma, f\bar{f})} = c\, e^{\frac{1}{4}(f\bar{f}, K f\bar{f})}$$

4. $$\int [d\Pi^a(x,y)]\, e^{(\Pi^a, K^{-1}_{ab} \Pi^b) + (\Pi^a, f\tau^a\bar{f})} = c\, e^{\frac{1}{4}(f\tau^a\bar{f}, K_{ab} f\tau^a\bar{f})}$$

5. The following integral over the fermion fields

$$\int [df][d\bar{f}]\, \exp\left[(f,\bar{f})\, M \begin{pmatrix} f \\ \bar{f} \end{pmatrix} + (f,\bar{f})\begin{pmatrix} \eta \\ \bar{\eta} \end{pmatrix} + (\eta,\bar{\eta})\begin{pmatrix} f \\ \bar{f} \end{pmatrix}\right]$$
$$= -i\, (\det M)^{1/2} \exp\left[-\frac{i}{2}(f,\bar{f})\, M^{-1} \begin{pmatrix} f \\ \bar{f} \end{pmatrix}\right] \tag{6.6}$$

For the case in question

$$(\det M)^{1/2} = \exp\left[\tfrac{1}{2}\,\mathrm{Tr}\,\ln\left[(S_F^{o\,-1}+\Sigma)^2+\Pi^2\right]\right]$$

In this way we obtain the effective action

$$Z = \int [d\Sigma][d\Pi^a]\,\exp\Big\{(\Sigma, K^{-1}\Sigma) + (\pi^a, K_{ab}^{-1}\pi^b) + \tfrac{1}{2}\mathrm{Tr}\ln\left[(S_F^{o\,-1}+\Sigma)^2+\Pi^2\right]\Big\}\, Z_{Source} \tag{6.7}$$

where

$$Z_{Source} = \exp\Big\{\int d^2x\, d^2y\; \bar{\eta}_i(x)\left[(S_F^{o\,-1}+\Sigma)\mathbb{1}_{ij}+\Pi^a\tau^a_{ij}\right]^{-1}_{x,y}\eta_j(y)\Big\}$$

Now define the ground state of S_{eff} and quantum fluctuations about it by writing,

$$\begin{aligned}\Sigma &= \Sigma_o + \sigma(x,y)\\ \Pi^a &= \Pi^a_o + \pi^a(x,y)\end{aligned} \tag{6.8}$$

where

$$\frac{\delta S_{eff}}{\delta\Sigma}\bigg|_{\substack{\Sigma=\Sigma_o\\ \Pi=\Pi_o}} = 0\,; \qquad \frac{\delta S_{eff}}{\delta \Pi^a}\bigg|_{\substack{\Sigma=\Sigma_o\\ \pi=\pi_o}} = 0 \tag{6.9}$$

The solution of these equations yields $\pi^a_0 = 0$ and the Dyson equation

$$\left[S_F^{o\,-1} + \Sigma_o\right]^{-1} = K^{-1}\Sigma_o \tag{6.10}$$

Using the explicit form of $K = g^2\,(4\pi r^2\partial_t^2)^{-1}$ one obtains the system of equations:

1. Equation for self energy

$$\begin{aligned}\Sigma(x,y) &= (K\,S_F)_{x,y}\\ &= \frac{\alpha_s}{r^2}\,|t-t'|\,S_F(x,y)\end{aligned}$$

Diagrammatically we have

Σ = 

2. Equation for propagator S_F :

$$\int d^2x\left[\hat{\gamma}.\partial\,\delta^{(2)}(x-x') + \Sigma(x,x')\right]S_F(x',y) = \delta^{(2)}(x-y)$$

If we define the Fourier transform

$$\tilde{S}_F(\omega,r,r') = \int_{-\infty}^{\infty} dt\; e^{i\omega t}\, S_F(t,r,r')$$

we obtain the basic equation of the system by substituting the expression for Σ in the second equation

$$\left[\sigma_1\partial_r + \omega\sigma_3 + \frac{\alpha_s}{r^2}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\,\frac{1}{\omega'^2}\,\sigma_1 S_F(\omega-\omega',r,r)\sigma_1\right]S_F(\omega,r,r') = \delta(r-r') \tag{6.11}$$

This equation can in fact be solved in the massless limit, the result is

$$\tilde{S}_F(\omega,r,r') = e^{\alpha_S\left[\frac{1}{r} - \frac{1}{r'}\right]\left[\frac{1}{\omega} - \frac{1}{\lambda}\right]} \tilde{S}^0_F(\omega,r,r') \quad , \tag{6.12}$$

where λ is an infra-red cut off. If we insert such a propagator into the Rubakov vacuum pairing argument, then it simply gives zero. However, the situation is exactly analogous to the 't Hooft treatment of QCD_2, namely quarks or any colour carrying states do not propagate, instead colour singlet bound states form and these propagate. In a complete treatment [15], in which the underlying current algebra and bound state correlations are analyzed, one can show that the cluster argument goes through and colour singlet condensates are formed precisely along the line suggested by the original Rubakov argument, despite the presence of the non-Abelian forces. In Ref. 15 we also show how the above approach provides a means of calculating all the relevant fermion Green functions involved in the catalysis reaction. However even if we have an adequate description of s-wave fermion Green's functions, the problem of soft gluon emission (also of course soft photon effects) has not been considered. Further, if the interactions which induce the correct scattering transition occur some way from the monopole core at low energies, then the fermions involved will still experience the ordinary confining vacuum state of QCD, which is only weakly modified by the monopole. This means that they will have effective masses and anomalous magnetic moments, which will have some bearing on the detailed cross-sections and branching ratios. For this reason one should also consider other approaches more suited to the low energy environment. One such approach is being developed by Witten in collaboration with Callan, to which we now turn.

VII. [illegible] TOPOLOGICAL BARYON APPROACH

Witten [16] has proposed a novel framework in which the catalysis of proton decay can be understood directly at the level of proton monopole scattering without direct reference to the underlying quark interactions, except indirectly through a boundary condition at the monopole core. The essential idea of this approach rests on the fact that at low energies many properties of the hadron system can be understood in terms of the non-linear σ-model, namely

$$\mathcal{L}_{eff} = \frac{F_\pi^2}{16} \mathrm{Tr}\,(\partial_\mu U)^+ (\partial^\mu U) + \cdots \tag{7.1}$$

where

$$U = e^{\frac{2i}{F_\pi} \vec{\tau}.\vec{\pi}(x)}$$

This Lagrangian exhibits the SU(2) chiral symmetry of QCD. Now if one examines the box anomaly (Fig. 9) corresponding to the four point function $\langle T\ V^B_\mu A^3_\alpha A^+_\beta A^-_\nu \rangle$ then in $\mathcal{L}_{eff}$ it corresponds to the conserved vector topological current

$$V^B_\mu = \frac{1}{24\pi^2} \epsilon_{\mu\alpha\beta\gamma} \mathrm{Tr}\,[U^{-1}\partial_\alpha U U^{-1} \partial_\beta U U^{-1} \partial_\gamma U] \tag{7.2}$$

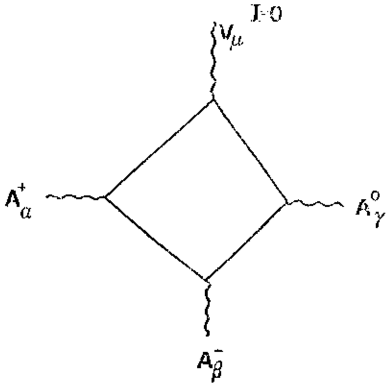


Fig. 9
Box anomaly

The charge corresponding to this current is given by

$$Q_B = \int d^3x \, V^B_0(x)$$

$$= \frac{1}{24\pi^2} \int d^3x \, \epsilon_{ijk} \, \mathrm{Tr}\{U^{-1}\partial_i U \, U^{-1}\partial_j U \, U^{-1}\partial_k U\} \tag{7.3}$$

i.e. the winding number associated with the group of transformations $\Pi_3(SU(2)) = Z$. For example if $\pi(x) \to \hat{x} F_\pi \to (h\pi)$ as $|\vec{x}| \to \infty$ then $Q_B = k$. Since this current can naturally be identified with baryon number, then baryons arise as topological excitations in the non-linear sigma models i.e. as Skyrme solitons [17]. One should add to $\mathcal{L}_{eff}$ some non-linear term like $(J{\cdot}J)^2$, where $J = U^{-1}\partial U$ in order to obtain stable soliton solutions.

Now one can couple electromagnetism to the system. However the current V^B must be modified in order to make it gauge invariant, i.e. $\partial_\mu \to D_\mu - e A_\mu(x)$. In order that the current remains conserved one has to add an additional piece to the current in the presence of a background electromagnetic field. Concentrating on the charge density, the extra piece for a monopole field is

$$V^B_0(x) = V^B_0(x)\Big|^{e=0} + e \, \vec{B}_{mon}(x) . \vec{\nabla}\pi^0(x) \tag{7.4}$$

where

$$V^B_0(x)\Big|^{e=0} \sim \epsilon_{ijk} \, \nabla_i \pi^+ \, \nabla_j \pi^- \, \nabla_k \pi^0$$

We note that the first part involves charged pion fields and consequently one would suppose should be suppressed near the monopole core because of the angular momentum barrier induced by the charge field interaction. On the other hand, the second term is neutral and can penetrate to the monopole core, experiencing the boundary condition at $r = 0$. Q_B remains a topological invariant (i.e. remains conserved) provided the proton does not reach the core of the monopole. However as the monopole passes through the proton, it can unravel the above winding number, leaving a state that can

dissipate into pions. To see what happens consider a Skyrme soliton corresponding to an incoming proton as it approaches GUT monopole. At first the topological charge is given entirely by the first term in Eq.(7.4). However, as the proton approaches the monopoles the second term gives the dominant contribution (i.e. the charge distribution is distorted or screened). The neutral component the pion field can be treated as a soliton wave, which can reach the core and feel the relevant boundary condition at r = 0.

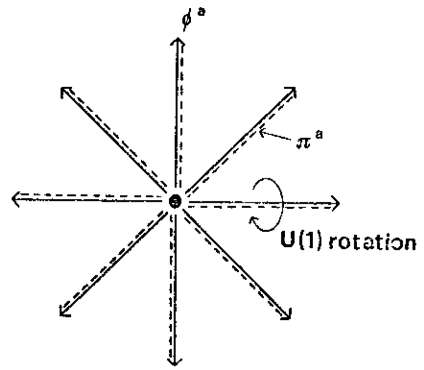


Fig.10

Superposition of a Skyrme hedgehog on a Higgs hedgehog .

Another way of understanding the above screening of the proton's topological charge by the monopole is to note that in a spherically symmetrical gauge the baryon is a Skyrme hedgehog of the form

$$\pi^a = \pi \, \hat{r}^a \qquad r \to \infty \, .$$

However in a non-singular spherically symmetric gauge the 't Hooft-Polyakov monopole corresponds to a Higgs hedgehog

$$\phi^a = \phi_0 \, \hat{r}^a \qquad \text{as} \quad r \to \infty \, .$$

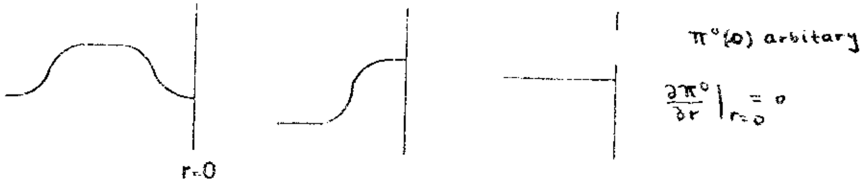
Electromagnetism corresponds to U(1) rotations about the Higgs field outside the core and we see from Fig. 9 , when the proton and monopole are superimposed, the proton is totally neutral and is described by the π^0 field

347

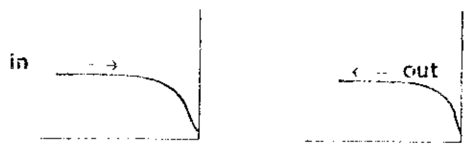
distribution. As this π^0 soliton wave reaches the monopole core, the boundary condition there can destroy the soliton structure, as only a net neutral flux of pions flows out and baryon number is lost. There is an explicit way of seeing that monopoles can unwind the Skyrme topological charge. One examines what happens in the unitarity gauge when the monopole is just like a point Dirac monopole. If we have an incoming Skyrme soliton corresponding to a proton along the negative z axis and the gauge is chosen so that the Dirac string singularity points along the positive z axis, then as the proton passes the core and moves along the positive z axis, one must make a singular gauge transformation to point the string along the negative z axis. This singular charge transformation unwinds the topological charge. This can be seen by noting that the incoming soliton in isospin space has the form $\vec{\pi} = \pi^0$ [$\sin\theta \cos\phi$, $\sin\theta \sin\phi$, $\cos\theta$]. However the singular gauge transformation referred to above corresponds in isospin space to $e^{i\phi}$ which leads to an outgoing configuration $\vec{\pi} = \pi^0$ [$\sin\theta$, 0, $\cos\theta$], which no longer corresponds to a Skyrme hedgehog.

One can try to complete this simple picture by adding an electron-wave so that one can define the correct SU(5) boundary condition $\Delta(B+L) \neq 0$, $\Delta(B-L) = 0$. Thus one considers the π^0 soliton and a soliton system and demands a mixed boundary condition at the monopole core. To explain this note that the effect of two possible boundary conditions are namely:

(a) A free boundary condition in which an incoming wave can be made to disappear



(b) In contrast, for a fixed boundary condition, i.e. $\pi_0 = 0$ at $r = 0$, we find the soliton bounces (i.e. is reflected)



Hence taking into account both the baryon and lepton waves one chooses

A) For the linear combination B-L a fixed boundary condition (so this linear combination is reflected),

b) While for B+L we take a free boundary condition so this linear combination disappears.

This ensures a proton evolves into a positron plus pions and respects the Coulomb energetics of the problem. Hence the two soliton picture describes the process $\Delta B = 1, \Delta(B-L) = 0$ and allows one, in principle, to calculate the cross-section for the proton decay catalysis reaction. However as it stands it has the drawback that although it by-passes the problem of confinement, it does not expose the underlying physics. Further, we do not see what other kinds of processes can compete with the decay reaction (see discussion in Sec.III)

NOTE ADDED:

Very recently Goldstone and Jaffe [18] have been doing some very interesting work which might allow us to combine Witten's approach with those developed by ourselves in Sec.VI, namely, one can combine the Skyrme soliton picture with the MIT bag model in the following way. The bag can be inserted into the Skyrme soliton as a topological defect. This will cause the winding number discussed above to no longer be an integer. On the other hand, the quarks inside the bag contribute to the total baryon number so with the correct bag boundary condition it remains unity. Seen the other way, if we define a bag Dirac boundary condition with a suitable chiral phase (described by the pion field on the boundary) one finds that the baryon number flows from inside the bag into the surrounding pion field, which at long distances appears to be a Skyrme soliton but taking into account all the contributions one preserves the baryon number. The extent to which the baryon number resides in the bag or the surrounding pion fields depends on the radius. The strategy is immediately clear, one simply opens up a hole (i.e. topological defect) and studies the possible monopole interactions at the SU(5) and quark level, one then studies the limit as $R_{bag} \to 0$ so as to define the appropriate boundary conditions and transformations acting on the Skyrme soliton structure as an SU(5) monopole passes through it. Whether such a programme can be realized in practice remains to be seen.

VIII. BRIEF COMMENTS ON OTHER WORKS AND CONCLUSIONS

There have been a number of suggestions of complications and other factors which have to be borne in mind and we briefly run through some of these here:

1. Question: Is the catalysis reaction suppressed by heavy flavours or electroweak effects as suggested by Grossmann, Lazarides and Sanda [19]. This seems to be unlikely because the anomaly should drive the same change in n_L for $M_W^{-1} << r << m_0^{-1}$, where m_0 refers to the light fermion masses.

2. Do anomalous magnetic moments play a role as suggested in Ref.20. Although this cannot change the boundary condition at r = 0 for a renormalizable gauge theory the effective anomalous magnetic moments of constituent quarks may play a role in the details of the catalysis reaction at low energies.

3. The question of selection rules [see Refs.3 and 21]. Particular channels may be suppressed by superselection rules and it is important to examine these carefully in any given GUT model.

4. A systematic study for other GUT theories as well as for super-GUTs models is needed. Some work in this direction has been done in Ref.22.

To conclude let me end by saying that after an extensive year of discussions there seems to be little doubt that if a GUT monopole exists, it will catalyze proton decays; if such processes occur inside their cores. However we are still far from being able to calculate cross-sections and branching ratios [23]. Nevertheless, there seems to be nothing to indicate that the correct order of magntiude is not given by the value obtained by s-wave unitarity, namely

$$\sigma_{\text{Rubakov}} \sim \pi/E^2 \quad .$$

ACKNOWLEDGMENTS

In preparing this review I have been helped by discussions with Sydney Coleman, Werner Nahm, Adam Schwimmer and Edward Witten. I am also grateful to Curt Callan for discussions after the completion of this manuscript. He has pointed out one perhaps important interpretational difference between the argument presented here regarding the region, in which the anomaly is effective to that obtained in his analysis. He would claim order unity charge in $n_L - n_R$ only occurs in a region r_0/α, where r_0 is the core radius and α is the fine structure constant.

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... RELATIVISTIC COSMOLOGICAL

MODELS NEAR THE SINGULARITY

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Invited lecture read by E.M.Lifshitz

The oscillatory mode of approach towards the singularity was first discovered for the homogeneous vacuum cosmological model of Bianchi type IX (cf. [1]). The character of the evolution of a model can be described by indicating three "scale functions" $a(t), b(t), c(t)$ which determine the temporal evolution of the lengths in three different directions in space. The oscillatory mode consists of an infinite sequence of successive periods (in [1] they were called eras) during which two of the scale functions oscillate and the third one decreases monotonically. On passing from one era to another (with decreasing time t) the monotonic decrease is transferred to another of the three scale functions. The amplitude of oscillations increases during each era but the increase is especially strong on passing from one era to another; however the product abc decreases monotonically - approximately as t. The eras become condensed with $t \to 0$; an adequate temporal variable for description of their replacements appears to be the "logarithmic time" $\Omega = -\ln t$.

We denote by $k_0, k_1, k_2, \ldots$ the "lengths" of successive eras (measured in terms of the number of oscillations they contain), beginning from a certain initial one. It turns out that this sequence of the lengths is determined by a

sequence of the numbers $x_0, x_1, x_2, \ldots$ $(0 < x < 1)$ each of which arises from the preceding one by the transformation

$$x_{s+1} = \{1/x_s\} \tag{1}$$

where the parentheses denote the fractional part of the number. The lengths $k_s = [1/x_{s-1}]$, the square brackets denoting the integer part of the number.

It was pointed out by I.M.Lifshitz and the two of us that the law of replacements of the lengths of the eras according to (1) leads to an important property: spontaneous stochastization of the behaviour of the model on approach to singularity ($t=0$) and the "loss of memory" of the initial conditions, prescribed at some moment of time $t=t_0>0$ ([2] , cited henceforth as I).

The importance of the oscillatory evolution in the homogeneous models stems from the fact that this model serves as a prototype for construction of a general inhomogeneous solution of the Einstein equations (in the neighbourhood of the singularity); the relevant work has been recently reviewed in [3] . Although the inhomogeneity and the presence of matter give rise to the appearance of certain new features (rotation of the axes to which the scale functions a, b, c refer), the law (1) remains unaltered. Thus, stochasticity in the vicinity of the singularity appears to be a most general property of cosmological models based on the classical Einstein equations.

The knowledge of the source of the stochastization makes it possible to construct with a considerable completeness a statistical theory of the evolution of the cosmological model in asymptotic closeness to singularity. However for a calculation of parameters of this theory an approximate method was devised

351

in I, the degree of exactness of which is difficult to estimate beforehand. The aim of the present work is to show that these parameters can be calculated exactly.

The starting point of the theory is the formula due to Gauss

$$w(x) = 1/(1+x)\ln 2 \tag{2}$$

which determines the probability distribution density of the values of $x_s \equiv x$ in the interval $[0,1]$ after many iterations of the transformation (1) (as we shall speak - in the stationary, i.e., independent of s limit) [1]. Hence follows the formula

$$W(k) = \frac{1}{\ln 2} \ln \frac{(k+1)^2}{k(k+2)} \tag{3}$$

for the probability distribution of the integer values of the era lengths. This function decreases with $k \to \infty$ merely as k^{-2} ; such slowness makes it necessary to use logarithmic physical quantities in order to obtain for them stable statistical distributions and mean values.

The basis of the following analysis constitute the recurrence formulas (obtained in I) for successive eras:

---

1) The regular evolution of the model according to the rule (1) can be interr pted by the appearance of "anomalous" eras (which were called in [1] the case of small oscillations). However it is important that in the asymptotic vicinity of the singularity (as $t \to 0$) the probability of occurence of such "dangerous" cases tends to zero as it was proved in I § 4.

$$\frac{\Omega_{s+1}}{\Omega_s} = 1 + \delta_s k_s \left(k_s + x_s + \frac{1}{x_s}\right) \equiv \exp \xi_s, \tag{4}$$

$$\delta_{s+1} = 1 - \frac{\delta_s (k_s/x_s + 1)}{1 + \delta_s k_s (k_s + x_s + 1/x_s)} \tag{5}$$

They are valid in asymptotic limit when $\ln \Omega/\Omega \to 0$ (in I formula (5) was given with a slip in the denominator). Here Ω_s is the moment of the beginning of the s-th era; the quantity δ_s is the measure (in units of Ω_s) of the initial (in the same era) amplitude α_s of the oscillations of the logarithms of the scale functions ($\ln a$, $\ln b$, $\ln c$): $\alpha_s = \delta_s \Omega_s$ ($0 \leq \delta_s \leq 1$). The quantity δ_s has a stable stationary statistical distribution $P(\delta)$ and a stable (small relative fluctuations) mean value. For their determination in I was used (with due reserve) an approximate method based on the assumption of statistical independence of the random quantity δ_s of the random quantities k_s, x_s . Now an exact solution of this problem is given.

Since we are interested in statistical properties in the stationary limit, it is reasonable to introduce the so-called natural expansion of the transformation (1) by continuing it indefinitely to negative indices. Such a "doubly-infinite" sequence $X = (\ldots, x_{-1}, x_0, x_1, x_2, \ldots)$ is uniform in its statistical properties over its entire length (and x_0 loses its meaning of an "initial" condition). The sequence X is equivalent to a sequence of integers $K = (\ldots, k_{-1}, k_0, k_1, k_2 \ldots)$, constructed by the rule $k_s = [1/x_{s-1}]$. Inversely, every number of X is determined by the integers of K as an infinite continuous fraction

$$x_s = \cfrac{1}{K_{s+1} + \cfrac{1}{K_{s+2} + \cfrac{1}{K_{s+3} + \dots}}} \equiv x^+_{s+1} \qquad (6)$$

We also introduce the quantities which are defined by a continuous fraction with a retrograde sequence of the denominators

$$x^-_s = \cfrac{1}{K_{s-1} + \cfrac{1}{K_{s-2} + \cfrac{1}{K_{s-3} + \dots}}} \qquad (7)$$

By means of some rearrangements (5) can be brought to the form

$$x_s \frac{1-\delta_{s+1}}{\delta_{s+1}} = \left[K_s + x_{s-1} \frac{1-\delta_s}{\delta_s} \right]^{-1} .$$

Hence by iterations: $x_s(1-\delta_{s+1})/\delta_{s+1} = x^-_{s+1}$ and finally

$$\delta_s = x^+_s / (x^+_s + x^-_s) \qquad (8)$$

The quantities x^+_s and x^-_s have a joint stationary distribution $P(x^+, x^-)$ which can be found starting from the joint transformation

$$x^+_{s+1} = \left\{ \frac{1}{x^+_s} \right\} , \quad x^-_{s+1} = \frac{1}{[1/x^+_s] + x^-_s} . \qquad (9)$$

In contrast to (1) it is a one-to-one mapping (in the unit square of variation of x^+ and x^-). Therefore the condition for the distribution to be stationary is expressed simply by the equation

$$P(x^+_{s+1}, x^-_{s+1}) = P(x^+_s, x^-_s)\, J(x^+_s, x^-_s) \qquad (10)$$

where J is the Jacobian of the transformation (9).

The normalized solution of this equation is

$$P(x^+, x^-) = 1/(1+x^+x^-)^2 \ln 2 \qquad (11)$$

(its integration over x^+ or x^- yields (2)). Since by (8) δ_s is expressed in terms of x^+_s and x^-_s , the knowledge of (11) makes it possible to find the distribution $P(\delta)$:

$$P(\delta) = 1/(|1-2\delta| + 1) \ln 2 \qquad (12)$$

The mean value $\langle \delta \rangle = 1/2$ already as a result of the symmetry of this function.

According to I the "doubly-logarithmic" time interval for a succession of a given number s of eras is

$$\tau_s \equiv \ln(\Omega_s / \Omega_0) = \sum_{p=1}^{s} \xi_p \qquad (13)$$

The mean value $\langle \tau_s \rangle = s \langle \xi \rangle$. The expression for ξ_s from (4) can be reduced to the form

$$\xi_s = \ln\left(\delta_s / (1-\delta_{s+1}) x_{s-1} x_s \right).$$

Since $\langle \ln \delta_s \rangle = \langle \ln(1-\delta_{s+1}) \rangle$ and $\langle \ln x_{s-1} \rangle = \langle \ln x_s \rangle$, we obtain

---

2) The reduction of the transformation to the one-to-one mapping was used already by Chernoff and Barrow [4] — for other variables and without applications to the problems which are considered here. As to the preceding papers by Barrow [5], they contain nothing beyond the main idea (taken from I) about the connexion of stochasticity in cosmological models with the transformation (1) and the distributions (2) and (3) (and the repetition of some well known statements of the general ergodic theory).

$$\langle \xi \rangle = -2 \langle \ln x \rangle = \pi^2 / 6 \ln 2 = 2{,}37. \qquad (14)$$

For large s the values of τ_s are distributed around $\langle \tau_s \rangle$ according to the Gauss' law with the density

$$\rho(\tau_s) = (2\pi D)^{-1/2} \exp\left\{ -(\tau_s - \langle \tau_s \rangle)^2 / 2D \right\} \qquad (15)$$

(cf. I § 4). The Calculation of the dispersion D is more complicated since it demands not only the knowledge of $\langle \xi^2 \rangle$ but also the mean values $\langle \xi_{p_1} \xi_{p_2} \rangle$ (which actually depend only in the difference $p = |p_1 - p_2|$). It appears to be useful to rearrange the terms in the sum (13) and omit the terms which do not increase with s. Thus one can obtain

$$\sum \xi_p = \sum \ln(1/x_p^+ x_p^-) \equiv \sum \eta_p$$

The dispersion

$$D = s \left\{ \langle \eta^2 \rangle - \langle \eta \rangle^2 + 2 \sum_{p=1}^{\infty} \left(\langle \eta_0 \eta_p \rangle - \langle \eta \rangle^2 \right) \right\}$$

The mean value $\langle \eta \rangle = \langle \xi \rangle$, and for the mean square one can obtain $\langle \eta^2 \rangle = 9\zeta(3)/2\ln 2 = 7{,}80$. Without taking into account correlations we would obtain $D = 2{,}17\, s$. By taking into account correlations with $p = 1, 2, 3, 4$ (calculated with the aid of an electronic computer) we arrive at the value $D = (3{,}5 \pm 0{,}1)\, s$.

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VI. LATTICE GAUGE THEORIES

SOLITONS AND THE PHASE STRUCTURE OF LATTICE GAUGE THEORIES

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1) Monopoles in Compact $U(1)$

2) Monopoles in $\frac{SU(N)}{Z(N)}$

3) "Monopoles" and "Vortex lines" in $SU(N)$

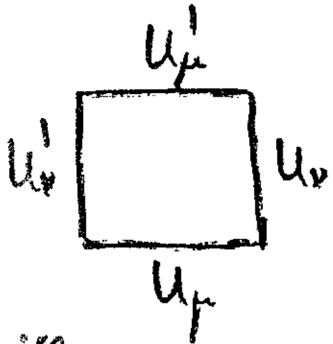
4) Large monopoles, special configurations, ...

Banks Kogut Myers Peskin

1) Compact $U(1)$

$$S = \beta \sum_{\mu\nu} \{ \mathrm{Tr} [U_\mu U_\nu U_{\mu'}^{+} U_{\nu'}^{+}] + c.c. \}$$

$\beta = \frac{1}{g^2}$ $\quad U_\mu \in H$



if $H = U(1)$ $\quad U_\mu = e^{i\varphi_\mu}$ $\quad S = 2\beta \cos\varphi$

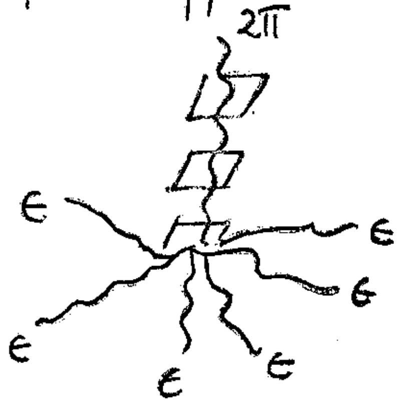
using covering group $(-\infty < \varphi_\mu < +\infty)$ makes use of Bianchi identity simpler:

$$\sum_{\text{cube}} \varphi_{\mu\nu} = 0 \quad \text{where } \varphi_{\mu\nu} \equiv \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu$$

compactness $\longrightarrow$ additional (K-R) gauge inv

$$\varphi_\mu \longrightarrow \varphi_\mu + 2\pi n_\mu$$

monopoles appear ~ G-G monopoles (Polyakov)



$\sum \epsilon = 2\pi \cdot n$

string "invisible"

$M = c\,\frac{\beta}{a}$.

$\Pi_1(H) = Z$

in space-time it describes closed loops of length L

- energy $\sim \beta \frac{L}{a^2}$
- entropy $\sim 7^L$

monopole condensation possible for

$\beta \lesssim \log 7$

the Villain form makes monopoles explicit:

$$Z = \sum_{m_{\mu\nu}} \int d\varphi_\mu \, \exp\left[-\beta\left(\varphi_{\mu\nu} + 2\pi m_{\mu\nu}\right)^2\right]$$

$m_{\mu\nu}$ — sum over it: periodic function

$m_{\mu\nu}$ — indep variable: GI $\begin{cases} \varphi_\mu \to \varphi_\mu + 2\pi n_\mu \\ m_{\mu\nu} \to m_{\mu\nu} - \partial_\mu n_\nu + \partial_\nu n_\mu \end{cases}$

decompose $m_{\mu\nu} = \partial_\mu N_\nu - \partial_\nu N_\mu + \epsilon_{\mu\nu\rho\sigma} \frac{\partial^\rho M^\sigma}{\partial^2}$

$\partial_\sigma M^\sigma = 0$, $M_\sigma = \epsilon_{\sigma\rho\mu\nu} \partial^\rho m^{\mu\nu}$

M_σ = monopole current = $\prod_{\text{cube}} m_{\mu\nu}$

One can add terms consistent with G.I., e.g.: $\gamma\left(\epsilon_{\mu\nu\rho\sigma} \partial^\nu m^{\rho\sigma}\right)^2$

then duality transformation gives:

$$Z = \sum_{m_{\mu\nu}} \int d\varphi_\mu \exp\left[-\beta(\varphi_{\mu\nu} + 2\pi m_{\mu\nu})^2 - \delta(\epsilon_{\mu\nu\rho\sigma}\partial^\nu m^{\rho\sigma})\right]$$

$$= \sum_{m_{\mu\nu}} \int d\varphi_\mu \int_{-\pi}^{\pi} d\theta_\mu \int_{-\infty}^{+\infty} d\psi_{\mu\nu} \exp\{i\chi^{\mu\nu}(\varphi_{\mu\nu} + 2\pi m_{\mu\nu}) +$$

$$+ i\theta^\mu \epsilon_{\mu\nu\rho\sigma}\partial^\nu m^{\rho\sigma}\} V(\theta_\mu) \exp\left[-\frac{1}{4\beta}\psi_{\mu\nu}^2\right]$$

- integrate φ, sum m
- solve δ-functions
- replace $\theta_\mu \to A_\mu - \partial_\mu \alpha$

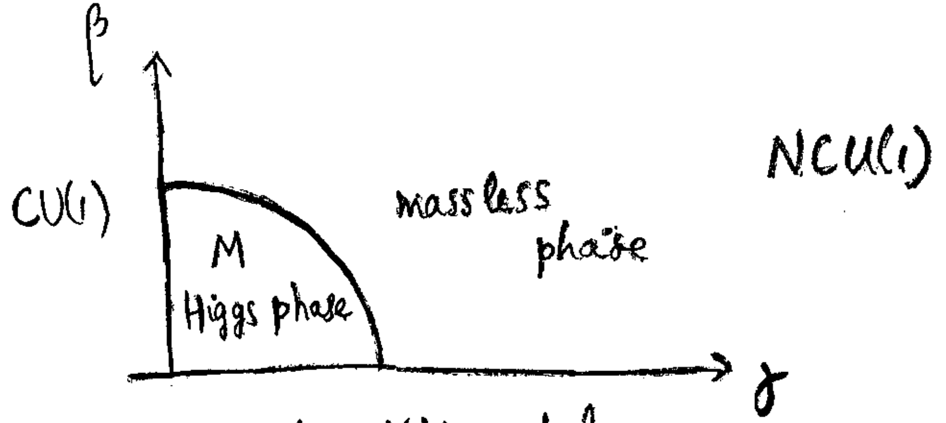
one obtains:

$$Z = \sum_{k_\mu} \int dA_\mu \, d\alpha \, \exp\left\{-\frac{1}{4\beta}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{4\gamma}(A_\mu - \partial_\mu \alpha + 2\pi k_\mu)^2\right\}$$

i.e. non-compact magnetic gauge field A_μ in interaction with monopole field $\Phi = \frac{1}{2\sqrt{\gamma}} e^{i\alpha}$ through standard Higgs coupling

$$D_\mu \Phi (D_\mu \Phi)^*$$

One expects the phase diagram:



4d X-Y model

i.e dual superconductor :
't Hooft–Mandelstam mechanism is realized:

$F_{\mu\nu} \equiv \varphi_{\mu\nu} + 2\pi M_{\mu\nu}$:

$\epsilon_{\mu\nu\rho\sigma}\,\partial^\nu F^{\rho\sigma} = 2\pi M_\mu =$ ↗ 0 – massless phase ; ↘ unconstrained – M-Higgs phase

$\Rightarrow \langle W(C)\rangle = \langle \exp i \sum F_{\mu\nu} \rangle$

$\sim \left[\int dF_{\mu\nu} \, \exp\left(i F_{\mu\nu} - \beta F_{\mu\nu}^2\right)\right]^{\text{area}}$

area law for the Wilson loop

2) Z(N) monopoles

$\Pi_1\left(\frac{SU(N)}{Z(N)}\right) = Z(N)$

i.e. in $\frac{SU(N)}{Z(N)}$, Z(N) monopoles are topologically stable.

i.e the analogy is:

Covering group: SU(N)

non compact U(1) → SU(N)

compact U(1) → $\frac{SU(N)}{Z(N)}$

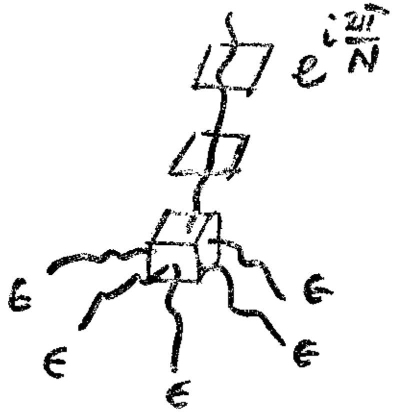
$\varphi_\mu \to \varphi_\mu + 2\pi n_\mu$ inv. ⟶ $U_\mu \to U_\mu \exp\left(i\frac{2\pi}{N} n_\mu\right)$

simplest inv. action: $\mathrm{Tr}_{adj}\left[U_\mu U_\nu U_\mu^{\prime +} U_\nu^{\prime +}\right] =$

$= \left\{\mathrm{Tr}_f\left[U_\mu U_\nu U_\mu^{\prime +} U_\nu^{\prime +}\right]\right\}^2 - 1$

for $\frac{SU(2)}{Z(2)} = SO(3)$

Z(N) monopole



if $\eta(p) = \text{sign}\, \text{tr}[U_\mu U_\nu U_\mu'^{+} U_\nu'^{+}]$

$$\overline{M}(c) = 1 - \prod_{p \in cube} \eta(p) = \text{monopole current density}$$

"Villain type" action

$$Z = \sum_{\sigma(p)=\pm 1} \int dU_\mu \exp\{\beta\, \text{Tr}[U_\mu U_\nu U_\mu^{+\prime} U_\nu^{+\prime}]\cdot\sigma + \gamma \prod_{p \in cube} \sigma(p)\}$$

has the additional gauge invariance

then $M(\beta) = 1 - \prod_{p \in cube} \sigma(\beta)$ also measures the monopole current

Dual transformation:

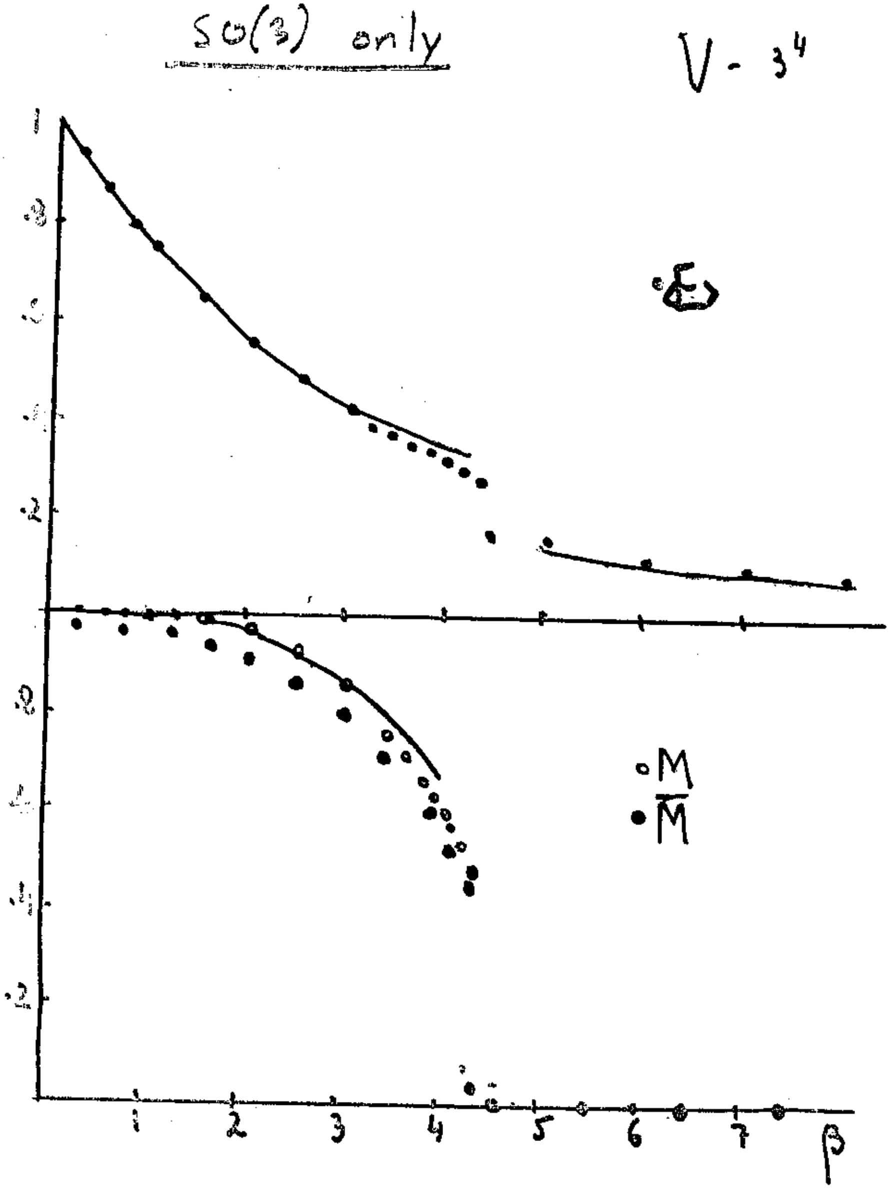
Z(2) monopoles coupled to a Z(2) magnetic gauge field in a SU(2) background

Higgs expectation value ↔ spin magnetization

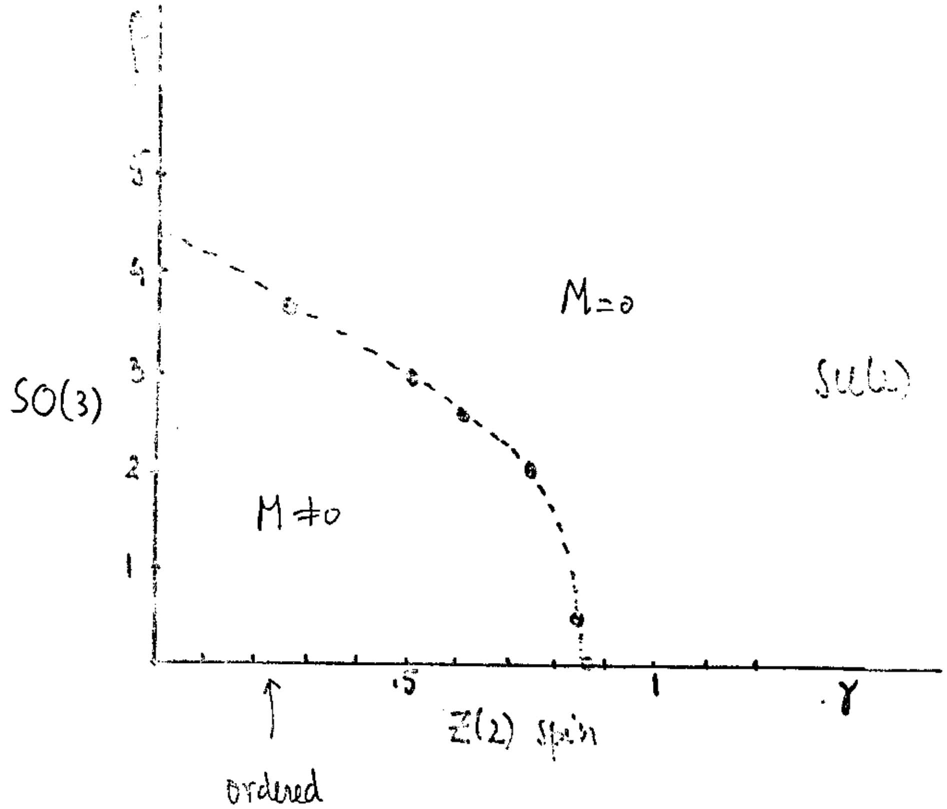
$$\overline{M} \nearrow SC: 1 - 4\left[\frac{I_2(\beta)}{I_1(\beta)}\right]^6 + O(\beta^{18})$$

$$\overline{M} \searrow WC: = 0 \text{ all orders in } \frac{1}{\beta}$$

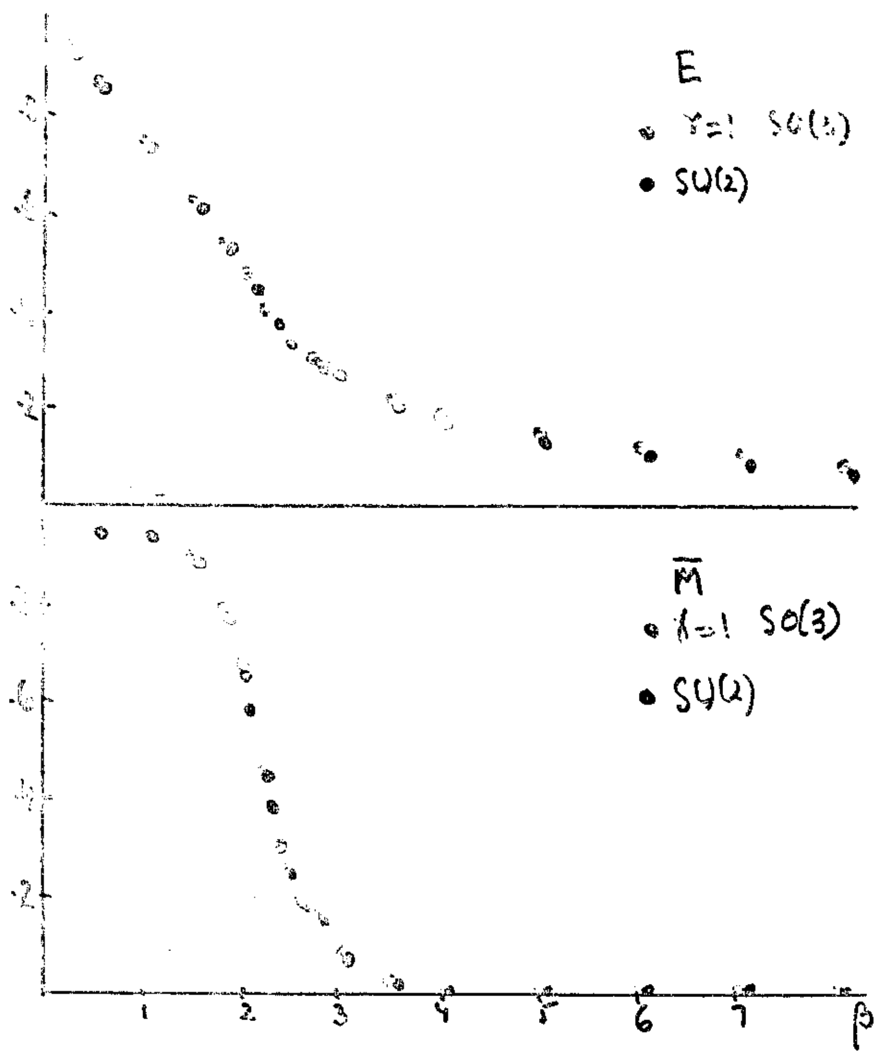
Ph-Tr. seen for $\frac{SU(2)}{Z(2)} \longrightarrow \frac{SU(6)}{Z(6)}$



Free energy/plaquette and monopole density for an SO(3)-action

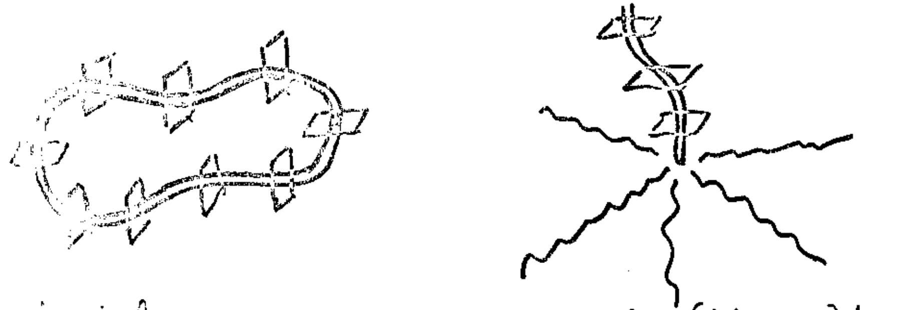


The phase diagram for SO(3) action with monopole chemical potential



SO(3) in the weak-coupling phase compared with SU(2)

Configurations related to the center in SU(N)



closed loops $= Z(N)$ monopoles (Mack-Petkova

energy $\sim$ length (compensated by entropy)

for $\eta(p) = \text{sign}\ \mathrm{Tr}\, U(\partial p)$

$M(c) = 1 - \prod_{p \in \partial c} \eta(p)$ monopoles

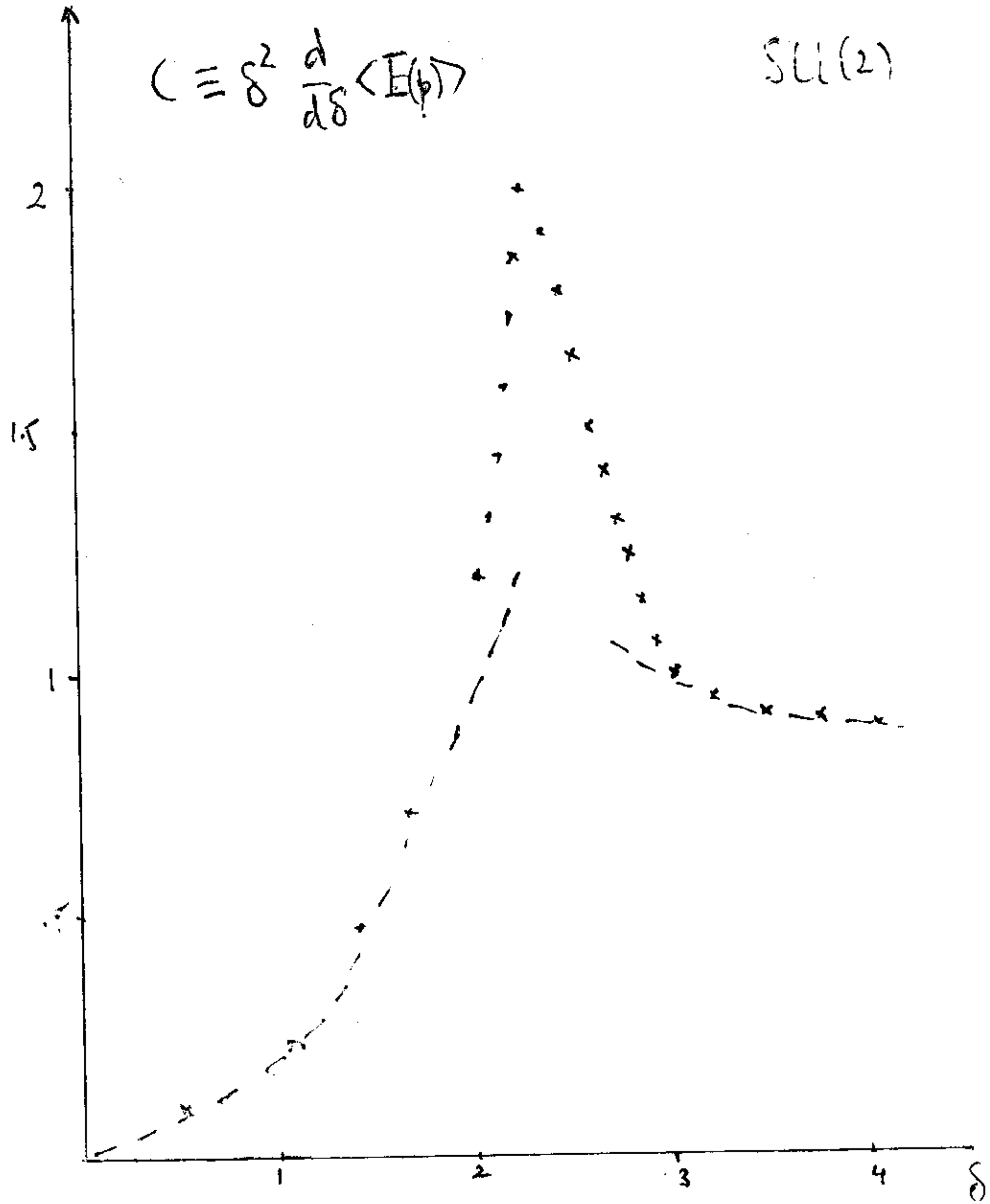
$E(b) = 1 - \prod_{b \in \partial p} \eta(p)$ loops

Their interplay appears in the mixed action:

$S = \beta X^2(p) + \delta X(p)$ $X = \mathrm{Tr}_f U(\partial p)$

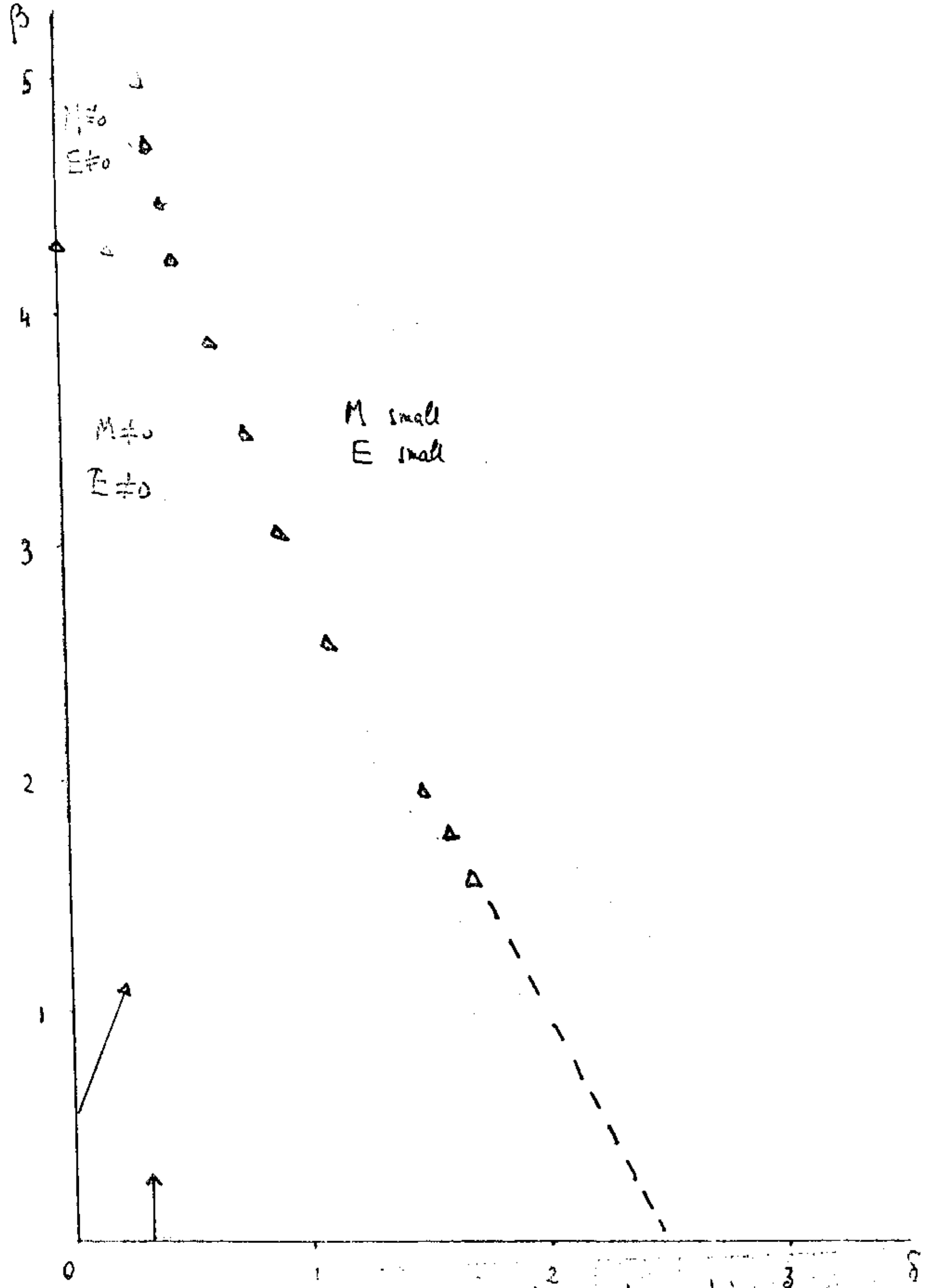
(Makeenko; Bhanot + Creu

↑ SO(3) ↑ SU(2)



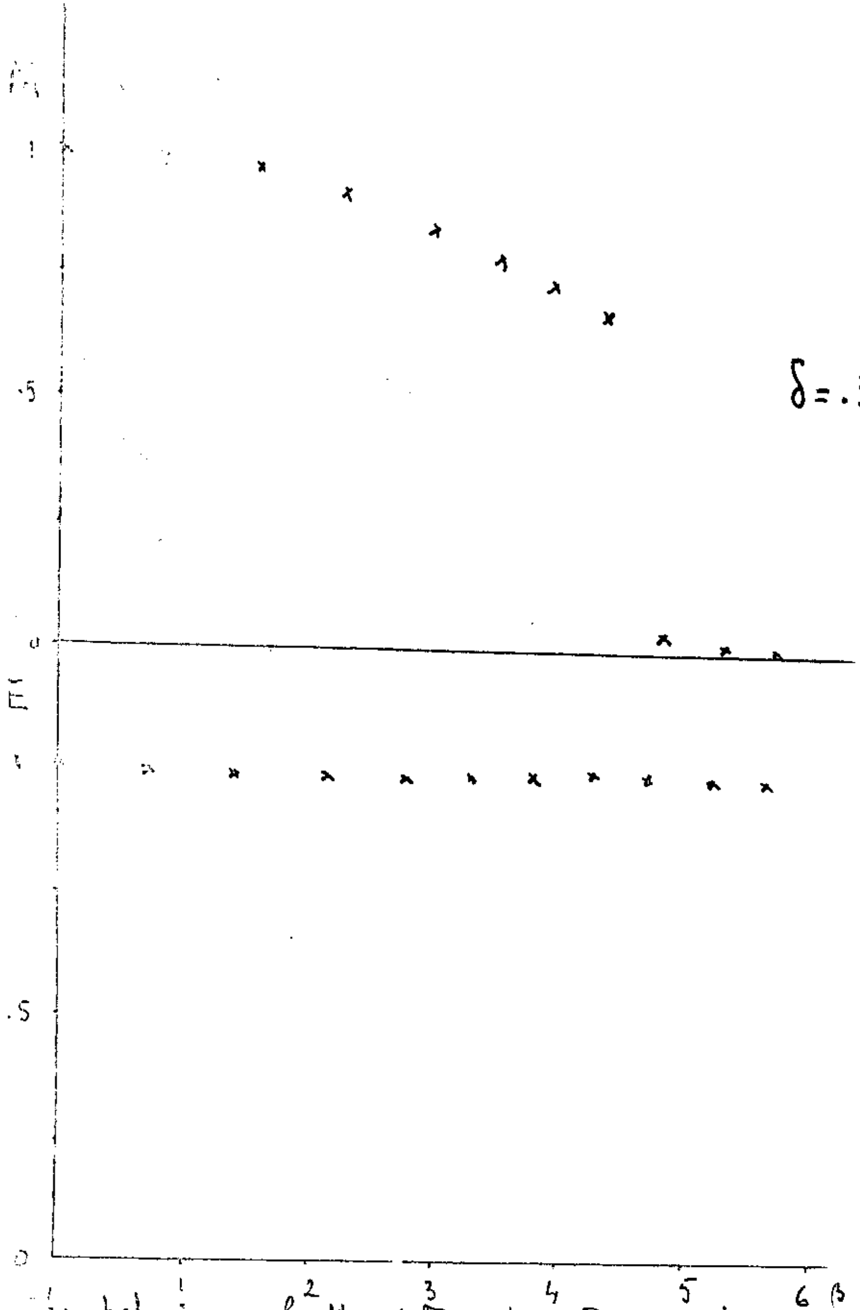
Specific heat in SU(2) (Lautrup + Nauenberg)

369

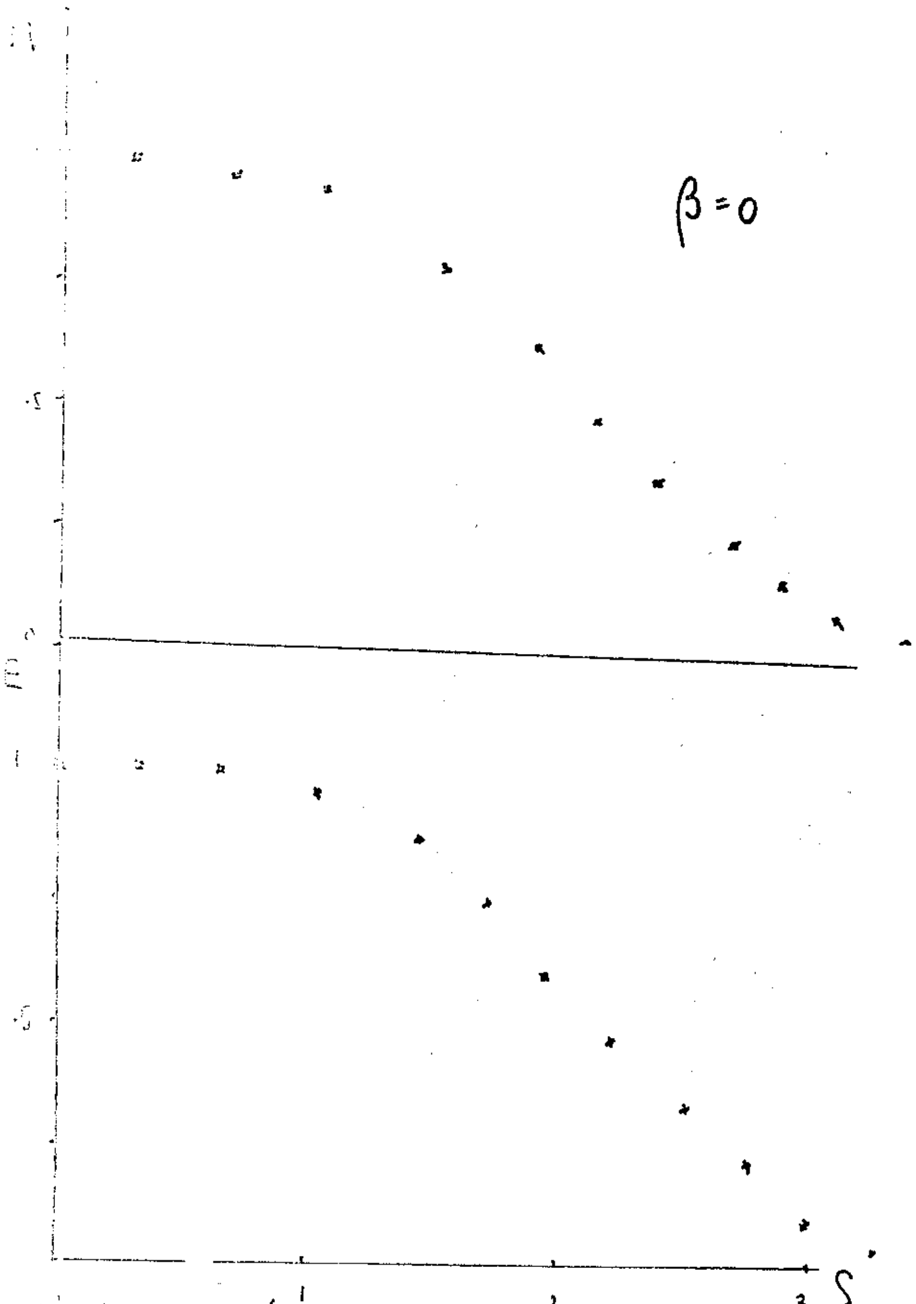


SU(2) - SO(3) mixed action phase diagram

370



... behaviour of M and E along δ = .35 line



... iour of M and E on the SU(2)-Wilson line

Center-configurations of non-minimal size

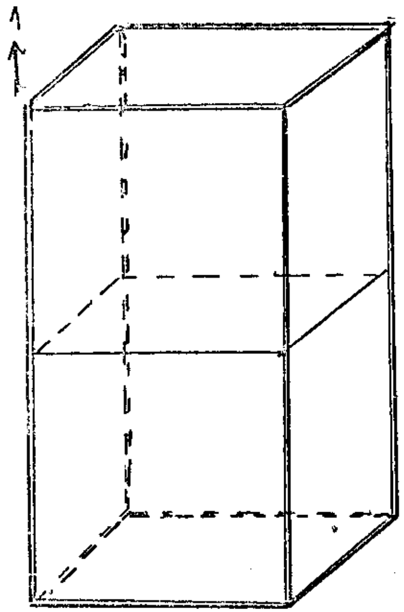
loops and monopoles with the string on $m \times n$ plaquette: i.e

$$\mathrm{Tr}\, U_{mn}(\partial p) \equiv \mathrm{Tr}\Big[\prod_{b \in p_{mn}} U(b)\Big] = -1$$

for $\eta_{mn}(p) = \mathrm{sign}\, \mathrm{Tr}\, U_{mn}(\partial p)$

monopole current $M_{211} = 1 - \prod_{p \in \text{elementary cube}} \eta(p)$

Correlations $\Delta^2 = \langle (M - \langle M \rangle)^2 \rangle$



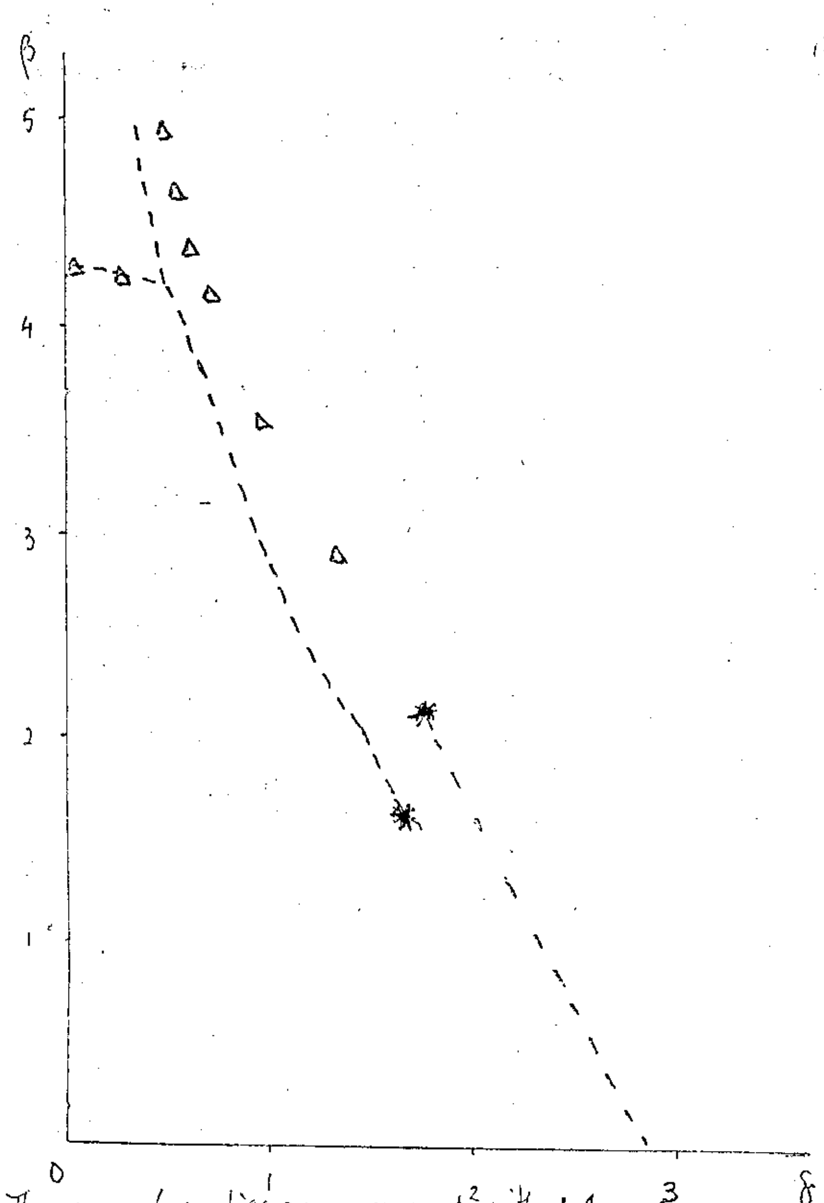
$$S = \beta\big[\mathrm{Tr}\, U(\partial p)\sigma(\partial p)\big] + \delta\, \mathrm{Tr}\, U(\partial p)$$

6 links or 4 links; 4 links

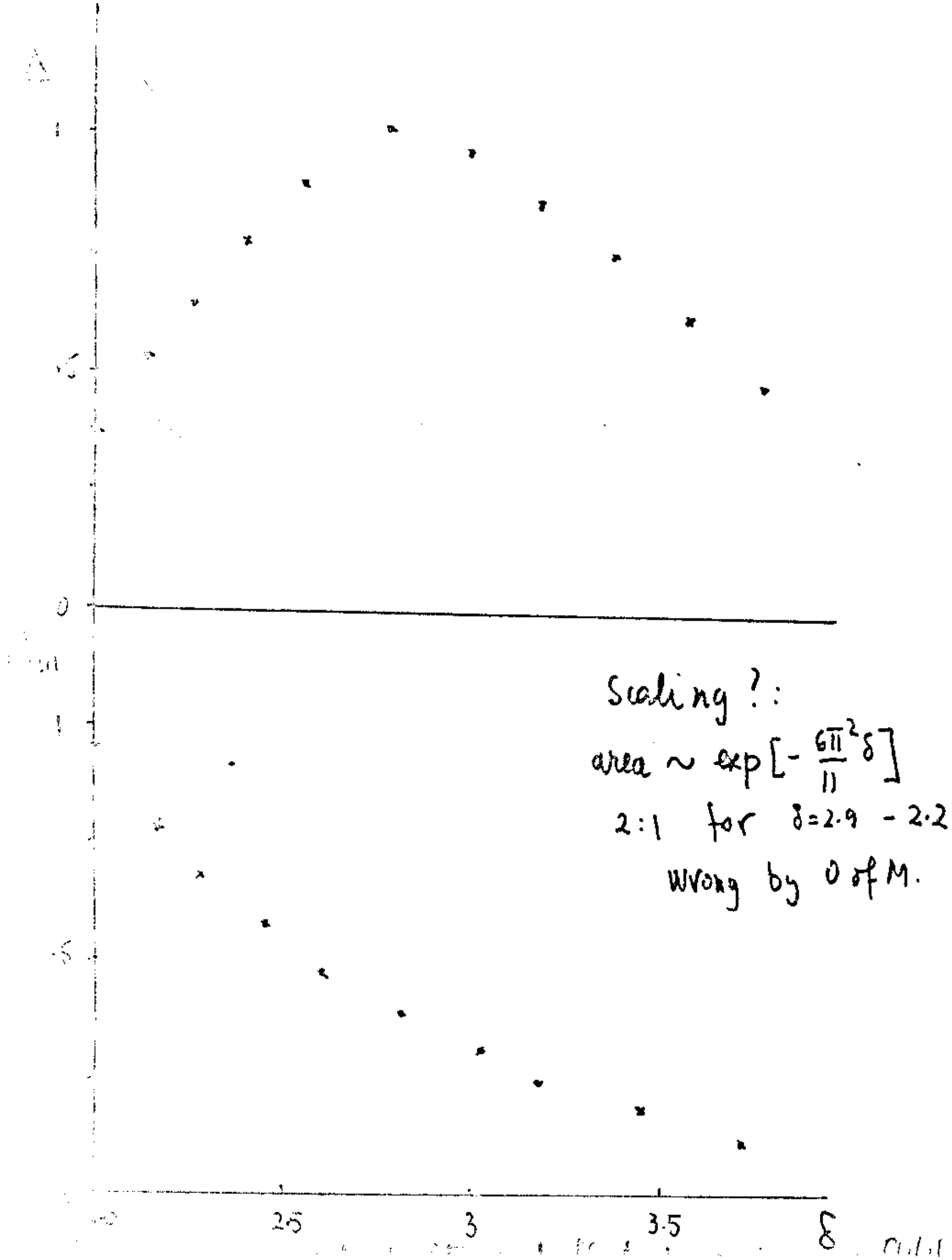
$\delta = 0$ ordinary SO(3)

$\beta = 0$ SU(2)

$\beta \to \infty$ "Z(2) on 2x1" in non abelian background



The 2x1 phase diagram compared with 1x1



Configurations not related to the center

for $SU(2)$: $S = \delta_3 [\mathrm{Tr}\, U(\partial p)]^3$

Anthony: → first order transition

Caneschi, Fox, Solomon

$S = \delta_2 x^2 + \delta_3 x^3 \qquad x \equiv \mathrm{Tr}\, U(\partial p)$

$\delta_3 = 0 \to SO(3)$

$\delta_2 = 0 \to A$

$\delta_2 \to \infty \to Z(2)$

$\delta_3 = -\delta_2 + \epsilon$, $\delta_2 \to \infty$ forces $x = 1$ or 0

i.e. the conjugacy class of $\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$

For $\delta_2 \to \infty$ P.T at $\epsilon \to \infty$

since $x=0$ has infinite entropy compared to $x=$

For δ_2 finite $x=0$ flux tubes!

Along $\delta_3 = -\delta_2$ vacuum changes

from $x=1$ to $x=0$.

choosing $S' = \delta_1 x + \delta_3 x^3$

for, $\delta_3 = -\frac{4}{3}\delta_1 + \epsilon \qquad \delta_1 \to \infty$

$x=1$ and $x = -\frac{1}{2}$ selected

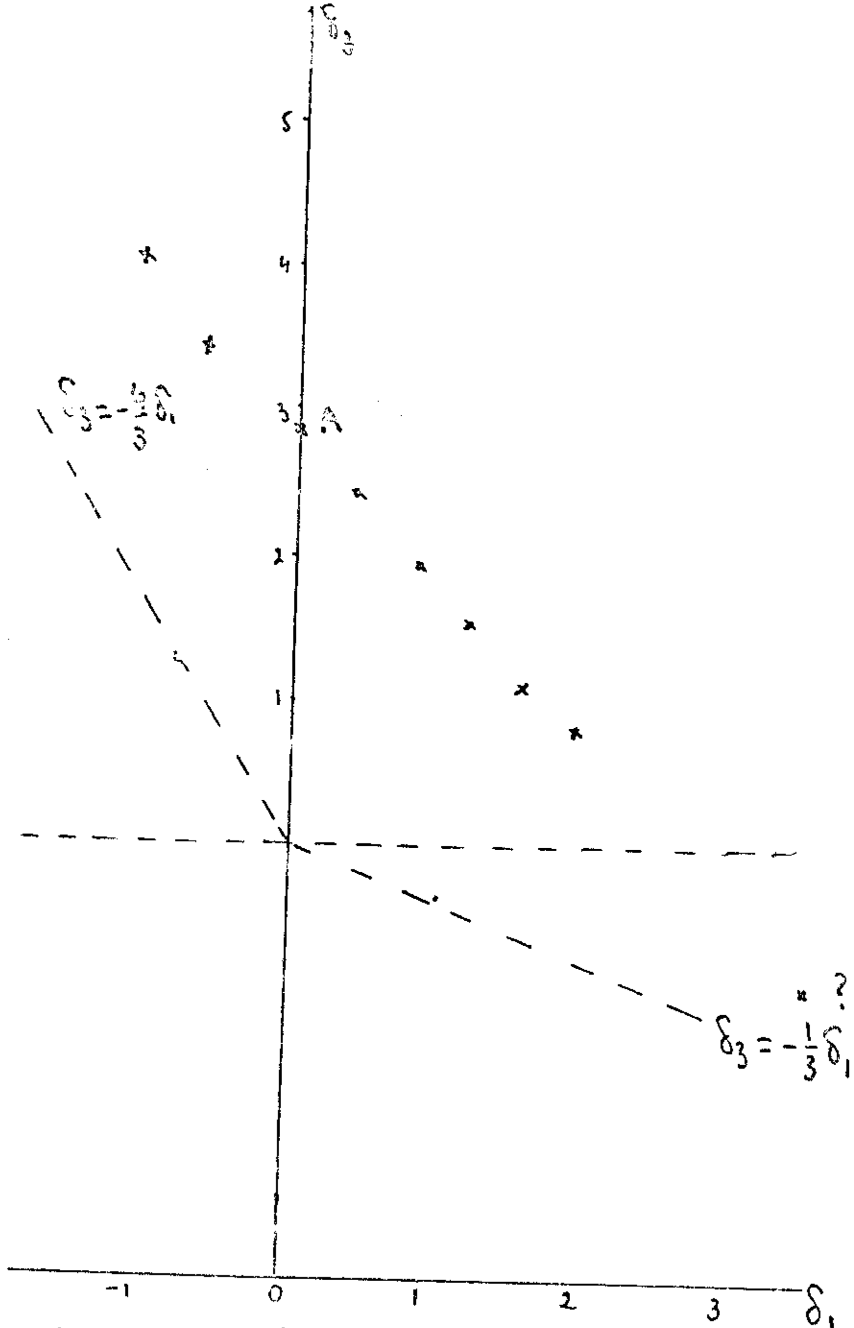
along $\delta_3 = -\frac{4}{3}\delta_1$ vacuum changes from

$x=1$ to $x \in (-1, -1/2)$

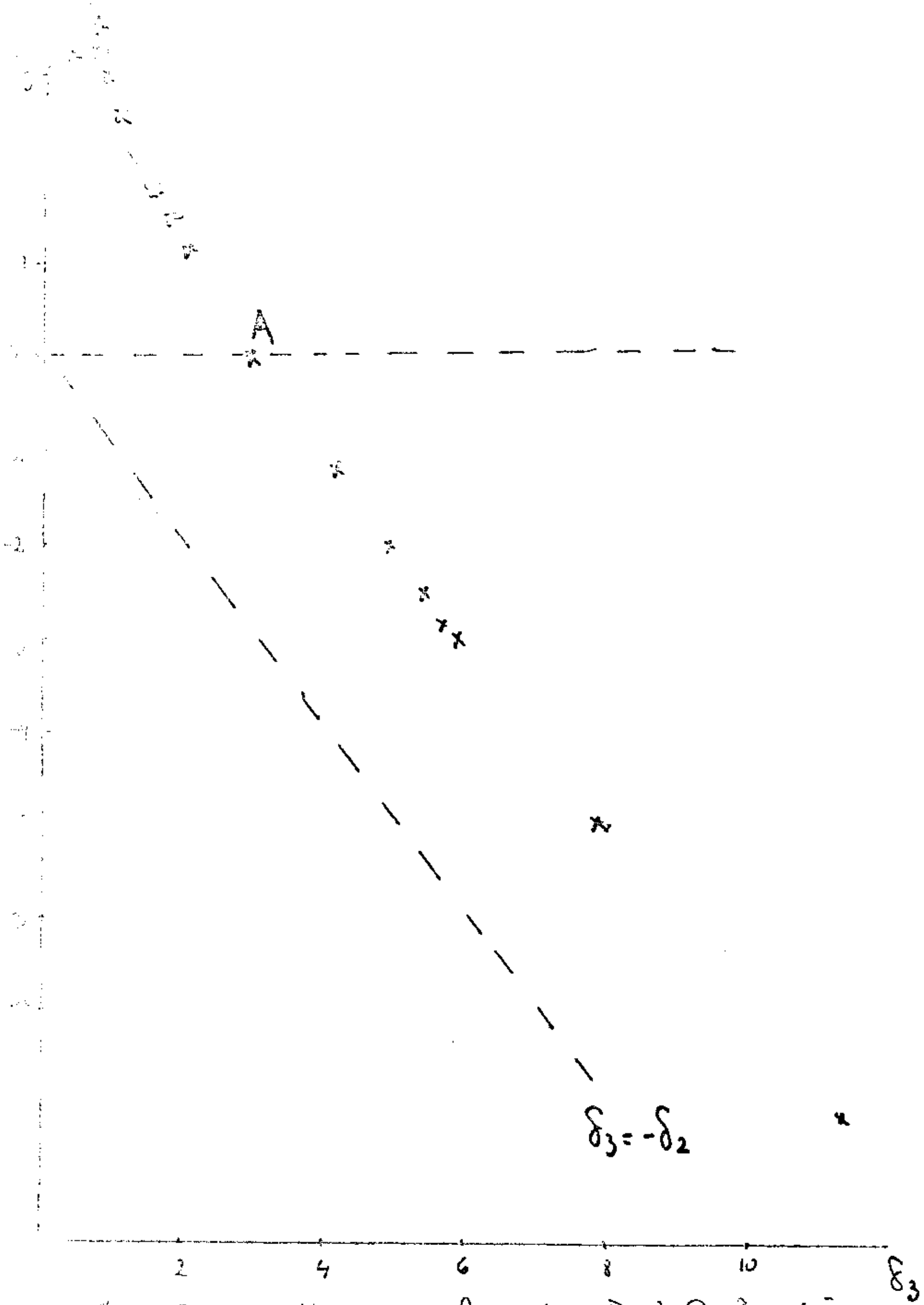
along $\delta_3 = -\frac{1}{3}\delta_1$ vacuum changes from

$x=1$ to $x \in (1/2, 1)$ → Higher order transition?

(Drouffe)



The phase diagram for $\delta_1 x + \delta_3 x^3$ action



The phase diagram for the $\delta_2 x^2 + \delta_3 x^3$ action

Conclusions:

- Topological configurations exist

- Minimal size loops and monopoles are responsible for the transition region

- Loops and Monopoles do not scale according to the RG for small sizes

- Configurations not related to the center may play a role in the phase structure.

VII. PREONS AND UNIFIED THEORIES

TESTING GRAND UNIFICATION THROUGH NON-CONSERVATIONS OF BARYON AND LEPTON NUMBERS*

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ABSTRACT

It is stressed that a large class of models of grand unification, which includes SU(5), SO(10) and the maximal one family symmetry SU(16), leads to essentially identical predictions for the grand unification mass M, the weak angle $\sin^2\theta_W$ and proton lifetime τ_p, if the respective grand unification symmetry descends in one step to SU(2) x U(1) x $SU(3)^c$ and if it has the minimal Higgs content. There is an enhancement by a factor of 4 to 5 for proton life-time for SU(16) relative to SO(10) and SU(5) owing to the presence of the mirror fermions for SU(16). Theoretical predictions of these models are compared with the present experimental status on proton decay. The possibility of intermediate mass scales of order $10^3 - 10^6$ GeV and $10^8 - 10^{12}$ GeV for maximal symmetries and their family extensions is stressed and the implications of these mass scales on the complexions of B,L - nonconservations are noted. It is further stressed that searches for the variety of proton decay modes (i.e. $p \to e^- +$ mesons, $p \to 3\ell$ or $3\bar{\ell}$ + mesons, $p \to \mu K$, $p \to \nu K$ etc.), neutron oscillations and neutrinoless double β-decay can provide the window for viewing high mass scales, if they exist. The need for pursuing new directions beyond grand unification is noted. Some remarks are made regarding three such directions: (i) Compositeness of quarks and leptons, (ii) supersymmetry and supergravity, and (iii) space-time with dimensions higher than four.

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384

International Atomic Energy Agency
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SPONTANEOUSLY BROKEN GAUGE THEORIES AS WEAKLY COUPLED LOW ENERGY EFFECTIVE LAGRANGIANS FOR ASF CONFINING PREON GAUGE THEORIES *

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MIRAMARE - TRIESTE
September 1983

* Talk given at the ICTP Summer Workshop in Particle Physics, 21 June - end July 1983. This work was done in collaboration with Jan Stern.

The talk is divided into the following parts:

(A) Introduction: Framework of our discussion, namely the link $\mathcal{L}_{preon} \longleftrightarrow \mathcal{L}_{eff}$ and the question of mass scales, M_G, Λ_{ASF}, $r_0^{-2} \sim \Delta M^2$.

(B) Correct way to interpret the global symmetry classification of $\mathcal{L}_{preon}$ bound states and an approximate gauge symmetry of $\mathcal{L}_{eff}$.

(C) How does $\mathcal{L}_{preon}$ "talk" to $\mathcal{L}_{eff}$ and in which sense can a renormalizable $\mathcal{L}_{eff}$ emerge from the properties of $\mathcal{L}_{preon}$.

(D) The price of breaking the L-R symmetry spontaneously in $\mathcal{L}_{preon}$ through preon condensates.

(E) Some examples with a Glashow-Salam-Weinberg $SU(2)_L \otimes U(1)$ structure.

(F) Summary and prospect of supersymmetric models.

A. INTRODUCTION

Let us begin with a vague general remark which we believe in some form is due to 't Hooft in connection with his meta colour ideas, namely: perhaps due to some quite general principles about the way quanta propagate and interact in a causal fashion, the relevant charges or quantum degrees of freedom at a given scale might always be describable in terms of a gauge theory if the spectrum contains spin-1 particles. Specifically, one is thinking of asymptotically free non-Abelian gauge theories or very weakly coupled Abelian theories. Let us consider the former and ask what happens if we view the behaviour of the system at much lower energies, in which only the ground state sector is relevant, then the effective Lagrangian for the latter should to a good approximation be another gauge theory if the low lying spectrum contains spin-1 states, for the same reasons the first Lagrangian was. In this way one might imagine that if we go to ever increasing energy scales, one gauge theory evolves into the next.

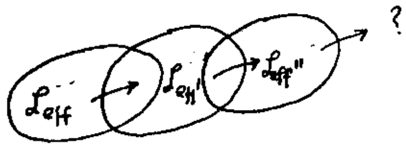


Fig.1

There will be an overlap region, in which life might be complicated and both the low energy and high energy effective Lagrangians too difficult to use, although one might hope for properties like precocious scaling and duality to help in such transition regions as appears to happen in QCD.

In this talk based on the work I have been doing with Jan Stern [1), we try to give a partial realization of the above vague idea, by concentrating on a definite framework. Namely, consider a class of preon gauge theories of the following general form, involving some meta colour gauge group

$$\mathcal{L}_{preon} = -\frac{1}{4} F_{..} F^{..} + \sum_{i=1}^{2} [\bar{f}_L^i \not{D} f_L^i + \bar{f}_R^i \not{D} f_R^i] + \mathcal{L}_x , \qquad (1)$$

where $\mathcal{L}_x$ is left unspecified for the moment. It may contain meta colour scalars or simply some fermions in some other representation of the gauge group.

The essential point is that the manifest chiral symmetry exhibited in the above definition of $\mathcal{L}_{preon}$ is not affected by $\mathcal{L}_x$. We might note that one could also choose $\mathcal{L}_x$ to be the supersymmetric completion of $\mathcal{L}_{preon}$. Many of our remarks will either carry over or be generalizable to supersymmetric preon gauge theories. For the moment let us note that the basic classification symmetry of the low lying bound states containing the spin-1/2 preons f^i is $SU(2)_L \otimes SU(2)_R \otimes U(1)_V$. The axial symmetry being broken by a meta colour anomaly.

The existence of conserved currents whose charges are the generators of this symmetry puts some severe constraints on any low energy effective Lagrangian, which gives a realization of the same basic symmetry group or any part of it. If the short distance expansion of products of these currents can be computed, then these constraints go way beyond the current algebra alone. The claim we want to demonstrate and pursue in this talk is the following. If the scale of ground state masses which are non-zero is

$$M_G^2 \sim \Lambda_{ASF}^2 << r_0^2 \sim (\alpha')^{-1} \sim \Delta M^2 \quad \text{(the spacing or Regge recurrence scale)}$$

then the constraints we are referring to above tell us that $\mathcal{L}_{eff}$ must be well approximated by a renormalizable theory. Further, if it contains spin-1 bound states, then $\mathcal{L}_{eff}$ must be well approximated by a gauge theory. But the existence of such a scenario is not immediately obvious because of an observation made by Weinberg and Witten. Namely, if we take matrix elements of Noether currents of some global symmetry between massless gauge particle states, then all such matrix elements must vanish. I have yet to see the proof of this result for confined gauge fields, however assuming this to be true, what about the states of a spontaneously broken gauge theory. Here there is a simple and elegant answer, which tells us a great deal about the possible link between $\mathcal{L}_{preon}$ and $\mathcal{L}_{eff}$. To expose this, we have to add to the Weinberg-Witten observation an observation of our own, which we will call the screening theorem, for want of a better word.

Before turning to the latter, let me add one important remark, at least in my mind. It is clear that if we were to develop $\mathcal{L}_{eff}$ systematically in its perturbation theory and ask for it to reproduce exactly all the properties of the $\mathcal{L}_{preon}$ perturbation theory, say at short distances, then this would amount to asking the Wilson OPE of both theories to be identical and we would be led to a tautology or absurdity. Hence it is also important

to spell out precisely the sense in which $\mathcal{L}_{eff}$ approximates $\mathcal{L}_{preon}$ and in what energy range. We should recall the [illegible] from current algebra, namely that any effective Lagrangian which gave a realisation of the basic global symmetry, reproduced all the Ward identities and zero energy theorems at the tree graph level. However the results are only valid for tree graphs and very low energy (actually vanishing energies). We want something more, we would like the corresponding low energy theorems to be satisfied over an energy range and at the one or two loop levels. Hence from the outset we know that $\mathcal{L}_{eff}$ better be weakly coupled and its underlying gauge symmetry good to the same approximation. Higher orders in g_{eff} should compete with effects of Reggeization in $\mathcal{L}_{preon}$.

Let us summarize by saying that any gauge symmetry in $\mathcal{L}_{eff}$ will be approximate and dynamical in origin. If the Regge recurrences are not too far away on the energy scale we are interested in, then the gauge symmetry will most probably be a poor approximation.

Let us now try to clarify these points and develop some simple models as illustrations. Many more issues will be raised as we go along, not all of which I will have time to give answers to, even if they exist.

THE CORRECT WAY TO INTERPRET THE GLOBAL SYMMETRY CLASSIFICATION OF $\mathcal{L}_{PREON}$ BOUND STATES AND AN APPROXIMATE GAUGE SYMMETRY OF $\mathcal{L}_{eff}$

I will not repeat the proof of the Weinberg-Witten claim, which [illegible] applies to massless gauge particles. Let us simply note that since the global symmetries of the preon Lagrangian we have in mind are generated by conserved currents, which are Lorentz 4-vectors, all mass spin-1 bound [illegible] singlet under the group of global preon symmetries. We are [illegible] interested in massive spin-1 particles, which Lee and Zumino pointed [illegible] can even couple directly in the form of current field [illegible] conserved symmetry currents. However the possible structure of [illegible] effective Lagrangian is restricted by the following theorem [illegible]

[illegible] screening theorem

Let a gauge theory possess a symmetry

$$G \otimes \underline{G'} ,$$

where G is the gauge group and G' is a maximal global symmetry group that commutes with G. Further, for simplicity, let the whole gauge symmetry G be spontaneously broken by Higgs scalars so that all the gauge particles become massive. Then the global symmetry group of the theory that remains unbroken spontaneously is necessarily a sub-group of G'.

Corollary: If there is no global symmetry to begin with, then there will be no global symmetry left after spontaneously breaking the gauge symmetry.

This claim is a direct consequence of the fact that in the Higgs phase the gauge charges of all particles are screened by the gauge charge of the Goldstone bosons that have been eaten up in the Higgs mechanism.

Let us give a brief proof of the above claim. Consider Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2}\sum_{r=1}^{N} (D_\mu \phi_r)^+ (D^\mu \phi_r) - V(\phi) + \text{fermion etc}$$

where $F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\, c^{abc} W^b_\mu W^c_\nu$. The Noether current generating global symmetry transformation of the group G is *)

$$J^a_\mu = c^{abc} F^b_{\mu\nu} W^{c\nu} + j^a_\mu \quad \text{(scalars plus fermions)} .$$

However from equation of motion

$$\partial^\nu F^a_{\mu\nu} + g\, c^{abc} F^b_{\mu\nu} W^{c\nu} = -g\, j^a_\mu$$

so that

$$J^a_\mu = -\frac{1}{g}\partial^\nu F^a_{\mu\nu}$$

$$\therefore Q^a = \int_{V(R)} d^3\vec{x}\, J^a_0 = \int_{V(R)} d^3x\, \partial_i E^a_i$$

which vanishes by virtue of the Gauss theorem for large radius R because W^a is massive.

*) $J^a_\mu = c^{abc} \dfrac{\delta \mathcal{L}}{\delta(\partial_\mu W^b_\nu)} W_c{}^\nu$ where $W^a_\mu \to W^a_\mu + c^{abc}\theta^b W^c_\mu$.

Example: The Salam-Weinberg model ($\theta_W = 0$)

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2}(D_\mu\phi)^+(D^\mu\phi) - \frac{\lambda}{4}(\phi^+\phi - \mu^2)^2$$

$\phi = \begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix}$ is a doublet of complex fields and the gauge group is SU(2). In the unitary gauge $\phi(x) = \begin{pmatrix} h(x) \\ 0\end{pmatrix}$, $h^+(x) = h(x)$ and

$$\mathcal{L}^U = -\frac{1}{4}\hat{F}^a_{\mu\nu}\hat{F}^{a\mu\nu} + \frac{1}{2} M_W^2 \hat{W}^2 + \frac{1}{2}(\partial_\mu h)^2 + \text{etc}$$

where $\langle h \rangle = \mu$ and $M_W^2 = \frac{1}{2} g^2\mu^2$. Does this contradict our claim, since the W^1, W^2, W^3 form a degenerate triplet $\Longrightarrow$ SU(2) global symmetry? The answer is no, since for an SU(2) gauge group, if $\phi \to \Omega(x)\,\phi$ then also $\phi_c \to \Omega(x)\,\phi_c$ where

$$\phi_c = \begin{bmatrix} -\phi_2^+ \\ \phi_1^+ \end{bmatrix} \quad \text{(charge conjugate doublet)} .$$

Hence the 2 x 2 matrix

$$\Sigma = \begin{pmatrix} \phi_1 & -\phi_2^+ \\ \phi_2 & \phi_1^+ \end{pmatrix}$$

transforms like $\Sigma \to \Omega(x)\,\Sigma$. However it also has a global SU(2) symmetry under the transformation

$$\phi \to a\phi + b\phi_c \quad ; \quad |a|^2 + |b|^2 = 1$$

which can be generated by

$$\Sigma \to \Sigma U^+ \quad \text{where } U \in SU(2)_{\text{global}} .$$

Hence $\Sigma \to \Omega(x)\,\Sigma\, U_L^+$, i.e. like a (2,2) representation of

$$SU(2)_{loc} \otimes SU(2)_{global} .$$

[illegible] and spontaneous symmetry breaking corresponds to a massless triplet of $\pi^{i,0}$ Goldstone bosons.

ii) The existence of a hidden "$SU(2)_L$" global symmetry in the Salam-Weinberg theory gives us hope of finding a suitable preon model leading to the electroweak theory at scales less than 1 TeV.

What are the lessons we have learned from the proceeding remarks.

1) The physical spectrum is seen in the unitarity gauges, in which spin-1 particles can carry global symmetry charges. In this respect an effective Lagrangian involving spin-1 bound states is like one involving spin-0 or spin-1/2. At low energies it will give a realization of current algebra, which we will come to later. However in the unitarity gauge $\mathcal{L}^{U}_{eff}$ is not the most general effective Lagrangian involving spin-1 particles we could have written down. It is in fact by construction only the one which could arise from a spontaneously broken gauge symmetry. The fact that the non-gauge couplings happen to be small must be a dynamical property of the preon theory. We shall show that this will necessarily follow from the Wilson OPE of the Noether currents of $\mathcal{L}_{preon}$, if the scales are as we supposed in the introduction, namely $M_G^2,\ \Lambda^2_{ASF} \ll \Delta M^2 \sim r_0^{-2}$.

2) If we were to ask what is the form of the field W (in the gauge invariant form of $\mathcal{L}_{eff}$) in terms of preons this may be a meaningless question, since W is a complex transformation of the fields $\hat{W}$ and h. However $\hat{W}_\mu$ can be loosely associated with $\bar{f}\gamma_\mu f$ of our preon theory.

Let us end these remarks by constructing the Noether currents of our supposed $\mathcal{L}_{eff}$, which we take to be the Salam-Weinberg Lagrangian (however dropping for the moment the Abelian factor):

$$\mathcal{L}^{inv}_{eff} = -\tfrac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} + \mathrm{Tr}\,(D^\mu\Sigma_L)^+ (D_\mu\Sigma_L) + V(\Sigma^+_L \Sigma_L) + \sum_i (\bar{\psi}^i_L \not{D} \psi^i_L + \bar{\psi}^i_R \not{\partial} \psi^i_R)$$

$$W_\mu \to \Omega(x)\, D_\mu(W)\, \Omega^+(x) \quad ; \quad \Omega(x) \in SU(2)_{Loc}$$

$$\Sigma_L \to \Omega(x)\, \Sigma_L\, U^+_L \quad ; \quad U_L \in SU(2)_L$$

$$\psi^i_L \to \Omega(x)\, \psi_L$$

$$\psi_R \to U_R\, \psi_R \qquad U_R \in SU(2)_R$$

$$(\Sigma_L)_{ij} \to h\, \mathbb{1}_{ij}$$

and

$$\langle \Sigma_L \rangle = \langle h \rangle \neq 0$$

$$\hat{\Sigma}_L = \Lambda^+(x)\, \Sigma_L = h\, \mathbb{1}$$

$$\hat{W} = \Lambda^+(x)\, D_\mu(W)\, \Lambda(x)$$

$$\hat{\psi}_L = \Lambda^+(x)\, \psi_L$$

$$\hat{\psi}_R = \psi_R$$

What remains is the following $SU(2)_L \otimes SU(2)_R$ transformation:

$\hat{W} \to U_L \hat{W} U^+_L$	corresponding to $SU(2)_L$ triplet
$\hat{\psi}_L \to U_L \psi_L$	corresponding to $SU(2)_L$ doublet
$\hat{\psi}_R \to U_R \psi_R$	corresponding to $SU(2)_R$ doublet
$h \to h$	corresponding to $SU(2)_L \times SU(2)_R$ singlet

In the unitarity gauge

$$\mathcal{L}^{U}_{eff} = -\tfrac{1}{4} \hat{W}^a_{\mu\nu} \hat{W}^{a\mu\nu} + \tfrac{1}{2} M_W^2 \hat{W}^2 + (\partial_\mu H)^2 + M_H^2 H^2 + (M_W H + \tfrac{g^2}{4}) \hat{W}^2 + \bar{\psi}_L \not{D} \psi_L + \bar{\psi}_R \not{\partial} \psi_R$$

The symmetry currents can be obtained in the ways illustrated schematically by the following diagram:

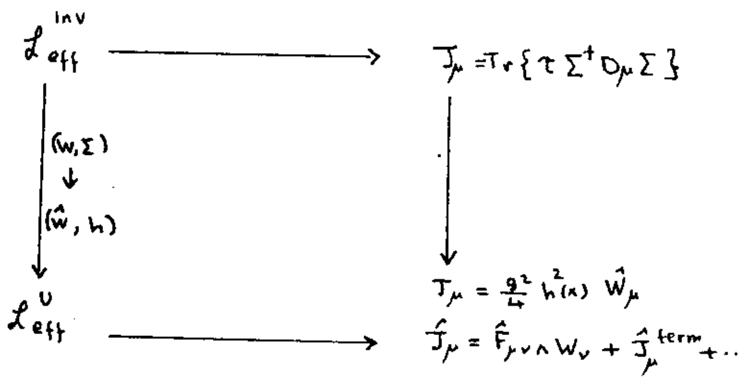


Fig.2

The two currents J_μ , $\hat{J}_\mu$ in Fig.2 are related by the equation of motion

$$\partial_\mu \hat{F}_{\mu\nu} + g\, \hat{W}_\mu \wedge \hat{F}_{\mu\nu} = \hat{J}_\nu^{ferm} + \cdots$$

This implies a current field identity of the Lee-Zumino type, namely

$$J^a_{L,\mu} = \frac{M_W^2}{g} W^a_{L,\mu} + \left[M_W H(x) + \frac{g^2}{4} H^2(x) \right] W^a_{L,\mu}$$

Note: a) This current gives a realization of local current algebra; b) unlike Lee-Zumino we have a renormalizable $\mathcal{L}_{eff}$ with a current field identity. Hence summarizing we note that the left and right-handed currents in the above simple model are given by very different structures, namely

$$J^a_L = \text{Const.} \times W^a_\mu + \cdots$$

and $$J^a_R = \bar{\psi}_R \gamma \tau^a \psi_R$$

in which we note ψ^i_L decouple from the current J_L via the equation of motion.This [illegible] which measure the breakdown of the L-R symmetry, i.e. based on the two-point function

$$\Delta = i\int d^4x\, e^{iq\cdot x} \langle 0 | T\{ J_L(x) J_L(0) - J_R(x) J_R(0) \} | 0 \rangle \quad .$$

C. HOW DOES $\mathcal{L}_{preon}$ "TALK" TO $\mathcal{L}_{eff}$

The existence of a set of conserved currents with the properties:

i) The corresponding charges $Q^a = \int d^3x\, J_0^a$ are generators of the $SU(2)_L \times SU(2)_R \times U(1)$ global symmetry;

ii) they satisfy a local current algebra

$$[J_0^a(x), J_\nu^b(y)]_{ET} = C^{abc} J^c_\nu(x)\, \delta^{(3)}(x-y) \quad ;$$

iii) their products at short distances can be estimated by the Wilson OPE and $\mathcal{L}_{preon}$ perturbation theory

$$J^a(x) J^b(0) \underset{x^2 \to 0}{=} C^{abc}(x^2, g) J^c(0) + \cdots ;$$

iv) matrix elements involving the bound states of $\mathcal{L}_{preon}$ or the vacuum and chronological products of these currents are causal.

Implies:

a) a set of Ward identities and low energy theorems,

b) a set of anomalous Ward identities

c) a large class of light cone sum rules and superconvergence relations.

More precisely we have:

(a) Normal Ward identities

If $$\Gamma_\mu = i\int dx\, e^{iq\cdot x} \langle A | T\{ J_\mu(x)\, O_p(0) \} | B \rangle$$

then

$$q^\mu \Gamma_\mu = \langle A | [Q, \mathcal{O}_5(0)] | B \rangle$$

(b) Anomalous Ward identity

For $SU(2)_L \otimes SU(2)_R \otimes U_V(1)$ one has

$$q^\omega \int dx\, dy\, e^{-i((q-k)x + ky)} \langle 0 | T \{ V_\mu^a(x) J_\nu^{U(1)}(y) A_\omega^b(0) \} | 0 \rangle$$

$$= \delta^{ab} \frac{N_{MC}}{\pi} \epsilon_{\mu\nu\alpha\beta}\, q^\alpha k^\beta$$

where the gauge group of $\mathcal{L}_{preon}$ is $SU(N_{MC})$.

(c) Light cone sum rules

Define

$$T_{AB}(q, P, \cdots) = \int dz\, e^{iq.z} \langle A | T \{ J(\tfrac{z}{2}) J(-\tfrac{z}{2}) \} | B \rangle$$

$$= \Gamma^i_{AB}(q, P)\; T^i(q^2, q.P, \cdots)$$

then

$$\underset{Q^2 \to \infty}{Lt} \frac{1}{\pi} \int \frac{ds}{s + Q^2} \, Im\, T^i(s, \omega, \cdots) = \left(\frac{1}{Q^2}\right)^d \sum_n C_n(Q^2, g) \langle A | O_n | B \rangle \times P_n(\omega, \cdots)$$

A,B = W, φ_L, h bound states (in the above example) or the vacuum state.

For the effective Lagrangian $\mathcal{L}_{eff}$ these have the following consequences:

(1) They are automatically satisfied provided we construct $\mathcal{L}_{eff}$ with the right symmetry and we work only at the tree graph levels at vanishing energies. However if $\mathcal{L}_{eff}$ is renormalizable then these Ward identities will be satisfied beyond the tree graph level and over a low energy range $\ll r_0^{-1}$ (the composite or Reggeization scale). However notice that a normal Ward identity (defining $\Gamma_\mu = \langle \cdots T \{ J_\mu \cdots \} \cdots \rangle$)

$$q^\mu \Gamma_\mu = \Delta\Gamma$$

always has the trivial solution $\Gamma_\mu = \Delta\Gamma = 0$, i.e. the whole global symmetry need not be manifest in $\mathcal{L}_{eff}$ and at low energies.

(2) However the anomalous Ward identities have the structure

$$q^\mu \Gamma_{\mu \cdots} = \mathbb{1} \qquad \forall\ q ,$$

where

$$\Gamma_{\mu \cdots} = \langle 0 | T \{ J_\mu \; J_\cdot \; J_\cdot \} | 0 \rangle$$

Hence the anomalies tell us which part of the spectrum must necessarily be present in $\mathcal{L}_{eff}$.

In particular, if the chiral symmetry of $\mathcal{L}_{preon}$ remains unbroken then they tell us a lot about the spectrum of massless fermion bound states. The exercise of matching the anomaly as seen by $\mathcal{L}_{preon}$ and $\mathcal{L}_{eff}$ is called the 't Hooft consistency condition and it must necessarily be satisfied. In the class of preon models we are considering, the 't Hooft condition will tell us something about $\mathcal{L}_X$. For example, if we follow Pati and Salam and let $\mathcal{L}_X$ involve scalar preons, which can form with the fermions f_L^i, the bound states $\psi_L^i = C^+ f_L^i$, then we trivially map the flavour symmetry of $\mathcal{L}_{preon}$ into the fermion sector of $\mathcal{L}_{eff}$. In this case the anomaly matching simply tells us

$$N_D = N_{MC} \quad ,$$

where N_D is the number of degenerate massless fermion doublets in $\mathcal{L}_{eff}$.

(3) The renormalizability and relationship between couplings and masses come from the light cone sum rules. Consider

$$\Pi(q^2) = \int dx\, e^{iq\cdot x} \langle T\{J(x)J(0)\}\rangle$$

then

$$\Pi(-Q^2) = \frac{1}{\pi}\int \frac{ds}{s+Q^2}\,\mathrm{Im}\,\Pi(s) + \text{up to possible subtractions.}$$

From OPE

$$J(x)J(0) \underset{x^2\to 0}{\simeq} C_0 \mathbb{1} + C_1 x^4 J_1 + \cdots + C_n O_n$$

This implies

$$\Pi(-Q^2) \sim C_0 \mathbb{1} + \frac{C_1}{Q^4} + \cdots$$

Now if $\mathcal{L}_{eff}$ is renormalizable

$$\mathrm{Im}\,\Pi(s) = A_0 \mathbb{1} + \frac{A_1}{s} + \frac{A_2}{s^2} + \cdots$$

and

$$\underset{Q^2 \text{ large}}{\mathrm{Lt}} \int \frac{ds}{s+Q^2}\,\mathrm{Im}\,\Pi(s) = A_0\mathbb{1} + \frac{A_1}{Q^2} + \cdots \text{ up to Logs}$$

However if $\mathcal{L}_{eff}$ is un-renormalizable then $\mathrm{Im}\,\pi(s) = \Sigma_k B_k s^k$ and

$$\therefore \int \frac{ds}{s+Q^2}\,\mathrm{Im}\,\Pi(s)$$

is not defined. After making subtractions

$$\Pi(-Q^2) = B_k (Q^2)^k \int^{r_0^{-2}} \frac{ds}{s^k(s+Q^2)}\,\mathrm{Im}\,\Pi(s)$$

Hence $B_k \sim (r_0^2)^k$ in order to reproduce OPE results in the energy scale $M_G \ll Q^2 \ll r_0^{-2}$.

D. THE PRICE OF BREAKING L-R SYMMETRY THROUGH PREON CONDENSATES

We assume (without any a priori justification) that the L-R symmetry of $\mathcal{L}_{preon}$ is broken spontaneously by preon condensates. However in order that our fermions remain massless we want to preserve the chiral symmetry. The only CP invariant condensates with this property are dimension 6, four preon condensates. From the Wilson OPE of $J_L(x)\, J_L(0) - J_R(x)\, J_R(0)$, the appropriate operator or order parameter is associated with the diagram

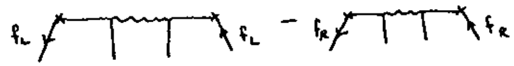


Fig.3

i.e $$\mu^6 = \langle :\bar{f}_L \gamma_\mu T^A_{HC} f_L\, \bar{f}_R \gamma^\mu T^A f_L : - : \bar{f}_R \gamma^\mu T^A f_R\, \bar{f}_R \gamma_\mu T^A_{HC} f_R : \rangle_{VAC}.$$

This means that if we examine the superconvergent sum rule

$$T_{\mu\nu}(q) = i\int d^4z\, e^{iq\cdot z} \langle 0|T\{J_{L,\mu}(z)J_{L,\nu}(0) - J_{R,\mu}(z)J_{R,\nu}(0)\}|0\rangle$$

$$= (g_{\mu\nu}q^2 - q_\mu q_\nu)\,\Pi(q^2)$$

Then $$\underset{Q^2\to\infty}{\mathrm{Lt}} \frac{1}{\pi}\int \frac{ds}{s+Q^2}\,\mathrm{Im}\,\Pi(s) = \mu^6/Q^6.$$

392

The L-R sum rule: Summary

Consider

$$\Pi_{\mu\nu} = i\int d^4x\, e^{iq\cdot x}\langle 0|T\{J_{L,\mu}(x)J_{L,\nu}(0) - J_{R\mu}(x)J_{R\nu}(0)\}|0\rangle$$

$$= (g_{\mu\nu}q^2 - q_\mu q_\nu)\,\Pi(q^2)$$

From the preon theory and the Wilson OPE ($J_LJ_L - J_RJ_R \sim x^4\, \bar f\Gamma^V f\, \bar f\Gamma^A f$) one finds $\Pi(-Q^2) = \Pi_{LL}(-Q^2) - \Pi_{RR}(-Q^2) \sim \mu^6/Q^6$ as $Q^2\to\infty$. This implies the following sum rule or superconvergence relation:

$$\frac{1}{\pi}\int \frac{ds}{s+Q^2}\, \mathrm{Im}\,\Pi(s) \to \mu^6/Q^6 \qquad \text{as } Q^2\to\infty .$$

From which we have

$$1)\ \int ds\ \mathrm{Im}\,\Pi(s) = 0\ ; \qquad 2)\ \int ds\ s\ \mathrm{Im}\,\Pi(s) = 0$$

$$\mathrm{Im}\,\Pi(s) = c_0 + \frac{c_1}{s} + \frac{c_2}{s^2} + \frac{c_3}{s^3} + \cdots$$

with

$$3)\ c_0 = 0; \qquad 4)\ c_1 = 0; \qquad 5)\ c_2 = 0 .$$

If we use the effective Lagrangian discussed earlier, except that we add an additional $SU(2)_L$ scalar multiplet $\Sigma_L^{(2)} = \sigma\mathbb{1} + i\,\pi\cdot\vec\tau$, then to lowest in g one obtains the following contributions (see Fig.4). Thus to order O(1) in g one obtains the following contributions:

$$\mathrm{Im}\,\Pi^{WH} = \frac{1}{96\pi}\,\frac{\Delta^{1/2}(s,M_W^2,M_H^2)}{s}\left[\frac{\Delta(s,M_W^2,M_H^2)+12M_W^2 s}{(s-M_W^2)^2}\right]\theta(s-(M_W+M_H)^2)$$

$$\mathrm{Im}\,\Pi^{\pi\pi} = \frac{1}{96\pi}\left[1-\frac{4M_\pi^2}{s}\right]^{1/2}\left[1-\frac{M_W^2}{s-M_W^2}\right]^2\theta(s-4M_\pi^2)$$

$$\mathrm{Im}\,\Pi^{\pi\sigma} = \frac{1}{96\pi}\left(\frac{s}{s-M_W^2}\right)^2\frac{\Delta^{3/2}(s,M_\pi^2,M_\sigma^2)}{s^3}\,\theta(s-(M_\pi+M_\sigma)^2)$$

$$\mathrm{Im}\,\Pi^{4_L\bar 4_L + W} = \frac{N_D}{24\pi}\left(\frac{M_W^2}{s-M_W^2}\right)^2\theta(s) + \left(\frac{M_W^2}{g}\right)^2\pi\,\delta^+(s-M_W^2)$$

$$\mathrm{Im}\,\Pi^{4_R\bar 4_R} = -\frac{N_D}{24\pi}\,\theta(s)$$

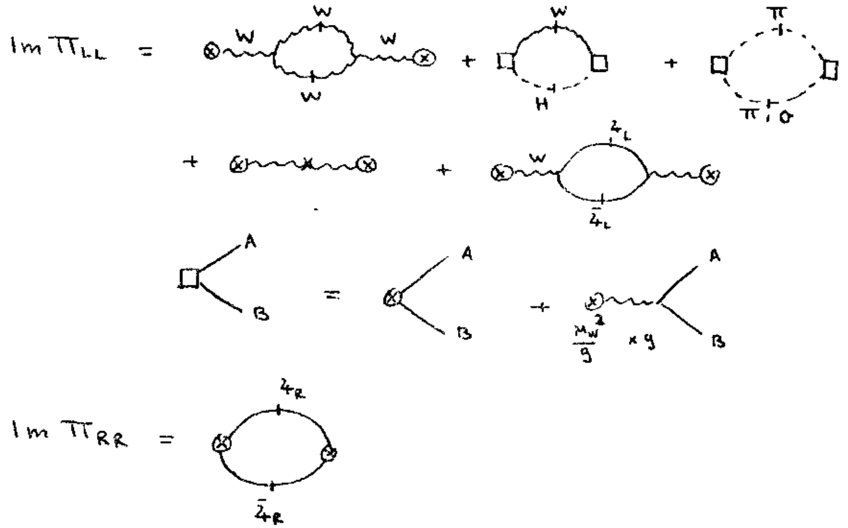


Fig.4

The asymptotic expansions are summarized in the following table:

state	O(1)	$O(M^2/Q^2)$	$O(M^4/Q^4)$
W-W	1	$16M_W^2$	$-71M_W^4$
W-H	1	$11M_W^2 - 3M_H^2$	$12M_W^2 - 18M_W^2M_H^2 + 3M_H^2$
π-π	1	$-2M_W^2 - 6M_\pi^2$	$-M_W^4 + 6M_\pi^2 + 12M_\pi^2M_W^2$
π-σ	1	$2M_W^2 - 3M_\pi^2 - 3M_\sigma^2$	$3M_W^4 + 3M_\pi^2 + 3M_\sigma^4$ $-6M_W^2(M_\pi^2 + M_\sigma^2)$
$W+\bar{\psi}_L\psi_L$	0	0	$4N_DM_W^4$
$\bar{\psi}_R\psi_R$	$-4N_D$	0	0

Table I

For a single doublet, we have the mass formulae:

1) $M_H^2 + M_\sigma^2 + 3M_\pi^2 = 9M_W^2$

2) $3M_H^4 + 3M_\sigma^4 + 9M_\pi^4 - 18\, M_W M_H^2 - 6M_W^2 M_\sigma^2 M_\pi^2 - 53M_W^4 = 0$.

Note, if we keep on adding scalar multiplets Σ^i, then the mass formulae would read:

1) $M_H^2 - 9\, M_W^2 + N_D\bar{M}^2 = 0$

2) $3M_H^2 - 18M_W^2M_H^2 - 53M_W^2 + 12N_D^2\bar{M}^4 - 6N_D M_W^2\, \Delta M^2 = 0$

plus two other constraints. This means that the sum rules would put limits on N_D .

The left-right symmetric model

[illegible] constraints, the following effective Lagrangian exhibiting a symmetry in the left and right-handed degrees of freedom seems an appropriate one to examine. We assume that $\mathcal{L}_{preon}$ leads to

$$\mathcal{L}_{eff}^{inv.} = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} W'^a_{\mu\nu} W'^{a\mu\nu} + (D^\mu\Sigma_L)^\dagger (D_\mu \Sigma_L)$$
$$+ (D^\mu\Sigma_R)^\dagger (D_\mu\Sigma_R) + V(\Sigma_L, \Sigma_R)$$
$$+ i\,\bar{\varphi}_L \not{D}\, \varphi_L + i\,\bar{\varphi}_R \not{D}'\, \varphi_R + \text{Yukawa couplings}$$

where the symmetry is defined by

$$\Sigma_L \to \Omega(x)\, \Sigma_L(x)\, U_L^\dagger \quad ; \quad \varphi_L \to \Omega(x)\, \varphi_L(x)$$
$$\Sigma_R \to \Omega'(x)\, \Sigma_R(x)\, U_R^\dagger \quad ; \quad \varphi_R \to \Omega'(x)\, \varphi_R(x) \; .$$

Now if we either break the L-R symmetry in $V(\Sigma_L,\Sigma_R)$ softly or induce it by loop corrections a la Coleman-Weinberg (Ref. Cvetic, Maryland preprint 1983), so that the stable minimum corresponds to

$$\langle \Sigma_R \rangle \gg \langle \Sigma_L \rangle \neq 0$$

then this leads to the following physical spectrum in the unitary gauge:

Particle	Mass	Global symmetry classification
W_L	M_L	spin 1 $SU(2)_L$ triplet
W_R	M_R	spin 1 $SU(2)_R$ triplet
$\varphi_L^{(i)}$	0	fermion $SU(2)_L$ doublet
$\varphi_R^{(i)}$	0	fermion $SU(2)_R$ doublet
h_L	m_L	scalar $SU(2)_L \times SU(2)_R$ singlet
h_R	m_R	scalar $SU(2)_L \times SU(2)_R$ singlet

In this case the parity violating sum rule gives the following contributions to the L-R sum rule, summarized in the table below.

Table: Asymptotic contribution for the symmetric model

States	$\frac{1}{96\pi}$	$\frac{1}{96\pi}\ \frac{1}{Q^2}$	$\frac{1}{96\pi}\ \frac{1}{Q^4}$
$W_L W_L$	1	$16M_L^2$	$-71M_L^4$
$W_L H_L$	1	$11M_L^2 - 3m_L^2$	$12M_L^4 - 18M_L m_L + 3m_L^4$
$\bar{\varphi}_L \varphi_L$	0	0	$4N_D M_L^4$
$W_R W_R$	-1	$-16M_R^2$	$-71M_R^4$
$W_R H_R$	-1	$-11M_R^2 + 3m_R^2$	$-12M_R^4 + 18M_R m_R - 3m_R^4$
$\varphi_R \varphi_R$	0		$-4N_D M_R^4$

Table II

This leads to the following mass formulae: ($M(W_L) = M_L$, $M(H_L) = m_L$ etc.)

1) $m_R^2 - m_L^2 = 9\ (M_R^2 - M_L^2)$

2) $(59-4N)\ M_L^4 + 18M_L^2 m_L^2 - 3m_L^4 = (59-4N)\ M_R^4 + 18\ M_R^2 m_R - 3m_R^4$.

1) and 2) lead to the following bound from the positivity of m_L^2:

$$M_R^2 < \left(\frac{151 - 2N_D}{11 + 2N_D}\right) M_L^2$$

Note for $N_D = 1$, $M_R < 3.4\ M_L$; while for $N_D = 4$, $M_R < 2.7\ M_L$.

4. MODELS WITH A GLASHOW-SALAM-WEINBERG $SU(2)_L \otimes U(1)$ STRUCTURE

Consider

$$\mathcal{L}_{preon} \longrightarrow \mathcal{L}_{preon} + \text{elementary } U(1),$$

i.e. where we couple weakly an elementary Abelian field; then the symmetry is broken softly according to

$$SU(2)_L \otimes SU(2)_R \otimes U_V(1) \longrightarrow U(1)_{I_{3L}} \otimes U(1)_{I_{3R}} \otimes U_V(1) .$$

Note

i) The symmetry currents of $\mathcal{L}_{preon}$ still develop ABJ anomaly required for massless fermions;

ii) One couples the elementary U(1) gauge field B_μ to preon fields $f^i_{L,R}$ so that associated charge is identified with $Q_f = I_3$. I_3 being the generator of the diagonal sub-group $SU(2)_{L+R}$.

The Lagrangians we thus start with take the form

$$\mathcal{L}_{preon} = -\frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \sum_{i=1}^{2} (\bar{f}^i_L \not{D}_{ij} f^j_L + \bar{f}^i_R \not{D}_{ij} f^j_R) + \mathcal{L}_X ,$$

where

$$(D_\mu)_{ij} = D_\mu^{H.C}\, \mathbb{1}_{ij} + g' \tau_{3ij} B_\mu .$$

$\mathcal{L}_X$ can also carry U(1) charges. In $\mathcal{L}_{eff}$ the rule for introducing B_μ field is simply obtained by modifying the basic transformation to:

$$\Omega_{ij}(x) \rightarrow e^{iQ\omega(x)}\, \Omega_{ij}(x)$$

$$U_{L,R} \rightarrow e^{i(Q + \tau_3)\omega(x)}$$

where Q = a central charge. Hence if i) $\Sigma \rightarrow \Omega(x)\, \Sigma\, U_L^+$ then the covariant derivative is given by

$$D_\mu \Sigma = (\partial_\mu - ig \frac{\tau^a}{2} W^a_\mu) \Sigma + ig' \Sigma \tau_{3/2} B_\mu .$$

Similarly if

ii) $\varphi \to \Omega(x)\varphi$ then

$$D_\mu \varphi = (\partial_\mu - i g W_\mu + i g' Q B_\mu)\varphi$$

Finally,

iii) If $\chi \to U_{L,R}\chi$ then $D_\mu = \partial_\mu - i g' (Q + \tau_3/2) B_\mu$.

Let us turn to the symmetry-breaking pattern: If

$$\langle \Sigma \rangle = \mu \, \mathbb{1}$$

then

$$\frac{1}{4} Tr \{ (D^\mu \Sigma)^\dagger (D_\mu \Sigma) \} = \frac{1}{8} g^2 \mu^2 [(W^1_\mu)^2 + (W^2_\mu)^2] + \frac{\mu^2}{8} [g W^3_\mu - g' B_\mu]^2$$

$$= \frac{1}{2} M_W^2 [(W^+_\mu)^2 + (W^-_\mu)^2] + \frac{1}{2} M_Z^2 (Z_\mu)^2$$

i.e. precisely the Salam-Weinberg mixing

$$Z_\mu = \cos\theta \, W^3_\mu - \sin\theta \, B_\mu$$

$$A_\mu = \sin\theta \, W^3_\mu + \cos\theta \, B_\mu$$

$$\cos\theta = \frac{g}{\sqrt{g^2 + g'^2}} \quad ; \quad \sin\theta = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$M_W^2 = \frac{1}{4} g^2 \mu^2 \quad ; \quad M_Z^2 = \frac{1}{4} G^2 \mu^2 ; \quad G = \sqrt{g^2 + g'^2}$$

$$\mathcal{L}_{eff} \Big|_{fermion} = K.E. + \sum_{r = \pm} W^{(r)}_\mu J^{(r)\mu}_{c.c} + Z_\mu J^\mu_{N.C} + A_\mu J^\mu_{em} .$$

For the prototype model one has:

$$J^{\pm\mu}_{c.c} = g \bar{\varphi}_L \gamma^\mu \frac{\tau^\pm}{2} \varphi_L$$

$$J^\mu_{N.C} = G [- \sin^2\theta \, \bar{\varphi} \gamma^\mu Q_f \varphi + \bar{\varphi}_L \gamma^\mu \frac{\tau^3}{2} \varphi_L]$$

$$J^\mu_{em} = e [\bar{\varphi} \gamma^\mu Q_f \varphi] .$$

While for a L-R symmetric model with only W_L one has

$$J^{\pm\mu}_{c.c} = g \bar{\varphi}_L \gamma^\mu \frac{\tau^\pm}{2} \varphi_L + g \bar{\Phi}_R \gamma^\mu \frac{\tau^\pm}{2} \Phi_R$$

$$J^\mu_{N.C} = G [- \sin^2\theta \, \bar{\varphi} \gamma^\mu Q_f \varphi + \bar{\varphi}_L \gamma^\mu \frac{\tau^3}{2} \varphi_L]$$

$$J^\mu_{em} = e [\bar{\varphi} \gamma^\mu Q_f \varphi + \bar{\Phi} \gamma^\mu Q_f \Phi]$$

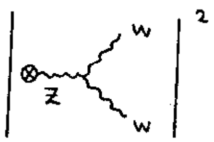
$\Phi_{L,R}$ is a heavy right-handed mirror of $\varphi_{L,R}$.

Spectrum in the latter model is:

W^+	M_W
Z	$M_Z = M_W / \cos\theta$
γ	0
φ^i	0
Φ^i	M_i
$\Sigma = (\sigma, \pi)$	$m_\sigma = m_\pi = m$

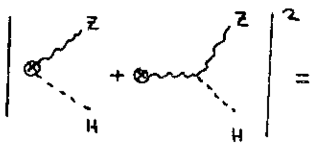
In the latter Σ transforms like a (2,2) representation of $SU(2)_{LOC} \otimes SU(2)_R$. In evaluating the parity violating sum rule to lowest order in G we have the following contributions:

1) W-W intermediate state to $\mathrm{Im}\ \pi_{LL}$



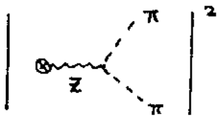
$$= \frac{1}{96\pi}\left[1-\frac{4M_W^2}{s}\right]^{1/2}\frac{1}{(s-M_Z^2)^2}\left[s^2+16sM_W^2-68sM_W^4-48M_W^6/s\right] \times \theta(s-4M_W^2)$$

2) Z-H intermediate state to $\mathrm{Im}\ \pi_{LL}$



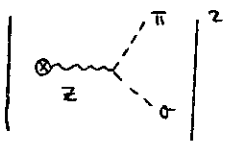
$$= \frac{1}{96\pi}\,\frac{\Delta^{1/2}(s,M_Z^2,M_H^2)}{s}\left[\frac{\Delta^{1/2}(s,M_Z^2,M_H^2)+12M_Z^2 s}{(s-M_Z^2)^2}\right] \times \theta(s-(M_Z+M_H)^2)$$

3) π-π intermediate state to $\mathrm{Im}\ \pi_{LL}$



$$= \frac{1}{96\pi}\left(\frac{M_Z^2\cos 2\theta}{s}\right)^2\theta(s-4M_\pi^2) + O\left(\frac{1}{s}\right)^3$$

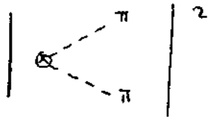
4) π-σ intermediate state to $\mathrm{Im}\ \pi_{LL}$



$$= \frac{1}{96\pi}\left(\frac{M_Z^2\cos 2\theta}{s}\right)^2\theta(s-(M_\sigma+M_\pi)^2) + O\left(\frac{1}{s}\right)^3$$

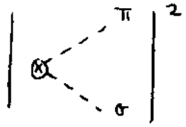
396

5) π-π intermediate state to $\mathrm{Im}\ \pi_{RR}$



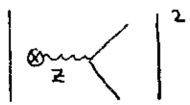
$$= \frac{1}{96\pi}\left[1-\frac{4M_\pi^2}{s}\right]^{3/2}\theta(s-4M_\pi^2)$$

6) π-σ intermediate state to $\mathrm{Im}\ \pi_{RR}$



$$= \frac{1}{96\pi}\,\frac{\Delta^{3/2}(s,M_\pi^2,M_\sigma^2)}{s^3}\,\theta(s-(M_\pi+M_\sigma)^2)$$

7) Light fermion contribution to $\mathrm{Im}\ \pi_{LL}$

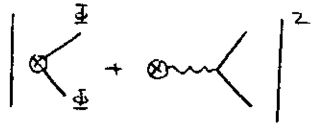


$$= \frac{1}{24\pi}\,\frac{M_Z^4}{s^2}\left[1-2\sin^2\theta\,\mathrm{Tr}\{T_3 Q_f\}+4\sin^4\theta\,\mathrm{Tr}\{Q_f^2\}\right] \times \theta(s)$$

8) Light fermion contribution to $\mathrm{Im}\ \pi_{RR}$

$$= \frac{1}{24\pi}\,\theta(s)$$

9) Heavy fermion contribution to $\mathrm{Im}\ \pi_{LL}$



$$= \frac{1}{24\pi}\left[1-\frac{4M^2}{s}\right]^{1/2}\left[\left[(1+\rho^2)\left(1-\frac{M^2}{s}\right)+\frac{4M^2}{s}\rho\right]+8\left(1-\frac{2M^2}{s}\right)\rho(1-\rho)\sin^2\theta\,\mathrm{Tr}\left\{Q_f\frac{\tau^3}{2}\right\} + 16\rho^2\sin^4\theta\,\mathrm{Tr}\{Q_f^2\}\right]$$

where $\rho = M_Z^2/(S - M_Z^2)$.

These contributions are summarized in the following table, from which we again [illegible] that there is no restriction on the number of fermion doublets, from the order 1 contribution.

Intermediate state	$\frac{1}{96\pi}$	$O(M^2/s)$	$O(M^4/s^2)$
W-W	+1	$2M_Z^2 + 16M_W^2$	$3M_Z^4 - 102M_W^4 + 28M_W^2M_Z^2$
W-H	+1	$11M_Z^2 - 3M_H^2$	$12M_Z^4 - 18M_Z^2M_H^2 + 3M_H^4$
π-π	-1	$6M_\pi^2$	$-6M_\pi^4 + M_Z^4\ (\cos 2\theta)^2$
π-σ	-1	$3M_\sigma^2 + 3M_\pi^2$	$-3(M_\sigma^4 + M_\pi^4) + M_Z^4\ (\cos 2\theta)^2$
Light fermions	$-4N_D$	0	$M_Z^4\ N_D\ [1-4\ \sin^2\theta \mathrm{Tr}\{Q_f\ \frac{\tau^3}{2}\} + 4\sin^2\ \mathrm{Tr}\{Q_f^2\}$
Heavy fermions	$+4N_D$	$[4M_Z^2\ \sin^2\theta \mathrm{Tr}\{Q_F\ \frac{\tau^3}{2}\} - 3M^2]\ N_D$	$[M_Z^4\ [1+4\sin^4\theta \mathrm{Tr}\ \{Q_F^2\}] - 6M_Z^2M^2]\ \times N_D$

Table III

The table shows the asymptotic contributions to the parity violating sum rule for L-R symmetric Lagrangian with elementary photon [N_D denotes number l fermion doublets].

From the table we read off in this case the quadratic mass formula:

$$13 M_Z^2 + 16 M_W^2 - 3M_H^2 + 3M_\sigma^2 + 9M_\pi^2 - 3M^2 + 4M_Z^2 \sin^2\theta\ \mathrm{Tr}\{Q_f \tfrac{\tau^3}{2}\} = 0$$

and the following quartic mass formula:

$$[15 + 2(\cos 2\theta)^2 + 2N_D F(\theta)] M_Z^4 - 102 M_W^4 + 3M_H^4 - 3M_\sigma^4 - 9M_\pi^4$$
$$+ 28 M_W^2 M_Z^2 - 18 M_Z^2 M_H^2 - 6M_Z^2 M^2)$$

where

$$F(\theta) = [1 - 2\sin^2\theta\ \mathrm{Tr}\{Q_f\ \tau^3/2\} + 4\sin^2\theta\ \mathrm{Tr}\{Q_f^2\}]\ .$$

[Note added: actually $M_\pi = M_\sigma$ in the model we have considered.]

In addition we have the following constraints coming from $\int \mathrm{Im}\Pi\ ds = 0$ and $\int s\ \mathrm{Im}\bar{\Pi}\ ds = 0$. Together, these comprise a formidable set of mass relations on a preon version of the electro-weak theory.

F. SUMMARY AND PROSPECT OF SUPERSYMMETRIC MODELS

Let me briefly conclude with the following remarks: If one starts with a well-defined framework in which we suppose an asymptotically free preon gauge theory leads to a weakly coupled gauge theory of the Glashow-Salam-Weinberg type as a low energy effective Lagrangian for the ground state sector, then the following points have emerged from our analysis which we hoped to have demonstrated by a few simple examples.

1. There is a definite approximation, in which the weakly coupled effective Lagrangian reproduces the current algebra and short-distance properties of $\mathcal{L}_{preon}$. Further, this approximation is valid only in some well-defined energy region between the ground state mass and scale of compositeness.

2. The particular choice of scales we advocated at the beginning of this talk, namely $M_G^2 \sim \Lambda_{ASF}^2 \ll \Delta M^2 \sim r_0^{-2}$ is the only one for which we see an analytical means of establishing a weakly coupled spontaneously broken gauge as a low energy effective Lagrangian. Here M_G is the ground state mass scale (other than zero mass states), Λ_{ASF} is the asymptotic freedom scale and ΔM^2 is the recurrence scale which is naturally linked to the compositeness scale r_0^{-2}. However since $\mathcal{L}_{preon}$ in principle only has the one scale Λ_{ASF}, one can only expect the numerical constants to be such that $\Delta M^2 \sim r_0^{-2} \gg \Lambda_{ASF}^2$, which is partially true even in QCD. This means that a firm prediction of the class of preon theories we have been discussing is that $\Delta M^2 \sim r_0^{-2}$ will not be too large. In fact since we are supposing $\Lambda_{ASF} \sim M_G \sim 100$ GeV, then we would expect $r_0^{-1} \sim 1$ to 10 TeV. The point being that one would not expect $\Delta M^2/\Lambda_{ASF}^2$ to be much larger than between a factor of 10 to 100.

3. The totality of current algebra, anomaly and sum rule constraints leads to a very restricted class of models, in which there exists numerous mass relations and restrictions on the allowed spectrum of particles.

4. Although we have not at all discussed a supersymmetric version of the class of preon models we have considered, there is one very good reason for doing so. Recall the low energy phenomenology of the $N = 1$ supergravity whose local supersymmetry breaking leads at low energies to a broken super-symmetric $SU(3)_{col} \otimes [SU(2)_L \otimes U(1)]_{broken}$, with definite mass predictions, namely $N_W \sim N_{\widetilde{W}} \sim m_{3/2}$, $m_{\tilde{g}} \sim \alpha_s\, m_{3/2}$ and $m_{\tilde{\gamma}} \sim \alpha\, m_{3/2}$, where $\widetilde{A}$ refers to the spin 1/2 partner to the gauge particle A. (See Pran Nath's talks on SUGR GUTS in this Workshop.) This phenomenology can be reproduced by the following preon model scenario:

$$\mathscr{L}_{preon} \Longleftrightarrow SU(3)_{col} \times U(1) \times SU(N_{MC})_{meta\ colour}$$

where $\mathscr{L}_{preon}$ is globally supersymmetric. Now assume that the meta colour theory has the properties we outlined in this talk and has massive effective gauge boson states, with their superpartners. Further, assume the mass generation, the breaking of L-R symmetry as well as the supersymmetry are all due to the formation of preon condensates. On the other hand, the $SU(3)_{col} \times U(1)$ sector does not have its supersymmetry intrinsically broken. The resulting low energy effective Lagrangian has all the features of the SUGRA GUTS scenario, outlined by Pran Nath. Namely since $SU(N_{MC})_{MC}$ has only one scale Λ_{MC}, then $M_W \sim M_{\widetilde{W}} \sim \Lambda_{MC}$, $\Delta M_W \sim \Lambda_{MC}$. However, since the fermion and scalar bound states of the meta colour theory will be split and both carry the elementary electromagnetic and colour gauge changes, there will be an induced supersymmetry breaking in the $SU(3)_{col} \times U(1)$ gauge sectors due to radiative corrections i.e. $m_{\tilde{g}} \sim \alpha_s \Lambda_{MC}$ and $m_{\tilde{\gamma}} \sim \alpha\, \Lambda_{MC}$. Such a scenario seems to be undistinguishable from the $N = 1$ supergravity theory, and is worth investigating. The machinery we have developed would be very useful here. Further, since $\mathscr{L}_{eff}$ will only apply at low energies, at TeV colliders, the SUGRA GUTS and preon scenarios will begin to depart from one another.

ACKNOWLEDGMENTS

In preparing this talk I have benefitted from discussions with Jogesh C. Pati, Pran Nath and Graham Ross.

REFERENCE

1) Most of the material presented will appear shortly in a preprint with J. Stern, in which a comprehensive list of references to relevant works will be given.

PREONS AND THEIR IMPLICATIONS FOR THE NEXT-GENERATION ACCELERATORS*

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1. In these few lectures, I would like to discuss the following topics:

(i) The need for "preons" as constituents of quarks and leptons.

(ii) Peculiarities of preon dynamics.

(iii) The flavon-chromon preonic models.

(iv) Preons, pre-preons and supersymmetry.

(v) Dynamical symmetry breaking through preons. – Advantages over technicolor theories – a prediction for the sizes of quarks and leptons – experimental signals.

(vi) The issue of chiral symmetry preservation for quantum preon dynamics (QPD). Here I would like to raise the possibility that chiral symmetry is in fact violated by QPD in a manner which is analogous, at least qualitatively, to that in QCD – contrary to commonly held belief.

(vii) The problem of fermion mass hierarchies.

(viii) Open problems.

2. The Need for Preons as Constituents of Quarks and Leptons:

It is useful to judge the need for preonic substructures in the light of the successes of unified gauge theories. Electroweak unification together with QCD, based on the gauge symmetry $SU(2)_L \times U(1) \times SU(3)^c$, is a successful low energy theory. Viewed as a fundamental theory, however, it is inadequate. Many of its inadequacies are removed by the idea of grand unification.[1] In particular, some of the desirable features of grand unification, above and beyond those of $SU(2)_L \times U(1) \times SU(3)^{col}$, are as follows:

(i) It provides a rationale for the existence of quarks and leptons by postulating that these particles are members of one multiplet of a symmetry group G.

(ii) It provides a rationale for the existence of weak, electromagnetic and strong interactions by postulating that these forces are aspects of a single force. It simultaneously accounts for the observed disparity between the strengths of these forces at low energies.

(iii) It provides a reason for the quantization of electric charge and accounts for the fact that it is the electron and the proton with equal and opposite charges, rather than the positron and the proton, which exhibit the same helicity in low energy weak interactions.

(iv) Simplest models of grand unification, subject to single stage descent (i.e. $G \xrightarrow{M} SU(2)_L \times U(1) \times SU(3)^c$) lead to a prediction for the weak angle $\sin^2\theta_W \approx 0.21$, which is in accord with experimental observation.

(v) Over the last decade it has helped remove the psychological inhibition which existed prior to 1973 against the idea of non-conservation of baryon and lepton numbers. Simplest models of grand unification predict proton decay. The judgement on whether this prediction is to be regarded as a definite success will have to await the findings of on-going proton decay searches. At present, the prediction of the minimal SU(5) model seems to be in conflict with the IMB searches. But grand unification through bigger symmetries like SU(16) or SO(10), which are aesthetically more desirable than SU(5), can tolerate longer proton life-times, exceeding 10^{32} years, especially if we permit intermediate mass scales. Even without the observation of proton-decay, there is strong support for baryon non-conservation from cosmology, in that it plays an essential role in the generation of matter-antimatter asymmetry,[2] which we see today.

(vi) Grand unification opens the door for an understanding of many other challenging problems of cosmology; e.g. the horizon, the flatness and the homogeneity problems. Understanding of these problems seems to be best addressed within an inflationary scenario.[3] The final picture regarding these issues is yet to emerge. But it seems that one needs a phase transition in the early universe at a high temperature $\sim 10^{12}$-10^{16} GeV, which grand unification naturally provides.

To sum up, the features (i) - (iv) are positive successes, which a future theory should somehow retain, and the successes of the features (v) and (vi) are yet to be judged. Despite these successes, there are a number of factors which point to the incompleteness of the idea of grand unification. These are:

(a) Proliferation of quarks and leptons without a resolution of the generation puzzle.

(b) Proliferation in the parameters of the Higgs sector.

(c) Proliferation in the Yukawa coupling parameters, which are needed to accomodate the observed bizarre fermion mass spectrum of the three families of quarks and leptons, including Cabbibo-like mixing angles and CP violating phases.

(d) Non-naturalness of the large ratio $(M_X/m_W) \sim 10^{12}$, or the so-called gauge hierarchy problem. Last but not least,

(e) Non-linkage of the electro-nuclear force with gravity.

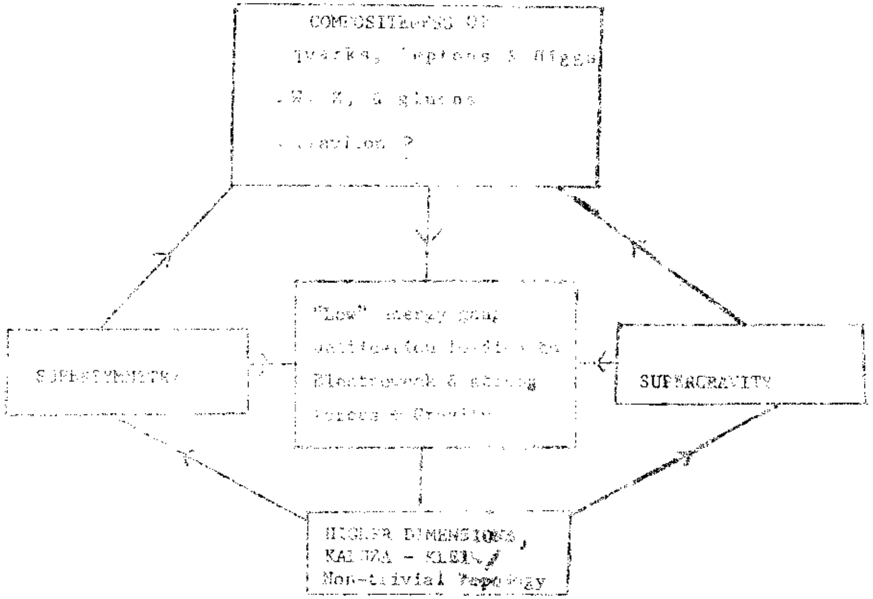
Leaving aside the fundamental question of unification of gravity, with the other forces, the single most important problem, it seems to me, involves an understanding of the Fermi-mass parameters - i.e. $m_e:m_\mu:m_\tau$; m_d-m_u, m_c-m_s, m_t-m_b, (m_e/m_d), (m_μ/m_s), (m_τ/m_b) and Cabibbo-angles etc. This, together with the proliferations listed under (a) and (b), appear to call naturally for a preonic substructure for quarks, leptons as well as the Higgs mesons. Within a preonic theory, one would hope to obtain an answer to the problem of the generation puzzle. One would also expect that the parameters of the Higgs sector including the Yukawa couplings of composite quarks and leptons and the composite Higgs bosons would be related to each other through quantum preon dynamics, just as the parameters of hadrons (i.e. π, K, ρ, N, Λ, N^* etc.) are related to each other through the single Λ-parameter of QCD.

Preons in one form or another also appear to be needed from other considerations. First of all supersymmetry, apart from its inherent beauty, is a concept which suggests itself primarily because it offers a scope for the desired linkage of gravity with the other forces.[4] Local supersymmetry, implemented within the maximal N=8 supergravity theory, provides a truly elegant structure, in which the graviton, gravitinos as well as spin-1, spin-1/2 and spin-o matter belong to one irreducible multiplet of the underlying symmetry.[5] This beautiful theory is yet to be cast in a form to correspond to observation. What is clear now is that if this theory is to describe nature in any way, not all known particles — quarks and leptons — and known forces generated by $SU(2)_L \times U(1) \times SU(3)^c$ can be the basic ingredients of this theory. One must assume that preonic substructures in some form precede the stage of quarks and leptons in terms of elementarity. Finally, higher dimensional Kaluza-Klein theories[6] also suggest the need for preons. This is because, these theories, in their purest form are likely to lead to vector-like four dimensional theories, while weak interactions of quarks and leptons are chiral. The problem may be resolved if we pass to the stage of quarks and leptons and their associated gauge particles (i.e. W_L's and W_R's) through a preceding stage or stages of substructures.[7]

To sum up, we see that the idea of compositeness of quarks, leptons and Higgs bosons (and perhaps even of the gauge particles), seems to be needed (a) to overcome the shortcomings of grand unification and (b) to realize the ideas of supergravity and Kaluza-Klein theories. Ultimately these three ideas may merge with each other in some appropriate fashion to provide one grand picture. This is indicated in a chart in the next page.

If quarks and leptons are composites of "preons", one may, in general, permit the possibility that preons themselves may be composites of more elementary objects — "pre-preons" — and so on. We believe that this chain of increasing elementarity will end, but only when one reaches a stage which is manifestly

economical, elegant and somehow "unique". After all, that is the primary goal of all searches for elementarity. In the context of supergravity or Kaluza-Klein theories, it seems natural that at least some stage of "elementarity" involving either preons, or pre-preons or pre-pre-preons must involve composites of Planck-size. In this case, there may be a hierarchy of sizes spanning from Planck size of 10^{-33} cm up to ~ 1 fermi.[8]



3. <u>A Peculiarity of Preon Dynamics:</u>

Having discussed the need for preons, let us note a very special feature of preon dynamics. The inverse sizes of electrons and muons, from g-2 experiments, exceed about 1/2 TeV, while those of quarks, from deep inelastic eN-scattering and e^-e^+ annihilation, exceed about 100 GeV. Let us assume, for simplifying our discussions, that the inverse sizes of quarks are the same as those of leptons. We notice now the peculiar fact that the inverse sizes of quarks and leptons - if they are composites - are very much greater than their masses:

$$(\Lambda_o)_{q,\ell} \equiv (1/r_o)_{q,\ell} \gtrsim 1/2 \text{ TeV} >> m_{q,\ell} \quad (1)$$

This feature is not encountered before in the history of composite particles, starting from molecules and ending with nucleons. For these latter composites, the inverse sizes are either smaller or atmost comparable to the masses of the corresponding composites (e.g. $r^{-1}_{Nucleon} \sim (2m_\pi \text{ to } m_\rho) \sim m_N$; $r^{-1}_{Nucleus} \sim 100$ MeV $\sim (10)\times$(B.E. of Nucleus) $< M_{Nucleus}$, etc.).

Ordinarily, one may expect that the preon-binding force, while making composites of small size ($r_o \lesssim 10^{-17}$cm), would give a mass of order $(1/r_o)$ to the composites. Compare with QCD, for which chiral symmetry of quarks is broken dynamically by the QCD-force leading to the formation of $\langle \bar{q}q \rangle$ condensate. This gives a dynamical or constituent mass to the quarks of order 300 MeV $\sim m_N/3 \sim (1/r_{Nucleon})$, while the current algebraic mass of the up and down quarks is nearly zero. One needs to understand why the preon binding force makes composites of small size (r_o) without making these composites as massive as order $(1/r_o)$. One idea which was suggested by t'Hooft[9] and is strongly prevalent is that somehow QPD differs characteristically from QCD in that it does not break chiral symmetry dynamically. This hypothesis brings with it the necessary constraint that the anomaly of global

chiral symmetry evaluated at the preonic level should match the same evaluated at the level of the massless composites. An alternative idea, which is recently put forth by Büchmuller, Peccei and Yanagida[10] is that composite quarks and leptons are supersymmetric partners of a set of massless Nambu-Goldstone bosons. We will have more to say about these ideas, and to what extent either one or both may be needed, later.

Whatever be the underlying mechanism which makes small size composites with small masses $m << (\frac{1}{r_o}) \equiv \Lambda_o$, once such composites are formed, consistency would demand that the effective interactions of these composites at momenta which are small compared to Λ_o must be given by a renormalizable lagrangian plus non-renormalizable terms, which should be damped by appropriate inverse powers of Λ_o.

$$\mathcal{L}_{eff} = \mathcal{L}_{Renormalizable} + \frac{1}{(\Lambda_o)^n} \sum \mathcal{L}^{(n)}_{Non-Renormalizable} \quad (2)$$

This consistency requirement, noted by Veltman and others, states that for spin-o, spin-1/2, and spin-1 composites, for which a renormalizable lagrangian can be constructed, it is this renormalizable piece which will govern their effective interactions at low momenta $<< \Lambda_o$. If the composites include "charged" spin-1 particles, the effective interaction must then be given by a Yang-Mills theory based on a local gauge symmetry, which is broken in no other way except spontaneously. In other words, for small size small-mass composites, the low energy effective interaction is not arbitrary, but necessarily rather special. This may be contrasted from the familiar case of QCD, where the masses of [illegible] and W are comparable to their inverse size. As a result, for momenta comparable to or exceeding the masses of these composites — i.e., 1 GeV — the non-renormalizable piece in (12) plays an important role. This brings in, for example, the [illegible] magnetic moment interaction of the neutron and the proton, of order $(1/M_N)$, where $M_N \sim$ [illegible].

Several [illegible] have considered the extreme case that W's and Z's are composites like [illegible], via they are formed [illegible] with their [illegible] nearly equal [illegible] mass. In this case, the [illegible] [illegible] [illegible] which [illegible] comparable [illegible] composites.

[illegible]

[illegible] view, are [illegible] in the sense that their dynamic size are very much [illegible] than their masses $[(\Lambda_c)^{-1}_{W,Z}$ [illegible] $(r_c)_{W,Z} \gg m_{W,Z}^{-1}]$, so the [illegible] momentum effective interactions are given by a renormalizable spontaneously broken gauge theory, [illegible] the first [illegible]

in Eq. (7). This point of view has been [illegible] some time ago.[11]

In the interest of removing ambiguities of quantum gravity, has furthermore been suggested that one may regard the spin-2 graviton itself to be a composite of small size. In this case, one can provide consistency arguments for the graviton to be massless and its effective interaction to be Einsteinian. Since graviton — interaction is not perturbatively renormalizable however, it must be given by the [illegible] of (7). This would [illegible] the Newtonian [illegible] a small [illegible] it is given by [illegible] by the small size r_c of the graviton.[12]

[illegible]

[illegible] which we need for [illegible] low energy physics [illegible]. In the extreme case, the graviton itself [illegible] a composite arising [illegible] theory [illegible] in the ultraviolet region.

In [illegible], we stress that these possibilities become worthy of consideration in the first place because of the peculiarity of [illegible] that it [illegible] to composites of small size

[illegible] r_o [illegible] r_o [illegible]
may well be a hierarchy of sizes of the composites spanning from Planck-size down to 1 fermi. One is thus left with the challenging task of finding an underlying theory, whose dynamics can lead to this peculiar behavior for the sizes versus the masses of the composites ($\frac{1}{r_o} >> m$), with possibly a hierarchy in the sizes of the composites. One needs to solve the dynamics of the theory well enough - perhaps by lattice gauge theory, 1/N or any other method - to be able to see whether the effective interactions of the composites can be pretty and well-behaved, so as to correspond to a renormalizable Yang-Mills theory. At present, this is a feature which we shall simply impose on the basis of consistency arguments only. The task of deriving such a behavior from an underlying dynamics remains a major burden of preonic theories, if the gauge particles of low energy - physics are indeed composites.[15]

4. Flavon-Chromon Preons and Variants:

To facilitate subsequent discussions, let us briefly recall the salient features of the "Flavon-Chromon" preonic model. The model was proposed in 1974 in the same paper where lepton number was suggested as the fourth color[16], and has been developed over the years.[17,18] The set of ideas behind the model has been used subsequently by a number of authors.[19]

In its simplest form, the model assumes that spin-$\frac{1}{2}$ quarks and leptons carrying flavor and color are made of two sets of entities (Preons):[20] (i) the flavons $(f^{i}_{a})_{L,R} = (u,d,\ldots)^{i}_{L,R}$ with spin-$\frac{1}{2}$, which carry flavor but no color and (ii) the chromons

$(C^{i}_{\alpha})^{*} = (r,y,b,\ell)^{(i)*}$ with spin-0, which carry color but no flavor. In other words, quarks and leptons are composites of a fermion (f) and a boson (C^*) carrying flavor and color respectively. Here "i" denotes representation-content with regard to a preonic gauge-symmetry G_b, which generates the preon binding force F_b; the indices "a" and "α" denote flavor and color respectively. The lightest spin-$\frac{1}{2}$ fC^*-composites identified with quarks and leptons, are assumed to be singlets of G_b, or "neutral" with respect to F_b. The small size together with the neutrality of quarks and leptons shields them from exhibiting a trace of the strong binding force F_b at low momenta $<< (1/r_o)$.

A variant of the model introduces three sets of entities: flavons, chromons and Somons ($S_\beta = 1,2\ldots$).[17] Either each of f, C and S have spin-$\frac{1}{2}$, or flavons have spin-$\frac{1}{2}$, but chromons and somons have spin-0. Somons are neutral with respect to flavor and color. Each of these entities are assumed to be non-neutral with respect to the binding force F_b. The representation - contents (or charges) of f, C and S with regard to the gauge-symmetry G_b are such that the lightest fCS-composites, identified with quarks and leptons, are singlets of G_b.

One characteristic feature of the flavon-chromon model is that quarks and leptons of a given family are made of the same flavons; they differ from each other only in respect of their chromon-contents (i.e., red, yellow, blue chromons for quarks versus lilac chromon for leptons). This goes together with the suggestion that lepton number is the fourth color. For the flavon-chromon-boson model quarks and leptons share the same flavons and the same bosons; once again they differ only in respect of their [illegible] contents.

Within this picture, one needs a minimum of two left-handed and two right-handed spin-$\frac{1}{2}$ flavons plus four spin-0 chromons to build the 16-component objects belonging to the [illegible] family.

$$(F_a)_{L,R} = (u,d)_{L,R};\ C=(r,y,b,\ell) \qquad (3)$$

The index with respect to the binding symmetry [illegible] is suppressed. Fermions belonging to the [illegible] and [illegible] families are in general [illegible] quantum pair-excitation excitations of the [illegible]: e.g., F_μ and/or F_τ = [illegible], or [illegible] etc. Alternatively, [illegible] may differ from [illegible] by an intrinsic quantum number, in this case two or [illegible] many [illegible] additional flavons (e.g., $(c,s)_{L,R}$ or $(c,s,t,b)_{L,R}$) may have to be introduced to build the [illegible] and [illegible] families. For the variant involving bosons, the three families may be built by introducing three different bosons S_e, S_μ and S_τ, with flavons and chromons having their minimal structure as in (3). These bosons play the role of "famileons".

As to the nature of the preon-binding force F_b, we shall assume that it is either QCD-like, viz. it is generated by a non-abelian local gauge symmetry (like SU(N)) with a scale parameter $\Lambda_b > 1$ TeV, or it is the simplest gauge force [illegible], i.e., an abelian or dual abelian force like electromagnetism, generated in the latter case by electric and magnetic type charges satisfying

[illegible]), one important feature is that neither flavor nor color gauge forces are introduced as components of the primodial force. These forces are to be viewed as effective residual forces arising at a composite (quark-lepton or preon) level with W, Z and even the gluons being composites of small size in the sense mentioned before. This point of view is necessitated by economy for building blocks and [illegible] at a fundamental level.[11] If the preons are composites of pre-preons, the preon-binding force F_b itself may be an effective residual force rather than primodial. (see remarks later in this regard).

5. Preons, Prepreons and Supersymmetry

In our attempts to construct composite particles and their interactions out of a minimal system of fundamental substructures, which I do not wish to discuss in any detail here, [illegible] and I were led to assume that the fundamental Lagrangian of the preons possess Fermi-Bose symmetry. [illegible] of this work may [illegible] in Ref. 8. Starting with a [illegible] system of pre-preons [illegible] of flavor and color, [illegible] set of preons [illegible] flavor and color [illegible] flavon-chromon model. For [illegible] starting with [illegible] left and right-handed supermultiplets Φ_+ and Φ_- of [illegible] we obtained at the level of [illegible] completion, subject to certain dynamical assumptions, and subsequent to supersymmetry breaking, six spin-$\frac{1}{2}$ [illegible] (left and right-handed) plus twelve spin-0 chromons. The six spin-$\frac{1}{2}$ flavons will give rise to three families [illegible] and the twelve chromons could define an effective [illegible] gauge symmetry which could break dynamically to [illegible] or $SU(3)^{col} \times U(1)_{B-L}$.

[illegible]

flavons plus four spin-0 chromons are needed to build a single family. Note that the fermionic degrees of freedom (2 + 2) precisely matches the bosonic degrees of freedom (4). Barring the possibility that this is merely a coincidence, it appears especially attractive to view flavons and chromons as supersymmetric partners of an underlying theory, e.g.

$$\Phi_{1+} = \binom{u_L}{r},\ \Phi_{2+} = \binom{d_L}{y},\ \Phi_{1-} = \binom{u_R}{b},\ \Phi_{2-} = \binom{d_R}{\ell} \qquad (4)$$

While one can write down a supersymmetric lagrangian involving the coupling of these superfields to local U(1) or U(1) x U(1), or SU(N) gauge fields, assigning the same "charge" or representation to all of them, it is quite clear that unless and until supersymmetry is broken, one cannot define flavor and color as distinct commuting symmetries. Even electromagnetism cannot be defined. It is only after supersymmetry is broken, with the gauginos becoming relatively heavy compared to the matter fields that one can define separate flavor and color-symmetries.[21] At this stage an effective anomaly-free gauge symmetry like $SU(2)_L$ x $SU(2)_R$ x $SU(4)^{col}$, which includes electromagnetism, could be born.

We may have the following scenario; starting with supersymmetric chiral multiplets of pre-preons coupled to some gauge fields, one may find that the dynamics permits of the emergence of supersymmetric preonic composite multiplets with inverse size Λ_p . Supersymmetry may break for energies $\Lambda_s < \Lambda_p$. At this stage, with the gauginos becoming heavy $(\sim \Lambda_s)$, flavor and color quantum numbers are born, synonymous with fermions and bosons of the preonic supermultiplets (4).[8] At the same time, flavor and color gauge particles (i.e. $W_{L,R}$'s and gluons) are formed as composites of preons (or pre-preons) with inverse sizes < Λ_s , defining an effective low-energy gauge symmetry of the type $SU(2)_L$ x $SU(2)_R$ x $SU(4)^{col}$, or its subgroups. In effect, the emergence of the concepts of flavor and color, in this minmal model, is synonymous with supersymmetry - breaking. Pictorially, the scenario is the following:

Pre-preons (M) → Preons (Λ_p) → Supersymmetry Breaking (Λ_s)

→• Emergence of flavor and color at preonic level.

• Emergence of an effective symmetry like

$SU(2)_L$ x $SU(2)_R$ x $SU(4)^C$

→ • Quarks and Leptons.

Since Λ_s -stage precedes the emergence of $SU(4)^C$ and since $SU(4)^C$ must break spontaneously above 3 x 10^5 GeV, we would expect Λ_s > 3 x 10^5 GeV. With the radiatively generated scale $(\alpha_b/2_\pi)\ (\Lambda_s^2/M)$ to correspond to electroweak scale of 100 GeV, the heavy scale M would be expected to lie above 10^8 GeV. In general Λ_p may lie between Λ_s and M. The extreme situation could be $M = \Lambda_p \approx$ Planck-Mass $\approx 10^{19}$ GeV and $\Lambda_s \approx 10^{10} - 10^{12}$ GeV.

One can envision an alternative scenario, in which flavons and chromons arise from a pre-preonic theory as Fermi and Bose - components of different superfields. The bosonic partners of the flavons and perhaps even the fermionic partners of the chromons acquire relatively heavy masses ~ Λ_s and get decoupled from the low energy theory, while the flavons and ch[illegible] light or needs a minimum of four flavonic plus eight chromonic superfields.[22] These define a global symmetry of the type $[SU(2)_L \times SU(2)_R]_{flavor} \times [SU(4)^C_L \times SU(4)^C_R]_{color}$, even with supersymmetry being intact. Since $SU(4)^C$ can now co-exist with supersymmetry, one can permit the scale Λ_s of supersymmetry-breaking to be even lower than 3×10^5 GeV, if needed.

Composite Supergravity

The N=8 supergravity theory is most attractive in that it puts graviton, gravitinos, spin-1, spin-$\frac{1}{2}$ and spin-0 objects into one multiplet. Treated as a fundamental theory, however, it does not appear to be renormalizable. Thus quantum gravity may still remain ill defined in these theories. One possible resolution of this problem could arise, as mentioned before, if the graviton could be generated as a composite of Planck-size from an underlying theory which is well-behaved in the ultraviolet region and is renormalizable. To this end, Salam and I observed[8] that the N=4 supersymmetric Yang-Mills theory, consisting of 16 "helicity" states (pre-preons), viz.

$$\text{one spin-1} + 4\ \text{spin-}\tfrac{1}{2} + 6\ \text{spin-0}$$

objects, each belonging to the adjoint representation of the Yang-Mills symmetry, will give rise through the Yang-Mills force to 16 x 16 = 256 bilinear S-wave composites (preons), which are singlets of the underlying Yang-Mills group. These 256 states have precisely the same helicity-structure as the N=8 supergravity theory, i.e.

$$\text{one spin-2} + 8\ \text{spin-}\tfrac{3}{2} + 28\ \text{spin-1} + 56\ \text{spin-}\tfrac{1}{2} + 70\ \text{spin-0 states}$$

In other words, bilinear composites of N=4 – fields, combined to make singlets of the underlying gauge group, give an irreducible N=8 – multiplet. (This situation is rather unique in that bilinear composites of N=1 fields give reducible multiplets of N=2 and likewise bilinear composites of N=2 give reducible multiplets of N=4.)

As is well known, N=4 supersymmetric Yang-Mills theory is not only renormalizable, but also (perhaps) finite. Furthermore, it is unique in its particle content and interaction. In order to introduce mass-scale into the theory, one may assume that the N=4 theory in 4 dimensions originates through spontaneous compactification of N=1 theory in 10 dimensions such a compactification will in general introduce masses (and therefore scales) into the theory. The 4-dimensional theory thus obtained is a broken N=4 theory, which is still renormalizable and may even be finite. Bilinear composites of this theory will lead to a broken N=8 supergravity theory. This scenario is attractive. But the extraction of low energy-states (quarks and leptons) and their interactions – through further compositeness is still a challenging task.

6. Dynamical Symmetry Breaking Through Preons; sizes of Quarks and Leptons

Elementary Higgs scalars are the most unaesthetic components of unified gauge theories in that the choice of their representations, their masses and their quartic and Yukawa hypotheses (TC)[23] attempts to remove this unwanted feature and simultaneously to provide a technically "natural" reason for the scale of electroweak symmetry-breaking to be around 1/2 TeV by assuming that the relevant Higgs particles are composites of a new set of fermions called "technifermions" (Q's), which possess not only the electroweak gauge force of $SU(2)_L \times U(1)$, but also a new QCD-like non-abelian gauge force, called techniforce (F_T), with a scale parameter Λ_T of order 1 TeV.[24] The simplest realistic models of technicolor appear to suffer, however, from an excessive magnitude for flavor-changing neutral current (FCNC) processes - not to mention the arbitrary proliferation in building blocks - which they invariably introduce. The difficulty with regard to FCNC-processes is sketched below.

Under the influence of F_T, the techniquarks are assumed to form condensates in pairs: $\langle \bar{Q}Q \rangle \equiv -\Lambda_Q^3 \neq 0$. Roughly speaking, Λ_Q is of the order of the renormalization - scale parameter Λ_T of F_T. The formation of the condensate breaks the flavor gauge symmetry $SU(2)_L \times U(1)$ dynamically. From the mass of $W_L^{\pm}$, or equivalently from the observed values of G_F and $\sin^2\theta_w$, one deduces $\Lambda_Q \sim \frac{1}{2}$ TeV. But a non-vanishing $\langle \bar{Q}Q \rangle$ does not necessarily break chiral symmetry of ordinary quarks and leptons. To give masses to ordinary fermions, one ends up introducing new extended technicolor (ETC) gauge interactions, which couple the ordinary fermions (q's and ℓ's) to the technifermions (Q's). These give rise, in second order, to an effective interaction of the type

where m_E is the mass of the ETC gauge boson and g_E their effective coupling constant. Substituting $\langle \bar{Q}Q \rangle = -\Lambda_Q^3 \approx -(\frac{1}{2}\ \text{TeV})^3$, $g_E \approx 1$ and $m_q \approx 5$ MeV, 300 MeV and 10 GeV for the typical "average" current algebraic masses of the e, μ and τ -families, we obtain $m_E \approx 150$ TeV, 20 TeV and 3 TeV for the e, μ and τ - families respectively. Taking a typical ETC-multiplet (Q_1, Q_2, ...Q_N, q_d, q_s), which is capable of giving diagonal as well as Cabibbo-mixed non-diagonal masses to the (q_d, q_s) - system, we obtain in second order of gauge interactions an effective interaction:

$$(g_{E'}^2/m_{E'}^2)\ (\bar{q}_d q_s)(\bar{q}_s q_d) + \text{h.c.} \qquad (6)$$

where q_d and q_s denote current eigen-states. Note that this amplitude does not have a GIM-invariant form. Expressing (q_d,q_s) in terms of the Cabibbo-rotated mass eigenstate (q'_d,q'_s), one, therefore, obtains from the effective interaction (6), a $|\Delta S|=2$ interaction.

$$(g_{E'}^2/m_{E'}^2)\ \cos^2\theta_c\ \sin^2\theta_c\ (\bar{q}_d q_s\ )^2 + \text{h.c.} \qquad (7)$$

with a strength $\approx (\frac{1}{2})\ (10^{-9}\ \text{TeV}^{-2})$ for $m_{E'} \approx 20$ TeV, $g_{E'} \approx 1$ and $\sin^2\theta_c \approx 1/16$. This is about 1000 times bigger than that permitted by the observed value of the K_L-K_S mass difference. Generation of excessive FCNC-processes thus appears to be a difficulty of at least the simplest technicolor models.

Now, turning to preonic models, it has been remarked[25] that these models score over standard technicolor models at least on grounds of economy: the same objects (preons), which make quarks and leptons, or suitable composites of these preons, serve as technifermions; while the same force (F_b) which binds preons, or effective residual forces generated by the preon binding force, serve as the techniforce. No new ingredients need to be postulated.

I now wish to point out that a class of preonic models have the added advantage that they permit the implementation of dynamical symmetry breaking (DSB) without running into the problem of FCNC-processes. These considerations lead one to predict that the inverse sizes of quarks and leptons are in the range of 3 TeV (only).[26]

We shall illustrate these remarks in the context of the simplest version of the flavon-chromon preon-model, though the mechanism for avoiding excessive FCNC processes is more general.

Assume that a set of spin-$\frac{1}{2}$ flavor carrying entities $\mathfrak{f}_{L,R}$ form condenstates in pairs $(\langle \bar{\mathfrak{f}}_L \mathfrak{f}_R \rangle \equiv -\Lambda^3_{\mathfrak{f}})$ under the influence of an attractive force F_c. The condensing fermions $\mathfrak{f}$'s may be the flavons f's themselves; alternatively they may be suitable composites of f's and a set of primed "chromons" C', for example.[27] The force F_c, which we shall refer to as the "condensing force" may in general be different from the binding force F_b, which binds the constituents of quarks and leptons. Under certain circumstances, though, F_c and F_b may be one and the same force. If $F_c \equiv F_b$, the scale $\Lambda_{\mathfrak{f}}$ of the condensate and the inverse size of the composites may be of the same order (as in QCD), but they need not be equal to each other. We shall elaborate on this point further later.

What concerns us here is that the condensate $\langle \bar{\mathfrak{f}}_L \mathfrak{f}_R \rangle \equiv -\Lambda^3_{\mathfrak{f}}$ breaks $SU(2)_L \times U(1)$. Using the scale of W_L-mass, we obtain $\Lambda_{\mathfrak{f}} \approx \frac{1}{2}$ TeV, as in the TC models.

Let us examine how ordinary quarks and leptons, which were massless in the limit of $SU(2)_L \times U(1)$, and which are singlets of F_c and F_b, acquire mass. If quarks and leptons were point particles and were not linked in any way to the condensing fermions $\mathfrak{f}$'s, they would still remain massless. However, with composite quarks and leptons, despite their neutrality, the $\bar{q}^b_R$ q^a_L pair can make a transition to $\bar{\mathfrak{f}}^b_R$ $\mathfrak{f}^a_L$-pair (See Fig. 1) utilizing the binding force F_b, consistent with the conservation laws. Here (a,b) denote flavor-indices. We expect that the amplitude would be governed by the inverse size Λ_o of the relevant quark or lepton. This

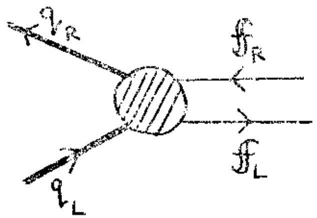


Fig. 1

would lead to an effective interaction:

$$(\kappa_q/\Lambda_o^2)\ (\bar{q}^b_R\ q^a_L)(\bar{\mathfrak{f}}^a_L \mathfrak{f}^b_R) + \text{h.c.} \tag{8}$$

where κ_q is an effective coupling constant of order unity. If we replace $\bar{\mathfrak{f}}\mathfrak{f}$ by the condensate $-\Lambda^3_{\mathfrak{f}}$, we obtain a mass for the quark:

$$m_q = (\kappa_q)(\Lambda^3_{\mathfrak{f}}/\Lambda_o^2) \tag{9}$$

If we assume that the same condensate, i.e., same $\Lambda_{\mathfrak{f}}$ controls masses of all quark flavors - i.e., from "top" to "up" - we would be led to assign different inverse sizes (Λ_o) to different quark-flavors. For example, taking "average" masses of quarks of e, μ and τ-families to be 5 MeV, 300 MeV and 10 GeV, we obtain $\Lambda_o \approx 150$ TeV, 20 TeV and 3 TeV for e, μ and τ-quarks respectively. It is perfectly possible that the inverse sizes of quarks and leptons of different flavors are the same; but the relevant condensates $\langle \bar{\mathfrak{f}}_\tau \mathfrak{f}_\tau \rangle$, $\langle \bar{\mathfrak{f}}_\mu \mathfrak{f}_\mu \rangle$ and $\langle \bar{\mathfrak{f}}_e \mathfrak{f}_e \rangle$ etc. differ from each other by radiative factors of order $\sqrt{\alpha}$, even though the condensing force in all three channels is the same. This line of approach, which we motivate later, leads to the conclusion that the common inverse size of quarks and leptons is about 2-3 TeV. This is determined by the mass of the heaviest quark flavor, which may be given by the top quark mass of about 30 GeV.

For examining the strength of $|\Delta S = 2|$ neutral current processes, let us proceed by taking the inverse size Λ_o of the quark belonging to the μ--family to be around 10-20 TeV. (We comment later on the case for which quarks and leptons possess a common inverse size $\Lambda_o \approx 2$-3 TeV).

The mechanism for generation of quark-masses outlined above provides diagonal masses for a=b, as well as non-diagonal ones for a≠b. For example, the condensate $\langle \bar{\mathfrak{f}}_d \mathfrak{f}_s \rangle$ would provide Cabibbo-mixing and thereby generate familiar strangeness - changing charged current weak interactions. Since the binding force is flavor-independent, we expect $\langle \bar{\mathfrak{f}}_d \mathfrak{f}_s \rangle \approx -(\Lambda_{\mathfrak{f}})^3_{ds}$ to be at most of the same order as $\langle \bar{\mathfrak{f}}_\tau \mathfrak{f}_\tau \rangle \approx \Lambda^3_{\mathfrak{f}\tau}(-1)$, where $\Lambda_{\mathfrak{f}\tau} \approx \frac{1}{2}$ TeV.

Now, consider the amplitude for the process: $q_d + q_d \to q_s + q_s$, which manifestly leads to $|\Delta S| = 2$ NC-processes even after Cabibbo rotation. Let us recall that each individual flavon number (likewise each individual chromon number) is conserved by the binding force F_b as well as the condensing force F_c. Now observe that if the strange quark q_s were were just a quantum pair excitation of the down quark q_d (in the sense mentioned before),

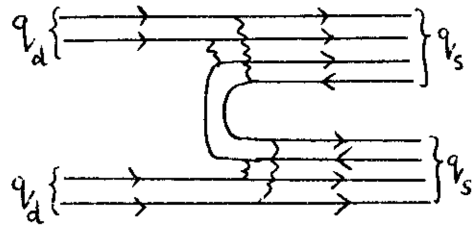


Fig. 2

as suggested by several authors, one could easily excite an overall singlet or neutral pair (see Fig. 2) to go with an incoming system of 2 down quarks. This would induce the process $q_d + q_d \to q_s + q_s$ with an amplitude of order $(1/\Lambda_o)^2 \approx (10 \text{ to } 20 \text{ TeV})^{-2} \approx 10^{-8}$-$10^{-9}$ GeV^{-2}, which is too large to be compatible with the observed K_L-K_S mass difference.

If, on the other hand, q_s differs from q_d by an attribute (quantum number), for example by its flavon constituent s being different from d, and this quantum number is conserved perturbatively by the binding force, the $|\Delta S| = 2$ NC-process $q_d + q_d \to q_s + q_s$ can occur only by utilizing the $|\Delta S| = 1$ condensate $\langle \bar{\mathfrak{f}}_s \mathfrak{f}_d \rangle$, generated non-perturbatively, twice. The corresponding amplitude should then be proportional to $(\Lambda_{\mathfrak{f}})^6_{ds}$. From dimensional considerations, the four fermion amplitude $q_d + q_d$

$\to q_s+q_s$ would [illegible] be of order $(\Lambda_{\mathcal{F}})^6_{ds}/\Lambda_o^8$, [illegible] 10^{-15} GeV^{-2}, for $(\Lambda_{\mathcal{F}})_{ds} \approx \frac{1}{2}$ TeV and $\Lambda_o \approx (10\text{-}20)$ TeV. This is certainly safe for $K_L - K_S$ mass difference. [The case where e, μ and τ fermions have a universal size parameter $\Lambda_o \approx 2$ to 3 TeV, but the e:μ:τ mass hierarchy is accounted for by a radiatively generated hierarchy for the condensates of different flavons, the amplitude $q_d+q_d \to q_s+q_s$ will also be suppressed as above, because $\langle \bar{\mathcal{F}}_s \mathcal{F}_d \rangle$ condensate will be radiatively damped compared to the maximum of $(1/2\ \text{TeV})^3$] .

Let us, therefore, consider those preonic models for which d≠e. We still need to consider the ΔS=0 process $q_d+q_s \to q_d+q_s$, which is what caused the problem for TC models, subsequent to Cabibbo rotation. Such a process can indeed occur within the preonic models through the intermediacy of preonic gluons: See e.g. Fig. 3, where $\bar{f}_\mu f_e$ composite plays the role of the ETC gauge boson. Since the preon binding force is invariant under rotations in the (e,μ) flavor space, however, the above amplitude is necessarily part of a GIM invariant form $\propto (\bar{q}_d q_d + \bar{q}_s q_s)^2$, which arises via a GIM invariant preonic amplitude $(\bar{f}_d f_d + \bar{f}_s f_s)^2$.

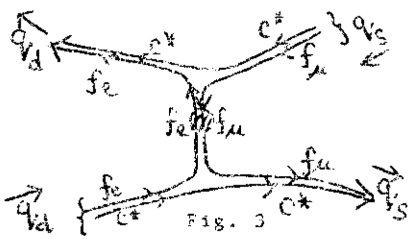


Fig. 3

This is what makes the combined set of processes (i) $q_d+q_s \to q_d+q_s$, (ii) $q_d+q_d \to q_d+q_d$ and (iii) $q_s+q_s \to q_s+q_s$ safe with respect to Cabibbo rotations within the class of preonic models described above. Note that the analog of GIM invariance is non-existent for the simplest ETC models. For a [illegible] on the other hand it is an automatic bonus.

To summarize, we see that the notion of dynamical symmetry breaking can live fruitfully in the context of preonic models without encountering the problem of excessive FCNC processes. Believing in DSB, this appears to call for a need for preons. Our mechanism to avoid FCNC processes in the context of DSB is more general than the preonic model considered here. Two ingredients have played important roles: First, the muon family must differ from the electron family in respect of some attributes (like (c,s)≠ (u,d)). [This incidentally leaves the door open that τ may still be the quantum pair excitation of e or μ. It is important to study $\tau \to e\gamma$ and $\tau \to \mu\gamma$ branching ratios up to a level of about 10^{-6} to judge on this issue]. Second, the preon dynamics must lead to a GIM invariant form for four quark transitions.

One non-trivial consequence of realizing DSB through preons is this: the inverse size of at least the heaviest quark flavor, which tentatively one may identify with a top quark with a mass of 30 GeV, must be equal to nearly 2-3 TeV. More precisely, $(\Lambda_o/\sqrt{\kappa}) \approx 2\text{-}3$ TeV, where κ entering into eq. (9) is of order unity.

If we believe in a radiative origin for the full fermion mass hierarchy and thereby in a universal size for all quarks and leptons - an idea which we motivate further later - we will be led to the prediction that the inverse sizes of quarks and leptons of even the electron family are nearly 2-3 TeV.[26]

$$\left(\frac{1}{r_o}\right)_{q,\ell} = (\Lambda_o)_{q,\ell} \approx 2\text{-}3 \text{ TeV} \tag{10}$$

This generates exciting new possibilities for accelerators of the next two decades.

We note that the model of DSB presented above with four or six flavons is expected to generate, subsequent to DSB, light pseudogoldstone bosons (PGB's), like in familiar TC models. This is because of the presence of the global flavor symmetry prior to DSB. We believe that the problem of light PGB's will be resolved by introducing the full flavor symmetry as a gauge symmetry, which may even be an effective symmetry. This needs to be examined further.

Meanwhile, it is worth noting experimental consequences of the prediction of a relatively low inverse size $\Lambda_o \approx 2$ to 3 TeV for quarks and leptons.

Experimental Signals:

There are two types of signals, which we expect would appear if quarks and leptons are composites:

(1) Appearance of Form Factors; Departures from OCD - Predictions at Large Momenta ~ Λ_o:

Composite quarks would exhibit power form factors, rather than logarithmic, when probe-mementa associated with photon or gluon probes is of the order of their inverse size Λ_o:

$$F_q(Q^2) \sim \left[\frac{\Lambda_o^2}{q^2-\Lambda_o^2}\right] \quad n=1 \text{ or } 2$$

These form factors, which would show in deep inelastic ep or

μp scatterings and in $e^-e^+ \rightarrow q\bar{q}$, would lead to a 50% departure in the cross sections for $Q^2 = -q^2 \approx \Lambda_o^2/4$, even if n=1. Similar departures would be expected in time-like processes like $e^- e^+ \rightarrow q\bar{q}$ and $pp \rightarrow \ell\bar{\ell}$ X. For $\Lambda_o \approx$ 2-3 TeV, such a departure would require probe mementa Q's $\approx \Lambda_o/2 \approx$ 1 to 2 TeV, say.

2) Production of Excess Lepton-Pairs in Hadronic Collisions:

Even more striking than the appearance of form factors is the production of excess lepton pairs by hadronic collisions. Such excess, above and beyond the expectations based on QCD and QED interactions of point-like quarks and leptns, would come about if quarks and leptons are composites sharing common constituents, and/or if the constituents of quarks and leptons are held together by

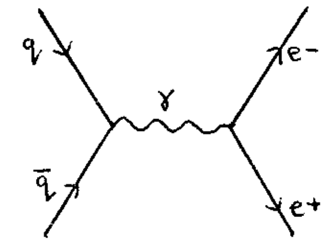


Fig. 5

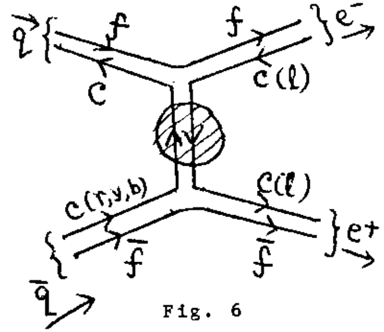


Fig. 6

Common binding force. Thus processes like $q\bar{q} \rightarrow e^-e^+$ would be generated by electrodynamics (Fig. 4) as well as by quantum preon dynamics (Fig. 5), with amplitudes of order:

$$A(\text{Fig. } 4) \sim \left(\frac{e^2}{q^2}\right)(\bar{q}q)(\bar{\ell}\ell) \tag{11}$$

$$A(\text{Fig. } 5) \sim (\kappa/\Lambda_o^2)(\bar{q}q)(\bar{\ell}\ell) \tag{12}$$

κ , appearing here, is related though not identical to "κ_q" appearing in eq. (8); we expect $\kappa \approx \mathcal{O}(1)$. Note that the interference between the two amplitudes would exceed more than 50% of the square of the electromagnetic amplitude (11) for

$$q^2 \gtrsim (\frac{e^2}{\kappa})(\Lambda_o^2/2)$$

or $$|q| \gtrsim (\Lambda_o/4) \sim 500\text{-}700 \text{ GeV. for } \kappa \approx 1, \Lambda_o \approx 2\text{-}3 \text{ TeV} \quad (13)$$

Quite clearly the momentum dependence of (12) is such that it would lead to large excess in e^-e^+ production over the expected amount once invariant mass $|q|$ equals or exceeds about $(\Lambda_o/4) \approx (1/2\text{-}3/4)$ TeV. This striking feature is a prediction of the preonic model together with ideas of DSB presented here.[26] To test this prediction one would need $\bar{p}p$ or pp machines in the 5-10 TeV range with high luminosity.

7. The Question of Chiral Symmetry Preservation for QPD:

The lightness of quarks and leptons, compared to their inverse sizes, has widely been attributed to the preservation of chiral symmetry by quantum preon dynamics (QPD). We now discuss to what extent chiral symmetry actually needs to be preserved by QPD. To judge on this issue, we must focus attention on the heaviest known family τ . (Analogous discussion will apply if a heavier family, which we expect would still be lighter than 100-200 GeV, is discovered). From our discussions of DSB, we recall:

$$\langle \bar{f}_\tau f_\tau \rangle \equiv -\Lambda^3_{f\tau}$$

$$m_{q_\tau} \approx \kappa (\Lambda^3_{f\tau} / \Lambda_o^2) \quad (14)$$

Here, Λ_o denotes the inverse size of the composite quark, while κ is an effective coupling constant of order unity (See eq. (9)). We stress that it is the condensate parameter $\Lambda_{f\tau}$, rather than the quark masses, which directly characterizes the strength of chiral symmetry breaking.

From the known scale of $SU(2)_L \times U(1)$ breaking, one deduces $\Lambda_{f\tau} \approx \frac{1}{2}$ TeV. Now, substituting a representative value for the heaviest flavor $m_{q_\tau} \approx 30$ GeV, and $\kappa \approx 1$, one obtains $\Lambda_o \approx 2$ TeV. Allowing for κ to vary between 1/4 to 4, say, we obtain:

$$\Lambda_{f\tau} \approx (1/2 \text{ to } 1/8)\, \Lambda_o \quad (15)$$

In other words, the chiral symmetry breaking parameter $\Lambda_{f\tau}$ is not so different, after all, from the inverse size Λ_o of the composites. They are comparable to each other within factors $\approx$ 1 to 1/10. Note, had we compared the quark mass of even the heaviest flavor, i.e. 30 GeV, with a $\Lambda_o \approx 2$-3 TeV, we would have noticed a much bigger ratio $\approx$ 60-100, between the two scales. Traditionally, this comparison, plus the fact that Λ_o can be permitted to be in general much larger than 2 TeV (even for the τ-family), have led to the view that chiral symmetry needs to be preserved by QPD, unlike the case in QCD.

We wish to suggest that this view is not warranted, and that chiral symmetry may indeed be broken dynamically by QPD very much like the case in QCD - barring minor numerical factors perhaps between the two cases (see elaboration below). This suggestion is based first on our deduction of the inverse size $\Lambda_o \approx 2$-3 TeV for quarks and leptons, and second, on our comparison (cf. eq.

15)) of Λ_o with the condensate parameter $\Lambda_{\mathcal{F}\tau}$ rather than with the quark masses. From eq. (14), we see that m_q is proportional to the third power of $\Lambda_{\mathcal{F}}$; thus a difference by only a factor of 5 between $\Lambda_{\mathcal{F}}$ and Λ_o , which is perfectly feasible within a QCD like theory, will magnify into a difference by a factor of 125 between m_q and Λ_o . Quite clearly it is the comparison of $\Lambda_{\mathcal{F}}$ versus Λ_o which provides a more direct information on the issue of chiral symmetry breaking. The additional reason for our suggestion is the qualitative feeling that dynamical considerations based on large N (see ref. 28) and lattice gauge theory calculations (see e.g. Ref.29) suggest that it is hard to preserve chiral symmetry at least in a QCD-like theory. Finally, it needs to be stressed that the constraint of anomaly-matching, noted by t' Hooft, invariably has the tendency of introducing a certain degree of richness into preonic theories; yet it is only necessary condition for the realization of massless spin 1/2 composites; it is never sufficient.

It is on these grounds that we are led to suggest - contrary to prevailing notion - that QPD does break chiral symmetry dynamically very much like QCD, at least in so far as we focuss attention on the heaviest quark-flavor only. To see more precisely how alike or unlike these two theories are, let us recall a few facts about QCD. Partly empirically and partly on the basis of theoretical calculations, we know the following facts about QCD: It is based on the gauge symmetry $SU(3)^c$ (i.e. $n_c = 3$), operating on 6 or more flavors ($n_f \geq 6$). Ignoring current quark masses, it is defined by just one scale parameter Λ_{QCD}, which controls the variation of the running coupling constant as a function of the running momentum, and is determined by deep inelastic phenomena:

$$\Lambda_{QCD} \approx 100 - 300 \text{ MeV} \tag{16}$$

QCD generates a number of other dynamical parameters which are determined by Λ_{QCD} . These are, e.g.

$$\begin{aligned} \Lambda_q &\equiv [-\langle\bar{q}q\rangle]^{1/3} \approx 250 \text{ MeV} \\ (m_q)_{dyn} &\approx \text{constituent up and down quark masses} \\ &\approx 300 \text{ MeV.} \\ (\Lambda_o)_{QCD} &= \text{Inverse Sizes of N and } \pi \\ &\approx 2m_\pi \text{ (From NN-Scattering)} \end{aligned} \tag{17}$$

Note, for QCD, it so happens that the condensate parameter Λ_q , the scale parameter Λ_{QCD} and the inverse sizes of composite nucleons and pions are all nearly equal to each other within a factor of two. As a result, the dynamical masses of the quark and the nucleon, calculated via the analog of Fig. 1, yields:

$$(m_q)_{dyn} = \text{"}\kappa\text{"}_q \, (\Lambda_q^3/\Lambda_{QCD}^2) \approx \Lambda_q \text{ (for "}\kappa_q\text{"} \approx 1),$$

$$\text{and } m_N = \kappa_N \, (\Lambda_q^3/\Lambda_{oN})^2) \approx \kappa_N \, \Lambda_q \, .$$

The near equality between the renormalization scale parameter, the condensate parameter and the inverse sizes of the composites for the case of QCD need not hold-to this extent-, however, for all QCD like theories. For instance, even if QPD with massless preons is defined by just one scale parameter Λp , depending upon the number of "flavors" (N_f) for the preonic space and the number of "colors" (N_c) defining the preonic gauge symmetry, the condensate parameter $\Lambda_{\mathcal{F}}$, the inverse size Λ_o and the scale parameter Λ_p of QPD may well differ from each other by factors[30] nearly 1 to as much as even 10. In general, we expect,

$$\Lambda_{\mathcal{F}} = \Lambda_p \, \Phi \, (N_c, \text{"}N_{\mathcal{F}}\text{"}, \ldots)$$
$$\Lambda_o = \Lambda_p \, \xi \, (N_c, \text{"}N_{\mathcal{F}}\text{"}, \ldots)$$
$$(m_{preon})_{dynamical} = \Lambda_p \chi \, (N_c, \text{"}N_{\mathcal{F}}\text{"}, \ldots) \qquad (17)$$

where the functions Φ, ξ and χ are dimensionless numbers of order unity. A future dynamical calculation should hopefully determine these functions Φ, ξ and χ. Meanwhile, it appears reasonable to assume - especially in the context of the prediction that the inverse sizes of quarks and leptons are nearly 2-3 TeV, that QPD breaks chiral symmetry almost like QCD.

In judging on the issue of chiral symmetry breaking through QPD, we focussed attention on the heaviest quark flavor only. The discussion of chiral symmetry breaking would not be complete, however, until we consider the lighter quark flavors belonging to the e and μ families and understand why they are so much lighter than the τ family. The understanding of fermion mass hierarchy, i.e. why $m_\tau >> m_\mu >> m_e$, is a problem, which arises regardless, of course, of whether QPD preserves or breaks chiral symmetry. We address to this problem in the next section.

Empirically, the six quark flavors exhibit the following hierarchies in their "bare" or current algebraic masses:

$$m_t >> m_b >> m_c >> m_s >> m_d > m_u \qquad (18)$$

or, equivalently

$$m_t - m_b >> m_c >> m_c - m_s >> m_s - m_d >> m_d - m_u \qquad (19)$$

The successive mass ratios are:

$$(m_b/m_t) \simeq 1/6 \quad , (m_c/m_b) \simeq 1/4$$
$$(m_s/m_c) \simeq 1/6 \quad , (m_d/m_s) \simeq 1/20$$
$$(m_u/m_d) \simeq 1/2$$

The intra-family mass-splitting are given by:

$$m_t - m_b \simeq 25 \text{ GeV} \simeq m_t \, (1- \mathcal{O}(\sqrt{\varepsilon})) \qquad (21a)$$

$$m_c - m_s \simeq 1 \text{ GeV} \simeq m_c \, (1- \mathcal{O}(\sqrt{\varepsilon'})) \qquad (21b)$$

$$m_d - m_u \simeq \text{Few MeV} \sim \mathcal{O}(m_d) \qquad (21c)$$

In writing these relations, we have assumed that the top quark has a mass of nearly 30 GeV. Here $\mathcal{O}(\sqrt{\varepsilon})$ and $\mathcal{O}(\sqrt{\varepsilon'})$ are positive numbers which are much less than unity (i.e. $\sqrt{\varepsilon} \sim \sqrt{\varepsilon'} \simeq 1/4 - 1/6$).

Note that the mass difference between any two successive members, representing either inter-family or intra-family mass-splitting, is comparable to the mass of the heavier one of the two. In other words, the mass-splitting between two successive members is, within a factor of two, equal to the average mass of the two members. For example, $(m_t - m_b) \sim m_t \sim (m_t + m_b)/2$; $(m_b - m_c) \sim m_b \sim (m_b + m_c)/2$, etc. This shows that neither the inter

[illegible]

<u>attributed to perturbative electroweak effects only</u>. This is because, such effects would in general induce mass splittings between two members, which are other wise degenerate, of order α (or at best $\sqrt{\alpha}$) only, compared to the average mass of the two members. The situation at hand strongly suggests that both inter as well as intra-family mass-splittings have their origins, one way or another, in the preon dynamics itself - involving perhaps non-perturbative phenomena, whose full complexion may well involve radiative quantum effects.

Two alternative scenarios, having bearing on the problem, suggest themselves. The truth may in fact involve a blend of both. These are:

(i) <u>A Hierarchy in the Sizes of the Composites</u>: Different families e, μ and τ, owing to their differing constituents and correspondingly differing binding forces, have varying inverse sizes, and thereby, following our discussions of sec. 6 on dynamical symmetry breaking, have varying masses. Consider, for example, the case, where

$$\Lambda_{oe} >> \Lambda_{o\mu} >> \Lambda_{o\tau} \qquad (22)$$

with $\Lambda_{oe} \sim 200$ TeV, $\Lambda_{o\mu} \sim 20$ TeV and $\Lambda_{o\tau} \sim 2$ TeV . In this case, even for a universal chiral symmetry breaking condensate parameter (i.e. $\langle \bar{f}_e f_e \rangle = \langle \bar{f}_\mu f_\mu \rangle = \langle \bar{f}_\tau f_\tau \rangle = -\Lambda^3{}_f$), e, μ and τ families would have differing masses given by $(\Lambda^3{}_f / \Lambda^2{}_{oe,\,\mu,\,\tau})$. Such a hierarchy in sizes can not by itself account for the hierarchy of the six quark flavors, however; unless one considers the unlikely possibility of a six-fold hierarchy in quark-lepton sizes.

Notwithstanding the problem of the six flavor system, one might ask: are there certain "natural" possibilities (scenarios) for which a three or even a two-fold hierarchy in sizes could emerge? Let us offer one such possibility[26] for generating a two-fold hierarchy. (The extension to three fold hierarchy can be implemented following similar ideas).

Consider a primodial gauge symmetry G_b generating a gauge force F_b, which is characterized by a scale parameter Λ_b . Assume a set of spin-1/2 flavons with a minimum of two flavons:

$$(f^i_a)_{L,R} = (u,d,..)^i_{L,R}, \qquad (23)$$

plus a set of spin-o chromons with a minimum of 8 chromons:

$$C^i_\alpha \equiv (C_I | C_{II}) = (r,y,b,\ell | r',y',b',\ell')^i \qquad (24)$$

The index "i" represents the representation label with respect to the primodial symmetry G_b. As mentioned before, such a proliferated set of preons can have its origin within an economical set of pre-preons (see discussions in sec. 5 and ref. 8).

Assume that the flavons and the chromons are associated with an <u>effective gauge symmetry[31]</u> like,

$$G_{eff} = SU(2)_L \times SU(2)_R \times SU(8)^c \qquad (25)$$

Now assume that $\langle CC^* \rangle$ condensates form under the influence of F_b. These transform as singlets of G_b, but as $\underset{\sim}{63} + \underset{\sim}{1}$ of $SU(8)^c$. Vacuum expectation value of $\underset{\sim}{63}$ will break $SU(8)^c$ into $SU(4)^c \times SU(4)^{HC} \times U(1)$, where $SU(4)^c$ acts on (r,y,b,ℓ), while $SU(4)^{HC}$ on (r', y', b', ℓ'). Assume furthermore that $\langle f_R f_R C^*_I C^*_I \rangle$ condensates form, also under the influence of F_b. These transform as $(1, \underset{\sim}{3}_R, \underset{\sim}{10}_c)$ of $SU(2)_L \times SU(2)_R \times SU(4)^c$, but as singlets of G_b as well as of $SU(4)_{HC}$. These break $SU(2)_R \times SU(4)^c \times U(1)$ into $U(1)_Y \times SU(3)^c$. Thus we have the pattern of SSB given by:

$$SU(2)_L \times SU(2)_R \times SU(8)^c \xrightarrow[\sim 63^c]{\langle CC^* \rangle} SU(2)_L \times SU(2)_R \times SU(4)^c \times U(1) \times SU(4)^{HC}$$

$$\xrightarrow{\langle f_R f_R C_I^* C_I^* \rangle} SU(2)_L \times U(1)_Y \times SU(3)^c \times SU(4)^{HC} \qquad (26)$$

The two stages of SSB exhibited above may essentially be one and the same stage, as they are both induced by F_b. Let us asume that neither $SU(3)^c$ nor $SU(4)^{HC}$ are broken dynamically[32]. Quite clearly, owing to the bigger "size" of $SU(4)^{HC}$ relative to $SU(3)^c$, the scale parameter Λ_{HC} of $SU(4)^{HC}$ representing the momentum scale, where $SU(4)^{HC}$ coupling $\alpha_{HC} \approx 1$, should be much bigger than that of $SU(3)^c$. On the other hand, we expect Λ_{HC} to be much less than the scale Λ_b of F_b. Thus,

$$\Lambda_{QCD} \ll \Lambda_{HC} \ll \Lambda_b \qquad (27)$$

(In so far as $SU(8)^c$ gauge force is a residual force, generated from F_b, we see how new effective scales Λ_{QCD} and Λ_{HC} may arise out of one given scale Λ_b).

First Stage of Compositeness (Due to F_b):

Let us now list the set of two body composites of the (f,C) system, which may form as singlets of the primodial gauge symmetry G_b under the influence of F_b. We classify them according to their representation with respect to $G_b \times SU(4)^{HC}$ $S(4)^c$ (although $SU(4)^c$ is broken, it is convenient for most purposes to use representation labels with respect to $SU(4)^c$):

	Remarks
$\mathfrak{f}_I = (f C_I^*)^{\Lambda_b}_{1_b,\, 1_{HC},\, 4^*_c}$	Can identify these with the fermions of the e-family
$\mathfrak{f}_{II} = (f C_{II}^*)^{\Lambda_b}_{1_b,\, 4^*_{HC},\, 1_c}$	Can identify these with the technifermions and also with the flavons for a second family (see remarks below).
$\mathbb{C} = (C_I C_{II}^*)^{\Lambda_b}_{1_b,\, 4^*_{HC},\, 4_c}$	Can play the role of chromons for some families
$\mathbb{D} = (C_{II} C_{II}^*)^{\Lambda_b}_{1_b,\, (15+1)_{HC},\, 1_c}$	Assume, these do not acquire VEV's.
$\mathbb{E} = (C_I C_I^*)^{\Lambda_b}_{1_b,\, 1_{HC},\, (15+1)_c}$	These may acquire VEV and break $SU(4)^c$ into $SU(3)^c \times U(1)_{B-L}$
$\phi = (\bar{f}_L f_R)^{\Lambda_b}_{1_b,\, 1_{HC},\, 1_c}$	Assume G_b does not break chiral symmetry; so ϕ's have zero VEV.

The superscript Λ_b denotes the order of magnitude of the inverse sizes of the composites. Let us assume that the primodial force F_b does not break the chiral symmetry defined by the flavons, so that $\mathfrak{f}_I$ and $\mathfrak{f}_{II}$, measured in the scale of Λ_b, are massless. This assumption would necessitate that we satisfy t'Hooft's anomaly matching condition. This condition is trivially satisfied for our model[32'] for any G_b for the residual "flavor"-symmetry being given by (26). Now, as noted above, the composites $\mathfrak{f}_I$ carrying flavor and color, but no primodial or hypercolor, can be identified with the quarks and leptons of the electron family. These would have inverse sizes of order Λ_b. From our discussions in sec. 6, Λ_b would then be of order few hundred TeV.

The Second Stage in the Formation of the Composites and the Breaking of Chiral Symmetry Through $SU(4)^{HC}$:

We expect new composites of singlets of G_b like $\mathfrak{f}_{II}$ and $\mathbb{C}$ to form under the influence of the hypercolor force. These would have inverse sizes of order $\Lambda_{HC} << \Lambda_b$. Of particular interest are the spin-1/2 composites:

$$\mathbb{F}_{II} = \left(\mathfrak{f}_{II}\mathbb{C}^*\right)^{\Lambda_{HC}}_{1_b,\,1_{HC},\,4^*_c} \qquad (28)$$

which carry flavor and color, but are singlets of G_b as well as of hypercolor. We may identify the composites F_{II} with the fermions of a heavier family[33] like the τ.

Let us now assume that unlike G_b, the hypercolor force generated by $SU(4)^{HC}$ does break chiral symmetry through the formation of the hypercolor singlet condensates:

$$\langle \bar{\mathfrak{f}}_{II}\mathfrak{f}_{II} \rangle = -\Lambda^3_{\mathfrak{f}} \neq 0 \qquad (29)$$

On the one hand, we expect $\Lambda_{\mathfrak{f}}$ to be of the order of Λ_{HC}, which is of the order of the inverse size Λ_{II} of the composites $\mathbb{F}_{II}$. On the other hand, by identifying $\Lambda_{\mathfrak{f}}$ with the scale of $SU(2)_L \times U(1)$ breaking and the composites F_{II} with the fermions of the τ family having a mass of 10–30 GeV $\approx \kappa\,(\Lambda^3_{\mathfrak{f}}/\Lambda^2_{oII})$, see sec. 5, we have $\Lambda_{\mathfrak{f}} \approx 1/2$ TeV and $\Lambda_{oII} \approx 2$–3 TeV (for $\kappa \approx 1$). This in turn says that the scale of the hypercolor force Λ_{HC} must be of the order of 1 to few TeV ($\Lambda_{HC} \approx$ (1–few) TeV).

The same condensate (29) will give a mass to the fermions of the electron family, identified with $\mathfrak{f}_I$. This mass is $m_I \approx \kappa\,(\Lambda^3_{\mathfrak{f}}/\Lambda^2_{oI})$, where $\Lambda_{oI} \sim \Lambda_b >> \Lambda_{II}$, and thus the electron family is much lighter than the τ family. As noted before, we need $\Lambda_{oI} \sim$ few hundred TeV, to account for $m_I \sim 1$–10 MeV.

We thus see how a two-fold hierarchy in sizes could emerge – somewhat naturally – within a preonic model with 8 chromons and induce a hierarchy in composite masses in spite of a universal condensate parameter. Starting with a system of 12 chromons, possessing a $SU(12)^c$ symmetry, one can extend this mechanism to create a 3-fold hierarchy in sizes, if needed.

(ii) A Hierarchy in the Magnitudes of the Condensates:

Even if composite quarks and leptons of different flavors have a universal size, and the underlying preon dynamics treats all six flavors on a completely symmetrical footing, their masses may still exhibit a hierarchy because of non-perturbative phenomena; the full complexion of the hierarchy may, of course, involve radiative effects. All this may come about as follows: The minimum of the effective potential of the scalars ϕ_i's which are, for example, $\bar{f}_i f_i$ composites of different flavons ($i = e, \mu, \tau$ etc.), despite the discrete symmetry $e \leftrightarrow \mu \leftrightarrow \tau$, may lead to a flavor asymmetrical solution for the vacuum expectation values $\langle\phi_i\rangle$'s of these scalars, or equivalently for the magnitudes of the condensates $\langle\bar{f}_i f_i\rangle$. For example, consider a system of 3 flavors $i = a,b,c$, with the full lagrangian respecting the discrete symmetry $a \leftrightarrow b \leftrightarrow c$. Ignoring condensates which mix different flavors and thereby induce Cabibbo rotation, for the sake of simplicity in the first round of considerations, it can happen that only one of the scalars, say $\phi_c = \bar{f}_c f_c$ develop a nonzero VEV at the tree level of the effective potential with $\langle\phi_b\rangle$ and $\langle\phi_a\rangle$ remaining zero at this level. Inclusion of quantum corrections of the Coleman-Weinberg type may lead to a nonzero $\langle\phi_b\rangle \approx O(\sqrt{\text{"}\alpha\text{"}})\langle\phi_c\rangle$, with $\langle\phi_a\rangle$ still remaining zero. This in turn will generate a hierarchy in the quark masses m_c ... m_b ... m_a. ...

constant as such, but symbolizes the strength of radiative effects arising from gauge, Yukawa and scalar-quartic interactions. Numerically $\sqrt{\text{"}\alpha\text{"}}$ is a small quantity $\approx (\frac{1}{4}) - (\frac{1}{20})$, as suggested by the observed mass-ratios (20).

We are encouraged to consider such a radiative origin for the mass hierarchy because of a recent work of Mirjam Cvetic and myself[34] which showed that a hierarchy in the pattern of VEV's of scalar fields, which are otherwise introduced symmetrically, can indeed arise non-perturbatively in the background of radiative quantum corrections. Let me describe this work briefly.

Assume for simplicity only three quark families ψ_e, ψ_μ and ψ_τ each having an "up" and "down" member. Assume furthermore, a distinct flavor gauge symmetry for each family:

$$G_{flavor} = [SU(2)_{Le} \times SU(2)_{L\mu} \times SU(2)_{L\tau}] \times (L \to R) \qquad (30)$$

subject to the discrete symmetry $e \leftrightarrow \mu \leftrightarrow \tau$. The physical $SU(2)_L$ is the diagonal sum $[SU(2)_L]_{e+\mu+\tau}$; likewise, for $SU(2)_R$. Introduce a set of scalar fields χ_i whose VEV's would break $SU(2)_{Le} \times SU(2)_{L\mu} \times SU(2)_{L\tau}$ spontaneously into the physical $SU(2)_L$. (A similar mechanism could also be invoked for the right-handed sector). Now introduce three distinct Higgs fields ϕ_e, ϕ_μ and ϕ_τ, each transforming as a $(\underset{\sim}{2}, \underset{\sim}{2})$ under the respective $SU(2)_L \times SU(2)_R$ group, and having Yukawa coupling as follows:

$$\mathcal{L}_{Yukawa} = h\,(\bar{\psi}_e\, \psi_e\, \phi_e + \bar{\psi}_\mu\, \psi_\mu \phi_\mu + \bar{\psi}_\tau\, \psi_\tau\, \phi_\tau) + h.c. \qquad (31)$$

The coupling of $\tilde{\phi}_i = \tau_2\, \phi^* \,\tau_2$ may be introduced likewise. Note that

... symmetrical footing. Subject to the discrete symmetry $e \leftrightarrow \mu \leftrightarrow \tau$, gauge symmetry and renormalizability, the effective potential of the Higgs scalars is:

$$\begin{aligned} V(\phi_e, \phi_\mu, \phi_\tau) &= -\mu^2\, Tr\,(\phi_e^+\phi_e + \phi_\mu^+\phi_\mu + \phi_\tau^+\phi_\tau) \\ &+ \lambda_1\,[Tr\,(\phi_e^+\phi_e + (e\to\mu) + (\mu\to\tau))]^2 \\ &+ \lambda'_1\,[Tr\,(\phi_e^+\phi_e\phi_e^+\phi_e + (e\to\mu) + (\mu\to\tau))] \\ &+ (\lambda_2 + 2\lambda'_1)[(Tr\,\phi_e^+\,\phi_e)\;\; Tr\,(\phi_\mu^+\,\phi_\mu) + e\to\mu\to\tau] \\ &+ (\text{Analagous terms involving } \tilde{\phi}_i\text{'s}) \end{aligned} \qquad (32)$$

Note that the complete Lagrangian including gauge plus Yukawa interactions as well as the potential V is invariant under $e \leftrightarrow \mu \leftrightarrow \tau$. Note, furthermore that the potential in the absence of the λ'_1 and $(\lambda_2 + 2\lambda'_1)$ terms has a rotational invariance in the space of e, μ, τ. In fact the invariance is SO(12). The gauge interactions, the Yukawa interactions as well as the λ'_1 and $(\lambda_2 + 2\lambda'_1)$ terms do not respect this bigger symmetry.

One finds that as long as $\lambda_2 > 0$, the minimum of the potential at the tree level leads to a flavor-asymmetrical solution:

$$\langle\phi_\tau\rangle \neq 0,\; \langle\phi_\mu\rangle = \langle\phi_e\rangle = 0 \qquad (33)$$

Inclusion of one loop radiative corrections of the Coleman-Weinberg type alters[35] the pattern of this VEV for a range of values of the parameters (e.g., $\lambda_1 \sim \lambda'_1 \sim (\alpha)$, $0 < \lambda_2 < (\alpha^2\phi^2/\chi^2, h^4/16\pi^2)$) to the following form:

$$\langle\phi_\tau\rangle = \langle\phi_\tau^0\rangle\,(1 + \mathcal{O}(\sqrt{\text{"}\alpha\text{"}}))$$

$$\langle\phi_\mu\rangle \doteq \langle\phi_\tau^0\rangle\, \mathcal{O}(\sqrt{\text{"}\alpha\text{"}})$$

$$\langle\phi_e\rangle = 0 \tag{34}$$

As mentioned before, "α" denotes a radiative parameter of order 1/20 - 1/100.

We see that an asymmetrical solution exhibiting a hierarchy of the desired sort has emerged radiatively but nonperturbatively, from a basic theory which is symmetrical.

The model exhibited above is not realistic, because (a) it does not allow for Cabibbo-mixings, (b) by itself, it does not account for the observed 6-fold hiearchy of the six quark flavors, and (c) it generates light charged pseudogoldstone bosons with masses of order 3-10 GeV, which are presumably excluded experimentally from PETRA experiments. Yet it is instructive and lends hope that such a radiative mechanism could possibly play a major role in the final picture, which would account for the full fermion mass spectrum.

The following comments are in order:

(1) We have exhibited the radiative mechanism in the context of "elementary" fermions and Higgs scalars. Believing in preons, these should, however, be interpreted as preonic composites which can be described in terms of effective local fields at low momenta. Owing to their small sizes, these should still be described in terms of a renormalizable theory, whose mass and coupling parameters (like h, λ_1, λ'_1, λ_2 etc.) are related to each other through QPD.

(2) We have imposed that ϕ_e which is a composite of say $\bar{f}_e f_e$ couples only to ψ_e, but not to ψ_μ. This may receive some

justification within a preonic theory by noting that for such a theory with vectorial or chiral gauge interactions, a spin-zero $\bar{f}_e f_e$ system cannot make transition to $\bar{f}_\mu f_\mu$ system in any order of perturbation theory in the preonic gauge interactions, as long as f_e and/or f_μ is massless.

(3) We have presented two alternative mechanisms for generating a hierarchy in fermion masses; these involve a hierarchy in the sizes of the composites on the one hand and in the magnitudes of the Higgs- VEV's or the condensates of different flavors on the other. For both cases, however, we have illustrated as though they generate a hierarchy only to split the three families from each other ($m_e \neq m_\mu \neq m_\tau$). This was done only for convenience of discussion. The true picture may well utilize both mechanisms so as to account for the full mass spectrum of six flavors involving inter as well as intra-family mass splittings, in addition to Cabibbo-like angles and CP-violating phases. For instance, the mechanism involving a hierarchy in sizes could be involved primarily in the inter-family splitting, while that involving a hierarchy in the VEV's could be primarily responsible for the intra-family splittings, or vice-versa. Our treatment and discussions would go through for all such cases with just a change of labelling of flavors.

(4) QCD breaks chiral flavor symmetry dynamically, but, by itself, it does not break dynamically the vectorial flavor symmetry like isospin. Vafa and Witten[36] have recently argued that in fact vector-like gauge theories (like QCD) cannot break spontaneously vector-like global symmetries. Our radiative mechanism for a

preonic theory leading to $\langle \bar{f}_e f_e \rangle \neq \langle \bar{f}_\mu f_\mu \rangle \neq \langle \bar{f}_\tau f_\tau \rangle$, for example, if it has to work, calls for a departure from this situation. Given that the underlying preon binding force does not distinguish in any way between the different flavors, the question is: can such a distinction be brought about dynamically spontaneously, for example through differing magnitudes for the condensates of different flavors? Following VW one must conclude that OPD must be non-vector-like for this to happen. It is conceivable that scalar exchanges arising perhaps from a supersymmetric theory, superposed on vector-exchanges, provide the desired trigger (i.e., $\lambda_2 \neq 0$ in eqn. (32)) for a breakdown of vectorial flavor-symmetry. In this respect, OPD may be drastically different from QCD. This question needs to be explored further.

(5) While we have addressed primarily to inter-family as well as intra-family mass-splittings involving "up" and "down" members, the full mass spectrum involves the question of quark-lepton mass splittings within a given family. As is well known, starting with a situation, which yields $m_e^{(o)} = m_d^{(o)}$, $m_\mu^{(o)} = m_s^{(o)}$ and $m_\tau^{(o)} \approx m_b^{(o)}$ at some high momentum limit, one can not account for the observed value of $(m_e/m_\mu)(m_d/m_s)^{-1}$, even after the inclusion of electroweak and OCD renormalizations. The quark-lepton mass-splitting within a family can, however, be accounted for with the inclusion of Higgs-scalar ξ , which transforms as a (2,2,15) under $SU(2)_L \times SU(2)_R \times SU(4)^C$. Such a scalar would arise as a composite of $(\bar{f}fC^*C)$ in our model.

Talking of the mass spectrum of leptons, it needs to be stressed that special considerations apply to neutrinos, as these can acquire not only Dirac but also Majorana masses. Following familiar suggestions, the most natural mechanism to account for the ultra-lightness of the observed neutrino of the electron-family ($m_{\nu_e} < 30$ eV) , in the context of a left-right symmetric theory, is to assume that ν_R acquires a heavy Majorana mass through VEV of a Higgs Δ_R transforming as a (1,3,10) of $SU(2)_L \times SU(2)_R \times SU(4)^C$. In the preonic context, such a Higgs would correspond to a $f_R f_R C^* C^*$ composite. The combined effect of Dirac and Majorana masses would be that the observed neutrino would have a mass $\sim(m^2_{Dirac}/M_{\nu_R}) \ll m_{Dirac}$.

(6) A Scenario: For what it is worth, let me present a scenario for the mass-matrix of the six quark-flavor system involving diagonal as well as non-diagonal elements. Such a matrix is motivated by the results of our toy model[34], which exhibited a radiative hierarchy in steps of $\mathcal{O}(\sqrt{"\alpha"}) \equiv \varepsilon^{1/2} \ll 1$. But, by no means have we derived such a matrix yet in the context of a realistic six flavor-model allowing for Cabibbo-mixings. [Note Cabibbo-mixings would arise by allowing for the formation of mixed condensate like $\langle \bar{f}_e f_\mu \rangle \neq o$, or equivalently introducing Higgs scalars which are composites of $\bar{f}_e f_\mu$ and allowing for such Higgs to have non-zero VeV's.]

The essence of the scenario is this. The top quark acquires a mass at the tree level of the Higgs potential through $\langle \mathfrak{F}_t \mathfrak{F}_t \rangle \neq o$ or $\langle \phi_t \rangle \neq o$. It is this mass, which generates radiatively a mass for the bottom quark (in the sense of the model of Ref. 34).

$$m_b^{(o)} = \mathcal{O}(\sqrt{\text{"}\alpha\text{"}})\, m_t^{(o)} \equiv \epsilon^{1/2}\, m_t^{(o)} \qquad (35)$$

We will impose $\epsilon^{1/2} \approx 1/5 - 1/6$. The remaining masses are generated in successive steps involving powers[37] of $\epsilon^{1/2}$ only via Cabibbo-like mixings between successive flavors.[38] The mass matrices, which we have in mind, for the "up" and "down" quark flavors are:

$$M_d = \begin{bmatrix} & d & s & b \\ & 0 & \epsilon^{3/2} m_b^{(o)} & 0 \\ & \epsilon^{3/2} m_b^{(o)} & 0 & \epsilon^{1/2} m_b^{(o)} \\ & 0 & \epsilon^{1/2} m_b^{(o)} & m_b^{(o)} \end{bmatrix}$$

$$\xrightarrow{\text{diagonalize}} \tilde{M}_d \simeq \begin{bmatrix} \epsilon^2 & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot m_b^{(o)}$$

$$M_u = \begin{bmatrix} u & c & t \\ 0 & \epsilon^2 & 0 \\ \epsilon^2 & 0 & \sqrt{\epsilon} \\ 0 & \sqrt{\epsilon} & 1 \end{bmatrix} \cdot m_t^{(o)}$$

$$\xrightarrow{\text{diagonalize}} \tilde{M}_u \simeq \begin{bmatrix} \epsilon^3 m_t^{(o)} & 0 & 0 \\ 0 & \epsilon\, m_t^{(o)} & 0 \\ 0 & 0 & m_t^{(o)} \end{bmatrix}$$

The masses, thus obtained, in terms of the single parameter $\sqrt{\epsilon} = \frac{1}{5}$ are quite sensible. For example, $m_t = 30$ GeV (input);

$m_b = \sqrt{\epsilon}\, m_t \approx 5$ GeV; $m_c = \epsilon\, m_t \approx 1.2$ GeV;

$m_s = \epsilon^{3/2}\, m_t \approx 200$ MeV; $m_d = \epsilon^{5/2} m_t \approx 8$ MeV;

$m_u = \epsilon^3\, m_t \approx 1.6$ MeV.

One can also obtain the Cabibbo-like angles:

$\theta_{ds} \approx \sqrt{\epsilon} \approx \sqrt{m_d/m_s} \approx 1/5$

$\theta_{uc} = \epsilon << \theta_{ds}$

$\theta_{bs} \sim \sqrt{\epsilon}$; $\theta_{tc} \sim \sqrt{\epsilon}$.

A similar matrix for the charged leptons can be considered with the added constraint that $m_t \propto (m_1 + m_{15})$, $m_b \propto \sqrt{\epsilon}.(m_1 + m_{15})$, while $m_\tau \propto \sqrt{\epsilon}(m_1 - 3m_{15})$; m_1 and m_{15} arise from VEV's of $\phi = (2,2,1)$ and $\xi = (2,2,15)$ respectively. These give $m_\tau = 1800$ MeV (input); $m_\mu = \epsilon\, m_\tau \approx 75$ MeV; $m_e \approx \epsilon^2\, m_\tau \approx 3$ MeV. For the neutrinos, the Dirac mass matrix can be written in a manner analogous to the up-quark mass matrix by including contributions from ϕ and ξ. These give, $m^D_{\nu_\tau} \approx 14$ GeV; $m^D_{\nu_\mu} \approx 1/2$ GeV; $m^D_{\nu_e} \approx 1$ MeV.

Combining with Majorana masses, and allowing for the same hierarchy in Majorana masses of ν_{eR}, $\nu_{\mu R}$ and $\nu_{\tau R}$, as the one for the Dirac masses of the neutrinos, we obtain:

$m_{\nu_{eL}} \approx 10^{-3}$eV, $m_{\nu_{\mu L}} \approx 1/2$ ev and

$\nu_{\tau L}$ ν_{eF}

The purpose of presenting a scenario for fermion mass-matrices is to motivate further work, which may help derive such matrices perhaps utilizing the mechanism of radiative hierarchy, as well as the one of hierarchy in composite sizes, as presented here.

9. Open Problems, Concluding Remarks:

For any preonic model, some of the most important questions and problems which one faces are:

(1) What is the nature of the "primodial" preon or pre-preon binding force? Is it entirely QCD-like, or does it differ from a QCD-like theory in some crucial aspect? We have suggested, on the basis of our considerations of dynamical symmetry breaking through preons that, on the one hand, QPD breaks chiral flavor symmetry very much like QCD, at least in so far as we focuss attention on the heaviest quark-flavor; but on the other hand it also breaks vectorial flavor symmetry non-perturbatively, unlike QCD. This would suggest that QPD has at least some intrinsic features which are not shared by QCD. These could involve perhaps scalar exchange forces superposed on vectorial gauge force, or perhaps a chiral primodial gauge force. If it would help.

(2) A related question: Do the observed electroweak strong gauge forces coexist with the primodial preon binding force at a fundamental level, or are they generated effectively only at some composite level? We have suggested in the interest of economy at a fundamental level coupled with consistency arguments for small size composites that these gauge forces are generated effectively only at a composite level. One needs to study by non-perturbative methods

energies.

(3) If quarks, leptons, Higgs mesons (ϕ), and the gauge bosons (W, Z, gluons) are composites of preons or pre-preons, and if the dominant aspect of QPD is QCD-like, one should be able to relate $\bar{q}q\phi$, $\bar{q}qW$, WWW, $W\phi\phi$, and ϕ^4 coupling constants to each other, once again by nonperturbative plus current algebraic methods. (The analogous couplings for QCD involve $\bar{N}N\sigma$, $\sigma\pi\pi$, $\bar{N}N\rho$, $\rho\rho\rho$, $\rho\pi\pi$, π^4 and σ^4 vertices). Derivation of such relations poses a challenging task for all preonic theories.

(4) Given the scenario of a hierarchy of compositeness, one is led to ask: do quarks, leptons, Higgs mesons and the spin-1 gauge bosons (W, Z, gluons etc.) - assuming that even these gauge bosons are composites - form at the same level of compositeness, and thereby have similar sizes, or do they form at different stages and thus have vastly differing sizes? In particular, do the quarks of different families or different flavors have the same size? We have suggested that the observed quark-lepton-mass-hierarchy owes its origin, at least in part, to a hierarchy in quark-lepton sizes, and have presented models where such a hierarchy could emerge, rather naturally. The success of this idea would, of course, have to await a successful derivation of the full fermion mass spectrum.

(5) Simple models of grand unification with elementary gauge bosons and quarks and leptons lead to a prediction for the weak angle $\sin^2\theta_W = 0.21$, which is in accord with experiments. The question arises (especially if this prediction is borne out accurately by future improved experiments): must preonic models

[illegible] effective grand unified theory – or at least a partial unified theory $SU(2)_L \times SU(2)_R \times SU(4)^c$ – to account for the observed value of $\sin^2\theta_W$ – or can they offer alternative explanations? While we do not have any alternative explanation, we have noted[26] that the standard one loop derivation of $\sin^2\theta_W$ requires that only the gauge bosons be point like up to say 10^{15} GeV. Similar restrictions do not apply to quarks and leptons, since their contributions finally drop out within the one-loop analysis. This leaves the door open that at least quarks and leptons may not be so point like.

(6) All preonic or pre-preonic models face the task of accounting suitably for cosmological problems including the problem of baryon excess and the horizon, the flatness and the homogeneity problems. Ideas developed in the context of grand unification for resolution of these problems can well apply to preonic models, even without the need for an effective grand unified theory, but these considerations would probably impose constraints on preonic or pre-preonic scales in that one may need suitable phase transitions at high temperatures, exceeding perhaps 10^{11} GeV.

(7) Needless to say, one of the most challenging tasks for preonic ideas is a successful derivation of the full fermion mass matrix in terms of a minimal number of parameters, which may be needed to set the relevant scale or scales. In other words, one is waiting for the equivalent of the Gell-Mann-Okubo formula for quarks and leptons to be derived from an underlying preonic theory. This will be the major test for preonic ideas especially in the absence of experimental indications.

(8) Equally challenging is the task of putting together the ideas of supersymmetry, supergravity, higher dimensions and compositeness – some or all of them – in one unified framework, which must, at the very least, reproduce observed phenomena.

The most important task at hand is experimental. Based on our considerations of dynamical symmetry breaking through preons and the assumption of universal size for all quarks and leptons, we are led to predict that quarks and leptons have an inverse size of the order of 2-3 TeV. This can be tested by high luminosity 5-10 TeV machines in the near future by looking for form factors for quarks and leptons and for excess production of lepton pairs in pp or $\bar{p}p$ collisions.

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References and Footnotes

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7. For example, consider a vector-like preonic gauge theory with left and right-handed preons ($P_{L,R}$) coupled to the [illegible] gauge particles (V): $g(\bar{P}_L \gamma_\mu P_L + \bar{P}_R \gamma_\mu P_R) \cdot V_\mu$. Here, internal symmetry indices are suppressed. Assume, for example, that quarks $q_{L,R}$ are made of $P_{L,R}$ and a set of spin-o preons, which are also coupled to V. Now, if W_L and W_R are spin-1 composites having the compositions $\bar{P}_L \gamma_\mu P_L$ and $\bar{P}_R \gamma_\mu P_R$ respectively, it is clear that W_L will couple to q_L, while W_R will couple to q_R. In this way, the interactions of the composites can be chiral, even though the preonic theory is vector-like.

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11. J. C. Pati, Proc. Workshop on Unification of Fundamental Interactions, Erice, March (1980), P. 267; Proc. Neutrino Conf. Erice, June (1980), p. 275; Phys. Lett. <u>98B</u>, 40 (1981).

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18. K. Matumoto, Progr. Theor. Phys. 52, 1973 (1974); O. W. Greenberg, Phys. Rev. Lett. 35, 1120 (1975). These papers contain the flavon-chromon idea partially in that they apply it to quarks only.

19. See e.g., H. Terazawa, Y. Chikashige and K. Akama, Phys. Rev. D. 15, 480 (1977); O. W. Greenberg and J. Sucher, Phys. Lett 99B, 339 (1981); H. Fritzsch and G. Mandelbaum, Phys. Lett. 102B, 319 (1981); R. Barbieri, R. N. Mohapatra and A. Masiero, Phys. Lett. 105B, 369 (1981). D. Wu, Harvard papers (1981); R. W. Robinett, Wisconsin preprint (1982).

20. A set is defined by the spin and binding "charge", or representation-content, with respect to the internal gauge symmetry, which must be the same for all members of the set.

21. See the second paper of Ref. 8, where two recently suggested alternative ideas for making gauginos superheavy in the context of N=1 supergravity theories are utilized.

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23. See e.g., E. Farhi and L. Susskind, Phys. Reports 74, No. 3 (1981); for a review of "Technicolor" and references.

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25. J. C. Pati (Ref. 11); E. Farhi and L. Susskind (Ref. 23); H. Harari, SLAC Summer School Lecture Notes (1981).

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27. We have in mind the following possibility. Imagine a minimal pre-preonic theory with or without supersymmetry which at the first level of compositeness, yields 8 (or even 12) spin-0 chromons $C^i_\alpha = (r,y,b,\ell,r',y',b',\ell')^i$ plus a set of spin 1/2-flavons $(f^i_a)_{L,R}$; "i" refers to the index for the primodial gauge symmetry G_b generating the binding force F_b of pre-preons. Examples of this kind have been exhibited in Ref. 8 (see discussion in text). In line with our discussion in the text, assume furthermore that the 8 chromons C^i_α are associated with an <u>effective gauge symmetry</u> $SU(8)^c$, which is generated through the formation of composite $SU(8)^c$-gluons. In other words, the $SU(8)^c$-force, in this picture, is an effective <u>residual</u> force generated by the primodial force F_b. Now, under the influence of F_b, which could form $\langle C^*C\rangle$-condensates, $SU(8)^C$ could break dynamically into $SU(4)^{col} \times SU(4)^{col'} \times U(1)$; and $SU(2)^{flav}_R \times SU(4)^{col}$ could further break into $SU(3)^{col} \times U(1)_Y$ through $\langle f_R f_R C^* C^*\rangle$ condensates,

... $SU(4)^{col'}$ ... color force. The scale-parameter Λ_c' of $SU(4)^{col'}$ would be naturally much bigger than Λ_{QCD}. The condensing fermions $\mathfrak{F}$'s could be composites of fC'^*, which are neutral with respect to the binding force F_b, but non-neutral with respect to both flavor and $SU(4)^{col'}$. In this case, $SU(4)^{col'}$ could provide the condensing force F_c, which would be distinct from the binding force F_b. This model is elaborated further in sec. 8.

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30. We conjecture that for a given N_c, if we progressively increase N_f starting with small values of $N_f = 1$, say, the chiral symmetry breaking parameter Λ will be progressively suppressed relative to the scale parameter Λ_p of QCD, at least over a range of values of N_f. It is important to examine by non-perturbative methods whether this happens for QCD itself.

31. For the sake of economy, we would assume, of course that neither flavor nor color exist even as global symmetries at the primodial level and that these symmetries in their local form (25) are generated only at a composite level through the primodial force F_b. But our discussions here would apply even if G_{eff} were a primodial symmetry.

32. We assume that the minimum of the potential does not permit the formation of condensates involving $f_L f_L C_{II}^* C_{II}^*$, $f_R f_R C_{II}^* C_{II}^*$, $f_L f_L C_I^* C_{II}^*$ and $f_R f_R C_I^* C_I^*$ even though these condensates are on par with the one of $f_R f_R C_I^* C_I^*$ from the viewpoint of the primodial force F_b. Such a selective and

symmetry.

32'. If we take the relevant chiral symmetry to be $SU(2)_L \times SU(2)_R \times U(1)_f$, the anomaly would match for the 8-chromon model if $G_b = SU(8)$. However, because of the fact that we use the condensate $\langle f_R f_R C^* C^* \rangle \neq 0$, both flavon and chromon number symmetries (i.e. $U(1)_f$ and $U(1)_C$) are broken spontaneously at the same time that the composites $\mathfrak{F}_I$ and $\mathfrak{F}_{II}$ are formed. The anomaly for the residual "global" symmetry given by (26) vanishes at the constituent as well as the composite level for any G_b. We, therefore, suggest that G_b is the minimal non-abelian symmetry $SU(2)$. This may explain why G_b operating on 2 flavons and 8 chromons does not break chiral symmetry, while the bigger symmetry $SU(4)_{HC}$ does. Note that the spontaneous breaking of $U(1)_f$ and $U(1)_C$ will generate two neutral massless goldstone bosons. These, however, will be coupled very weakly and do not contradict known facts. See, Y. Chikashige, R.N. Mohapatra and R.D. Peccei, Phys. Lett. 98B, 265 (1981), and G.B. Gelmini, S. Nussinov and T. Yanagida, Trieste preprint IC/82/157 for discussion on constraints on such massless particles.

33. Note, even if $\mathbb{F}_{II}$ as a composite of $(\mathfrak{F}_{II}\mathbb{C}^*) = (fC_{II}^*)(C_I^* C_{II})$ has the constituents $fC_I^* C_{II} C_{II}^*$, it would be misleading to view $\mathbb{F}_{II}$ as a quantum-pair excitation of fC_I^*. This is because, for any probe with momentum $\ll \Lambda_b$, the composite $\mathbb{F}_{II}$ would appear as a legitimate two-body composite with constituents $\mathfrak{F}_{II}$ and $\mathbb{C}^*$, and not as a four-body composite involving quantum pair-excitation of fC_I^*. We note that for this model, the diagonal masses of $\mathbb{F}_{II}$ and $\mathfrak{F}_I$ as well as the non-diagonal mixing between $\mathbb{F}_{II}$ and $\mathfrak{F}_I$ will be generated by the same condensate $\langle \bar{\mathfrak{F}}_{II} \mathfrak{F}_{II} \rangle$. The corresponding effective four-fermion amplitudes (compare with fig. 1, sec. 6) are: (i) $\bar{\mathbb{F}}_{II} + \mathbb{F}_{II} \longrightarrow \bar{\mathfrak{F}}_{II} + \mathfrak{F}_{II}$, (ii) $\bar{\mathfrak{F}}_I + \mathfrak{F}_I \longrightarrow \bar{\mathfrak{F}}_{II} + \mathfrak{F}_{II}$ and

(iii) $\mathbb{F}_{II} + \mathfrak{f}_I \longrightarrow \mathfrak{F}_{II} + \mathfrak{f}_{II}$; all three processes involve an exchange of $\mathbb{C}$ in the crossed channel. The processes (ii) and (iii) are expected to be damped however relative to (i) owing to the difference between the sizes of $\mathbb{F}_{II}$ and $\mathfrak{f}_I$, which would reflect itself in the respective vertices controlling these transitions. More precisely, the damping would depend upon the ratio η of the strengths of the vertices $\mathfrak{f}_{II} + \mathbb{C}^* \longrightarrow \mathfrak{f}_I$ versus $\mathfrak{f}_{II} + \mathbb{C}^* \longrightarrow \mathbb{F}_{II}$. It is remarkable that this model would lead to a 2×2 mass-matrix whose diagonal elements are proportional to 1 and η^2, while the off-diagonal element is proportional to η. This is qualitatively in accord (for $\eta \ll 1$) with the pattern of the mass-matrices M_u and M_d, which we suggest later, in this section, for the case of 3 families. The full picture would have to involve, of course, small corrections on the pattern mentioned above, which may perhaps be of radiative origin. Note furthermore that for this rather special 8-chromon model with $\mathfrak{f}_I$ and $\mathbb{F}_{II}$ having different sizes but same intrinsic quantum numbers, the simple dimensional argument of sec. 6 leading to eq. (9), which was developed for the case where only one scale (or size) is involved, does not hold simultaneously for $\mathfrak{f}_I$ and $\mathbb{F}_{II}$. As noted above, all three processes are controlled primarily by $(M_{\mathbb{C}})^{-2}$ (ignoring differences between vertices). In the case at hand, we assume that $\mathbb{C}$ has a mass of order $\Lambda_{HC} \sim$ few TeV, which is of the order of the inverse size of $\mathbb{F}_{II}$ rather than of $\mathfrak{f}_I$. Details of these considerations will be discussed in a paper.

34. M. Cvetic and J.C. Pati, to be published.

35. We argue (Ref. 34) that quantum corrections can alter the symmetry of the tree level solution for the vacuum, even in the absence of "accidental" symmetries, contrary to the commonly held view in this regard. This comes about provided certain non-zero eigenvalues of the scalar mass matrix are of the order of or smaller than quantum corrections.

36. C. Vafa and E. Witten, Princeton Univ. preprint, 1983.

37. In general, the off-diagonal elements could involve a new parameter $\sqrt{\varepsilon'}$, especially if these have a different origin compared to the diagonal elements. One may conceive of the two mechanisms described in the text being responsible for generating the diagonal and off-diagonal entries respectively. For simplicity, we put $\varepsilon' = \varepsilon$. Note that the idea that non-perturbative solutions can lead to different diagonal masses for the "up" and "down" members of the heaviest family (like $0 \neq m_t \neq m_b \neq 0$, or equivalently $0 \neq \langle \bar{\mathfrak{F}}_t \mathfrak{f}_t \rangle \neq \langle \bar{\mathfrak{F}}_b \mathfrak{f}_b \rangle \neq 0$), inspite of the $SU(2)_L \times SU(2)_R$-flavor-symmetry of the force that is responsible for the formation of the condensates, can be motivated naturally within an effective Higgs-mechanism. For example, recall that the scalar $\phi(2,2,0)$ of a $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$-gauge theory can have VEV at the tree-level with differing and non-vanishing diagonal elements κ and κ', which generate different masses for the "up" and "down" members of a fermion-family.

38. I believe that a mass matrix having such a pattern was first consdiered by H. Fritzsch, though without the specific relative entries exhibited here, which are motivated in part by the size-effect and in part by the radiative mechanism, presented here. The "Zeroes" appearing in the mass-matrices are to be interpreted as entities which are small relative to other respective relevant entries, rather than as exact zeroes.

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