INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

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"COLLEGE ON SOIL PHYSICS"

2 - 20 November 1987

"Flow of Water in Saturated Soils"

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Flow of Water in Saturated Soils - E.G. Youngs

Lecture 1: Thursday, 5 November, 9.00 - 10.15.

Lecture 2: Thursday, 5 November, 10.45 - 12.00.

Lecture 3: Friday, 6 November, 9.00 - 10.15.

Textbooks

E.C. Childs. The Physical Basis of Soil-water Phenomena, Wiley, 1969.

T. J. Marshall and J. W. Holmes. Soil Physics. Cambridge University Press, 1979.

D. Hillel. Fundamentals of Soil Physics. Academic Press, 1980.

J.N. Luthin (Ed.). Drainage of Agricultural Lands. American Society of Agronomy, 1957.

J. van Schilfgaarde (Ed.). Drainage for Agriculture. American Society of Agronomy, 1974.

J. Bear, D. Zaslavsky and S. Irmay. Physical Principles of Water Percolation and Seepage. UNESCO, 1968.

M. E. Harr. Groundwater and Seepage. McGraw-Hill, 1962

A. Verruijt. Theory of Groundwater Flow. Macmillan, 1970.

M. Muskat. The Flow of Homogeneous Fluids through Porous Media. McGraw-Hill, 1937.

Water Movement in Soils

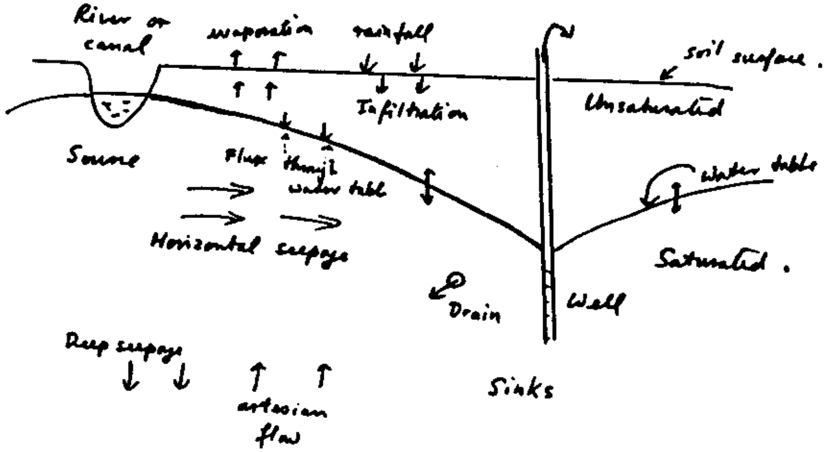
The groundwater zone in which the soil is saturated is just part of the flow region for soil-water movement. The complete flow region comprises of the groundwater zone and the unsaturated soil-water zone.

The groundwater zone gains water from:

1. Sources such as rivers and canals;
2. Surface precipitation percolating through unsaturated soil;
3. Drainage from the unsaturated soil-water zone above;
4. Artesian flow from below (foreign water).

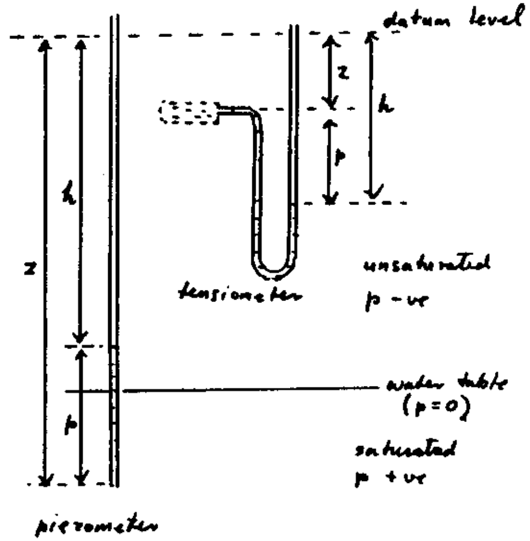
It loses water by:

1. Sinks such as drains and pumped wells;
2. Capillary rise, wetting soil above; also supplying water to crops and losing water by evaporation.
3. Deep seepage.



Soil-water Pressure and Hydraulic Head

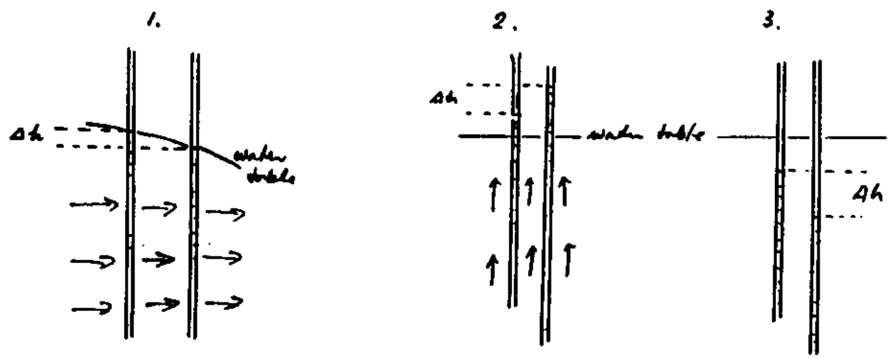
The soil-water pressure is the pressure of bulk water in equilibrium with the water in the soil pores. The soil-water pressure head is thus given by the height of water in the manometer arm of a tensiometer or height of water in a piezometer relative to the position of measurement. The hydraulic head (or potential) is the height of water in the manometer relative to a datum level.



Thus $h = p + z$.

Water moves from positions where the hydraulic head is high to those where it is low.

Examples



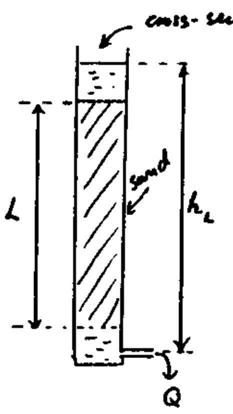
Horizontal water movement.

Upward movement

Downward movement

Darcy's Law (1856)

Established from experiments on sand columns:



$$Q = -KA\frac{h_L}{L}$$

with the potential measured as the hydraulic head, K is the hydraulic conductivity with dimensions LT^{-1}!

Hydraulic conductivity values of saturated soils:

Fine-textured soils:	$K < 10\ mm\ d^{-1}$
Soils with well-defined structure:	$10\ mm\ d^{-1} < K < 1\ m\ d^{-1}$
Coarse-textured soils:	$K > 1\ m\ d^{-1}$.

Darcy's law describes the flow of water in porous materials at low velocities when viscous forces opposing the flow are much greater than the inertial forces. The ratio of viscous forces to inertial forces is represented by the Reynold's Number Re, which may be defined as

$$Re = \frac{v d \rho}{\eta}$$

where v = mean flow velocity, d = characteristic length (mean pore diameter ?), ρ = density of water, η = viscosity of water. When Re > ~1.0, Darcy's law no longer holds.

Darcy's law is an empirical equation describing the macroscopic flow of water — an average flow as if the water pervaded all space. It does not address the complex microscopic flow pattern in the pore network.

It assumes soils to be "uniform" porous media. At the microscale the pore network makes soils inherently heterogeneous, but at a larger scale a volume can contain a representative configuration. Darcy's law assumes that flow takes place in volumes consisting of many representative volumes.

Darcy's law was established for inert, non-swelling porous materials.

Darcy's law can be expressed in differential form as:

$$v = -K\ \mathrm{grad}\, h$$

where v = flow velocity, h = hydraulic head. The principle of conservation of matter gives div v = 0, so that

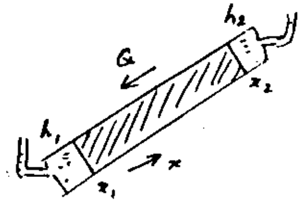
$$\nabla^2 h = 0$$

where ∇^2 is the Laplacian operator $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Thus the flow of water in soils is analogous to flow of electricity in conductors, and so we can use potential theory to obtain solutions to groundwater problems.

Examples of Flow in Soils

(1) Linear flow:



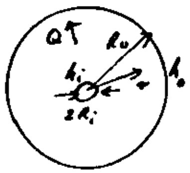
$$|Q| = +KA\frac{\partial h}{\partial x}$$

or $|Q|(x_2 - x_1) = KA(h_2 - h_1)$

$|Q| = KA\ \frac{h_2 - h_1}{\ }$

(2) Radial flow

Total outward flow Q at any radial distance r is:



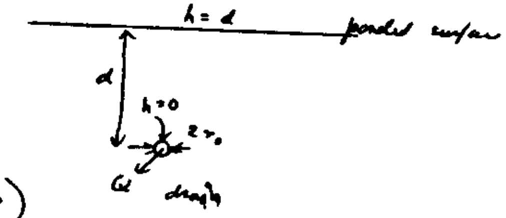
$$Q = -2\pi r K \frac{dh}{dr}$$

$$\therefore \quad \frac{Q}{2\pi K}\int_{R_i}^{R_0} \frac{dr}{r} = -\int_{h_i}^{h_0} dh$$

$$\text{or} \quad Q = -(h_0 - h_i)\, 2\pi K \Big/ \ln\left(\frac{R_0}{R_i}\right)$$

(3) Flow from ponded soil surface to cylindrical drain

The problem is solved by potential theory using method of images.



$$Q = (d - r_0)\, 2\pi K \Big/ \ln\left(\frac{2d - r_0}{r_0}\right)$$

$$\doteq \frac{2\pi K d}{\ln\left(\frac{2d}{r_0}\right)}.$$

Groundwater Flow

Groundwater may be contained in unconfined aquifers, that is groundwater regions bounded at the top by a phreatic surface and are continuous with the unsaturated soil-water zone; or in confined aquifers, where the water is confined (under pressure) by an impermeable stratum. In both situations Darcy's law:

$$v = -K \operatorname{grad} h$$

gives the flow with h described by Laplace's equation:

$$\nabla^2 h = 0.$$

The problem is solved with the known boundary conditions of the situation.

Boundary conditions at a water table (phreatic surface):

(a) $p = 0$, $h = \text{constant} + z$

(b) given flux derived from (i) surface flux and (ii) change in soil-water content of unsaturated zone in non-steady state conditions.

Flows are generally 3-dimensional so solutions to particular problems are often complicated.

General methods of solution

1. Analytical

Mathematical solutions to Laplace's equation are sought with exact (or nearly exact) boundary conditions. Alternatively, solutions are obtained using Seepage Analysis:

or the Dupuit-Forchheimer approximate analyses (see later).

2. Numerical

Finite difference and finite element methods can be used to give numerical solutions to Laplace's equation with the given boundary conditions.

3. Analogues

Since flow of electricity in conductors behaves the same as water flow in soils, electric analogues can be constructed to give solutions to groundwater problems. 2-dimensional problems are solved with graphited paper; 3-dimensional ones use electrolytes.

4. Hydraulic models

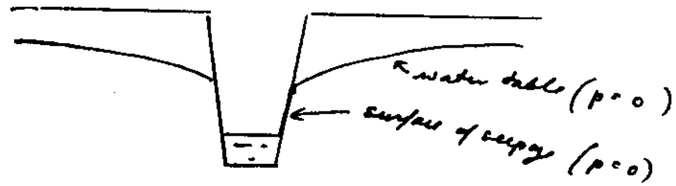
Hydraulic models can be constructed to scale, and groundwater behaviour observed on these models.

Groundwater Seepage

Many groundwater problems are concerned with horizontal seepage, and this was the first application of Darcy's Law to groundwater flow by Dupuit and Forchheimer last century. The seepage analysis is an exact analysis, whereas the Dupuit-Forchheimer analysis is approximate. Both reduce the dimensions of the problem by one.

Surface of Seepage

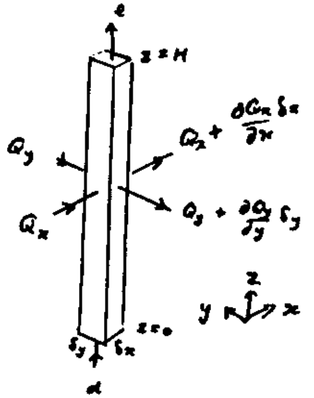
Surfaces of seepage occur above free water levels in ditches when drainage from the soil is occurring. The water table is not drawn down to the free water level, but to some height above, with water seeping out of the soil at zero soil-water pressure.



The existence of surfaces of seepage, as well as the existence of phreatic surfaces, make groundwater flow problems different from other potential problems. Calculating the groundwater seepage avoids complicated solutions to the potential problem.

Horizontal Seepage in Unconfined Aquifers.

Consider a vertical prism of a groundwater aquifer with base $\delta x\,\delta y$ from a horizontal base at $z=0$ to the water table at $z=H$. We consider vertical fluxes (+ve upwards) d and e at $z=0$ and $z=H$, respectively.



By continuity, flows in = flow out.

Thus

[Q_x, Q_y seepage per unit width in x + y directions]

$$Q_x\,\delta y + Q_y\,\delta x + d\,\delta x\,\delta y$$

$$= \left[Q_x + \frac{\partial Q_x}{\partial x}\delta x\right]\delta y + \left[Q_y + \frac{\partial Q_y}{\partial y}\delta y\right]\delta x + e\,\delta x\,\delta y.$$

or
$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = d - e \qquad \text{(Continuity equation).}$$

By Darcy's law

$$Q_x = -\int_0^H \left\{K(z)\frac{\partial h}{\partial x}\right\}dz ,$$

and
$$Q_y = -\int_0^H \left\{K(z)\frac{\partial h}{\partial y}\right\}dz$$

considering hydraulic conductivity $K(z)$ to vary with height. Note that $h = h(x,y,z)$.

Now the rule of differentiation of a definite integral gives:

$$\frac{d}{dx}\left[\int_0^H \{K(z)\,h\}\,dz\right] = \int_0^H \left\{K(z)\frac{\partial h}{\partial x}\right\}dz + H\,K(H)\frac{\partial H}{\partial x}$$

or
$$\int_0^H \left\{K(z)\frac{\partial h}{\partial x}\right\}dz = \frac{d}{dx}\left[\int_0^H \{K(z)h\}\,dz\right] - H\,K(H)\frac{\partial H}{\partial x}$$

$$= \frac{d}{dx}\left[\int_0^H \{K(z)\,(h-z)\}\,dz\right]$$

$$= \frac{d}{dx}\left[\int_0^H \{K(z)\,p\}\,dz\right]$$

Now defining $E = \int_0^H \{K(z)\,p\}\,dz$

$$Q_x = -\frac{\partial E}{\partial x} , \qquad Q_y = -\frac{\partial E}{\partial y}$$

so that
$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = -d + e$$

Thus we have reduced the three-dimensional equation (Laplace's equation) for groundwater flow to a two-dimensional equation for the horizontal groundwater seepage. E may be ~~as~~ treated as a potential to find the seepage. (Often referred to as the "seepage potential).

Boundary conditions for E

(1) At vertical boundaries extending in depth from $z=H$ to $z=0$.

(i) water at level H_1, entering soil ..

$$E_1 = \int_0^{H_1} \{K(z)\,(H_1 - z)\}\,dz$$

(ii) water at level H_2, leaving soil; seepage surface to a height H_2'

$$E_2 = \int_0^{H_2} \{K(z)\,(H_2 - z)\}\,dz + \int_{H_2}^{H_2'} \{K(z)\,(z - z)\}\,dz$$

$$= \int_0^{H_2} \{K(z)\,(H_2 - z)\}\,dz.$$

(2) Horizontal seepage rates given: $((Q_x)_p$ + $(Q_y)_p$ given)

$$(Q_x)_p = -\left(\frac{\partial E}{\partial x}\right)_p \quad ; \quad (Q_y)_p = -\left(\frac{\partial E}{\partial y}\right)_p$$

For wells being pumped at a rate Q_w, at well walls when $r = r_0$,
$$\left(\frac{\partial E}{\partial r}\right)_w = \frac{Q_w}{2\pi r}$$

(3) Vertical fluxes $d(x,y)$, $e(x,y)$ given.

d is upward flow from artesian supplies.

e is +ve if there is upward movement (e.g., due to plant consumption and evaporation); e is -ve for rainfall seeping to water table and drainage of unsaturated zone.

Note that equation for the horizontal seepage gives exact seepage rates when boundary conditions are accurately given. It gives values of $E(x,y)$ at (x,y) throughout the flow region, not $H(x,y)$. $H(x,y)$ may be estimated and estimates given with a calculated precision, in certain situations.

The Dupuit - Forchheimer Analysis

Analysis developed by Dupuit (1863) and Forchheimer (1886) to describe water movement in shallow aquifers.

Assumptions: (1) flow is horizontal ($\equiv$ vertical equipotentials)

(2) hydraulic head gradient = water-table slope

(3) No surfaces of seepage.

For uniform soil, flow is therefore given by

$$Q_x = -KH\frac{\partial H}{\partial x}, \quad Q_y = -KH\frac{\partial H}{\partial y}$$

and with $d=0$,

$$\frac{\partial}{\partial x}\left(KH\frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(KH\frac{\partial H}{\partial y}\right) = e$$

— (an approximation of seepage equation)

i.e.,
$$\frac{\partial^2 H^2}{\partial x^2} + \frac{\partial^2 H^2}{\partial y^2} = \frac{2e}{K}.$$

Allows an estimate of water-table height H. This is clearly in error near surfaces of seepage where no surface of seepage is recognised and water table is drawn down to level of water in ditch.

If we apply the Dupuit - Forchheimer assumptions to the case of hydraulic conductivity varying with depth ($K = K(z)$):

$$Q_x = -\int_0^H K(z)\frac{\partial H}{\partial x}\,dz = -\frac{\partial H}{\partial x}\int_0^H K(z)\,dz \qquad Q_y = -\frac{\partial H}{\partial y}\int_0^H K(z)\,dz$$

Introducing the function G (the Girinskii potential)

$$G = \int_0^H K(z)(H-z)\,dz$$

$$\frac{\partial G}{\partial x} = \int_0^H K(z)\frac{\partial}{\partial x}(H-z)\,dz - (H-H)K(H)\frac{\partial H}{\partial x}$$

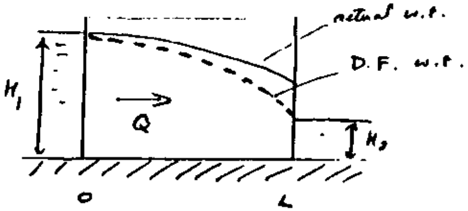
$$= \int_0^H K(z)\frac{\partial H}{\partial x}\,dz = -Q_x.$$

Similarly $Q_y = -\dfrac{\partial G}{\partial y}$

and $$\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} = e.$$

Examples of Horizontal Seepage

(1) Earth dam:



Flow per unit width

$$Q_x = -\frac{\partial E}{\partial x}.$$

Integrating
$$\int_0^L Q_x\,dx = E_1 - E_2$$
$$= \int_0^{H_1} K(z)\,(H_1 - z)\,dz - \int_0^{H_2} K(z)\,(H_2 - z)\,dz$$
$$= \int_{H_2}^{H_1} \left\{ \int_0^H K(z)\,dz \right\} dH$$

or
$$Q_x = \frac{1}{L} \int_{H_2}^{H_1} \left\{ \int_0^H K(z)\,dz \right\} dH$$

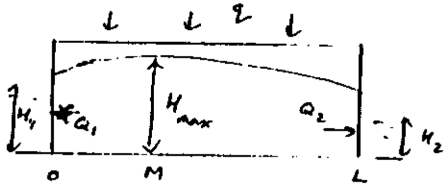
For K = constant

$$Q_x = \frac{K}{2L}\left(H_1^2 - H_2^2\right) \quad \text{— Dupuit's equation.}$$

Dupuit-Forchheimer analysis gives fortuitously <u>correct</u> flow into through earth bank. Seems that assumption of vertical equipotentials offsets assumption that no seepage surface exists.

(2) Earth bank with precipitation (or evaporation).

$$\frac{\partial^2 E}{\partial x^2} = -q$$



Integrating twice:

$$\frac{\partial E}{\partial x} = -qx + A$$
$$E = -q\frac{x^2}{2} + Ax + B \qquad (A \,\&\, B \text{ constants})$$

At $x = 0$, $E = E_1$; at $x = L$, $E = E_2$; At watershed M, $\frac{\partial E}{\partial x} = 0$.

Thus $B = E_1$, $A = \left(E_2 - E_1 + q\frac{L^2}{2}\right)/L$

so that at $x = M$ where $\frac{\partial E}{\partial x} = 0$,

$$M = \frac{L}{2} - \frac{E_1 - E_2}{qL}$$

Flow into ditch 1, $-Q_1 = \left(\frac{\partial E}{\partial x}\right)_{x=0}$

$$= A = \frac{qL}{2} - \frac{E_1 - E_2}{L}$$

and flow into ditch 2, $Q_2 = -\left(\frac{\partial E}{\partial x}\right)_{x=L}$

$$= qL - A = \frac{qL}{2} + \frac{E_1 - E_2}{L}.$$

E_1 and E_2 are known if $K(z)$ is known.

For K constant,

$$M = \frac{L}{2} - \frac{K(H_1^2 - H_2^2)}{2qL}$$
$$-Q_1 = \frac{qL}{2} - \frac{K(H_1^2 - H_2^2)}{2L}$$
$$Q_2 = \frac{qL}{2} + \frac{K(H_1^2 - H_2^2)}{2L}.$$

For $K = K(z)$, approximate water table obtained using Girinskii's potential. When K = constant, Dupuit-Forchheimer analysis gives water table $H(x)$ as:

$$H = \sqrt{\left[H_1^2 - \left(H_1^2 - H_2^2\right)\frac{x}{L} + \frac{q}{K}x(L-x)\right]}$$

when $H_1 = H_2$, at mid spacing the water table height is H_M, $M = \frac{L}{2}$

$$\frac{4\left(H_M^2 - H_1^2\right)}{L^2} = \frac{q}{K}.$$

Good approximation since at midspacing equipotential is nearly vertical if depth of aquifer not too great.

(3) Radial flow into completely penetrating pumped wells.

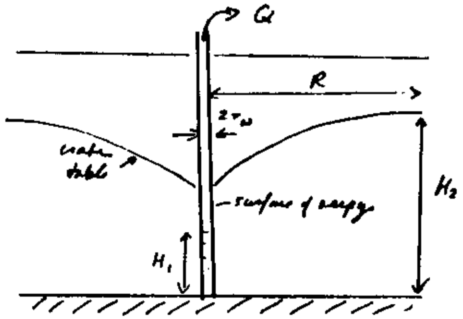
For steady flow at any radial distance

$$Q = 2\pi r \frac{\partial E}{\partial r}$$

$$\therefore \frac{Q}{2\pi}\int_{r_w}^{R}\frac{dr}{r} = \frac{Q}{2\pi}\ln\frac{R}{r_w}$$

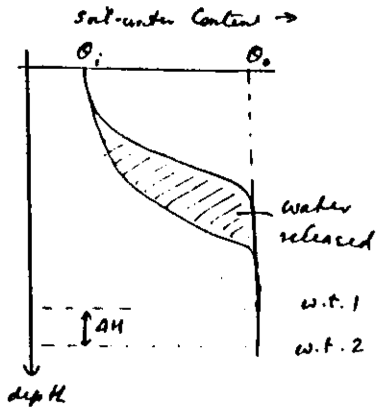
$$= E_2 - E_1$$

$$\text{or } Q/2\pi = \int_{H_1}^{H_2}\left[\int_0^H K(z)\,dz\right]dH \Big/ \ln\frac{R}{r_w}$$



The Moving Water Table – Specific Yield

When water tables fall, the soil above drains, giving rise to a flux of water through the water table. The "specific yield" is the amount of water released by the unsaturated soil for a unit fall of the water-table height. Thus:



Specific yield $S = \dfrac{\text{water released}}{\Delta H} = \theta_0 - \theta_i$ if profile maintains constant shape.

$$\text{Flux} = S\frac{dH}{dt}.$$

$$\left[\text{Generally } S = S(H,t)\right].$$

Unsteady Groundwater Problems: Boussinesq's Equation

The general non-steady state of water movement in soils gives a moving water-table situation with water either draining or wetting the unsaturated soil. The behaviour in the whole region can be described by Richards' equation

$$\frac{\partial \theta}{\partial t} = \nabla . K\nabla h + \frac{\partial K}{\partial z}$$

subject to the boundary conditions of the region, noting that in the saturated region this equation reduces to

$$\nabla^2 h = 0$$

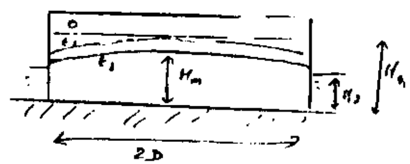
Numerical solutions are difficult. A simple approximate way of investigating the groundwater movement under non-steady state situation is to combine the concept of specific yield with the Dupuit-Forchheimer analyses. Thus we write the flux across the water table as $S\frac{\partial H}{\partial t}$ and write:

$$K\frac{\partial H^2}{\partial x^2} + K\frac{\partial H^2}{\partial y^2} = 2S\frac{\partial H}{\partial t} \qquad \text{(Boussinesq's equation)}$$

Various methods of linearising this equation ~~are~~ can be made to give solutions for simple boundary conditions.

Example: Drainage to ditches:

Ditch level changed from H_1 to H_2. We wish to follow fall of maximum water table.



We rewrite Boussinesq's equation as:

$$S\frac{\partial H}{\partial t} = K\bar{H}\frac{\partial^2 H}{\partial x^2} \qquad \text{where} \quad \bar{H} = \frac{H_1+H_2}{2}$$

This is a diffusion equation which can be solved with the boundary conditions of the problem to give

$$A_m \doteq \frac{4(H_1-H_0)}{\pi}\, e^{-\frac{K\bar{H}}{S}\frac{\pi^2 t}{4D^2}}$$

<u>Solution assuming a succession of steady states</u>

One way of considering the groundwater movement when the water table moves is to assume the water table to fall as a succession of steady states with the flux through the water table $= S\frac{dH}{dt}$.

<u>Example</u>: Drainage to ditches.

The steady state water table height H_m is given by the Dupuit-Forchheimer analysis as:

$$\frac{H_m}{D} = \sqrt{\frac{q}{K}}\,.$$

Putting $q = S\frac{dH_m}{dt}$ in this equation:

$$\frac{S}{K}\frac{dH_m}{dt} = \frac{H_m^2}{D^2}$$

$$\text{or} \quad t = \int_{(H_m)_0}^{H_m} \frac{SD^2}{K}\frac{1}{H_m^2}\, dH_m$$

$$= \frac{SD^2}{K}\left[\frac{1}{H_m^2} - \frac{1}{(H_m)_0^2}\right]$$

If $S = S(H_m)$ and $K = K(z)$ $[= K(H_m)]$, equation can easily be integrated numerically.

The Measurement of Hydraulic Conductivity of Saturated Soils.

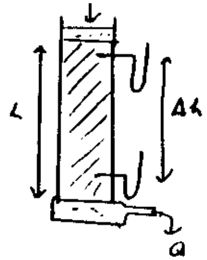
1. <u>Laboratory measurements</u>

Measurements essentially repeat Darcy's experiment on cores of soil collected from the field. Great care has to be taken in obtaining cores so that the soil is undisturbed. They are usually obtained in brass or pvc or glass fibre cylinders that form part of the <u>permeameter</u>.

(i) <u>Constant head permeameter</u>

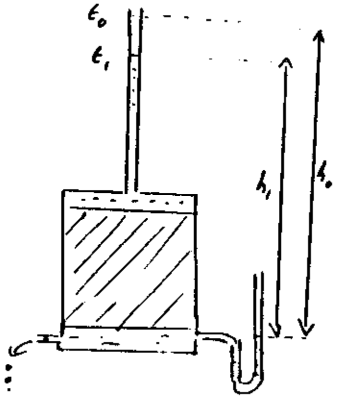
$$Q = KA\frac{\Delta h}{L} \quad \text{or} \quad K = \frac{QL}{A\,\Delta h}$$

(Δh measured by piezometers)



(ii) <u>Falling head permeameter</u>

No water is added after time t_0 when water level fall is observed.



$$K = \frac{A'L\ \ln(h_0/h_1)}{A\,(t_0 - t_1)}$$

(A' = cross sectional area of supply tube
A = cross sectional area of soil sample)

(iii) Infiltration Method

The Green and Ampt analysis of infiltration shows that the infiltration rate

$$\frac{di}{dt} = K\left(\frac{h_f}{z} + 1\right)$$

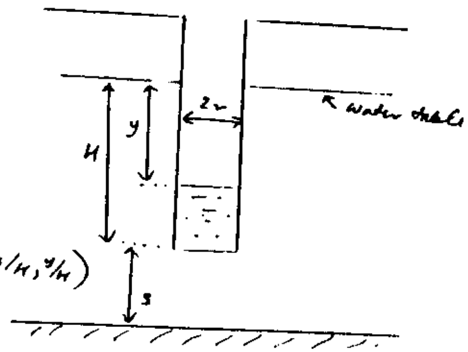
where z is the advance of the wetting front and $-h_f$ is the soil-water pressure head at this front. Thus, by observing di/dt as a function of z, a plot of di/dt against $1/z$ gives an intercept of K.

2. Field Measurements below a Water Table

These measurements employ unlined or lined wells sunk below the water table and involve measurements of flows into or out of the wells when the water level is disturbed from the equilibrium.

(i) Auger-hole Method

Water is bailed from a cylindrical unlined hole, and rate of rise of water level observed. Then



$$K = C\frac{dy}{dt}, \quad C = C(r/H, s/H, y/H)$$

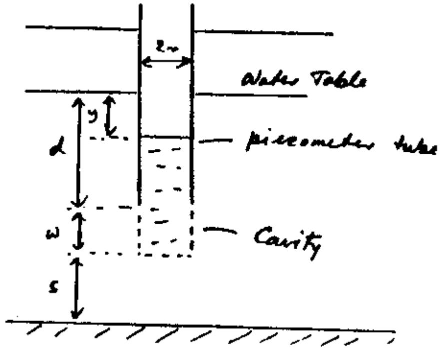
or

$$K = \frac{4.63}{(20 + H/r)(2 - y/H)} \cdot \frac{r}{y} \cdot \frac{dy}{dt} \qquad s > 0.5H$$

$$K = \frac{4.17}{(10 + H/r)(2 - y/H)} \cdot \frac{r}{y} \cdot \frac{dy}{dt} \qquad s = 0.$$

(ii) Piezometer Method

The piezometer method uses lined wells, sunk below the water table, with or without a cavity. As in the auger hole, the water level is depressed and the rate of return to equilibrium observed.

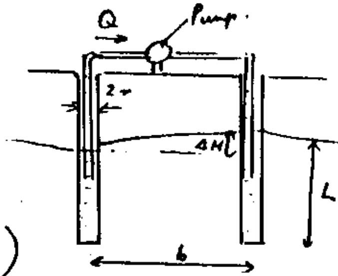


$$K = \frac{\pi r^2 \ln(y_0/y)}{A(t - t_0)} \qquad A = A(d/r, w/r, s/r).$$

A is a shape factor, usually obtained by electric analogue.

(iii) Two-well Method

The two-well method of Childs employs two un-lined wells sunk to the same depth below the water table.



$$K = \frac{Q}{\pi \Delta H (L + L_f)} \cosh^{-1}\left(\frac{b}{2r}\right)$$

L_f is an end correction to be added to take into account the flow beneath the wells and the flow in the capillary fringe.

(iv) Land Drains used as Permeameters

Drain flow and hydraulic conductivity related by drainage equations.

$$q/K = f(H_m/D)$$

H_m = maximum water-table height
D = half drain-spacing.

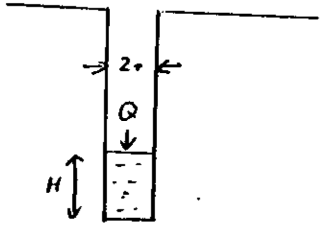
$$\text{or} \quad K = \frac{q}{f(H_m/D)}.$$

3. Field Measurements in the Absence of a Water Table.

These measure the water uptake by unsaturated soils from a saturated surface, and rely on infiltration theory. The measured flows usually depend on the capillary absorptive properties of the soil, as well as the hydraulic conductivity of the saturated soil. Exact interpretation of experimental data to give hydraulic conductivities is often difficult, and many formulas used are approximate.

(i) Borehole Permeameter

Cylindrical hole made in soil and level of water kept constant at H with input Q.



$$K = \frac{CQ}{2\pi H^2}$$

with $C = \sinh^{-1}\left(\frac{H}{r}\right) - 1$ for $H \gg r$.

(ii) Auger-hole Method

Similar to bore hole method, but level of water is allowed to fall without input Q after filling to H_0.

Then $$K = \frac{r \ln\left[\left(1 + 2H_0/r\right)/\left(1 + 2H/r\right)\right]}{2(t - t_0)}$$

(iii) Air-entry Permeameter

A column of soil is contained within an infiltration cylinder driven into the soil. Water under a pressure head infiltrates into the soil and the rate measured after the

wetting front has penetrated some distance down the isolated column of soil. The hydraulic conductivity is obtained from this rate and the hydraulic head gradient causing the flow, deduced from measurements on the depth of the wetting front and the pressure head at this front.

(iv) Ring-Infiltrometer Method

Analyses the 3-dimensional flow from infiltration rings. Steady-state flow Q (obtained very quickly) gives K:

$$K \doteq \frac{Q}{\pi R^2} \Big/ \left(1 + 2h_f/R\right)$$

(where R = ring radius, h_f = air entry value).

Alternatively K can be obtained from early non-steady flow from:

$$K = \frac{\rho g \eta R^4 \Delta\theta^2}{\sigma^2 t^2}\left[-0.365 + \left[0.133 + \frac{I}{R^3 \Delta\theta}\right]^{1/2}\right]^2$$

where I = infiltration after time t, $\Delta\theta$ = increase in water content, ρ = density of water, η = viscosity of water, σ = surface tension of water, g = acceleration due to gravity.