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The Infiltration Process

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I. INTRODUCTION TO THE INFILTRATION PROCESS

by

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1. DEFINITIONS

Infiltration is the process of entry of water into the soil through the soil surface.

Soil surface - plane: horizontal or inclined,
- concave or convex, combination,
- cavity: sferical or tubular.

Source of water - covers the whole surface,
- only a part /point, line.../

Solutions: one dimensional vertical
horizontal / = absorption/
two and three-dimensional

This introduction is restricted to the one-dimensional vertical infiltration through the horizontal plane. Soil will be considered as inert to water /no shrinkage, no swelling/.

2. NOTATION AND BASIC TERMS

A	parameter in Philip's algebraic equation of infiltration
A_2, A_3..	parameters in Philip's series solution
C_m	empirical constant in empirical infiltration equations
C_w	soil water capacity, $C_w = d\theta/dH$
D	soil water diffusivity
DBC	Dirichlet's boundary condition
F	flux concentration relation, $F = q/q_o$
H	soil water pressure head
H_A	air entry value of H
H_i	initial value of H
H_o	pressure head at $z = 0$
H_f	pressure head at the wetting front in Green and Ampt's approximate solution
I	cumulative infiltration
I_D	cumulative infiltration with DBC
K	hydraulic conductivity $K(\theta)$
K_i	hydraulic conductivity at the initial soil water content θ_i
K_s	saturated hydraulic conductivity
L_f	depth of the wetting front
S_1, S	sorptivity
q	flux
q_D	infiltration rate with DBC
q_e	effective rain intensity
q_o	flux at z=0, infiltration rate
q_{oc}	constant infiltration rate for DBC, theoretically at $t \to \infty$
K_A, K_B	approximates of K_s

q_{o1} infiltration rate at t=I, usually at t = 1 min

q_r rain intensity

t time

t_p ponding time

z vertical coordinate

$\alpha, \beta, \gamma, \varepsilon$ empirical exponents in empirical infiltration equations

η Boltzman´s variable

θ soil water content

θ_i initial soil water content

θ_o soil water content on the soil surface, at $z = 0$

θ_s saturated soil water content

3. ROLE OF INFILTRATION

A. In hydrology: Infiltration divides the precipitation in

(a) portion of the surface water excess:

(a1) immediate surface runoff,

(a2) surface storage etc.

(b) portion of water entering the soil:

(b1) storage of water in soil porous medium,

(b2) feeding the ground water,

(a1) and (b2) supply the river discharge.

B. In plant production: Continuous flux of water through the plant - Discontinuous source of water in precipitation Soil: storage reservoir with A.(b1).

C. In environmental production: Water = carrier of polutants. Rate of transport of polutants from the source of polutants: by A.(a1) $\gg$ A(b2).

D. In soil hydrology: One of the elementary processes.

A note: Soil hydrology = elementary processes (infiltration, redistribution, drainage to ground water, upward fluxes, evapotranspiration) + meteorologic situations and processes. The resulting storage of soil water is classified according to the degree of the excess or insufficiency and to the duration of such a period. The year´s and vegetational period is considered, too.

4. BOUNDARY CONDITIONS

Richards´ equation is solved either in the diffusive form (Klute, 1952)

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[D(\theta)\frac{\partial \theta}{\partial z}\right] - \frac{dK}{d\theta}\frac{\partial \theta}{\partial z} \tag{1.1}$$

or in the capacitance form

$$\frac{d\theta}{dH}\frac{\partial H}{\partial t} = \frac{\partial}{\partial z}\left[K(H)\frac{\partial H}{\partial z}\right] - \frac{\partial K(H)}{\partial z} \tag{1.2}$$

$d\theta/dH = C_w$ the soil water capacity.

Soil is considered as the semiinfinite column. Richards´ equation is solved for (a) the initial condition; in the semianalytic and approximative procedures:

$$\theta = \theta_i \qquad t = 0 \qquad z > 0 \tag{1.3}$$

In numerical methods $H = H_i$ instead of $\theta = \theta_i$. In the field: $\theta = \theta_i(z)$.

(b) the boundary conditions. We distinguish Dirichlet´s boundary condition and Neuman´s boundary condition.

4.1. Dirichlet´s boundary condition (DBC, concentration condition)

Formulated as either soil water content θ_i or the soil water pressure head H on the boundary:

$$\theta = \theta_0 \qquad z = 0 \qquad t > 0 \tag{1.4}$$

Usually: $\theta_0 = \theta_s$

$$H = H_0 \qquad z = 0 \qquad t > 0 \tag{1.5}$$

(exact)

Field situations: Floods.

Irrigation: Check-basin irrigation,
border irrigation,
furrow irrigation.

Infiltration tests: Double ring infiltration test,
basin infiltration test.

Note: $H = H_0(t)$ is formulated in some numerical procedures, too, e.g. for infiltration with falling water table.

Results of the tests and of the theoretical treatment of the problem:

The soil water content profile $\theta(z,t)$ is similar to piston-like flow (step-like profile), especially in sands. Inclination of the wetting front in clay increases with time (Fig.1 data from Haverkamp et al., 1977). If θ_i increases, the rate of advance of the wetting front increases, too.

Cumulative infiltration in time $I(t)$ is steep at short time, after long time $I(t)$ is a straight line (theoretically at $t \to \infty$). Infiltration rate q_o decreases first rapidly with time, at $t = 0$ is $q_o \to \infty$. After long time, $dq_o/dt \to$ const., theoretically $\lim_{t \to \infty} q_o(t) = K_s$. Constant rate q_o is denoted sometimes q_{oc} (Fig.2). The shape $I(t)$, $q_o(t)$ depends upon the hydraulic characteristics of the soil $K(\theta)$, $\theta(H)$ and upon θ_i, θ_s. With the increase of θ_i the rate q_o decreases (Fig.3, light clay, Philip, 1957). When $H_o > 0$, and the depth of water on the surface increases, the infiltration rate q_o increases, too (Fig.4, light clay, Philip, 1958).

The process can be partitioned into matric and gravitational components, Fig.5 (Kunze, Nielsen, 1982). The matric forces have the dominant influence at the early stage of infiltration, the gravitational force at large time. The relative difference of the position of the wetting front between the horizontal and vertical infiltration increases with time.

4.2. Neuman's boundary condition (NBC, flux condition)

On the boundary z_o instead of the soil water content (or pressure head) as in DBC, the flux q_o is defined. Either q_o = const., or $q_o(t)$:

$$q_o = -D(\theta)\frac{\partial \theta}{\partial z} + K(\theta) \qquad z=0 \qquad t>0 \tag{1.5}$$

or

$$q_o = -K(H)\frac{\partial H}{\partial z} + K(H) \qquad z=0 \qquad t>0$$

Field situations: Rain infiltration.

Irrigation: Sprinkler irrigation.

Infiltration tests: Rain simulators.

Since the infiltration rate q_o can be under certain circumstances lower than the rain intensity q_r, we have to use in the discussion these terms with the strict meaning:

q_r - rain intensity (= flux from the atmosphere),

q_o - infiltration rate (flux through the soil surface).

4.2.1. q_r = const.

Two classes of problems of rain infiltration exist, if q_r is the rain intensity:

A. $0 < q_r/K_s \leq 1$ and $q_o = q_r$

The boundary condition is (1.5). The profile of the soil water content is in Fig.6, the soil water content at the surface θ_o increases with time, approaching θ_c in $q_o = K(\theta_c)$.

B. $q_r/K_s > 1$

B.1. Time interval $0 < t \leq t_p$, $q_o = q_r$

The boundary condition is in the form of NBC:

$$q_o = -D(\theta)\frac{\partial \theta}{\partial z} + K(\theta) \qquad z=0 \qquad 0<t\leq t_p \tag{1.6}$$

The soil water content at the surface θ_0 increases with time, reaching $\theta_0 = \theta_s$ at ponding time $t = t_p$ (Fig.7).

B.2 Time $t > t_p$

The boundary condition is in the form of DBC with :

either no runoff of excess water

$$H = H_0(t) \qquad z = 0 \qquad t > t_p \tag{1.7}$$

or full runoff of excess water

$$\theta = \theta_s \quad \text{or} \quad H = 0 \qquad z = 0 \qquad t > t_p \tag{8}$$

Time of ponding t_p separates from the rain the effective rain which can cause the immediate surface runoff. The greater is the rain intensity, the shorter is the period without ponding and t_p decreases (Fig.8, Rubin, 1969).

4.2.2. $q_r(t)$

Note: Infiltration with DBC (from the ponded surface with $H_0 \to 0$) can be formulated as infiltration with the flux boundary condition and with $q_r/K_s \to \infty$ when the non-infiltrated excess water flows away (full runoff). The ponding time t_p for $q_r(t)$ is obtained from two equalities (Kutílek, 1982, Peschke and Kutílek, 1982 , based upon the development of Mls , 1980 and Kutílek , 1980):

$$\int_0^{t_p} q_r(t)\,dt = \int_0^{t_x} q_D(t)\,dt \tag{1.9}$$

$$q_r(t_p) = q_D(t_x) \tag{1.10}$$

where q_D is the infiltration rate with DBC (= ponded surface with $H_0 \to 0$). Graphical demonstration is in Fig.8. When q_r = const., and $q_r/K_s > 1$, we get

$$q_r t_p = \int_0^{t_x} q_D(t)\,dt \tag{1.11}$$

see Fig.8.

The infiltration rate q_o at $t > t_p$ is $q_D(\tau)$ with $\tau = t - (t_p - t_x)$, i.e. the q_D curve is shifted by $(t_p - t_x)$. Effective rain rate is

$$q_e = q_r(t) - q_D(\tau) \tag{1.12}$$

The relations between the infiltration with DBC and NBC as discussed above in eq.(1.9) to (1.12) are valid if the soil surface is not affected by the rain in a different way from the ponded type of infiltration. The sealing of the surface due to the kinetic energy of rain drops is not considered. As it follows from the development, the ponded infiltration test with DBC should offer sufficient information for the determination of the rainfall infiltration (with NBC) provided that the sealing by rain does not exist.

Graphical illustrations for rain intensities closer to reality and to the hydrological practice are in Fig.9 (Peschke, Kutílek, 1982).

5. CLASSES OF SOLUTIONS

5.1. Physically based solutions

The solution of the Richards' equation (1.1) to the initial and boundary conditions (1.3) and (1.4) is searched and the form $I(t)$ or $q_0(t)$ is preferred.

Philip's (1957) procedure starts with the solution of the horizontal infiltration and the result is corrected with regard to the gravitational term, Fig.5 is instructive. The solution is based on the similarity solution of the one-dimensional absorption of the gravity-free Eq.(1.1), i.e. of

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left[D(\theta)\frac{\partial \theta}{\partial x}\right] \qquad (1.13)$$

for conditions (1.3) and (1.4). Solution of Eq(1.13) is the first step of the solution of Eq.(1.1) and it has the form (Fig.10)

$$z_1(\theta, t) = \eta_1(\theta)\, t^{1/2} \qquad (1.14)$$

The cumulative volume of water thus infiltrated is

$$I_1 = \int_{\theta_i}^{\theta_0} z_1(\theta, t)\, d\theta \qquad (1.15)$$

When the term sorptivity S (Philip, 1957) is introduced

$$S_1 = \int_{\theta_i}^{\theta_0} \eta_1(\theta)\, d\theta \qquad (1.16)$$

we get

$$I_1 = S_1 t^{1/2} \tag{1.17}$$

As Eq(1.13) is equal to Eq.(1.1) when the gravitational term $\partial K/\partial z$ is introduced, the solution(1.14) has to be corrected by the term y and

$$z = z_1 + y \tag{1.18}$$

where again y (t, θ) . However, the solution allows to obtain the approximate of y only, denoted here y_1 and $y = y_1 + u_1$ where u_1 is the error of approximation and

$$z = z_1 + y_1 + u_1 + \cdots \tag{1.19}$$

with transformations analogic to(1.14). The final solution of (1.1) has the form of a time serie solution

$$z(\theta, t) = \eta_1 t^{1/2} + \eta_2 t + \eta_3 t^{3/2} + \cdots \tag{1.20}$$

In analogy to(1.15) and(1.17) Philip obtained

$$I = S_1 t^{1/2} + A_2 t + A_3 t^{3/2} + \cdots + A_n t^{n/2} + K_i t \tag{1.21}$$

The last term expresses the flux at $\theta = \theta_i$, since

$$\int_0^\infty (\theta - \theta_i)\, dz = \int_{\theta_i}^{\theta_0} z\, d\theta = I(t) - K_i t \tag{1.22}$$

Parlange´s (1971) approach was different. He integrated in the first step the equation of continuity, the result was combined with the Darcy-Buckingham equation, integrated again and this first approximate z_1 was used in the next second step etc.

The method was modified by Císler (1974) and by Philip and Knight (1974):

Philip (1973) first discussed the flux concentration relation (Morel-Seytoux, 1971)

$$F = \frac{\int_{\theta_i}^{\theta} \eta(\theta)\, d\theta}{\int_{\theta_i}^{\theta_0} \eta(\theta)\, d\theta} = \frac{q}{q_0} \tag{1.23}$$

in various types of the unsaturated flow processes and for the envelope of possible solutions:

(a) the linear soil, D = const.,

(b) soil with $D(\theta)$ according to Dirac δ -function.

Further on, Philip and Knight (1974) have proposed an iterative scheme for the solution of the horizontal infiltration (absorption) with sorptivity expressed as

$$S_n = \left[2 \int_{\theta_i}^{\theta_0} \frac{(\theta - \theta_i)\, D(\theta)}{F_n(\theta)}\, d\theta \right]^{1/2} \tag{1.24}$$

The index n denotes the n-th approximate, F_n is à priori not known except of linear soil and Dirac δ-function soil. Generally, $F(D, \theta, \theta_i, \theta_0)$. For the solution of the vertical infiltration, they started with an analogic development as for absorption and finally they obtained a relatively laborious iteration scheme. The principle disadvantage in applications is that I(t) is not explicitly formulated.

Parlange et al. (1982) have utilized all tools up to now developed in the solution of the vertical infiltration. The procedure was extended to the ponded infiltration (Parlange et al., 1985) and modified (Haverkamp and Parlange, 1987). The description of the solution of the "double integration method" will be subdivided into 5 steps:

1st step: Richard´s eq. with a changed dependent variable is integrated. The explicit expression for z is obtained and the flux concentration ratio is introduced.

2nd step: In the cumulative infiltration I = $\int z \, d\theta$ the previous expression (1st integral of Richards´ eq.) is substituted. D(θ) is subdivided into two domains: 1. domain with D(θ) continuous. 2. domain with D defined by the Diarc δ-function. This expression is transcribed into saturated conductivity and the water entry value H_w is introduced here.

3rd step: In the previous expression a difficult solvable integral occurs. It is treated now by a shape factor. This parameter occurs together with F(θ) when the relative conductivity is related to the relative sorptivity. After substitution of a new variable in the relative sorptivity and derivation, the integral from the 2nd step is integrated.

4th step: The correction owing to the saturated zone at the surface is performed by Darcy´s law.

5th step: The equation is rewritten to dimensionless terms and I is obtained as a function inter alia of q_o. After derivation, rearrangement and integration, the final solution is obtained with the shape factor from the 3rd step equal unity:

$$[I - K_i t] = \frac{K_s (H_o - H_A)(\theta_o - \theta_i)}{q_o - K_s} + \frac{S^2 + 2 H_A K_s (\theta_o - \theta_i)}{2(K_s - K_i)} \ln\left[1 + \frac{K_s - K_i}{q_o - K_s}\right] \qquad (1.25)$$

$$t = \frac{K_s (H_o - H_A)(\theta_o - \theta_i)}{(q_o - K_s)(K_s - K_i)} - \frac{S^2 + 2 K_s H_A (\theta_o - \theta_i)}{2(q_o - K_i)(K_s - K_i)} + \frac{S^2 - 2K_s (H_o - 2H_A)(\theta_o - \theta_i)}{2(K_s - K_i)^2} \ln\left[1 + \frac{K_s - K_i}{q_o - K_s}\right] \qquad (1.26)$$

5.2. Approximate solutions

When the convergence of Eq.(1.21) is considered, a simple Philip's algebraic equation (Philip, 1957) is obtained

$$I = S t^{1/2} + A t \qquad (1.27)$$

where A includes theoretically A_2, K_i and the truncation error. Philip (1969) has shown that A is related to K_s, generally $K_s/3 < A \leqslant 2/3 K_s$. However, for large time $A \to K_s$.

In the method of Green and Ampt (1911), the soil water content profile is simplified to the step-like profile (Fig.11) with H_f acting as the accelerating force on the wetting front, with (Neumann, 1976)

$$H_f = \frac{1}{K_s} \int_0^{H_i} K(H) dH = \frac{1}{K_s} \int_{\theta_0}^{\theta_i} D(\theta) d\theta \qquad (1.28)$$

or

$$H_f = \frac{1}{2} \int_0^{H_i} \frac{\theta_0 + \theta - 2\theta_i}{\theta_0 - \theta_i} \frac{K(H)}{K_s} dH \qquad (1.29)$$

As the whole step-soil water profile is the saturated domain of the soil, infiltration is solved by the application of Darcy's law:

$$q_0 = K_A \frac{H_0 - H_f + L_f}{L_f} \qquad (1.30)$$

Here, K_A is the approximate of K_S, $K_A \approx K_S$.

Philip (1957 and 1973) has shown that the step-like profile is exact when $D\theta$ is the Dirac's δ-function, and then $K_A = K_S$.

As $q_0(t)$ and $L_f(t)$, too, $q_0(t)$ is expressed as dI/dt, $I = L_f(\theta_s - \theta_i)$, and after integration

$$t^* = I^* - \ln(1 + I^*) \qquad (1.31)$$

with

$$t^* = \frac{K_A t}{(\theta_s - \theta_i)(H_0 - H_f)} \qquad I^* = \frac{I}{(\theta_s - \theta_i)(H_0 - H_f)} \qquad (1.32)$$

since for horizontal infiltration $I = St^{1/2}$ with

$$S = \left[2 K_A (H_0 - H_f)(\theta_s - \theta_i)\right]^{1/2} \qquad (1.33)$$

eq.(1.31) can be transcribed to

$$K_A t = I - \frac{S^2}{t} \ln\left[1 + \frac{2 I K_A}{S^2}\right] \qquad (1.34)$$

or

$$I = K_A t + \lambda \ln\left[1 + \frac{I}{\lambda}\right] \qquad (1.35)$$

with $\lambda = (H_0 - H_f)(\theta_s - \theta_i)$.

5.3. Empirical equations

The empirical relations were proposed for the gradual decrease of the infiltration rate q_o with time t:

$q_o(t)$ as the hyperbole:

Kostiakov's equation:

$$q_o = C_1 t^{-\alpha} \qquad (1.36)$$

$$I = \frac{C_1}{1-\alpha} t^{(1-\alpha)} \qquad (1.37)$$

where C_1, α are the empirical coefficients, C_1 should equal q_{o1}, the infiltration rate after the first time unit, usually at t = 1 min, $0 < \alpha < 1$. The equation is not appropriate to the description of infiltration at large time as $q_o \to 0$ at $t \to \infty$. To overcome this incovenience, Mezencev (1948) proposed the shift of q_o axis:

$$q_o = C_2 + C_3 t^{-\beta} \qquad (1.38)$$

$$I = C_2 t + \frac{1}{1-\beta} C_3 t^{(1-\beta)} \qquad (1.39)$$

where C_2, C_3, β are empirical coefficients. For $t \to \infty$ is $C_2 \to q_{oc}$, the constant infiltration rate, when the quasi-steady infiltration is reached. Theoretically $v_c = K_s$. $(C_2 + C_3)$ should equal q_{o1}, the infiltration rate after the first time unit and $0 < \beta < 1$.

Exponential decay of $q_o(t)$:

Horton's (1940) equation:

$$q_o = C_4 + C_5 \exp(-\gamma t) \qquad (1.40)$$

$$I = C_4 t + \frac{1}{\gamma} C_5 \left[1 - \exp(-\gamma t)\right] \qquad (1.41)$$

where C_4, C_5 and γ are empirical coefficients. At t = 0 has q_o a final value, in contradiction to the theory of flow in porous media. For $t \to \infty$ is $C_4 \to q_{oc}$ and $C_4 = K_s$ is frequently supposed.

Decay of q_o with I:

Holtan's (1961) equation:

$$q_o = C_6 (W - I)^{\varepsilon} + C_7 \qquad (1.42)$$

where C_6, C_7, ε are empirical coefficients, $C_7 = q_{oc}$, W is soil water storage above the impeding layer(!), ε is not integer, most frequently $\varepsilon > 1$.

II. APPLICATION OF SOLUTIONS TO THE FIELD INFILTRATION TESTS

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Field infiltration tests are routinely performed (double ring, or basin tests). The solutions developed for the Dirichlet's boundary condition are applicable and the solutions in the form of the algebraic equations are prefered for their simplicity. The set of the experimental data $I(t)$ is fitted to a certain infiltration algebraic equation and the parameters of the equation are obtained.

Questions:

1. Which of the algebraic equation is the most appropriate? How to interprete the parameters? How to predict the infiltration for the initial and boundary conditions different from those in the experiment?

2. What can we gain from the infiltration test? Is there an inverse solution of infiltration?

3. How to apply the results of the infiltration tests in the runoff hydrology and in agronomy?

In spite of separate discussions of the problems, the mutual links are clear and cannot be omitted.

1. TESTING OF ALGEBRAIC INFILTRATION EQUATIONS

1.1. Methodology

Criteria:

1. Time stability of parameters.
2. Physical interpretation of parameters.
3. Applicability of the equation to the extrapolation (to time limit different from time interval of the measurement).
4. Applicability of the ëquation for prediction of infiltration when the initial or boundary conditions are changed.

Detailed studies were performed and the algebraic equations were tested on generated and experimental data $I(t)$ obtained for light clay and sand (Kutílek et al., 1987, Haverkamp et al., 1987). Two procedures were applied:

In the first procedure the time stability of parameters and their applicability to extrapolation was studied by the modified method of the moving averages: The limiting time of convergence t_{lim} of Eq. (1.21) was computed. The time interval $\langle 0, t_{lim} \rangle$ was divided into 40 equal time steps Δt and the corresponding I_r was computed for each time step acc. to Eq. (1.21). From this basic set $\{I_r(t)\}$, the subset of the first five time steps $\{I_r(t)\}_{5i}$ was separated and on the data the examined equations were fitted. The evaluated parameters correspond to the average time $\bar{t}$ = 2.5 of the examined interval $\langle 0, 5\Delta t \rangle$. The procedure continues by increasing the subset to $\{I_r(t)\}_{7i}$ with $\bar{t}$ = 3.5 and so on up to the full set of $\{I_r(t)\}$ containing all 40 data regularly distributed in time $\langle 0, t_{lim} \rangle$. The parameters belong now to $\bar{t}$ = 20. The subscript 5i means that we have started with the initial five data of the set, the number of data in the subsets has been increased gradually each time by two. Each subset denoted by the subscript ni contains therefore n initial data from the start of infiltration while (40-n) final data are still lacking in the subset. After the full set $\{I_r(t)\}$ has been reached, the procedure continues by cutting off the first data and this subset is denoted by $\{I_r(t)\}_{39f}$, then the subset $\{I_r(t)\}_{37f}$ is obtained by slicing off two initial data from the previous subset and so on up to the last five time steps. The subscript nf means that the final n terms are included in the subset, while (40-n) initial data from the full set are lacking. If follows from the procedure that $\bar{t}$ is not a real time. When the method was applied to the experimental data, the number of time steps was reduced to the number of measured cumulative infiltration data at the equidistant Δt intervals.

Parameters of the infiltration equations in the time span $0 < \bar{t} < 20$ represent the situation when the measured data at the initial stage of infiltration are used for the extrapolation towards large time. Parameters in the range of $20 < \bar{t} < 40$ model the conditions for the extrapolation from the later measured data

towards zero time. The method offers the information, too, on the differences in the parameters of the infiltration equations when either only the start values or only the final values of the infiltration tests were available for the fitting procedure.

The degree of the instability of a certain parameter C in time is characterized by the maximum relative difference in the whole span of $\bar{t}$, $\vartheta = (C_{MAX} - C_{MIN})/C_{MAX}$ where the suffix MAX denotes the maximum and MIN denotes the minimum value of the parameter C in consideration. When the applicability of equations to the extrapolation in time is mutually compared, the value of ϑ in each of the parameters of the tested equation is considered as the main criterion.

In the first lecture, we have mentioned parameters which are frequently identified either with the hydraulic characteristics or with certain fluxes in the soil, as e.g.: K_s, S, or q_1. The exponents in the empirical equations of the hyperbolic type are comparable to the theoretical value 1/2 of the first term in Eq. (1.21). In order to demonstrate the degree of inaccuracy in the physical interpretation of the parameters, especially when dealing with a non-complete set of experimental data, the maximum error ε was evaluated. There is no need to emphasize that a physically based parameter should be stable in time and independent from the number of data in the evaluated set provided that the reference data are free from errors. The lowest value of ε in each of the parameters of the physically derived equations is the additional criterion for the selection of the most appropriate equation for the evaluation of infiltration.

The number of data in the regression analysis is gradually changed and the time stability together with the theoretical error of parameters is studied in the model which represents the fitting procedure and extrapolation of the limited number of the field infiltration data.

The values of parameters are determined from the subsets $\{I_r(t)\}_n$ by the least squares regression analysis.

In the second type of procedure, the precision of the cumulative infiltration was selected as the criterion of the suitability of the equation. It was expressed in terms of the variance between cumulative infiltration values obtained for the reference solution I_r and those calculated with the algebraic equation $\hat{I}$

$$\sigma(I) = \frac{\sum_{i=1}^{N} \left(I_{ri} - \hat{I}_i\right)^2}{N-1} \qquad (2.1)$$

where N is the total number of points for the generated data, N = 200 . The equations are analyzed by the fitting the equation on the reference data for gradually increasing time periods up to t_{lim}, starting at very short times. For the precision criterion $\sigma^2 \leq 0.005$ was selected, taking $\sigma^2(I)$ of the numerical simulation as the threshold value.

1.2. Results

Kostiakov´s equation

Both parameters in Eq. (1.36) and (1.37) are strongly time dependent, see the values of ϑ in Table 1 and their values are decreasing with the increase of the average time $\bar{t}$ as demonstrated in Fig.1. The instances with a roughly constant value of C_1 at the starting period of infiltration could exist, see the curve $C_1(\bar{t})$ for sand in the interval of $\bar{t} < 10$.

The exponent α when extrapolated to $\bar{t} \rightarrow 0$ is approaching the theoretically predicted value 0.5 in clay only, while in sand this tendency does not exist. C_1 should theoretically equal to q_1, the infiltration rate at the first time unit, here at t = 1 min. There is a non-unique relationship of C_1/q_1. In clay, $C_1/q_1 < 1$ all over the tested time range while in sand $C_1/q_1 > 1$ for $0 < \bar{t} \leq 18$ and $C_1/q_1 > 1$ for $\bar{t} > 18$. Assuming the theoretical equality $C_1 = q_1$, the maximum error was high, see the Table 1. Since the time dependency of both parameters, α and C_1 is strong and as their behaviour in time is unpredictable, the Kostiakov´s equation is not suitable for the extrapolation towards t = 0. The testing has confirmed that C_1 and α are fitting parameters only and their value depends in additon to soil and test conditions upon the number of data in the set $\{I(t)\}$.

Sometimes, the equation $I(t)$ linearized into the log-log form and then the linear regression analysis is performed.

The results of this type of fitting are presented in Fig.1, too. Owing to the linearization, the weight of parameters has been changed and the dotted curves demonstrate the alteration of parameters. At $\bar{t}$ = 20 a strong singularity has appeared. The relative time stability of C_1 in the interval $0 < \bar{t} < 20$ is exceptional. The linearization does not contribute to the increase of the time stability of the parameters and our conclusion on non-suitability of the equation for the extrapolation towards $\bar{t}$ = 0 kepps valid. We have mentioned earlier that this equation is mathematically not appropriate for the extrapolation towards great time.

A good precision in I is reached at short time, where the equation can be used for interpolation between the measured data. For medium and long time, the precision is very poor and the equation is not appropriate even for interpolation.

The equation is not appropriate for the prediction of infiltration when θ_i or H_o are different from the values at the time of the experiment.

Mezencev's equation

The increase in the number of parameters to three in Eqs (1.38) and (1.39) results in the expected lower variation of parameters with time, see the Table 1 and in the lowering of ϑ-values compared to the ϑ-values in Kostiakov's parameters. In spite of this relative increase of the time stability, the values of ϑ in C_2 and C_3 are still relatively high. In addition to it, the character of the time dependence of $C_3(\bar{t})$ is dissimilar in clay and sand, and so is $C_{1M}(\bar{t})$, see Fig.2.

The value of the exponent β is at the short time $\bar{t}$ close to the theoretical value 0.5 for both soils and then it rises slowly with the increase in $\bar{t}$. This indicates an improvement with regard to the Kostiakov's equation. When C_{1M} is related to q_1, the error ε in C_{1M} is obtained. It rises from very low values of 2% in clay and 0.5% in sand at the shortest time $\bar{t}$ to 13% in clay and 3.5 in sand for the full set of the data, i.e. at $\bar{t}$ = 20, and ends with maximum values shown in Table 1 at the largest $\bar{t}$. In spite of improvement in comparison to Kostiakov's equation the error is still high. The identification of C_2 with the saturated hydraulic conductivity K_s leads to a high error at the short time $\bar{t}$. With the increase in $\bar{t}$, the error decreases but even for the full set of data at $\bar{t}$ = 20 when we assume that the infiltration rate is close to quasi--steady conditions, the error in C_2 is still high. It means that the extrapolation of infiltration data to large time in order to obtain the estimate of K_s is erroneous.

We conclude that similarly to Kostiakov's equation the parameters C_2, C_3, C_{1M} and β cannot be interpreted physically, they are fitting parameters only and their behaviour in time is not predictable. The equation is not suited for the extrapolation.

When Eq.(1.38) was modified by putting $C_2 = K_s$, the maximum error of the remaining coefficients C_{1M} and β has increased and their time instability has increased, too, see the Table 1.

On the other hand side, the precision is remarkably good in the full time range, see Table 2.

The equation is therefore well suited for the interpolation between the measured data. However, owing to its empirical character and lack of physical meaning of the parameters, the equation cannot be used for the prediction of infiltration when either Θ_i or H_o are changed.

Horton´s equation

In spite of three parameters existing in Eqs (1.40) and (1.41), the time dependence of all three coefficients C_4, C_5 and γ was the strongest of all the tested equations, as it follows from Table 1. The value of all three parameters decreases with time $\overline{t}$, see Fig.3. Here, the parameters were calculated exceptionally in units of cm and hours owing to the exponential form of the equation. The coefficient C_4 is sometimes identified with the saturated hydraulic conductivity. It is instructive to see that the maximum error in C_4 exceeds the errors occuring in parameters of all other tested equation. All parameters have the character of fitting parameters only.

The Horton´s equation is the least appropriate for the extrapolation of the measured data in time and none of its coefficients can be interpreted physically. The precision is acceptable only for short time where the interpolation leads to errors below the chosen threshold value (Table 2).

Green and Ampt´s equation

As it follows from Fig.4, the coefficients K_A and λ of Eq. (1.35) are time dependent, K_A rises with $\overline{t}$, λ decreases

with the increase in $\overline{t}$ and the change is more significant at the short time $\overline{t}$. The time dependency according to values of ν^{P} in Table 1 is less strong than found for parameters of the pure empirical equations. However, the maximum error in K_A, related to the saturated hydraulic conductivity is still high. It decreases with the increase of time $\overline{t}$ but even at the largest $\overline{t}$ its value is 25% in clay and 10% in sand. This subset represents the situation when only the final values of the infiltration test are evaluated. For the full set of $\{I(t)\}$, the error is higher and it is 28% in clay and 13% in sand. The physical nature of the relationship of K_A to K cannot be found and the determination of the saturated conductivity from K_A is not feasible. The parameter λ should have the physical meaning, too, but its value is again not constant in time and if we assume $(\Theta_S - \Theta_i)$ as a constant, then H_f should change up to 25%. We confirm therefore that in spite of the physical derivation of Eq. (1.35), the coefficients behave like fitting parameters only. Or, in other words, the assumptions and approximations applied in the derivation of Eq. (1.35) were of such an extent that the physical reality was overshadowed by them and the parameters of the final equation behave consequently like empirical coefficients.

Owing to still high values of ν^{P} in the parameters, the equation is not appropriate to the extrapolation in time.

On the other hand side, the precision criteria $\sigma^2(I)$ in Table 2 demonstrate that the equation is appropriate for the interpolation of experimental data. When either Θ_i or H_o

are changed, the resulting values of K_A or H_f are altered, too, as it can be expected from the fitting character of parameters. The equation is therefore not appropriate for the prediction of infiltration when the initial or boundary conditions differ from the conditions at the time of the experiment.

Philip's algebraic equation

Both parameters S and A in Eq.(1.27) are time dependent, S decreases with time $\bar{t}$ and A increases with it (Fig.5). The maximum relative difference ϑ of S has substantially decreased when compared to ϑ of the parameters in the earlier discussed equations, see Table 1. Thus, the time stability of S is higher than in other parameters. However, the time dependence of the parameter A is significantly greater than that one of S, see the higher value of ϑ of A.

At the initial stage of infiltration, the value of S is close to the theoretical value of the sorptivity S_1 as it is anticipated and with the rise of $\bar{t}$ the value of S is deviating from S_1. The maximum error $\mathcal{E}$ in S is found at the largest time $\bar{t}$, i.e. when S is evaluated from the subset where the data at the early stage of infiltration are missing. Since the initial data are usually not available for the fitting procedure, the error in the time range $\bar{t} > 20$ is decisive for our conclusion on the applicability of the equation. For our soils, the error is in ranges 5% to 10% in clay and 9% to 16% in sand, respectively.

The error of the parameter A increases with time $\bar{t}$, too, reaching the maximum value above 50% at the largest time $\bar{t}$ if $(A_2 + k_1)$ is considered as the reference level. This increase of the error with time $\bar{t}$ is in accordance with the expected increase of the truncation error with time. Since the saturated hydraulic conductivity K_S is frequently assumed to be simply related to A by A = mK_S the dependence of the correction factor m upon $\bar{t}$ is plotted in Fig.6. A simple relationship between A and K_S was not found, m is not only time dependent, but it is strongly related to the hydraulic characteristics of the soil. Its dependence upon the initial soil water content was obtained, too, see Fig.7, where the error in S and A is shown as θ_i dependent, too. If we take K = 3/2 A (Youngs, 1968, Philip, 1969), the maximum error in the estimation of K is 32% in clay and 14% in sand. The upper boundary of the range of m value is above 2/3 even in the initially dry sand and when the initial soil water content rises, the value of m grows far above 2/3 in both soils even for $\bar{t}$ = 20, i.e. for the full set of the data.

It can be concluded that the S term is only approximating the real sorptivity S_1, its maximum error would not exceed 15% and when the I(t) data from the early till medium time of infiltration are evaluated, the error would be below 5%. The coefficient A keeps the character of the fitting parameter and its relation to K_S cannot be predicted without a significant error.

The precision criterion is satisfied for short and medium time. In this range, the interpolation of experimental data by Philip's equation is therefore applicable.

The predictive applicability of the equation is better than with the earlier equations but the results are still of low accuracy, especially due to the unknown dependence of A upon θ_i and H_o.

Three parameters equation of the Philip's type

As the increase of parameters leads to their higher stability in time, it is instructive to test the three terms equation derived from the time-series solution, Eq. (1.21):

$$I = C_8 t^{1/2} + C_9 t + C_{10} t^{3/2} \qquad (2.2)$$

With C_8, C_9, C_{10} approximating the exact parameters S_1, A_2, A_3 in Eq. (1.21). As at t_{lim} denoting the time of the end of the convergence of Eq. (1.21) is $dv/dt \to 0$, we get from the second derivative of Eq. (2.2) the approximation

$$t_{lim} \approx \frac{C_8}{3C_{10}} \qquad (2.3)$$

and approximating again $q_o(t_{lim}) \approx K_S$ we obtain the estimate of K_S, denoted as K_E

$$K_E = \sqrt{3 C_8 C_{10}} + C_9 \qquad (2.4)$$

The time dependence of the coefficients C_8, C_9 and C_{10} of Eq. (2.2) differs from the parameters in the previously

discussed equations by the increased time stability, see the distinct decrease in ϑ in Table 1.

The theoretical error of the parameter C_8 is substantially depressed and the identification of C_8 with S_1 offers a very good estimate of the sorptivity, much better than in the previous two-terms equation. The accuracy of the estimate increases when the early stage of infiltration is evaluated and even for the full set of $\{I(t)\}$, the error in clay is negligibly small and in sand it makes about 1% (Fig.8). The error in C_9 is in the tolerable ranges and only the error in C_{10} for sand is comparable to the earlier discussed parameters. Generally, the errors of coefficients found for sand are up to approximately one order higher than in clay. Judging from the time dependence of the coefficients, see Fig.8, the equation is better suited for the extrapolation towards large time than towards t = 0. However, even in the sets $\{I(t)\}_f$ where the early readings of the data are lacking, the estimate of S_1 from C_8 is still very good and the error in C_9 and C_{10} is distinctly lower than in all other equations.

When the estimate of the hydraulic conductivity K_E (Eq. (2.4)) is evaluated in our model of the extrapolation, we find that its time stability is very high, the value of ϑ is comparable to the maximum relative difference of C_8, i.e. the time stability is similar to the stability of the sorptivity estimate. The maximum error reaches 9% in sand at low $\bar{t}$ while in clay the maximum error is found at large $\bar{t}$ and its value is by one order lower than in sand (Fig.9). We recommend to evaluate K_E from the full set of $\{I(t)\}$ data in order to minimize the error.

Further on, a study was performed on the role of the initial soil water content θ_i upon the error in K_E. Similarly to the time dependence, the relationship of the error to the value θ_i/θ_S was not unique for clay and sand and it depends additionally upon the hydraulic characteristics of the soil (Fig.10). When $\theta_i/\theta_S > 0.75$, the error is distinctly decreasing with the increase of θ_i. When the maximum relative difference of K_E was evaluated for the variation in θ_i, we obtained 0.042 for sand and 0.039 for clay, respectively. We can assume therefore that the initial soil water content variation does not influence essentially the estimate of the saturated hydraulic conductivity.

The proofs on a good accuracy in the estimation of the saturated hydraulic conductivity have eliminated the necessity of the extrapolation of the measured data to the non-defined estimate of the quasi-steady infiltration rate.

The precision criteria in Table 2 show that the equation is of high accuracy and it is applicable for the interpolation of the experimental data in full range of time.

Parlanges´ equations

In the Parlanges´ equations (1.25) and (1.26) the time stability of H_A was tested by moving averages when the remaining hydraulic characteristics of the soil were taken constant. The value of ϑ Table 1 is very high as the errors of all approximation steps are included here. The physical interpretation of H_A

is therefore questionable and H_A should be considered as a fitting parameter.

The precision criteria (Table 2) demonstrate high accuracy of the equation in full range of time. The equation is applicable for the interpolation of the experimental data. The extrapolation beyond the measured data, especially towards t = 0 is feasible.

2. WHAT CAN WE GAIN FROM THE INFILTRATION TEST?

It has been shown in Chapter 1 that the majority of algebraic infiltration equations behave like empirical equations with fitting character of their parameters. Exceptions:

Sorptivity S can be estimated from Philip's algebraic two-terms eq.; with better accuracy from the three-terms equation of Philip's type and from Parlange's equation.

Saturated hydraulic conductivity K_S can be estimated from the three-terms eq. of Philip's type and from Parlange's eq. When estimated from Philip's A parameter in two-terms eq., the possible inaccuracy is great.

However, for modeling in the soil hydrology, irrigation and drainage, for runoff hydrology, the knowledge of the soil water retention curve $H(\theta)$ and of the unsaturated hydraulic conductivity function $K(\theta)$ is indispensable. Even the prediction of infiltration at variable θ_i is impossible without knowledge on $H(\theta)$, $K(\theta)$. The methods of their determination up to recently developed are restrictive for the routine use. If the characteristics are measured on undisturbed core samples, the representativeness of the sample is questionable and the data thus obtained are not applicable directly to the solution of the flow problems in the field or in the catchment. The other option is to measure the values of H and θ during a well defined flow process in the field experiment on a small plot equipped with the adequate instrumentation. The estimation of the soil hydraulic characteristics has been described in the literature

(e.g. Hillel et al., 1972, Nielsen et al., 1973, Jones and Wagenet, 1984, Green et al., 1986). These methods have two main disadvantages:

1. Expensive equipment. A skilled technician or researcher is required for the operation of the instrumentation.
2. Time consuming procedure of one single experiment on one single plot.

The space variability of soils and of their characteristics brings about the question on how to obtain greater number of data in the field or in the watershed during a short time period.

The idea of the estimation of the soil hydraulic characteristics from a simple field experiment is therefore attractive and very actual. The double ring infiltration test is the example of such a simple procedure.

In the first attempts, the analytical solutions together with a certain degree of approximation were used. E.g. the relation $K(H)$ was prescribed by the exponential form and when the Green and Ampt's (1911) approach was applied for the evaluation of the infiltration test, $K(H)$ was estimated. Or, the soil water diffusivity $D(\theta)$ in the exponential form is searched from the sorptivity term S. However, the theoretical error in the applied approximations can be non-negligible (Kutílek and Valentová, 1985).

Since we do not know à priori the function $D(\theta)$ which is searched, we cannot predict the most suitable approximation of sorptivity. The degree of accuracy of various approximations is in Fig.11 when $D(\theta)$ is modelled by

$$D = \alpha \exp(\beta\theta) \qquad (2.5)$$

where α, β are empirical parameters. The tested equations are:

Philip's (1955) approximation

$$S = \left[\frac{8}{\pi}\int_{\theta_i}^{\theta_s}(\theta - \theta_i)\, D(\theta)\, d\theta\right]^{1/2} \qquad (2.6)$$

Parlange's (1971) first approximation

$$S = \left[2\int_{\theta_i}^{\theta_s}(\theta - \theta_i) D(\theta) d\theta\right]^{1/2} \qquad (2.7)$$

Parlange's (1975) second approximation

$$S = \left[(\theta_s - \theta_i)\int_{\theta_i}^{\theta_s} D(\theta) d\theta + \int_{\theta_i}^{\theta_s}(\theta - \theta_i) D(\theta) d\theta\right]^{1/2} \qquad (2.8)$$

From the Philip and Knight's (1974) eq. (1.24) is if $F(\theta)$ is defined

$$S = \left[2\int_{\theta_i}^{\theta_s}\frac{(\theta - \theta_i)\, D(\theta)}{F(\theta)}\, d\theta\right]^{1/2} \qquad (2.9)$$

However, $D(\theta)$ is not known and thus $F(\theta)$ is unknown, too. Sometimes, the assumptions are made on D = const., or $D(\theta)$ = Dirac δ-function. For linear soil is then

$$S = \left[2D\int_{\theta_i}^{\theta_s}\frac{(\theta - \theta_i)\ d\theta}{\exp\{-(\mathrm{inverfc}[(\theta-\theta_i)/(\theta_s-\theta_i)])\}}\right]^{1/2} \qquad (2.10)$$

and for soil with $D(\theta)$ expressed by Dirac δ-function where $F = (\theta - \theta_i)/(\theta_s - \theta_i)$ is

$$S = \left[2(\theta_s - \theta_i)\int_{\theta_i}^{\theta_s} D(\theta)\, d\theta\right]^{1/2} \qquad (2.11)$$

In spite of exactness of equations (2.10), (2.11), they become approximative when the appropriate $D(\theta)$ is not applied. Fig.11 demonstrates that the frequent simplification of (2.9) to (2.11) leads to a maximum error of 25 to 30% in majority of soils.

Summarizing the approaches of the approximative-analytical character we find that the complete estimate on soil hydraulic functions of soils from the infiltration test has not been found yet. The degree of inaccuracy in prescribing the simple algebraic form of the searched functions can be high.

Parallel to the approximative-analytical type of the solutions, the combination of the analytical models of the soil hydraulic functions with the numerical solution exists.

It has been first proposed by Haverkamp et al. (1979).

They estimated $K(\Theta)$ by the optimization of the exponent b in the Mualem's (1976) model of relative conductivity $K_r(\Theta)$, using the measured $\Theta(H)$ function and the cumulative infiltration $I(t)$, where t is the time. However, the $K(\Theta)$ capillary model is less sensitive to the change in b than to the variation of other parameters in the $\Theta(H)$ function (Vogel et al., 1985).

An improved method (Šír et al., 1987) is based on the sensitivity analysis of $K(\Theta)$ relative to $\Theta(H)$ and of the Richards' (1931) equation relative to $\Theta(H)$ and $K(\Theta)$ (Šír et al., 1985, Vogel et al., 1985). For the procedure, the routine laboratory and field equipment together with the microcomputer facilities are needed, only. The infiltration rate is measured by the double ring method, the depth of wetting immediately after the infiltration test is obtained gravimetrically by sampling the soil with the auger or, alternatively, a more sophisticated method can be applied. The soil water retention curve has to be determined on the undisturbed soil sample by the routine procedure in the laboratory.

We define:

1. The small scale functions which are obtained on the undisturbed soil samples of the volume approximately 100 cm^3.
2. The large scale functions which are reflected by the infiltration test performed on the area of about 0.1 m^2 to 1 m^2 in the field.
3. The capillary model which is assumed to be valid in both, in the small scale as well as in the large scale functions (item 1 and 2). The general shape of the basic function $\Theta(H)$ is therefore kept in both scale functions and only a slight deformation is allowed.

For the solution of the one-dimensional infiltration of water in the soil, the Richards' (1931) equations is applied in its capacitance form, eq. (1.2), with the boundary conditions (1.3) and (1.4).

The infiltration is numerically simulated by the implicit final differences method. Thus, the data $I_N(t)$ and $\Theta_N(z,t)$ are obtained, the subscript N is for the numerically simulated data.

For the numerical solution of Eq. (1.2), the hydraulic functions $\Theta(H)$ and $K(\Theta)$ are needed and they are the matter of the optimization.

The soil water retention curve is expressed by the equation of van Genuchten (1978) modified by Šír et al. (1985) to the form

$$\Theta_E = \frac{1}{[1 + (\alpha |H|)^n]^m} \qquad (2.12)$$

with $m = 1 - 1/n$, $\alpha > 0$, $n > 1$, and Θ_E denoting the modified effective soil water content

$$\Theta_E = \frac{\Theta - \Theta_a}{\Theta_m - \Theta_a} \qquad (2.13)$$

which slightly differs from the commonly defined effective soil water content Θ_e

$$\Theta_e = \frac{\Theta - \Theta_r}{\Theta_s - \Theta_r} \qquad (2.14)$$

where θ_r is the residual, θ_S is the saturated soil water content, θ_a substitutes θ_r allowing $\theta_a < 0$, θ_m substitutes θ_S allowing $\theta_m \geq \theta_S$. The physically real part of $\theta(H)$ is limited by $\theta \in \langle \theta_r, \theta_S \rangle$. The greater flexibility of the Eq. (2.12) with θ_E to the experimental data is demonstrated in Fig.12.

The introduction of θ_E admits to identify the analytical expression (2.12) with soils where the air or water entry values exist. The terms α, n, θ_a, θ_m in Eq. (2.12) and (2.13) are fitting parameters only. The mutual conversion between θ_e and θ_E is

$$\theta_E = \frac{\theta_r - \theta_a}{\theta_m - \theta_a} + \frac{\theta_S - \theta_r}{\theta_m - \theta_a}\,\theta_e \qquad (2.15)$$

When Eq. (2.12) is applied to the Mualem's (1978) capillary model, the relative hydraulic conductivity $K_r = K(\theta)/K_S$ is obtained by

$$K_r(\theta_e) = \theta_e^{\,b}\left[\frac{d - \left(1-\theta_E^{\,1/m}\right)^m}{d - \left(1-\theta_E^{\,1/m}\right)^m\Big|_{\theta_e=1}}\right]^2 \qquad (2.16)$$

$$d = \left(1 - \theta_E^{\,1/m}\right)^m\Big|_{\theta_e=1}$$

The exponent b is taken as b = 0.5 (Mualem, 1978).

The detailed analysis has shown that the capillary model of the relative hydraulic conductivity is very sensitive to the value of θ_m (Vogel et al., 1985). As θ_S limits the physical validity of Eq. (2.12) and since K_S transforms K_r to the unsaturated hydraulic conductivity, the parameters θ_m, θ_S and K_S are defined as optimization parameters. Let us note that the shape of the $\theta(H)$ function is relatively less sensitive to the slight variation of θ_m and so is the relation C(H).

The values of optimalized parameters θ_m, θ_S and K_S are obtained from the conditions (2.17) and (2.18). The best coincidence between measured and simulated cumulative infiltration as the function of time is expressed by the condition (2.17). The agreement between measured and simulated value from the point of view of the position of the wetting front is guaranteed by the condition (2.18).

$$\left[I_E(t) - I_N(t)\right]^2 \to \min. \qquad (2.17)$$

$$\left[z_{SE} - z_{SN}\right]^2 \to \min. \qquad (2.18)$$

I_N are the values of the numerically simulated cumulative infiltration and I_E are the experimental data. z_S is the depth of wetting front and z_{SE} is the experimental one, z_{SN} is the numerical one, $z \leq z_S$, $\theta = \theta_S$ and $z > z_S$, $\theta < \theta_S$.

When θ_S is altered from θ_{S1} to θ_{S2} and the remaining parameters in Eq. (2.12) are not changed, we get for $I_{N1} = I_{N2}$

different moisture profiles with $K_{S1} = K_{S2}$, see the Fig.13. This is why z_S was introduced. It represents simply the soil water content profile.

The starting values of the parameters θ_m, θ_S are taken from the evaluation of the soil water retention curve of the undisturbed sample of the soil. The starting value of K_S is estimated from the field infiltration test either by Eq. (2.4) or by $K_S = 1.5\,A$, Eq. (1.27).

In the optimization process, the variation of parameters K_S and θ_S is limited by $K_S \in \langle K_S \rangle$ and $\theta_S \in \langle \theta_S \rangle$ where $\langle K_S \rangle$ and $\langle \theta_S \rangle$ denote the range of physically acceptable values. The value of θ_m is subdued fully to the optimization with the only one restriction $\theta_m \geq \theta_S$. The remaining parameters of the hydraulic functions of the soil, n, θ_a, θ_r, α are kept unaltered and numerically equal to the values obtained from the soil water retention curve of the undisturbed core sample.

When the first set of $I_N(t)$ data is obtained by the numerical simulation, the correction of the starting K_{S1} is obtained by (Vogel, 1985)

$$K_{S2} = K_{S1} \frac{t_1}{t_2}$$
$$I_N = I_E \qquad (2.19)$$

see Fig.14. Vogel obtained Eq. (2.19) from the linear scaling of the Richards' equation (1.2).

For the corrected K_{S2} the position of the depth of the wetting front z_{SN} is checked by the experimental value z_{SE}. If $z_{SN} > z_{SE}$, the value of θ_S should be increased and vice versa and the procedure of the numerical simulation should be repeated again. However, according to our experience, the correction of θ_S owing to the distinct inequality in z_S has not been usually needed. When neither θ_S nor K_S alteration lead to the equality of $I_N(t)$ and $I_E(t)$, the increase of θ_m is required.

<u>Practical procedure:</u>

1. Undisturbed soil sample is taken in the field. The initial soil water content θ_i is determined together with the soil water retention curve in the laboratory. The set of experimental data $\{\theta(H)\}$ is fitted to the Eq. (2.12) and the parameters are evaluated.
2. Double ring infiltration test is performed and the set of $\{I_E(t)\}$ data is fitted to the Eq. (2.2). From the evaluation of the parameters, K_S is estimated according to Eq. (2.4) and a smooth line $I_E(t)$ is plotted.
3. The depth of the wetting front z_{SE} is determined after infiltration by sampling the soil with the auger and using the gravimetrical method. The time related to z_{SE} is the time of the end of the infiltration test.
4. The infiltration is simulated numerically according to (1.2), (1.3) and (1.4), using the hydraulic functions obtained from the experiments, see items 1 and 2. The numerical procedure offers $\{I_N(t)\}$ and the soil water content

profiles $\Theta_N(z,t)$ from which the position of the wetting from z_{SN} is read at time corresponding to the time of the end of the field infiltration test.

5. Optimization according to Eqs. (2.17) and (2.18) is performed. As a useful tool, Eq. (2.19) should be applied.

3. APPLICATION OF INFILTRATION TESTS IN RUNOFF HYDROLOGY AND AGRONOMY

The application is linked to the problem of the combination of two different scale domains. Infiltration test is performed on the area of 10^0 m^2, the interest of agronomy and runoff hydrology starts at the area of $\sim 10^6$ m^2. The heterogeneity of the larger area, when defined by functions obtained on small area is to be found, defined and then operated with. The tools of geostatistics, scaling methods etc. will be used as it will be shown later on in the lectures on spatial variability.

Table 1. Evaluation of maximum differences of parameters ϑ and of their errors ε, eventually

Equation of Parameter	Maximum rel. difference, ϑ Clay	Maximum rel. difference, ϑ Sand	Reference characteristics	Maximum error, ε Clay	Maximum error, ε Sand
Kostiakov					
α	0.297	0.425	0.5	0.376	0.534
C_1	0.603	0.181	q_1	0.671	0.131
Mezencev					
β	0.163	0.190	0.5	0.216	0.278
C_2	0.344	0.258	K_S	0.484	0.330
C_3	0.371	0.163	-	-	-
$C_{1M}=C_2+C_3$	0.371	0.044	q_1	0.615	0.048
Mezencev with $C_2=K_S$					
β	0.289	0.248	0.5	0.636	0.575
$C_{1M}=K_S+C_3$	0.691	0.090	q_1	3.090	0.133
Horton					
γ	0.926	0.924	-	-	-
C_4	0.502	0.401	K_S	1.135	0.785
C_5	0.768	0.777	-	-	-
Green-Ampt					
K_A	0.172	0.122	K_S	0.365	0.208
λ	0.261	0.237	-	-	-
Philip					
S	0.092	0.143	S_1	0.101	0.160
A	0.251	0.226	A_2+K_i	0.594	0.578
3-parameters, Philip's type					
C_8	0.006	0.042	S_1	0.006	0.042
C_9	0.038	0.153	A_2+K_i	0.038	0.189
C_{10}	0.070	0.400	A_3	0.065	0.425
K_E	0.009	0.054	K_S	0.008	0.091
Parlange					
H_A	0.67	0.37			

Table 2. Precision of the cumulative infiltration in terms of $\sigma^2(I)$

CLAY

Time range [h] / Equation of:	1	5	50	100	150	200	250
Kostiakov	4.9E-7	1.3E-5	1.8E-3	8.5E-3	2.1E-2	4.1E-2	6.9E-2
Mezencev	6.5E-12	8.4E-10	9.2E-7	7.6E-6	2.6E-5	6.1E-5	1.2E-4
Horton	1.2E-4	6.0E-4	5.8E-3	1.1E-2	1.6E-2	2.1E-2	2.5E-2
Green-Ampt	5.4E-10	4.2E-9	3.9E-6	3.1E-5	1.0E-4	2.4E-4	4.6E-4
Philip 2 para	1.3E-10	1.6E-8	1.8E-5	1.4E-4	4.9E-4	1.2E-3	2.2E-2
3 para (Philip)	8.2E-11	4.2E-10	6.1E-8	3.0E-6	7.9E-5	9.2E-4	2.0E-4
Parlange	8.2E-8	1.3E-7	2.1E-6	5.8E-6	8.5E-6	1.1E-5	1.2E-5

SAND

Time range [h]	0.05	0.1	0.2	0.3	0.4	0.5	0.667
Kostiakov	3.0E-4	1.3E-3	5.9E-3	1.4E-2	2.7E-2	4.4E-2	8.0E-2
Mezencev	7.9E-8	6.0E-7	4.3E-6	1.3E-5	2.8E-5	5.0E-5	9.7E-5
Horton	9.4E-4	1.8E-3	3.5E-3	5.E-3	6.4E-3	7.7E-3	9.5E-3
Green-Ampt	2.2E-7	1.7E-6	1.2E-5	3.8E-5	8.3E-5	1.5E-4	3.0E-4
Philip 2 para	1.6E-6	1.2E-5	9.2E-5	2.9E-4	6.3E-4	1.2E-3	2.4E-3
3 para (Philip)	4.6E-9	7.5E-7	8.7E-6	5.2E-5	1.7E-5	9.2E-4	8.0E-4
Parlange	9.1E-6	2.5E-5	6.6E-5	1.2E-4	2.1E-4	2.4E-4	3.1E-4

L I T E R A T U R E

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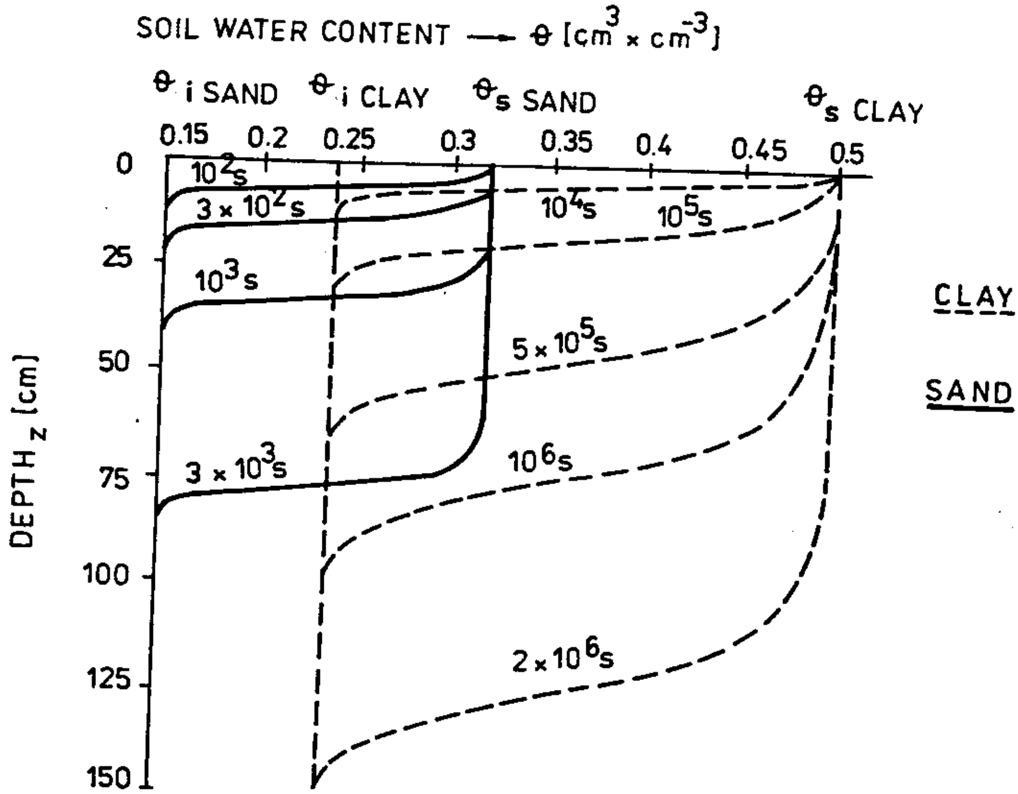

SOIL WATER CONTENT —→ θ [cm³ × cm⁻³]
θ i SAND
θ i CLAY
θs SAND
θs CLAY
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
DEPTH z [cm]
0
25
50
75
100
125
150
10²s
3 × 10²s
10³s
3 × 10³s
10⁴s
10⁵s
5 × 10⁵s
10⁶s
2 × 10⁶s
CLAY
SAND

FIG.1/I

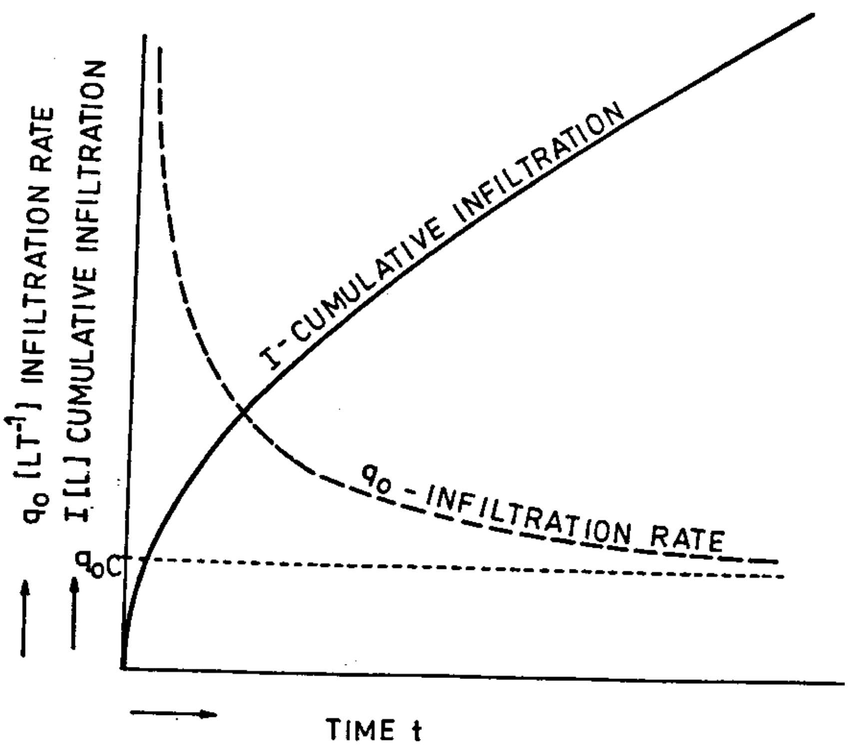

q₀ [LT⁻¹] INFILTRATION RATE
I [L] CUMULATIVE INFILTRATION
I - CUMULATIVE INFILTRATION
q₀ - INFILTRATION RATE
q₀C
TIME t

FIG. 2/I

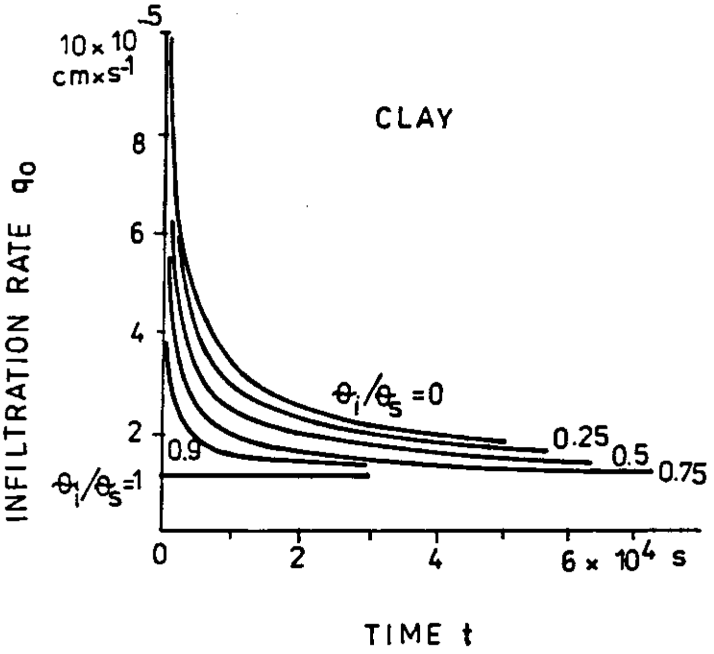


FIG 3/I

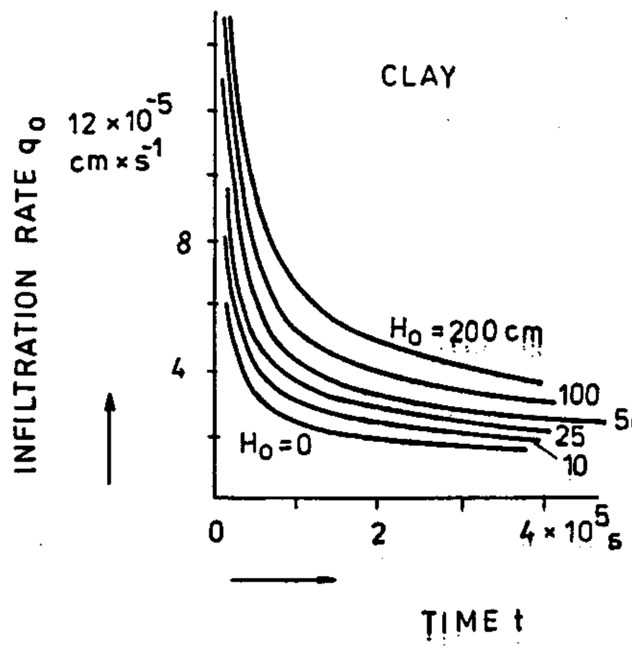


FIG. 4/I

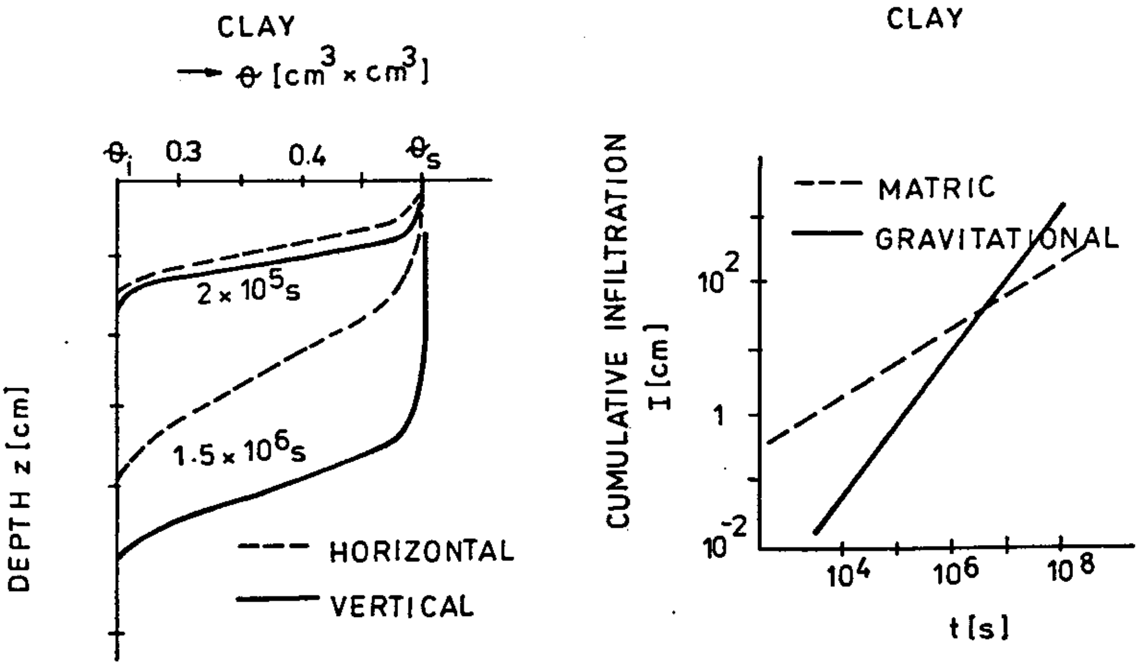


FIG. 5/I

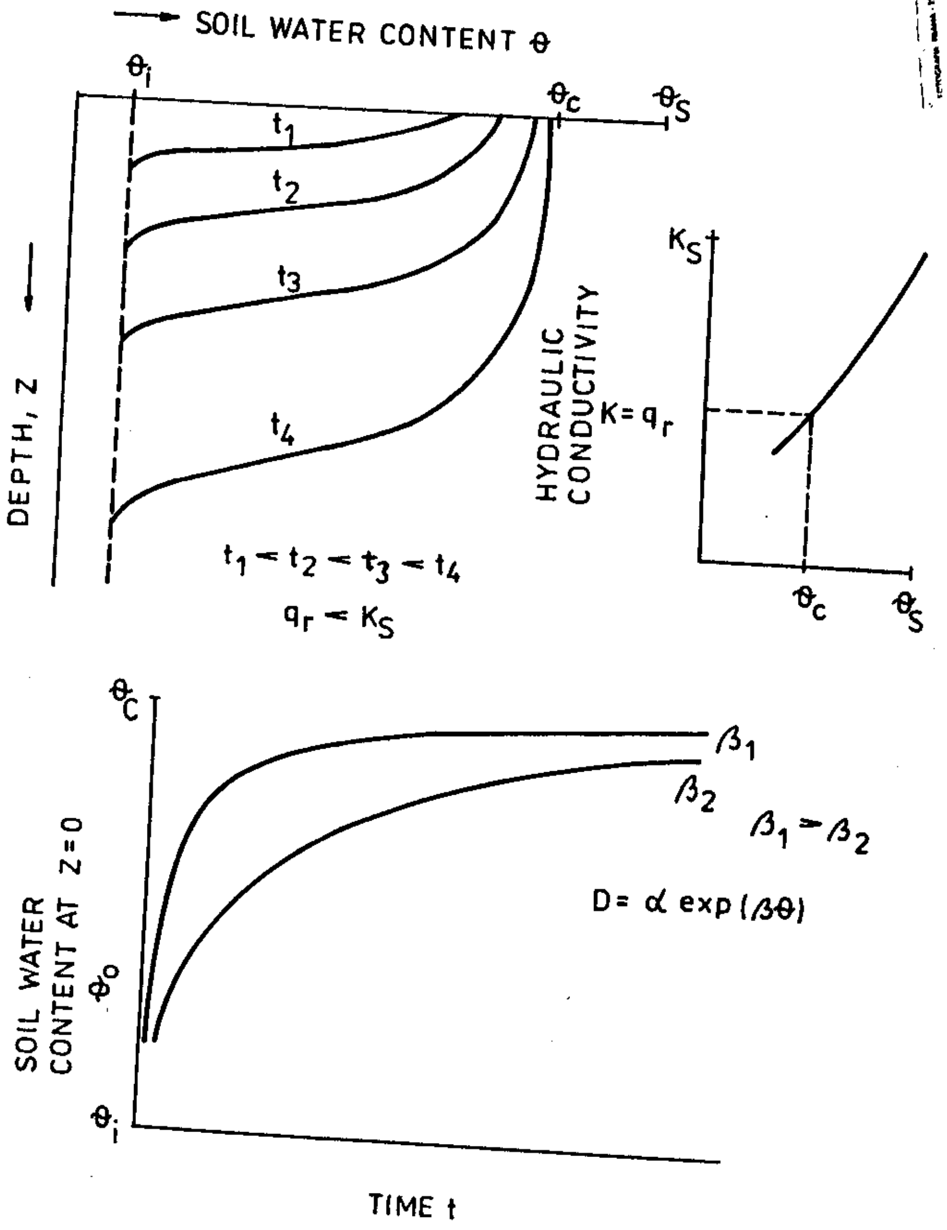

SOIL WATER CONTENT θ
θi
θc
θS
t1
t2
t3
t4
DEPTH, Z
t1 < t2 < t3 < t4
qr < KS
HYDRAULIC CONDUCTIVITY
KS
K = qr
θc
θS
θC
β1
β2
β1 > β2
D = α exp (βθ)
SOIL WATER CONTENT AT Z = 0
θo
θi
TIME t

FIG. 6/I

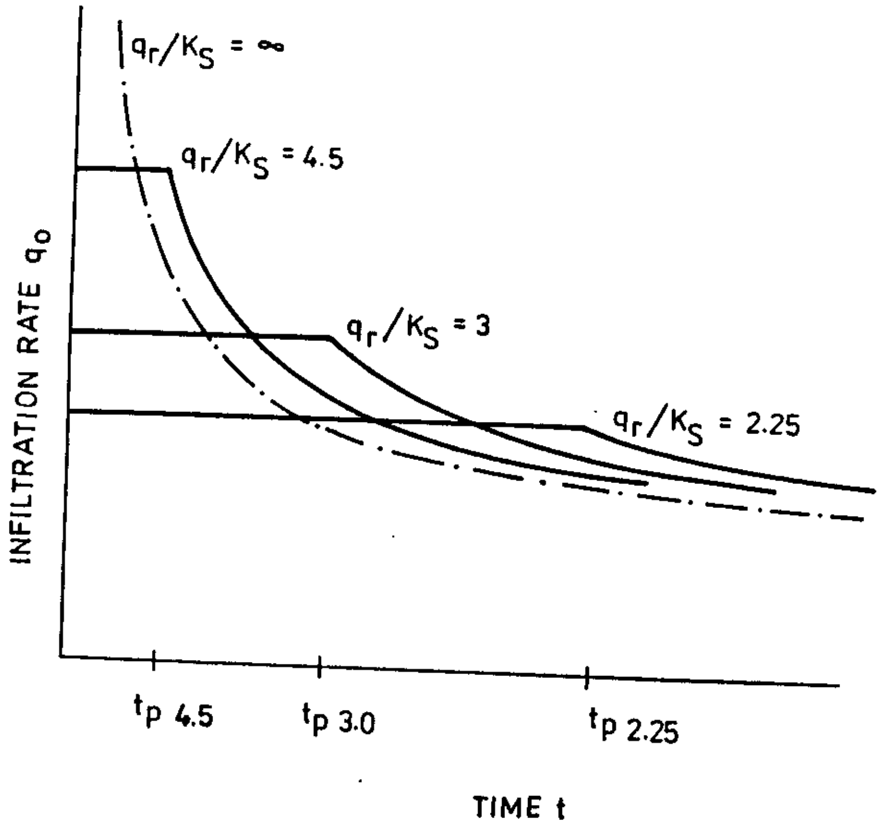
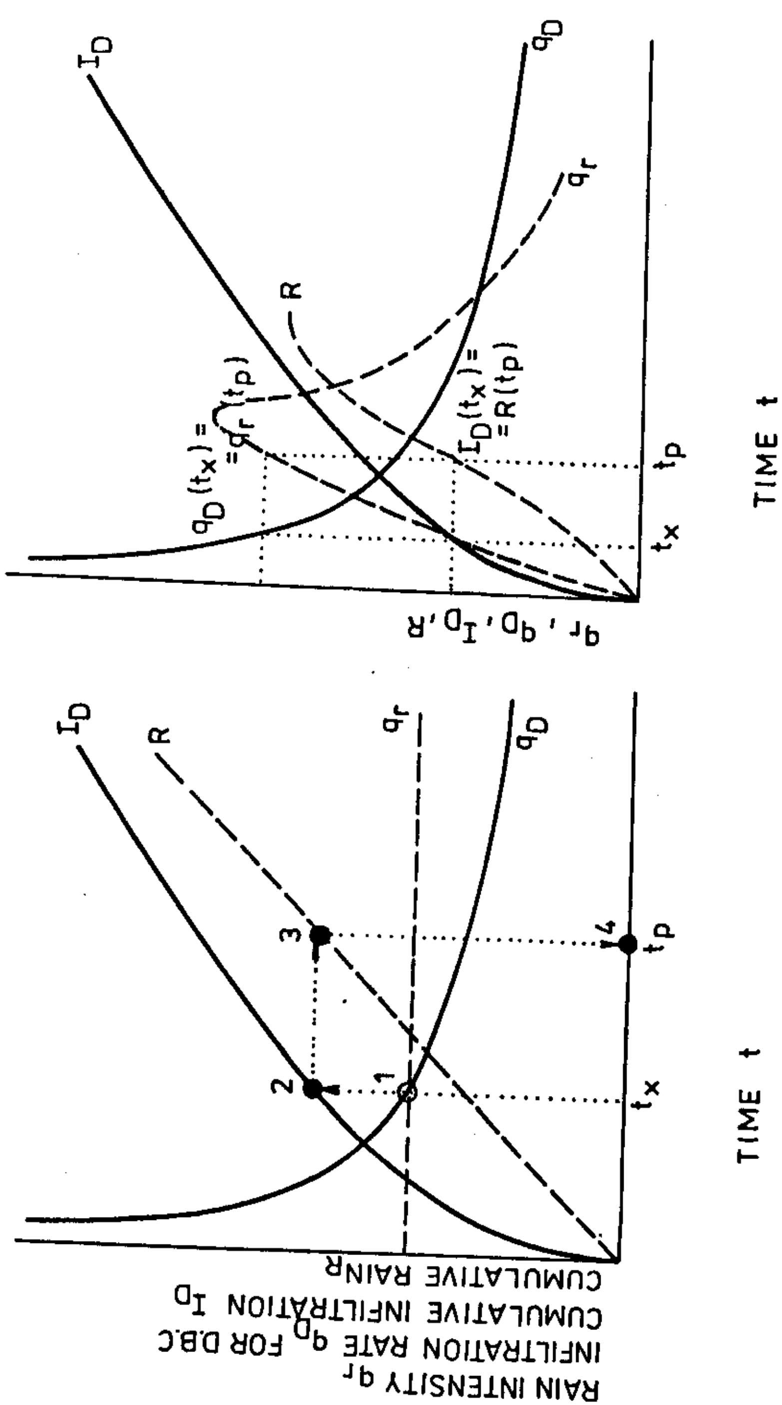
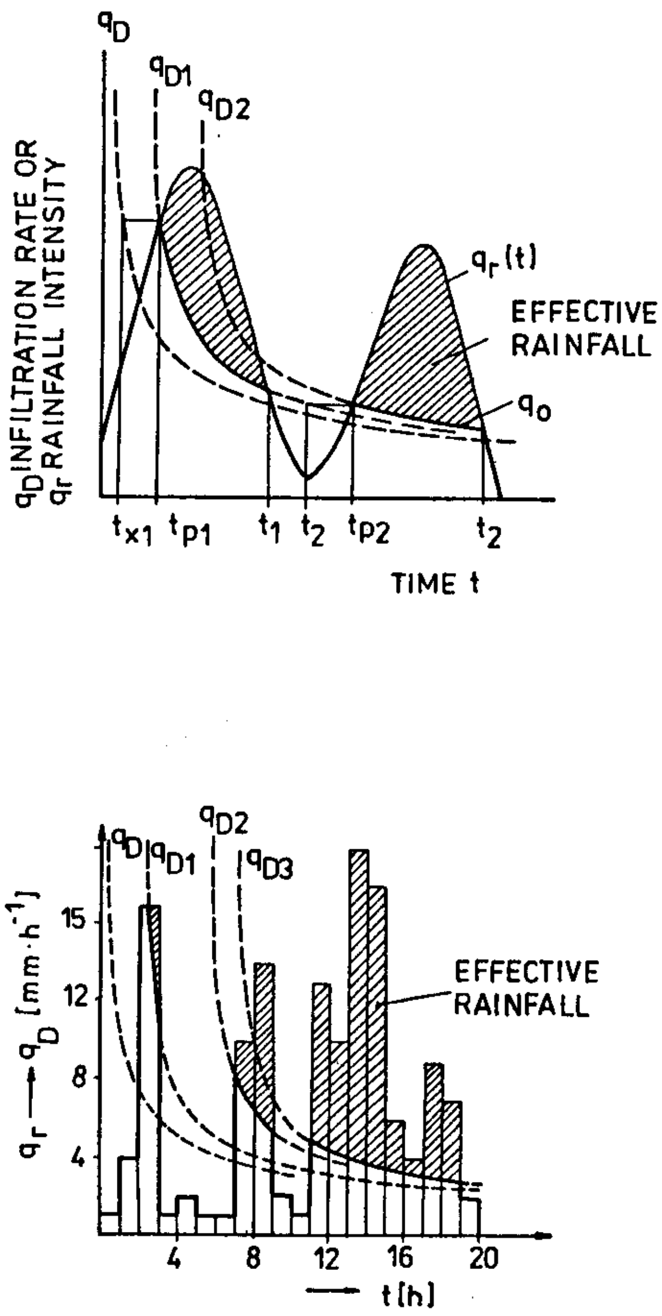

qr/KS = ∞
qr/KS = 4.5
qr/KS = 3
qr/KS = 2.25
INFILTRATION RATE qo
tp 4.5
tp 3.0
tp 2.25
TIME t

FIG. 7/I



q_r , q_D , I_D ,R
TIME t
$q_D(t_x) = q_r(t_p)$
$I_D(t_x) = R(t_p)$
RAIN INTENSITY q_r
INFILTRATION RATE q_D FOR D.B.C
CUMULATIVE INFILTRATION I_D
CUMULATIVE RAIN R

FIG 8 /..



q_D INFILTRATION RATE OR
q_r RAINFALL INTENSITY
$q_r(t)$
EFFECTIVE RAINFALL
q_o
TIME t
$q_r \longrightarrow q_D$ [mm·h^{-1}]
EFFECTIVE RAINFALL
$\longrightarrow$ t [h]

FIG. 9/I

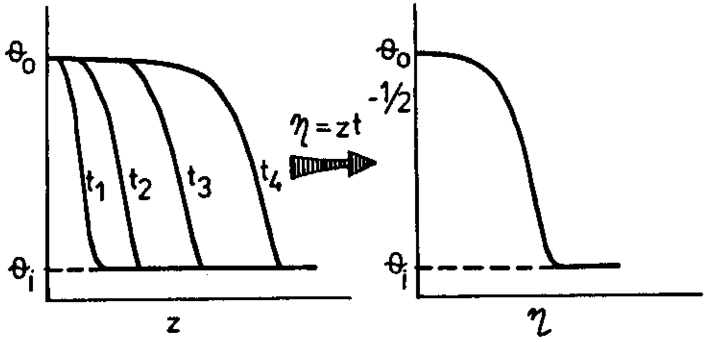


FIG. 10/I

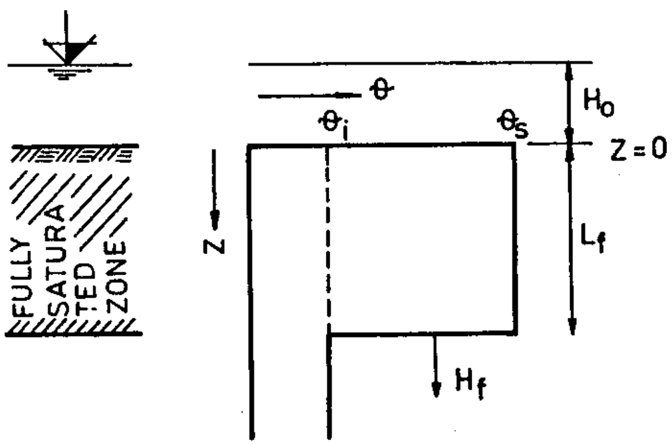


FIG. 11/I

FIG. 1/II

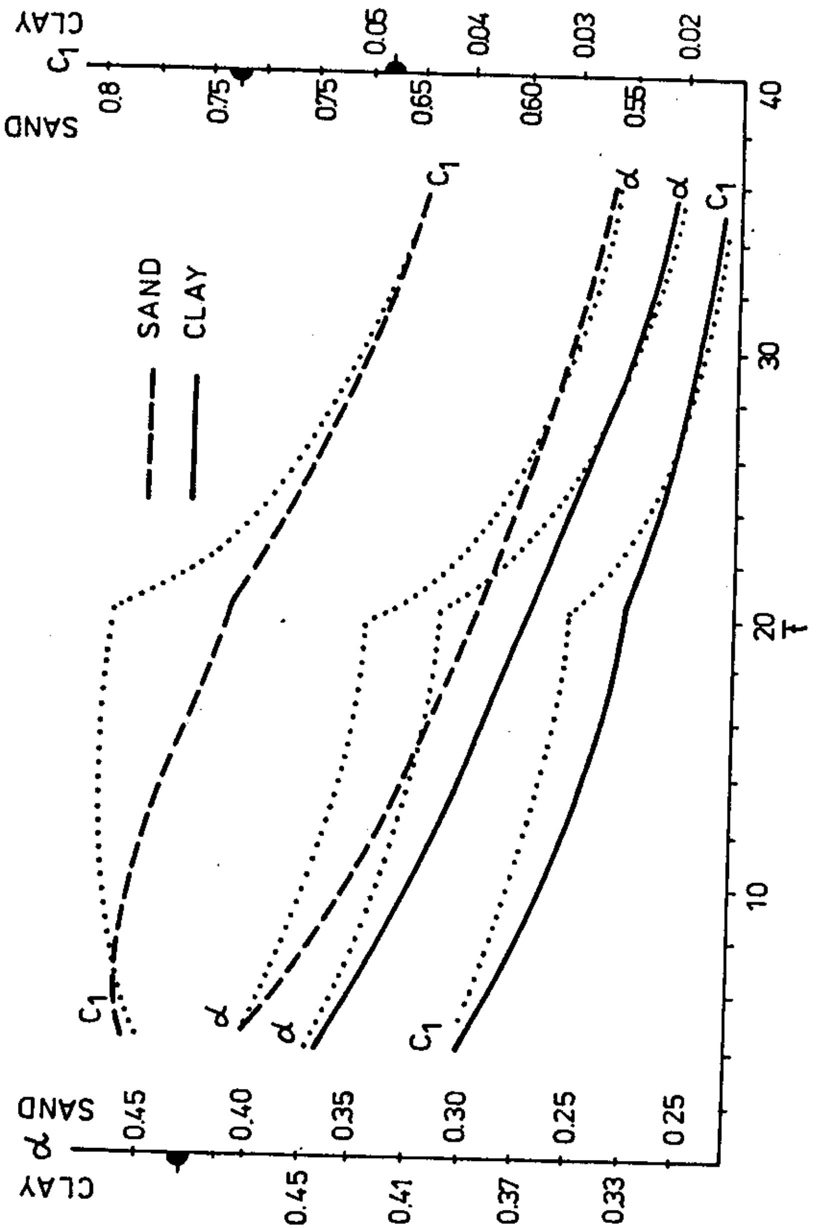
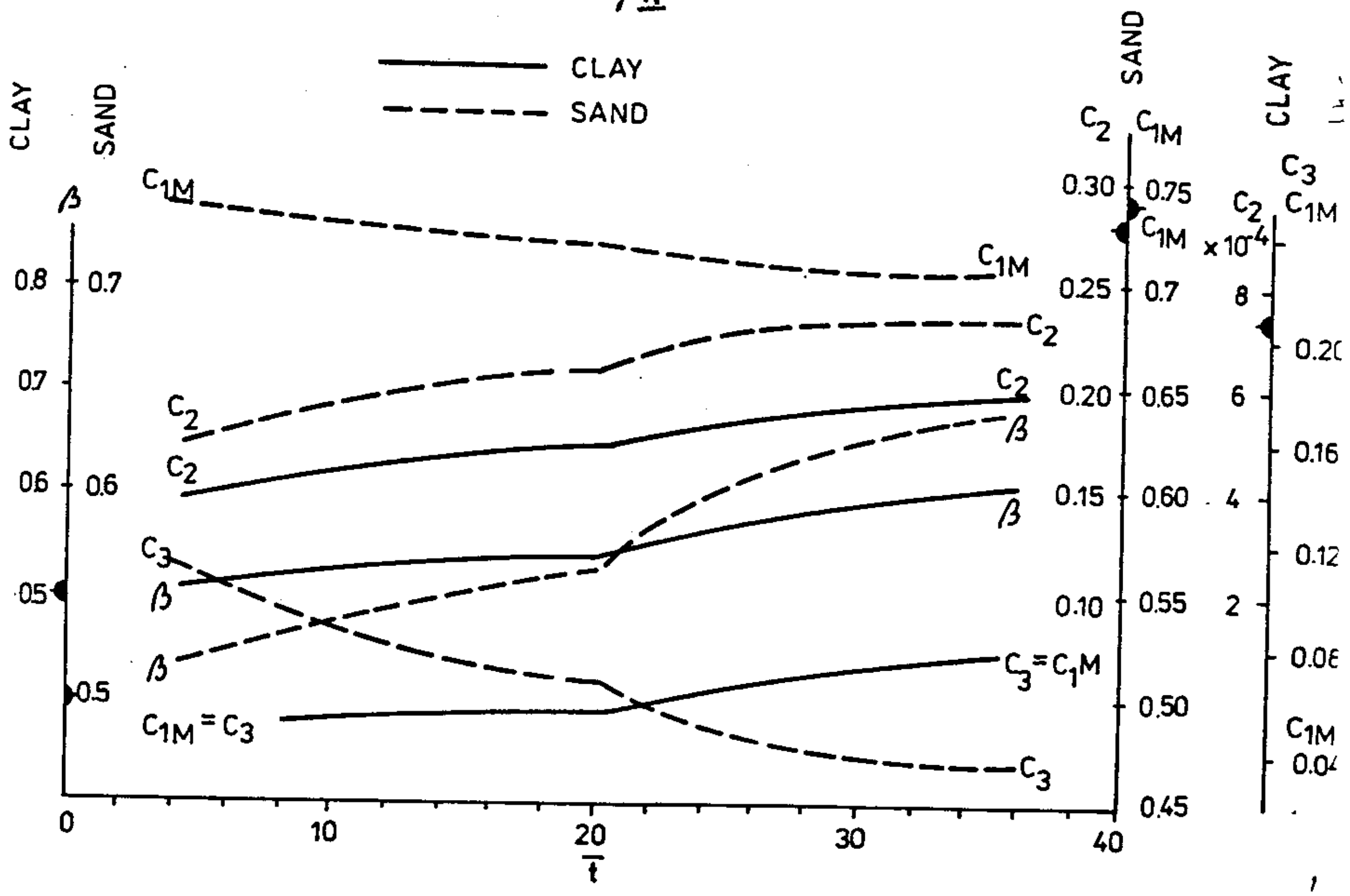


FIG. 2/II



- 47 -

FIG. 3/II

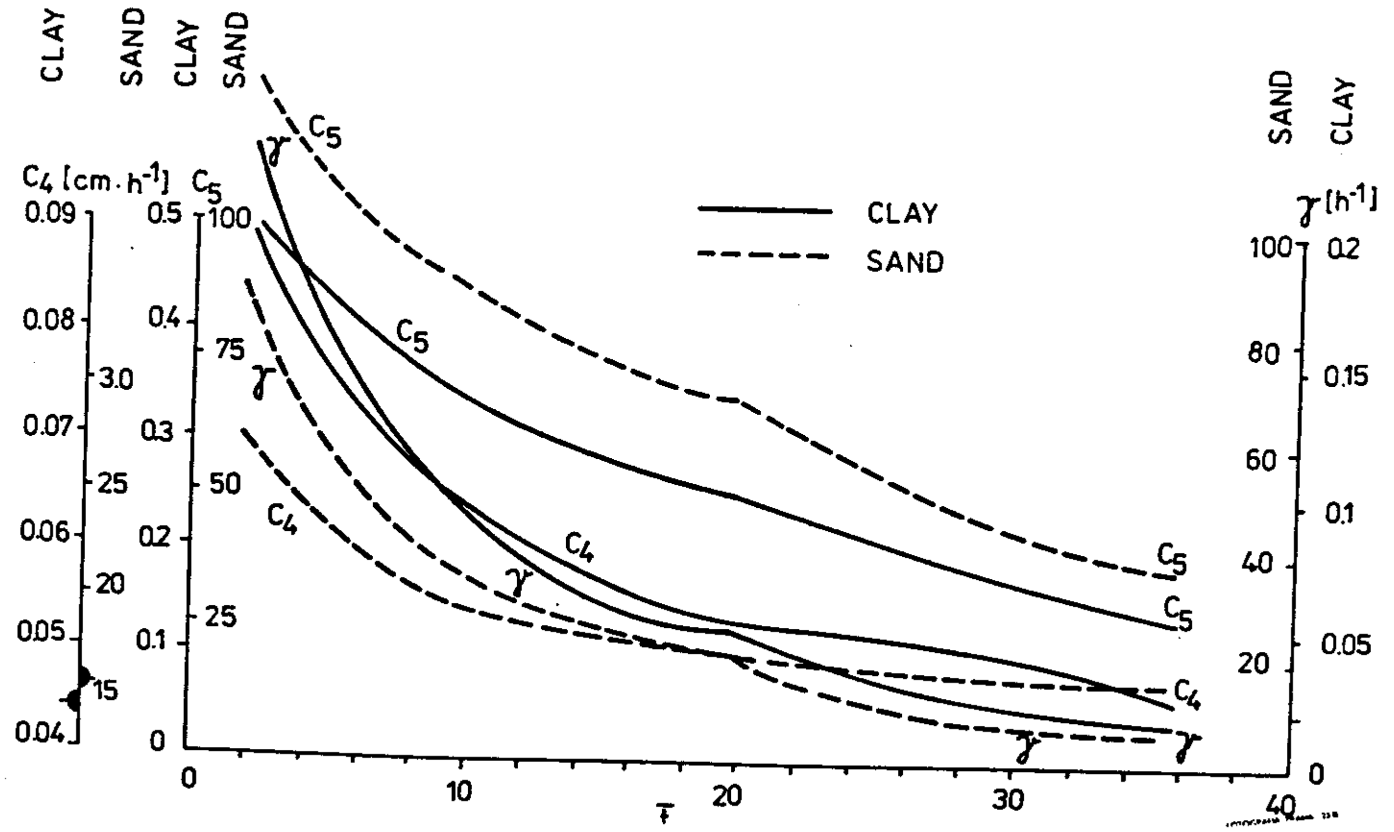


FIG. 4/II

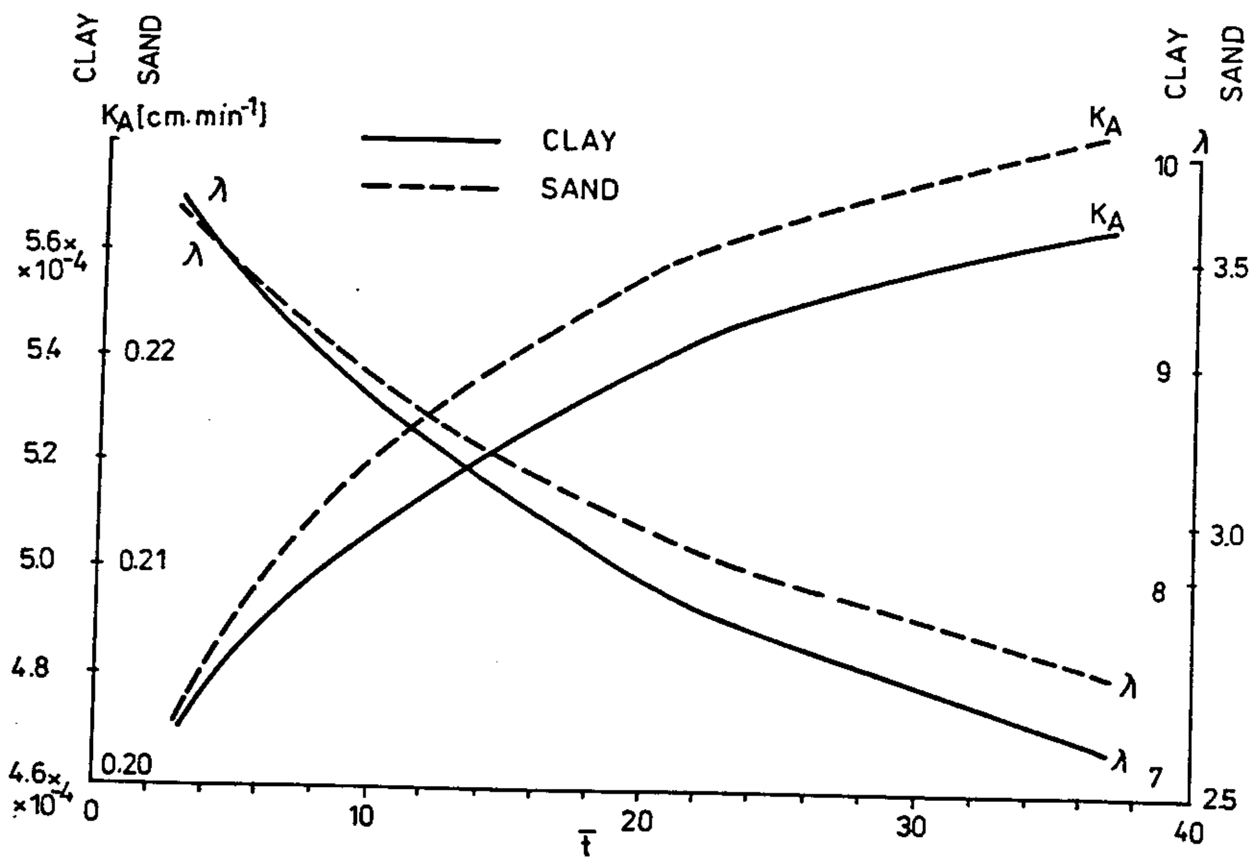


FIG. 5/II

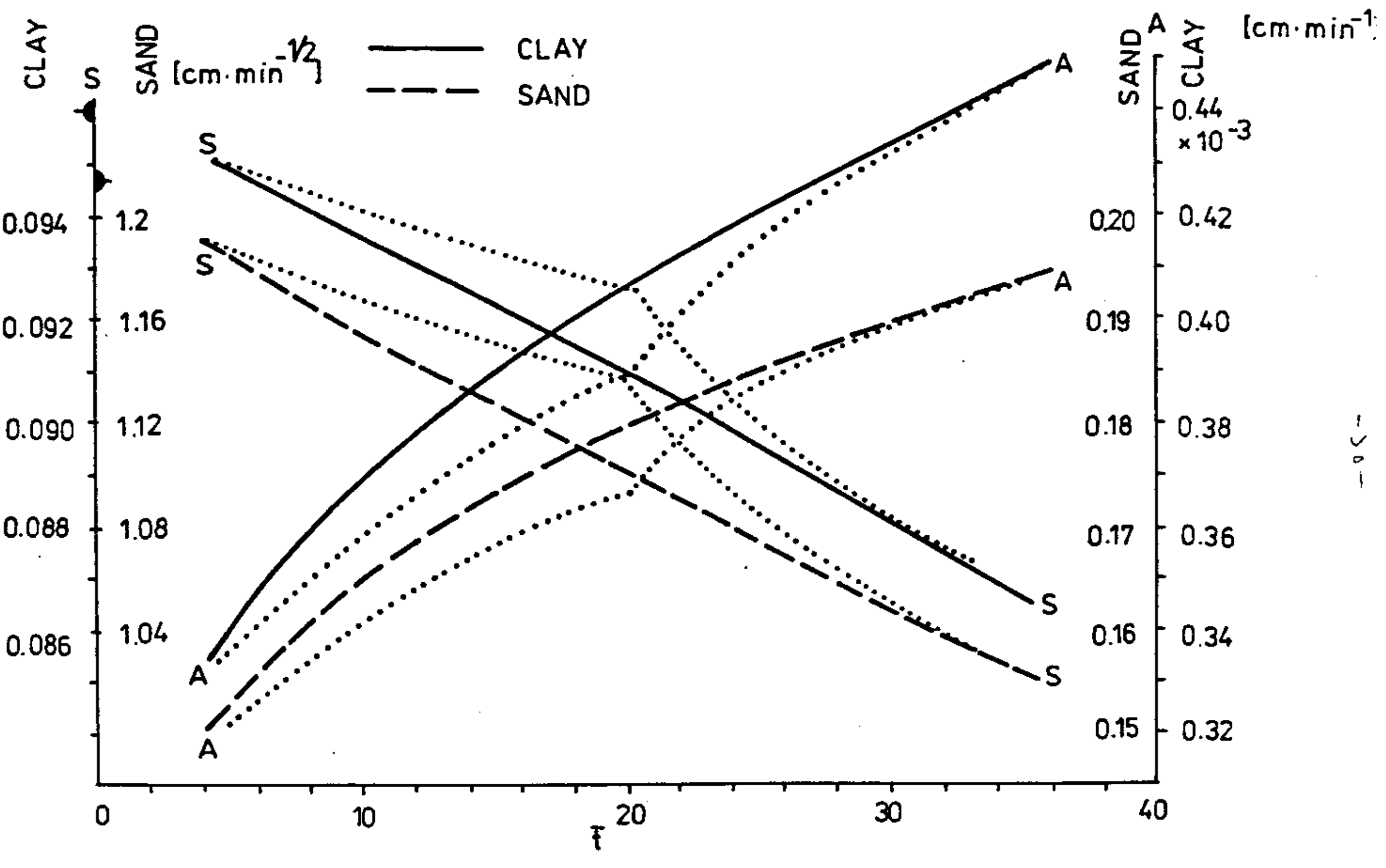


FIG. 6/II

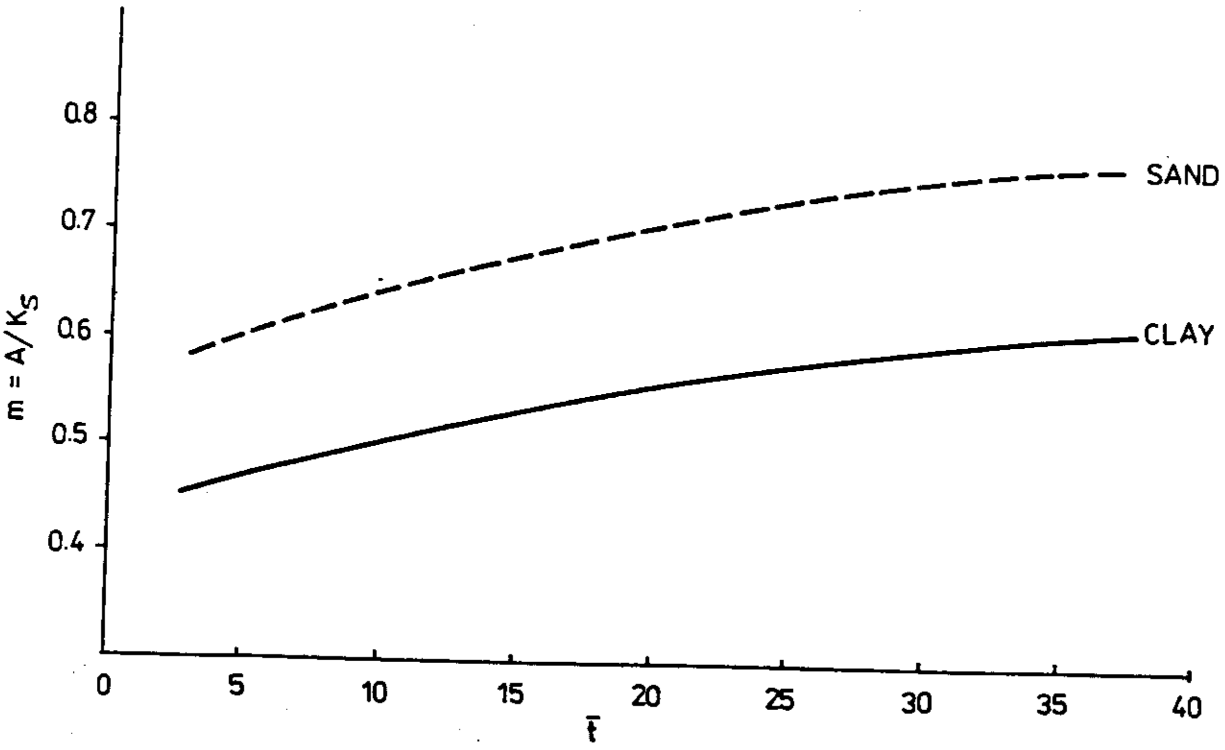


FIG. 7/II

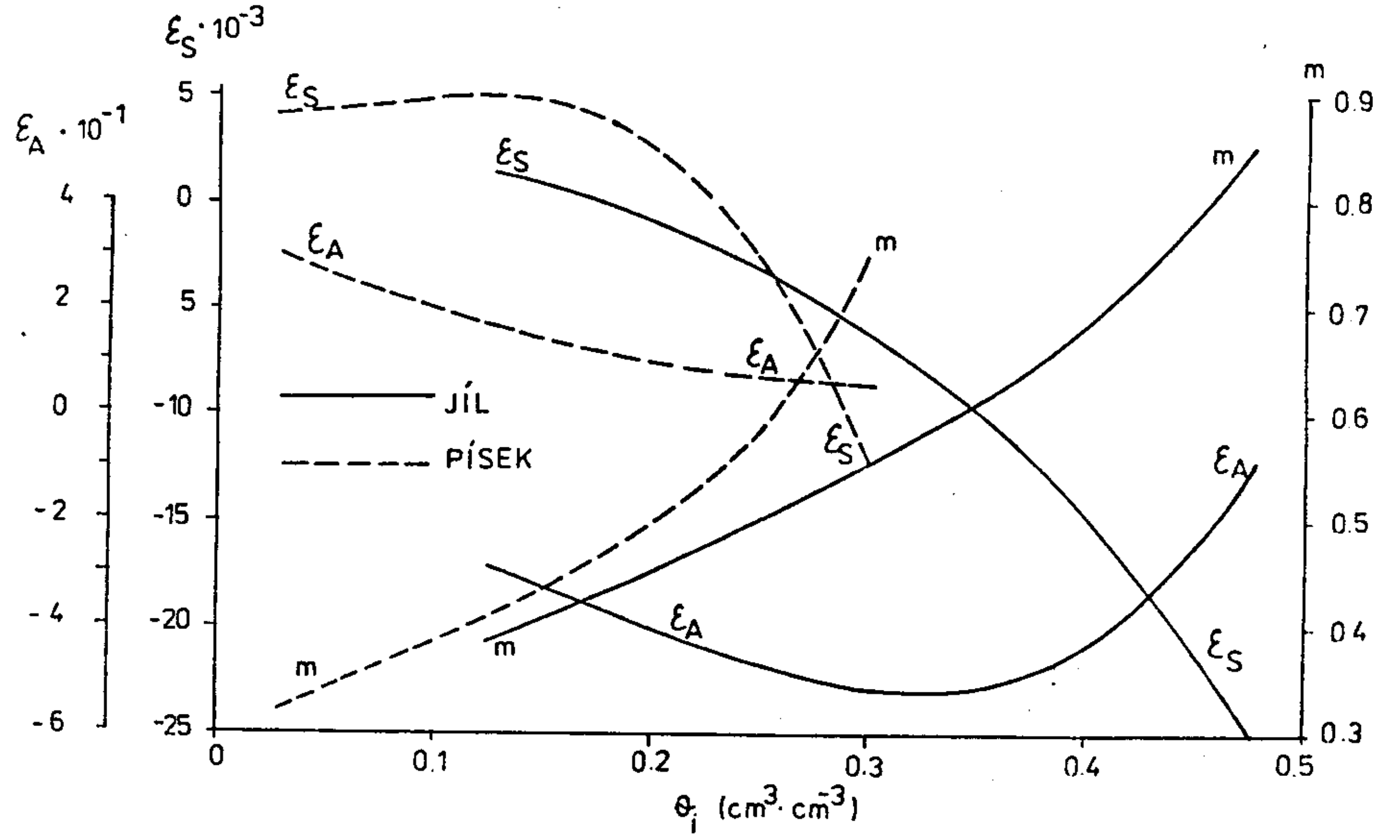


FIG. 8/II

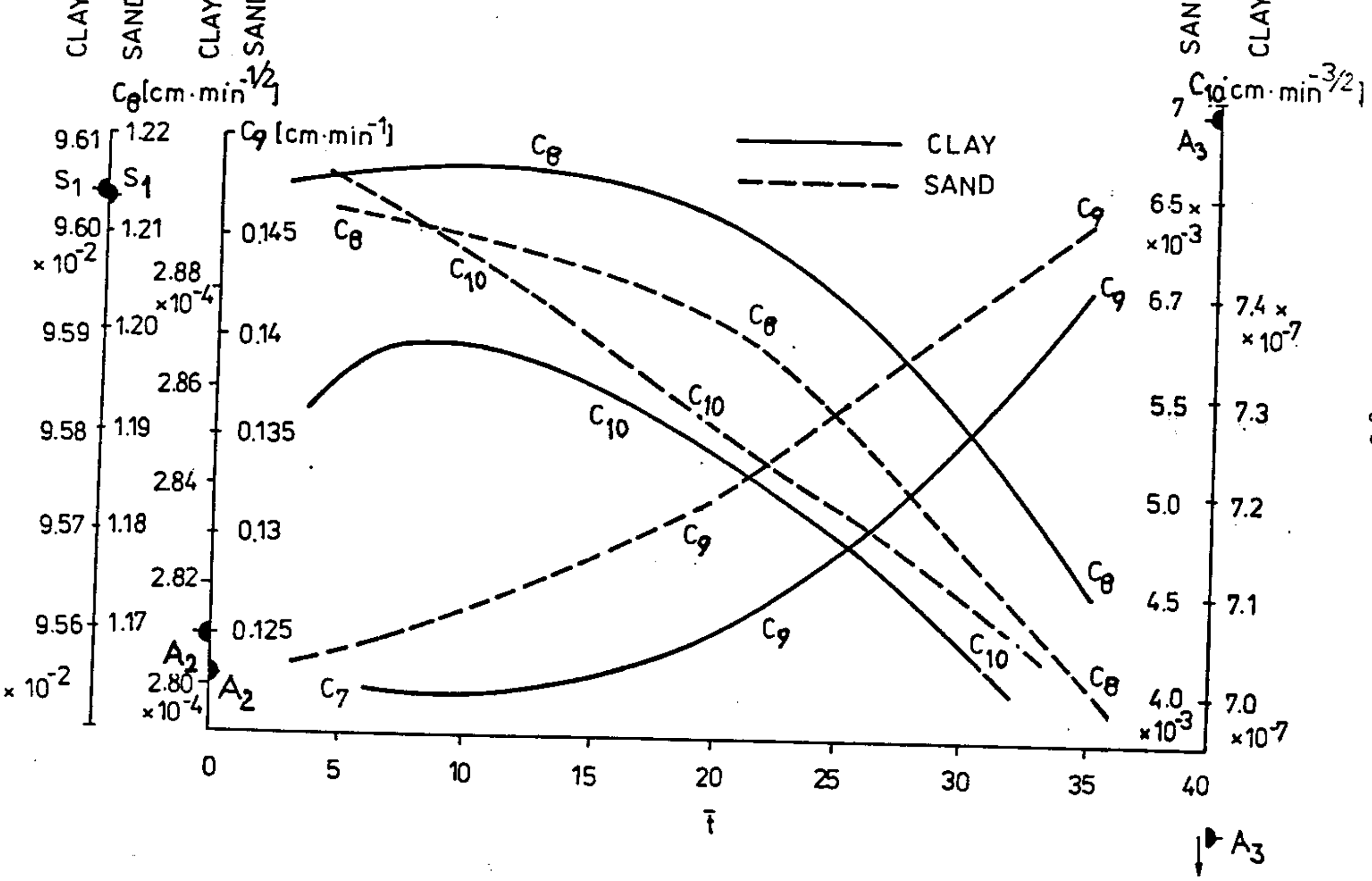


FIG. 9/II

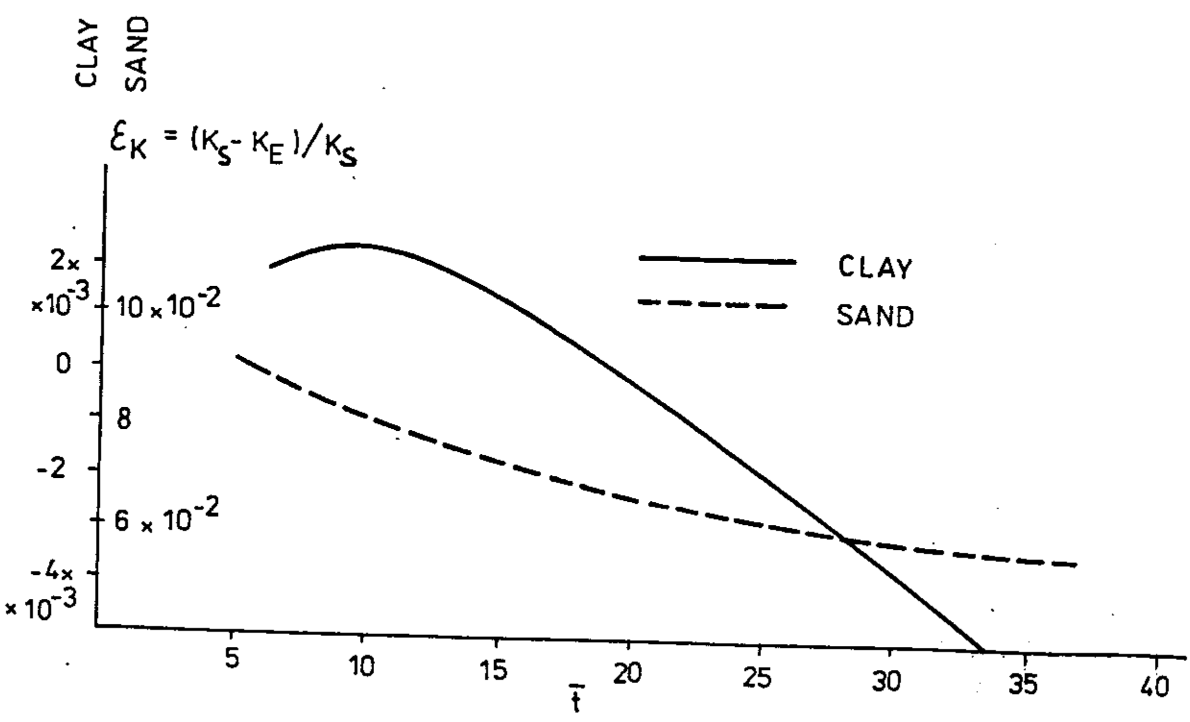
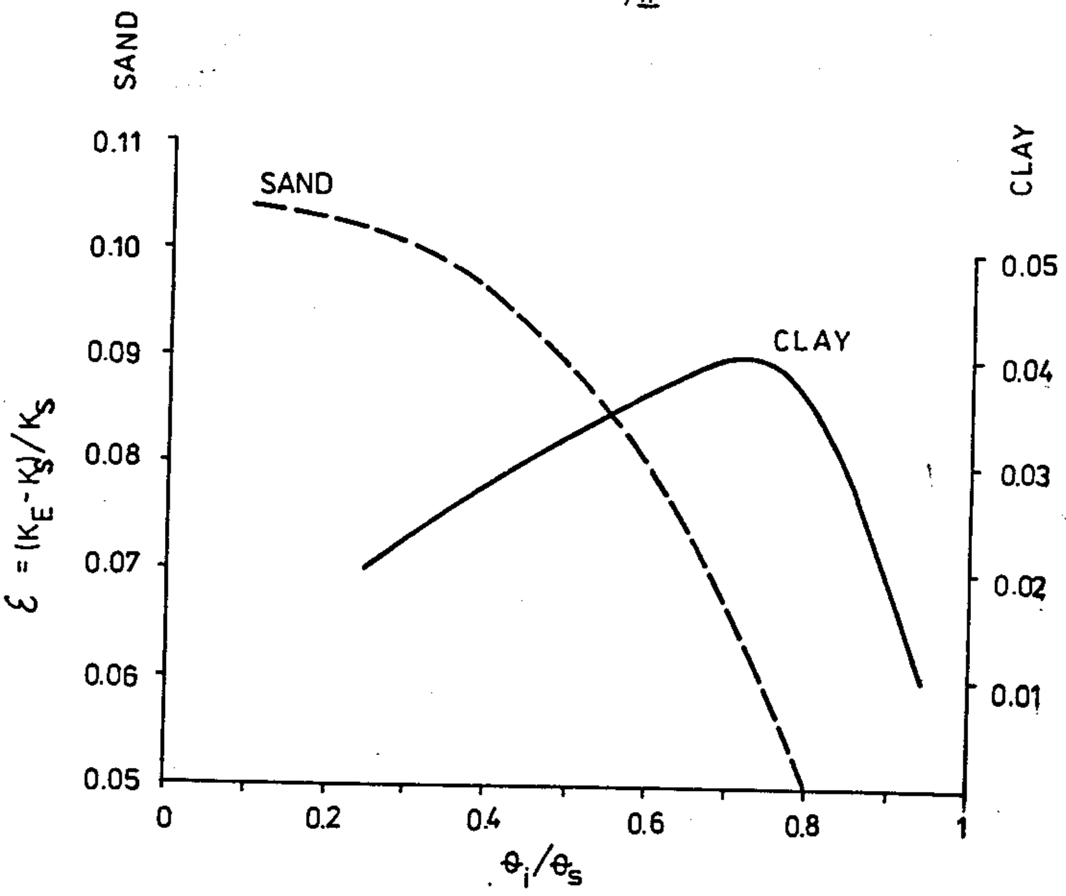


FIG. 10/II



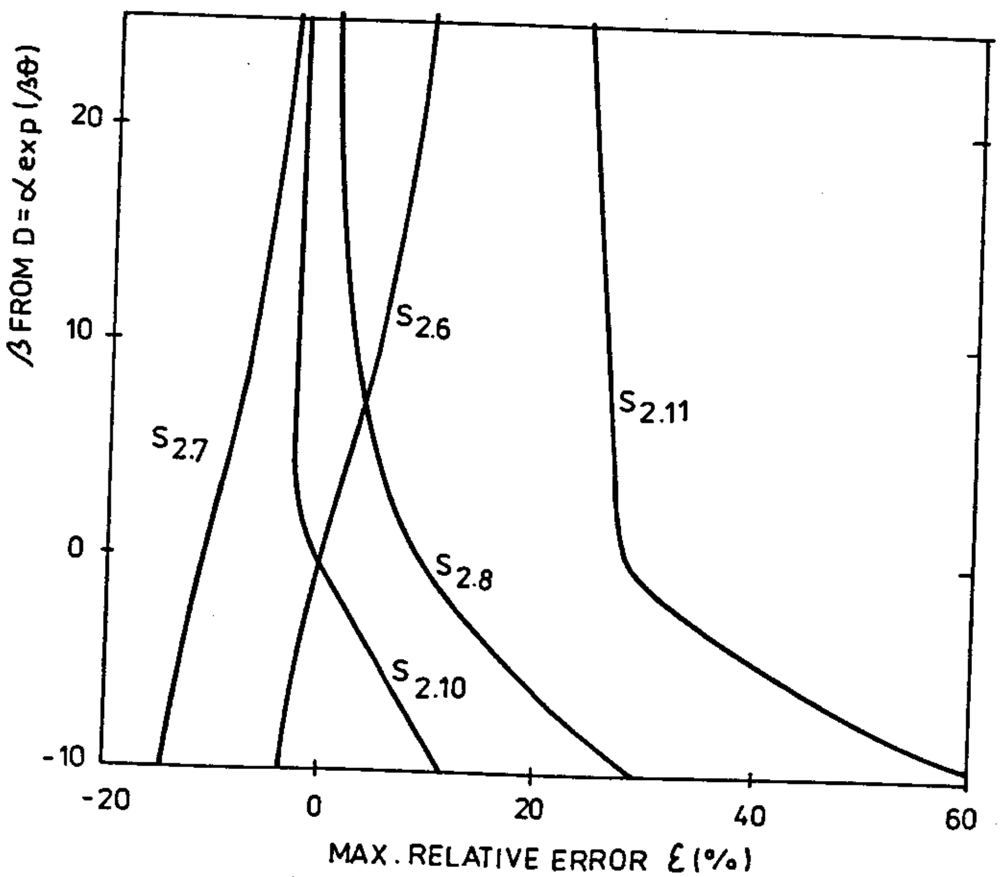


FIG.11/II

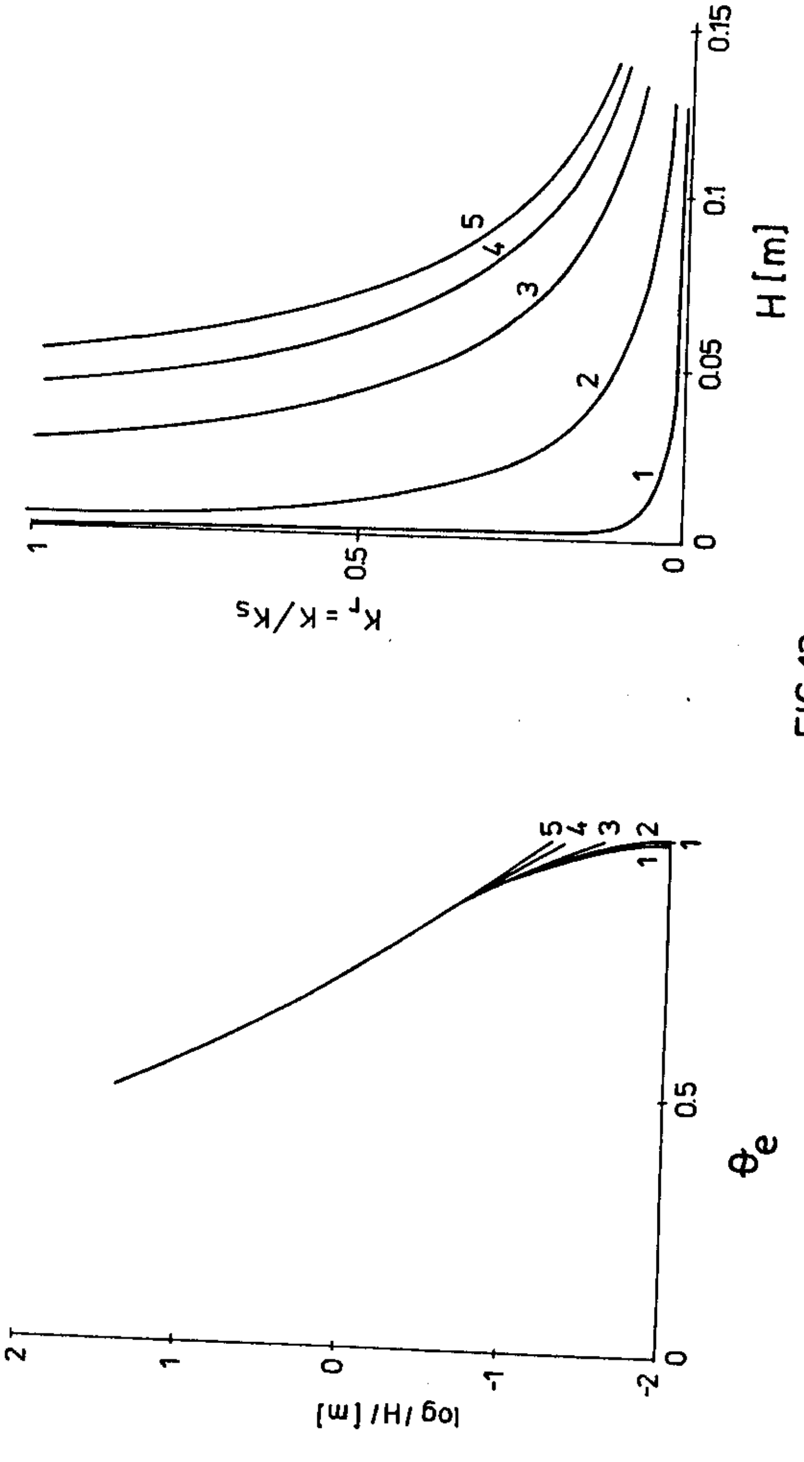


FIG.12/II

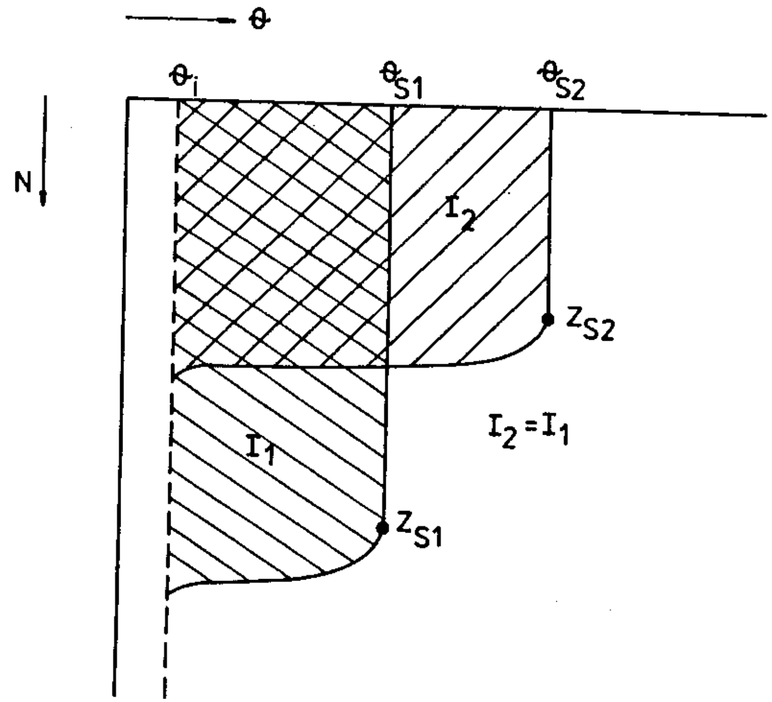


FIG. 13/II

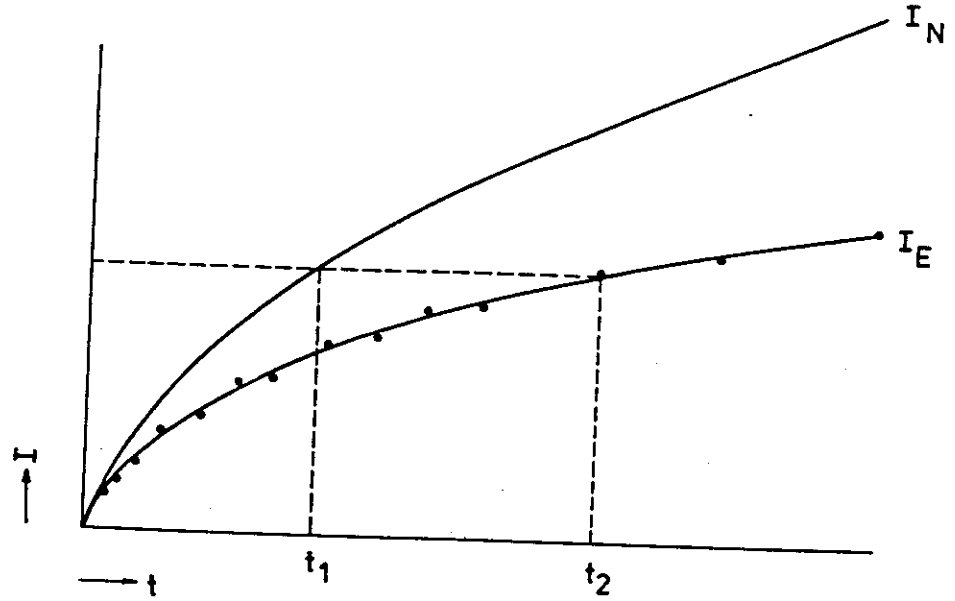


FIG. 14/II