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COLLEGE ON SOIL PHYSICS
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"Flow of Water in Unsaturated Soil"

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3. Flow of water in unsaturated soil.

3.1. Introduction.

Most of the processes involving soil water interactions in the field, and particularly the flow of water in the rooting zone of most crop plants, occur while the soil is in an unsaturated condition. Unsaturated flow processes are in general complicated and difficult to describe quantitatively, since they often entail changes in the state and content of soil water during flow. Such changes involve complex relations among the variable soil wetness, suction and conductivity, whose interrelations may be further complicated by hysteresis. The formulation and solution of unsaturated flow problems very often require the use of indirect methods of analysis, based on approximations or numerical techniques. For this reason the development of rigorous theoretical and experimental methods for treating these problems was rather late in coming. In recent decades, however, unsaturated flow has become one of the most important and active topics of research in soil physics, and this research has resulted in significant theoretical and practical advances.

In soil physics, soils of which the pore volume is only partially filled with water are called unsaturated.

3.2. Comparison of unsaturated versus saturated flow

We will see, that soil-water flow is caused by a driving force resulting from an effective potential gradient, that flow takes place in the direction of decreasing potential, and that the rate of flow (flux) is proportional to the potential gradient and is affected by the geometric properties of the pore channels through which flow takes place. These principles apply in unsaturated as well as in saturated soils.

The moving force in a saturated soil is the gradient of a positive pressure potential. On the other hand, water in an unsaturated soil is subject to a subatmospheric pressure, or suction, and the gradient of this suction likewise constitutes a moving force. The matric suction is due, as we have pointed out, to the physical affinity of the water

to the soil-particle surfaces and capillary pores. Water tends to be drawn from a zone where the hydration envelopes surrounding the particles are thicker, to where they are thinner, and from a zone where the capillary menisci are less curved to where they are more highly curved. In other words, water tends to flow from where suction is low to where it is high. When suction is uniform all along a horizontal column, that column is at equilibrium and there is no moving force. Not so when a suction gradient exists. In that case, water will flow in the pores which remain water-filled at the existing suction, and will creep along the hydration films over the particle surfaces, in a tendency to equilibrate the potential.

The moving force is greatest at the "wetting front" zone of water entry into an originally dry soil. In this zone, the suction gradient can be many bars per centimeter of soil. Such a gradient constitutes a moving force thousands of times greater than the gravitational force. Such strong forces are sometimes required (for a given flux) in view of the extremely low hydraulic conductivity which a relatively dry soil may exhibit.

The most important difference between unsaturated and saturated flow is in the hydraulic conductivity. When the soil is saturated, all of the pores are filled and conducting, so that conductivity is maximal. When the soil becomes unsaturated, some of the pores become airfilled and the conductive portion of the soil's cross-sectional area decreases correspondingly. Furthermore, as suction develops, the first pores to empty are the largest ones, which are the most conductive, thus leaving water to flow only in the smaller pores. The empty pores must be circumvented, so that, with desaturation, the tortuosity increases. In coarse-textured soils, water sometimes remains almost entirely in capillary wedges at the contact points of the particles, thus forming separate and discontinuous pockets of water. In aggregated soils, too, the large interaggregate spaces which confer high conductivity at saturation become (when emptied) barriers to liquid flow from one aggregate to its neighbors.

For these reasons, the transition from saturation to unsaturation generally entails a steep drop in hydraulic conductivity, which may decrease by several orders of magnitude (sometimes down to 1/100,000 of tis value at saturation) as suction increases from zeto to one bar. At still higher suctions, or lower water contents, the conductivity may be so low that very steep suction gradients, or very long times, are required from any appreciable flow to occur.

At saturation, the most conductive soils are those in which large and continuous pores constitute more of the overall pore volume, while the least conductive are the soils in which the pore volume consists of numerous micropores. Thus, as is well known, a sandy soil conducts water more rapidly than a clayey soil. However, the very opposite may be true when the soils are unsaturated. In a soil with large pores, these pores quickly empty and become nonconductive as suction develops, thus steeply decreasing the initially high conductivity.

In a soil with small pores, on the other hand, many of the pores remain full and conductive even at appreciable suction, so that the hydraulic conductivity does not decrease as steeply and may actually be greater than that of a soil with large pores subjected to the same suction.

Since in the field the soil is unsaturated most of the time, it often happens that flow is more appreciable and persists longer in clayey than in sandy soils. For this reason, the occurrence of a layer of sand in a fine-textured profile, far from enhancing flow, may actually impede unsaturated water movement until water accumulates above the sand and suction decreases sufficiently for water to enter the large pores of the sand. This simple principle is all too often misunderstood.

3.3. Relation of conductivity to suction and wetness

Let us consider an unsaturated soil in which water is flowing under suction. Such flow is illustrated schematically in the model of figure 9. In this model, the potential difference between the inflow and outflow ends is maintained not by different heads of positive hydrostatic pressure, but by different imposed suctions.

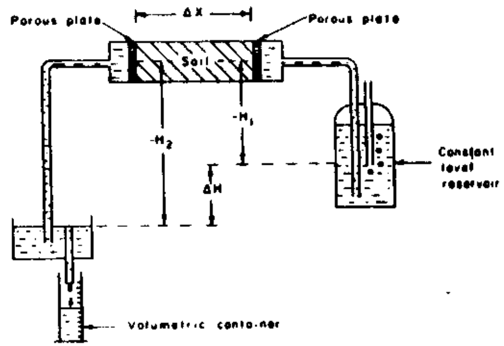


Figure 9 : A model illustrating unsaturated flow (under a suction gradient) in a horizontal column.

In general, as the suction varies along the sample, so will the wetness and the conductivity. If the suction heads at both ends of the sample are maintained constant, the flow processes will be steady and the suction gradient will increase as the condqctivity decreases with the increase in suction along the axis of the sample. This phenomenon is illustrated in figure10.

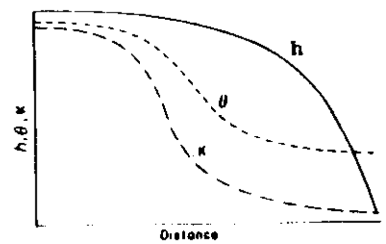


Figure10 : The variation of wetness θ, matric potential h_m, and conductivity K along a hypothetical column of unsaturated soil conducting a steady flow of water.

Since the gradient along the column is not constant, as it is in uniform saturated systems, it is not possible, strictly speaking, to divide the flux by the overall rat) of the head drop to the distance ($\Delta H/\Delta x$) to obtain the conductivity. Rather, it is necessary to divide the flux of the exact gradient at each point to evaluate the exact conductivity and its variation with suction. In the following treatment, however, we shall assume that the column of figure 9 is sufficiently short to allow us to evaluate at least an average conductivity for the sample as a whole (i.e. $K = q.\ \Delta x/\Delta H$).

The average negative head, or suction, acting in the column is :

$$-\bar{H} = \bar{h} = -\frac{H_1 + H_2}{2}$$

We assume that the suction everywhere exceeds the air-entry value so that the soil is unsaturated throughout.

Let us now make successive and sytematic measurements of flux versus suction gradient for different values of average suction. The results of such a series of measurements are shown schematically in figure 11. As in the case of saturated flow, we find that the flux is proportional to the gradient. However, the slope of the flux versus gradient line, being the hydraulic conductivity, varies with the average suction. In a saturated soil, by way of contrast, the hydraulic conductivity is generally independant of the magnitude of the water potential, or pressure.

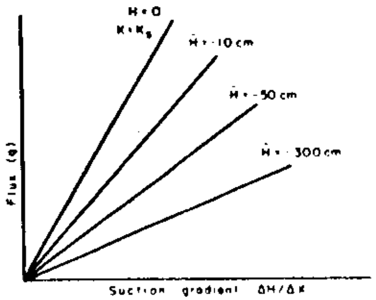


Figure 11 : The hydraulic conductivity, being the slope of the flux vs. gradient relation depends upon the average suction in an unsaturated soil.

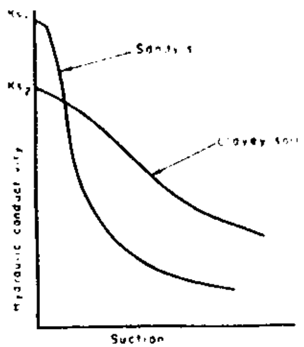


Figure 12 : Dependance of conductivity on suction in soils of different texture.

Figure 12 shows the general trend of the dependence of conductivity on suction in soils of different texture. It is seen that, although the saturated conductivity of the sandy soil Ks_1 is typically greater than that of the clayey soil Ks_2, the unsaturated conductivity of the former decreases more steeply with increasing suction and eventually becomes lower

No fundamentally based equation of general validity is available for the relation of conductivity to suction or to wetness, and existing knowledge does not allow the reliable prediction of unsaturated conductivity from basic soil properties. Various empirical equations have been proposed, however, including the following

$$K = \frac{a}{h^m} \quad (1)$$

$$K = \frac{a}{b + h^m} \quad (2)$$

$$K = \frac{K_s}{1 + (h/h_c)^m} \quad (3)$$

$$K = a\ .\ \theta^m \quad (4)$$

$$K = K_s . W_s^{\ m} \quad (5)$$

where :

- K – the hydraulic conductivity at any degree of saturation (or unsaturation),
- K_s – the saturated conductivity of the same soil,
- a, b and m = empirical constants (different in each equation),
- h = the matric suction head,
- θ = the volumetric water content,
- W_s = the degree of saturation,
- h_c = the suction head at which $K = 1/2\ K_s$.

Of these various equations, the most commonly employed are the first two (of which the first is the simplest, but cannot be used in the suction range approaching zero). In all of the equations, the most important parameter is the exponential constant, since it controls the steepness with which conductivity decreases with increasing suction or with decreasing water content. The m-value of the first two equations is about two or less for clayey soils, and may be four or more for sandy soils. For each soil, the equation of best fit, and the values of the parameters, must be determined experimentally.

The relation of conductivity to suction depends upon hysteresis, and is thus different in a wetting than in a drying soil. The reason is that, at a given suction, a drying soil contains more water than a wetting one. The relation of conductivity to water content, however, appears to be affected by hysteresis to a much lesser degree. The value of the exponent for the relation of K to θ (equation 4) can be as high as 10 or more.

3.4. Hydraulic conductivity.

The hydraulic conductivity of unsaturated soils is smaller than that of saturated soils, because only the pores which still contain water can contribute to the flow of water. Since the hydraulic conductivity of a pore is proportional to

square of its radius, the largest water-filled pores contribute most to that flow. When a saturated soil starts becoming unsaturated, the largest pores are emptied first. As a result, the unsaturated hydraulic conductivity decreases very fast with water content.

It has been found experimentally that Darcy's law is also valid for unsaturated soils. Thus, the flux density for the flow of water in unsaturated soil is:

$$q = -\ K(\theta)\ \frac{\partial H}{\partial s} \qquad (8)$$

where now the hydraulic conductivity K(θ) vary over many orders of magnitude with θ.

Figure 13 shows the hydraulic conductivity K of a medium fine sand and a loam as function of the volume fraction of water, θ. For instance, when in the sand θ decreases from θ = 0.34 to θ = 0.07, K changes from about $K = 1\ m\ d^{-1}$ to $K = 10^{-5}\ m\ d^{-1}$.

The relationship between K and θ is not linear, but in many soils log K is approximately proportional to θ, as indicated in figure 13 by the semi-logarithmic scale. Generally, the K-θ relationship is not influenced by hysteresis, in contrast with the K-h_m relationship. Therefore, the K-θ relationship is used most of the time.

The hydraulic conductivity of loam (figure 13B) is lower than that of sand (figure 13A) for the same θ.

At the same water content the water-filled pores of loam are smaller than those of sand. Thus the contact area per volume between the solid phase and the water is the largest for loam. This results in a higher resistance to flow, or a lower hydraulic conductivity.

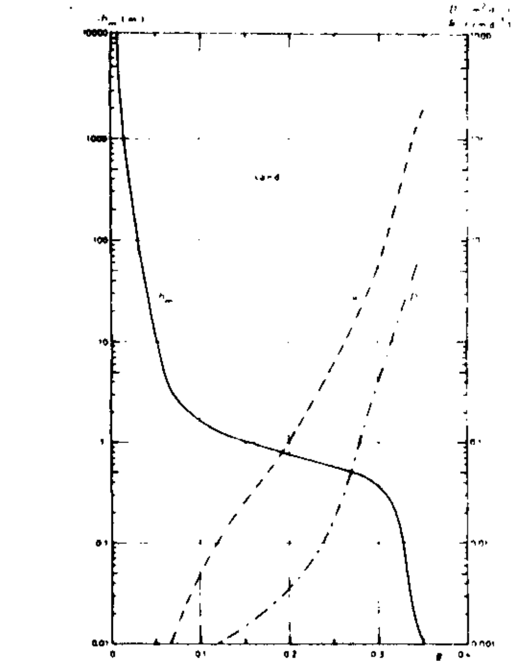


Figure 13A. Soil water characteristic h_m-θ (——), hydraulic conductivity k-θ (----) and hydraulic diffusivity D-θ (- — - —) for medium fine sand.

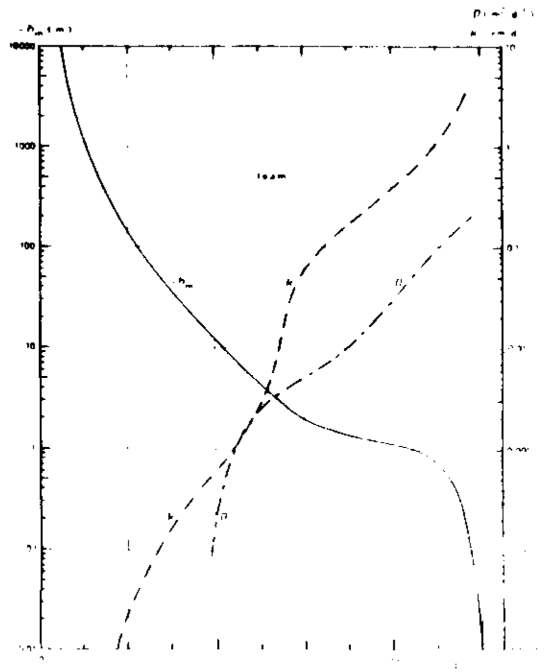


Figure 13B. Soil water characteristic h_m-θ (——), hydraulic conductivity k-θ (----) and hydraulic diffusivity D-θ (- — - —) for loam.

3.5. Steady unsaturated flow of water.

3.5.1. Steady upward flow.

To study steady, upward flow of water in unsaturated soil, we will first consider a situation as in figure 14. The water in the soil column is in static equilibrium with the simulated groundwater table, because a plastic cover prevents evaporation from the soil surface. Thus the water content profile shown is the same as the soil water characteristic of the soil for the particular range of h_m. Such an equilibrium situation may occur in a field with a constant, shallow water table during periods of fog, when the air is saturated with water vapour.

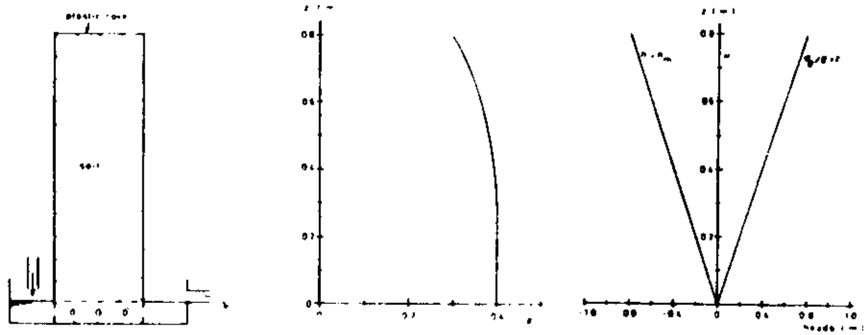


Figure 14. Static equilibrium with a groundwater table.

The soil is saturated below z = 0.3 m. Since at z = 0.3 m, h_m = - 0.3 m, the air-entry value is - 0.3 m.

If the plastic cover on top of the soil column in figure 14 is removed, evaporation will start (figure 15). If the evaporation rate is constant and the groundwater table is maintained at its original level, steady upward flow will be established. This means that θ does not vary in time and thus, according to the continuity equation:

$$q = -K\frac{dH}{dz} = \text{constant and uniform}$$

For the flow to be directed upwards, it must decrease with height, making dH/dz negative and q positive. Thus h, and with it θ, must decrease. Since K decreases fast with θ (figure 13) it decreases fast with height. To maintain the same flux at all heights, a certain rate of decrease of K with height must be compensated by the same rate of increase with height of the absolute value of dH/dz. Thus the H curve in the potential diagram must be increasingly flatter towards the soil surface, as indicated in figure 15.

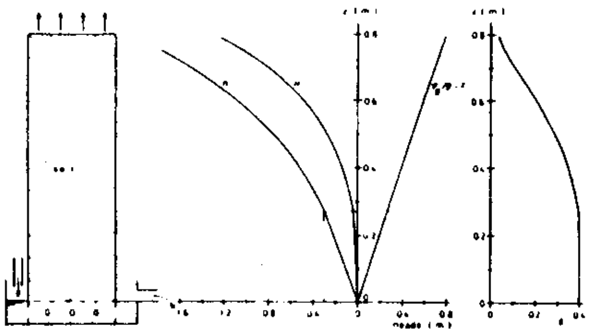


Figure 15. Steady upward flow of water, constant evaporation from a stable groundwater table.

The above situation is often found in the field during periods of drought, when a rather sharp transition from a dry upper layer to a moist lower zone is developed due to the sharp decrease of K with decreasing water content. The formation of such a sharp transition is the result of a self-accelerating mechanism, which can be explained as follows. When evaporation starts, θ as well as h and H decrease near the surface. But, because K decreases too, the new H gradient is too small to maintain the upward flow needed to satisfy the evaporative demand. Hence, θ decreases still further, causing a further decrease in H, but again not sufficient to off-set the accompanying decrease in K etc... This process continues until the surface

layer has become so dry that practically all flow of water ceases. The formation of this dry surface layer is of practical importance because it protects the soil against large evaporation losses. This so-called soil mulch is the principle upon which dry-land farming is practised.

The exact values of h and H in the steady situation represented in figure 15 are difficult to calculate, and then only when the k-θ and h_m-θ relationships are known accurately. The gradients in the lower, water-saturated part of the profile can be calculated easily, provided the saturated K, the evaporation rate and the air-entry value are known.

PROBLEM 7

Is the relationship of H and h with z curvilinear in the water-saturated part of the profile in figure 15?

ANSWER

In the water-saturated part of the profile, K is constant with height. In steady saturated water transport, the flux density is also constant with height. This implies that dH/dz is constant with height or that H changes linearly with height, and not curvilinear. The same is true for h.

PROBLEM 8

Estimate the evaporation rate in figure 15, assuming a steady state situation, a saturated K value of 0.05 m d^{-1} and h = - 0.22 m at z = 0.2 m.

ANSWER

Since q is constant with height, the evaporation rate at the surface is the same as the flux density in the saturated zone. If h = - 0.22 m at z = 0.2 m, then H = - 0.02 m at that height, and thus:

$$q = -K\frac{dH}{dz} = -0.05\ \text{m d}^{-1}\ \frac{-0.02}{0.20} = 5\ \text{mm d}^{-1}$$

P R O B L E M   9

a. Determine the height of the saturated zone for the situation in problem 8

b. Does this height remain constant if the evaporation rate changes?

A N S W E R

a. In the saturated zone is

$$\frac{dH}{dz} = \frac{q}{-k} = \frac{0.005\ \text{m d}^{-1}}{-0.05\ \text{m d}^{-1}} = -0.1\ \text{m m}^{-1}$$

Then $\frac{dh}{dz} = \frac{dH}{dz} - \frac{dz}{dz} = -0.1 - 1.0 = -1.1\ \text{m m}^{-1}$

or $h = -1.1\ z + C$

Since $h = 0$ at $z = 0$, $C = 0$

At the top of the saturated zone, h is equal to the air-entry value, or h = - 30 m (see text). Thus the height of the saturated zone is

$$z = \frac{h}{-1.1} = \frac{-0.3}{-1.1} \approx 0.27\ \text{m}$$

b. No. The larger the evaporation rate, the larger the absolute value of the hydraulic potential gradient, and thus also the h-gradient, the smaller the height over which the air-entry value will be reached, and thus the smaller the saturated zone

5.2. Steady downward flow.

Figure 16 shows the potential diagram for a field situation similar to figure 6. The water at the surface is applied by sprinklers and the groundwater table is stabilized by tile drains. The sprinkling rate is such that [illegible], the layer of water on the soil surface remains constant.

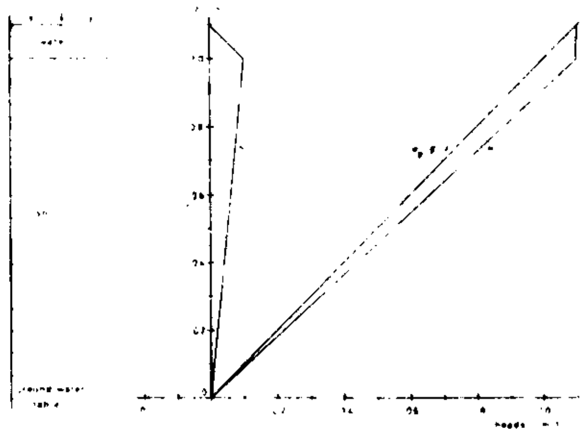


Figure 16. Steady downward flow of water at a high sprinkling rate (flooded soil).

P R O B L E M   1 0

Calculate the flux density in figure 16 if K = 10 cm d^{-1} and the water layer is 10 cm deep.

A N S W E R

$$q = -k\frac{dH}{dz} = -0.1\ \text{m d}^{-1} \times \frac{1.1}{1.0} = -0.11\ \text{m d}^{-1}$$

P R O B L E M 1 1

Calculate the sprinkling rate in the steady state situation of figure 16 if the depth of the water layer at the surface is negligible.

A N S W E R

In this situation h = 0 at the soil surface and in the whole profile, because h = 0 also at z = 0. This means that H = z and dH/dz = 1 m m^{-1}. The sprinkling rate then equals the value of the hydraulic conductivity, 10 cm d^{-1}.

If the sprinkling rate is decreased to another constant values, a new steady state situation arises, when the downward flux density still equals the sprinkling rate. This new situation is accompanied by a decrease in the H gradient, i.e. both H and h at the soil surface decrease. If the sprinkling rate falls below a certain critical value, the air-entry value of the soil will be exceeded and an unsaturated zone will develop in the top of the soil profile. Since K decreases with θ in the unsaturated zone and q is constant, the H gradient must increase with height, as shown in figure 17. Consequently, the h gradient must also increase with height, which means that the h curve becomes steeper towards the soil surface.

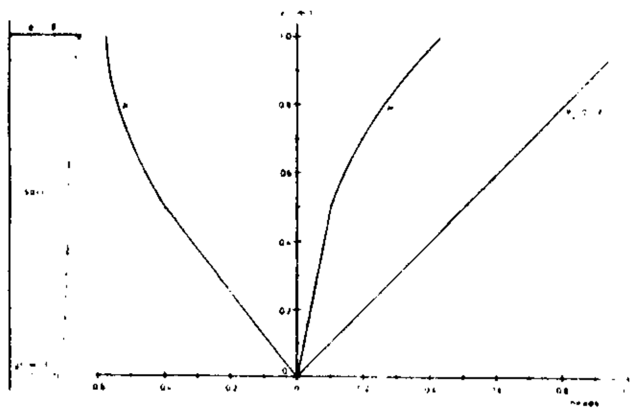


Figure 17. Steady downward flow of water at a low sprinkling rate.

P R O B L E M 1 2

Under what conditions are H and h linear functions of z throughout the profile?

A N S W E R

H and h are linear functions of z only when dH/dz = constant, i.e. when k is constant throughout the profile. Because the soil is saturated at the groundwater table, it should be saturated throughout the profile.

P R O B L E M 1 3

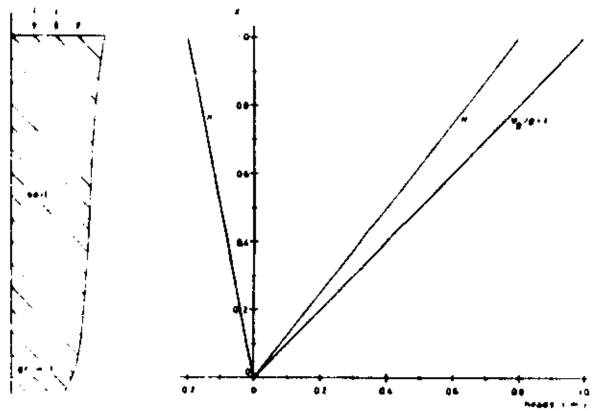
Assuming that the air-entry value is not exceeded, draw the diagram for the steady state situation with a sprinkling rate of 8 cm d^{-1}.

A N S W E R

A flux density of 8 cm d^{-1} corresponds to

$$dH/dz = - q/k = 0.8 \text{ m m}^{-1}$$

Hence, at the soil surface, H = 0.8 m and h = - 0.2 m.



PROBLEM 14

What is the highest possible H gradient that can be attained in a steady state downward flow as shown in figure 17.

ANSWER

In the unsaturated zone, where h is decreasing with height, k is also decreasing with height. Therefore, dH/dz must increase with height to satisfy the condition for steady state: $q = -K\frac{dH}{dz}$ = constant

Consequently, dh/dz must also increase with height, i.e. become less negative. Now suppose dh/dz would become positive at a certain height. Then both h and k would increase with height. To keep the flux constant, dH/dz would have to decrease with height, and thus also dh/dz. This is in conflict with the supposition made above. Thus, we can conclude that the highest possible value of dh/dz is zero. In this situation h has reached the constant value at which K has the same value as the imposed flux density, which then flows through the soil under the gravitational potential gradient of $1\ m\ m^{-1}$.

PROBLEM 15

a. Draw as logical as possible the potential diagram for steady downward flow with a sprinkling rate of $1\ cm\ d^{-1}$

b. Why is the phrase "as logical as possible" needed in a?

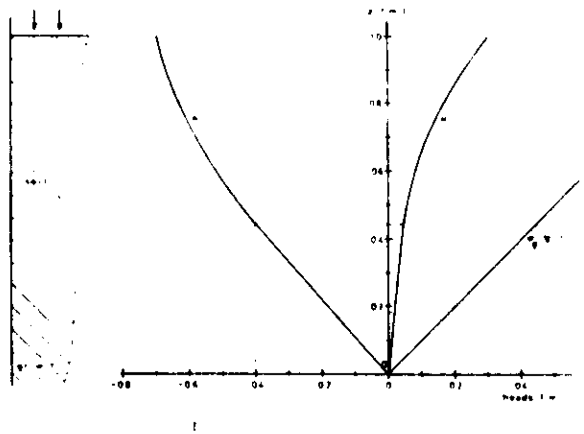
ANSWER

a. From $q = -1.0\ cm\ d^{-1}$ and $K = 10\ cm\ d^{-1}$ it follows:

$$\frac{dH}{dz} = -\frac{q}{K} = 0.1\ m\ m^{-1} \quad \text{and}$$

$$\frac{dh}{dz} = \frac{dH}{dz} - \frac{dz}{dz} = 0.1 - 1 = -0.9\ m\ m^{-1}$$

The air-entry value is h =-0.4 m. This value is reached at z = 0.44 m. Thus the soil is saturated below z = 0.44 m. Above z = 0.44 m the gradient dH/dz increases with height. but cannot exceed the value for which dh/dz = 0. With these considerations the potential diagram can be drawn.



b. Above z = 0.44 m the soil is unsaturated and θ as well as K vary. Since K is not given in the unsaturated zone, we only can guess the most probable shapes of the H-and h-profiles. Even if K were known, calculation of these profiles is complex because of the non-linear nature of the problem.

Figure 18 presents a summary of the h-profiles for the various cases of steady vertical flow of water discussed in this section. The numbers next to the curves are the values of the flux density. Upward flux densities (q > 0) represents evaporation and downward flux densities (q < 0) represents drainage. Zero flux density (q = 0) is the special case of hydrostatic equilibrium.

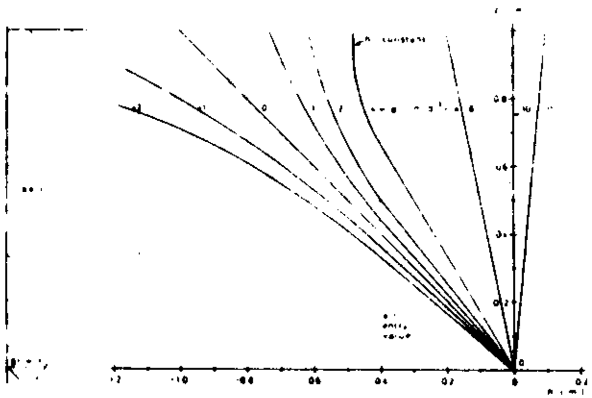


Figure 18. Summary of h-profiles for steady vertical flow of water with a stable groundwater table.

3. 6. Non-steady unsaturated flow of water.

3. 6.1. Introduction.

For steady state flow through a tube of uniform size (figure 19), the flux density is the same through every cross section of the tube. Consequently, water is not stored.

With non-steady or transient state flow, water is stored (or in some situations it is coming from storage) in the soil. Thus in transient state flow, the flux density entering the tube in figure 19 would not equal the flux density leaving the tube. The difference between that entering and that exiting is the storage. That is, the storage (which can be expressed as a change in volume water content with time, $\partial\theta_v/\partial t$) can be determined from the difference between inflow and outflow (which can be expressed as the change in flux density along the length of the tube $\delta Jw/\delta x$).

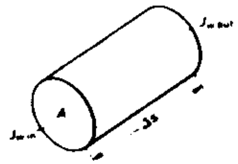


Figure 19. A tube of cross-sectional area A and length Δs is uniformly packed with a uniform soil. Water enters at a flux density J_{win} and leaves at a flux density J_{wout}

3. 6.2. Analysis of H-profiles.

In the field, water transport seldom is steady, because precipitation, evaporation, irrigation, drainage etc... change continually. Transport processes which change with time, are called non-steady. One can obtain insight into non-steady water flow processes in the field, qualitatively, by obtaining tensiometer data at different times and depths in the soil profile. With tensiometers one obtains information on the matric head h, as well as the hydraulic head $H = h + z$. From the matric head at different times one can derive whether the volume fraction of water is increasing or decreaseing. From the gradient of H one can deduce in which direction the water is moving, and possibly predict values of θ and directions of flow at future dates.

Which is the direction of flow if $\partial H/\partial z$ is positive, negative of zero?

Since z is defined positive upward, a positive flux density is directed upward and a negative flux density downward. Also $q = -k\ \partial H/\partial z$. Thus, when $\partial H/\partial z$ is positive, then q is negative, and thus directed downward. Similarly, when $\partial H/\partial z$ is negative, the flux density is directed upward. There is no flow when $\partial H/\partial z = 0$.

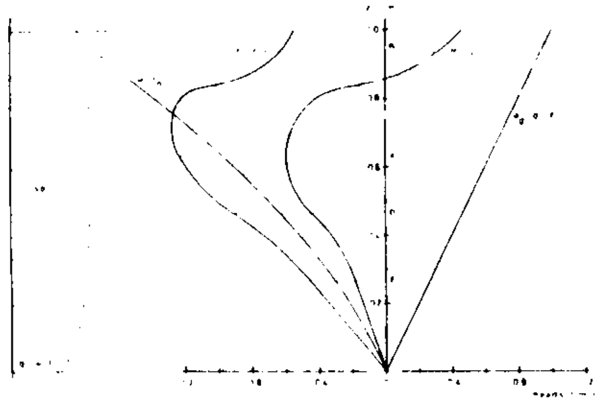


Figure 20. H- and h-profiles in a soil during steady evaporation (t_0) and some times after it started raining (t_1).

Figure 20 shows H-profiles at two different times. Initially at t_0, there is steady evaporation for which $\partial H/\partial z$ increases with height, since θ and K decrease with height. If then it starts to rain and water infiltrates into the soil profile, the H-profile may after some time have changed to that indicated by t_1. From this profile one can tell in which direction the water is moving, and whether $\partial\theta/\partial t > 0$ or $\partial\theta/\partial t < 0$. Below height A there is still upward movement, because $\partial H/\partial z < 0$. Above height A, the transport is now downward. At height A, there is no movement because $\partial H/\partial z = 0$.

P R O B L E M 1 6

Is there a water layer on top of the soil at time t_1?

A N S W E R

At the soil surface, $h < 0$. Thus, there can be no free water at the soil surface.

P R O B L E M 1 7

Explain, by using the continuity equation, whether at height A in figure 20 θ decreases, increases or remains after t_1.

A N S W E R

At some height h_1 above A the flux density q_1 is negative, because there $\partial H/\partial z$ is positive. At some height h_2 below A the flux density is positive, because $\partial H/\partial z$ is negative. Therefore, at height A,

$$\frac{\partial q}{\partial h} = \lim_{(h_1 \to A \text{ and } h_2 \to A)} \left(\frac{q_1 - q_2}{h_1 - h_2}\right) < 0,$$

because $(q_1 - q_2) < 0$ and $(h_1 - h_2) > 0$

Thus, at height A, $\frac{\partial\theta}{\partial t} = -\frac{\partial q}{\partial z} > 0$

This means that θ increases with time. Even without the above proof, it is easy to see that the water content at A must increase because water is moving towards A both from below and from above.

For height A it is easy to tell whether θ will increase or not, just by looking at the H-profile. For other heights it is not so easy to predict the changes in θ. Take for instance heights B and C in figure 20. At both heights the flux density is downward.

Since C is the inflection point of the H-profile above height A, the H-gradient at B is smaller than at C. However, θ is greater at B than at C, as can be interpreted from the h-profile. This means that K is greater at B than at C. Therefore, without knowing the exact numerical values of K and $\partial H/\partial z$, it cannot be concluded which height has the greater flux density. This reasoning is true for every height above C. Thus, it is not possible to conclude whether above C θ will increase or decrease, or even remain constant.

The same problem arises in the interval between heights D and E, where D is the inflection point of the H-profile below A, and E is the height where the H-profile becomes linear.

In contrast, between A and D, the absolute value of $\partial H/\partial z$ as well as K decrease with height, i.e. $\partial q/\partial z < 0$. Hence, $\partial\theta/\partial t$ at every height in this interval is positive. A similar reasoning can be given for the interval between C and A. Also in this region $\partial q/\partial z$ is negative. Therefore, $\partial\theta/\partial t$ will be positive for every height between A and C.

P R O B L E M 1 8

Suppose the weather becomes foggy for an extended period of time directly after the rain has stopped:

a. What will be the ultimate shape of the H-profile

b. Compare θ at different heights in this final stage with those at time t_1

A N S W E R

a. When there is fog, there is no evaporation. But, downward and upward water within the soil profile will continue until finally static equilibrium is attained. Since at static equilibrium $\partial H/\partial z = 0$, H is constant and zero throughout the soil profile, because $h = 0$ at $z = 0$.

b. At equilibrium, h is linear with height and equal to $-z$. A little above C, $H = 0$ at time t_1, and thus at that height h as this final stage will be the same as h at time t_1. Therefore the water content also will remain the same at that height. Above this point, h and θ will decrease compared to their values at t_1. Below this point, both h and θ will increase. At point E and below, the soil is saturated at time t_1. Although there h will increase with time, because the flux will vanish, the water content cannot increase any further.

By the preceding reasoning one can acquire only a qualitative understanding of the water flow processes in soil. Since in unsaturated soil K varies with θ, it is not possible to calculate q as a function of height directly from H-profiles as shown in figure 20. For this it is necessary to know, in addition, K-θ and θ-h relationships. The procedure that then should be followed to estimate $\partial\theta/\partial t$ is outlined in figure 21. In this figure, the subscript z indicates height, s stands for a small increment in height and θ is the average volume fraction of water.

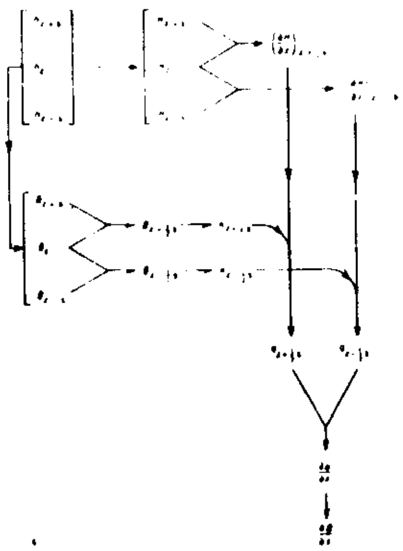


Figure 21. Calculation scheme for analysing a θ-profile.

In order to estimate $\partial\theta/\partial t$ at a height z in a soil profile, we will describe in words the steps followed in figure 21. By means of tensiometers, measure h at

By means of tensiometers, measure h at three different heights, i.e. at z, z + s and z - s. Add z to each to obtain H. Estimate $\partial H/\partial z$ at the heights z + 1/2 s and z - 1/2 s, assuming that H changes linearly with height in the intervals from z to z + s and from z - s to z. By means of a predetermined soil water characteristic curve, determine θ that corresponds to h at z, z + s and z - s. Assuming that θ changes linearly with height, find θ at the heights z + 1/2 s and z - 1/2 s. By means of a predetermined K-θ relationship, find the K-values at heights z + 1/2 s and z - 1/2 s. Now,

$$q_{z+1/2\,s} = -K_{z+1/2\,s}\left(\frac{\partial H}{\partial z}\right)_{z+1/2\,s} \quad \text{, and}$$

$$q_{z-1/2\,s} = -K_{z-1/2\,s}\left(\frac{\partial H}{\partial z}\right)_{z-1/2\,s}$$

Finally, a good estimate of $(\partial\theta/\partial t)_z$ is:

$$\left(\frac{\partial\theta}{\partial t}\right)_z = \frac{q_{z+1/2\,s} - q_{z-1/2\,s}}{(z + 1/2\,s) - (z - 1/2\,s)}$$

The procedure outlined in figure 21 makes use of average valies of θ, and not of hydraulic conductivity or pressure head. This amounts to assuming that θ changes linearly with height. This assumption is usually not far from reality and introduces small errors in ∂θ/∂t. An alternative approach would be to average the K-values for each of the pertinent θ values. This amounts to assuming that K changes linearly with θ, which obviously introduces much more serious errors in ∂θ/∂t because of the strong non-linear K-θ relationships. An intermediate approach will be to use average h-values.

P R O B L E M 1 9

In a medium fine sandy soil, for which the θ-h and K-θ relationships are given in figure 5 and 6, three tensiometers are installed at depths of 0.3, 0.4 and 0.5 m below the surface. At a certain moment the tensiometers indicate h-values of - 2.0, - 0.8 and - 0.4 m respectively. Estimate ∂θ/∂t at a depth of 0.4 m using the scheme given in figure 21. Hysteresis may be neglected.

A N S W E R

By reading the appropriate values from figure 13, the following scheme can be set up:

depth m	z m	h m	H m	θ	$\bar{\theta}$	K mm d^{-1}	$\frac{\partial H}{\partial z}$ m m^{-1}	q mm d^{-1}	$\frac{\partial\theta}{\partial t}$ d^{-1}
0.3	0.2	-2.0	-1.8	0.09					
					0.14	0.19	-11	2.1	
0.4	0.1	-0.8	-0.7	0.19					0.105
					0.24	4.20	-3	12.6	
0.5	0.0	-0.4	-0.4	0.295					

P R O B L E M 2 0

a. Also estimate ∂θ/∂t at a depth of 0.4 m using average h values. Do the same using average K values.

b. Compare the values for K and ∂θ/∂t obtained by the three calculation schemes.

A N S W E R

a. Using average h values the calculation scheme is:

depth m	z m	h m	H m	$\bar{h}$ m	θ	K mm d^{-1}	$\frac{\partial H}{\partial z}$ m m^{-1}	q mm d^{-1}	$\frac{\partial\theta}{\partial t}$ d^{-1}
0.3	0.2	-2.0	-1.8						
				-1.4	0.11	0.08	-11	0.88	
0.4	0.1	-0.8	-0.7						0.117
				-0.6	0.24	4.2	-3	12.6	
0.5	0.0	-0.4	-0.4						

Using average K values:

depth m	z m	h m	H m	θ	K mm d^{-1}	$\bar{K}$ mm d^{-1}	$\frac{\partial H}{\partial z}$ m m^{-1}	q mm d^{-1}	$\frac{\partial \theta}{\partial t}$ d^{-1}
0.3	0.2	-2.0	-1.8	0.09	0.03				
						0.39	-11	4.29	
0.4	0.1	-0.8	-0.7	0.19	0.75				0.57
						20.4	-3	61.2	
0.5	0.0	-0.4	-0.4	0.295	40.0				

b. The results in the previous problem show that the θ values derived from the measured h values vary nearly linearly with depth. We will, therefore, consider these results as standard against which the other two calculation schemes can be evaluated.

A comparison of the results of the first two schemes shows that using average values of h leads to an underestimation of K for the drier soil. The error in the final value of $\partial\theta/\partial t$ is about 10 %. The third scheme schows that using average K values leads to a value of $\partial\theta/\partial t$ which is almost six times too large. This illustrates the extreme non-linear character of unsaturated water transport.

P R O B L E M 2 1

In a loamy soil profile nine tensiometers are installed at depths of 0, 0.1, 0.2 ... and 0.8 m below the soil surface. Estimate $\partial\theta/\partial t$ at depths 0.1 ... and 0.7 m for the moment that the tensiometers indicate h values of 0, - 0.3, - 0.8, - 1.1, - 1.6, - 1.0, - 0.5, 0.2 and 0 m, respectively,. Use the θ-h and K-θ relationships in figure 13. Hysteresis may be neglected.

A N S W E R

The calculation scheme is as follows:

depth m	z m	h m	H m	θ	$\bar{\theta}$	K mm d^{-1}	$\frac{\partial H}{\partial z}$ m m^{-1}	q mm d^{-1}	$\frac{\partial \theta}{\partial t}$ d^{-1}
0.0	0.8	0.0	0.8	0.50					
					0.49	50	4.0	-200	
0.1	0.7	-0.3	0.4	0.48					1.10
					0.46	15	6.0	-90	
0.2	0.6	-0.8	-0.2	0.44					0.68
					0.42	5.5	4.0	-22	
0.3	0.5	-1.1	-0.6	0.4					0.10
					0.36	2.0	6.0	-12	
0.4	0.4	-1.6	-1.2	0.32					0.24
					0.37	2.4	-5.0	12	
0.5	0.3	-1.0	-0.7	0.42					0.24
					0.445	9.0	-4.0	36	
0.6	0.2	-0.5	-0.3	0.47					0.04
					0.475	20	-2.0	40	
0.7	0.1	-0.2	-0.1	0.48					0.10
					0.49	50	-1.0	50	
0.8	0.0	0.0	0.0	0.50					

3.6.3. Infiltration theory.

A typical example of non-steady unsaturated flow of water is the infiltration of water into soils. When water is applied at the soil surface (e.g. under flood irrigation or inundation), it enters the soil profile and changes the water content distribution with depth, also called water content profile. After irrigation has continued for some time, the following zones can usually be distinguished in the water content profile (figure 22):

- saturated zone
- transition zone
- transmission zone
- wetting zone
- wetting front

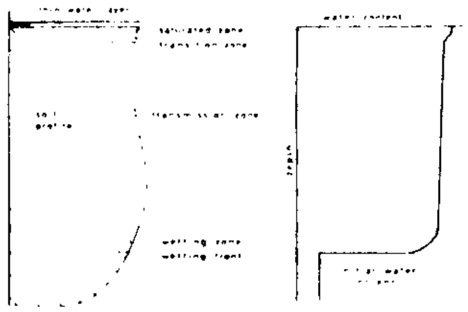


Figure 22. Zones of water content profiles some time after start of irrigation.

The saturated zone is a thin zone at the soil surface. The transition zone is a zone of decreasing water content between the saturated zone and the nearly saturated transmission zone. Its lower end may reach from a few millimeters to a few centimeters below the surface. These two upper zones are not always clearly distinguishable, especially in the laboratory. They are caused by structural changes at the soil surface and the entrapped air.

The transmission zone is the conveyance zone for the infiltrating water. While all the other zones remain clearly constant in thickness, this zone continues to elongate as long as water is supplied at the soil surface. Its water content, though slightly changing with depth, is rather constant and close to saturation.

The wetting zone is the normally thin zone where the water content changes from its initial value to the value of the transmission zone.

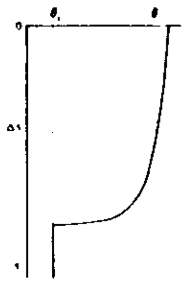
The wetting front is the visible limit of water penetration, where the gradient of the pressure head is very large.

The cumulative infiltration, I, which is the volume of water that has infiltrated into the soil divided by the surface area, is

$$I = \int_{s=0}^{\infty} (\theta - \theta_i)\, ds \qquad (9)$$

where s is the distance in the direction of flow (here depth in the soil profile), θ is the volume fraction of water at distance s and θ_i is the initial uniform value of θ. The cumulative infiltration is, of course, an increasing function of time.

A column of soil with surface area A can be divided into n slices, each with thickness Δs. The volume of water absorbed by each slice is then $A(\theta - \theta_i)\,\Delta s$, where θ is the average volume fraction of water of the slice.



The volume of water absorbed by the whole column then equals:

$$Q = A \sum_n (\theta - \theta_i)\,\Delta s$$

From $\Delta s \to 0$ this can be written as the integral:

$$Q = A \int_{s=0}^{\infty} (\theta - \theta_i)\, ds$$

The volume divided by the surface area is then:

$$I = \frac{Q}{A} = \int_{s=0}^{\infty} (\theta - \theta_i)\, ds$$

(whereas θ_i has been assumed constant, it could also vary with s, without changing equation (9). The integral would only get another value).

The driving forces for the water entering the soil are the gradient of the pressure head between the wetting front and the soil surface, and gravity. While the latter is constant, the gradient of the pressure head decreases with time, because of the advancing wetting front. As a result, the flux density through the soil surface, also called the infiltration rate i, decreases monotonically with time and approaches asymptotically a constant value, as gravity becomes the main driving force (figure 23).

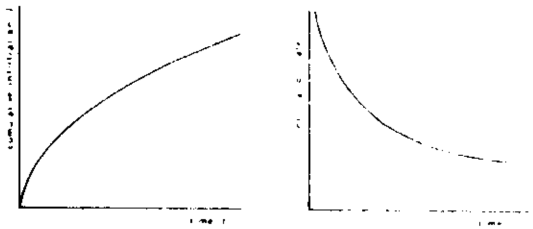


Figure 23. Cumulative infiltration I and infiltration rate i as function of time.

The infiltration rate may be expressed as:

$$i = \frac{dI}{dt}$$

Suppose at time t, the cumulative infiltration is I and at time t + Δt, I + ΔI. The mean flux density during the time interval Δt is then the increase of the cumulative infiltration ΔI, divided by Δt, i.e. $\frac{\Delta I}{\Delta t}$.

The flux density (= infiltration rate) at time t can [illegible] by letting Δt approach zero:

$$i = \lim_{\Delta t \to 0} \left(\frac{\Delta I}{\Delta t}\right) = \frac{dI}{dt}$$

Integration of equation (10) gives the cumulative infiltration as a function of time

$$I = \int_0^t i\,dt \qquad (11)$$

Equation 11 can be derived from equation (10) as follows: - equation (10) may be written as: dI = idt

- integration with boundary condition I = 0 for t = 0

gives: $$I = \int_0^t i\,dt$$

The cumulative infiltration as a function of time may be measured in the field, but such an experiment does not give any information about the water content distribution or the depth of the wetting front. To obtain the volume fraction of water as a function of depth and time, θ(s,t), a general flow equation must be solved, using appropriate intial and boundary conditions.

3.6.4. General flow equation.

3.6.4.1. Horizontal infiltration.

3.6.4.1.1. Flow equations

Darcy's law, through originally conceived for saturated flow only was extended to unsaturated flow, with the provision that the conductivity is now a function of the matric suction head (i.e. $K = K_{(h)}$):

$$q = - K_{(h)}\, \nabla H$$

where ∇H is the hydraulic head gradient, which may include both suction and gravitational components.

As pointed out in literature, this formulation fails to take into account the hysteresis of soil-water characteristics. In practice, the hysteresis problem can sometimes be evaded by limiting the use of Eq. 1-5 in 3.3 to cases in which the suction(or wetness) change is monotonic - that is, either increasing or decreasing continuously. In processes involving both wetting and drying phases, Eq. 1 in 3.3 is difficult to apply, as the $K_{(h)}$ function may be highly hysteretic.

The relation of conductivity to volumetric wetness $K(\theta)$ or the degree of saturation $K(W_s)$ is affected by hysteresis to a much lesser degree than is the $K_{(h)}$ function, at least in the media thus far examined.

Thus, Darcy's law for unsaturated soil can also be written as :

$$q = - K_{(\theta)} . \nabla H \qquad (13)$$

which, however, still leaves us with the problem of dealing with the hysteresis between h and θ.
To obtain the general flow equation and account for transient as well as steady flow processes, we must introduce the continuity equation :

$$\frac{\partial \theta}{\partial t} = - \frac{\partial V}{\partial x} \qquad (14)$$

where :
V = flux (cm/sec),
θ = volumetric moisture content (cm^3/cm^3),
t = time (sec),
x = distance (cm).

Thus :

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} (K_{(\theta)} . \frac{\partial H}{\partial x}) \qquad (15)$$

Remembering that the hydraulic head H is, in general, the sum of the pressure head or its negative, the suction head, h, and the gravitational head (or elevation) z,

$$H = h + z \qquad (16)$$

we can write :

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} (k_{(\theta)} \frac{\partial h}{\partial x} + k_{(\theta)} \frac{\partial z}{\partial x}) \qquad (17)$$

The two dependent variables of this equation, θ and h are connected by a "formal" relationship :

$$C_{(\theta)} \equiv \frac{d\theta}{dh} \quad (cm^{-1}) \qquad (18)$$

in which $C_{(\theta)}$ is termed the differential capacity of the medium.

Introducing (18) into (17) gives :

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} (\frac{k_{(\theta)}}{C_{(\theta)}} . \frac{\partial \theta}{\partial x} + k_{(\theta)} \frac{\partial z}{\partial x}) \qquad (19)$$

This equation can further be simplified by the introduction of the soil water diffusivity D, (also a function of the water content θ, defined as :

$$D_{(\theta)} = \frac{k_{(\theta)}}{C_{(\theta)}} \equiv k_{(\theta)} \frac{dh}{d\theta} \quad (cm^2.sek^{-1}) \qquad (20)$$

Equation (19) then becomes :

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} (D_{(\theta)} \frac{\partial \theta}{\partial x} + k_{(\theta)} \frac{\partial z}{\partial x}) \qquad (21)$$

For horizontal flow, vertical flow upward, and vertical flow downward, the value of $\partial z/\partial x$ is 0, 1 and -1 respectively.

For horizontal flow eq. (21) becomes :

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} (D_{(\theta)} \frac{\partial \theta}{\partial x}) \qquad (22)$$

For vertical flow eq. (21) becomes :

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x} \left(D_{(\theta)} \frac{\partial\theta}{\partial x} - k_{(\theta)}\right) \qquad (23)$$

In these equations we see that the diffusivity D is concentration-dependent (Θ).

The major advantage of the diffusion equation is that concentration gradients are more tractable than potential gradients. This fact in part explains recent popularity of the diffusion equation as a mathematical description of the flow process in unsaturated soils. As a result, several soil physicists have actively sought solutions for the diffusion equation.

In defining D it is assumed that K, h and dh/dΘ are unique functions of Θ. This cannot be strictly true because K, h and dh/dΘ will depend not only on Θ but on how the sample has been wetted -its past wetting history. Other nonuniqueness factors may be trapped air, variations with time of the air-water-solid contact angle, heat released during wetting, and (on vertical infiltration) pressure changes due to depth of overburden.

Thus we see the value of D at a given moisture content for a drying soil may be different than that for a wetting soil and that D may even be different for different initial moisture contents of the same drying soil. This hysteresis in D and K seems to parallel the hysteresis of moisture-tension curves. A given tension on the soil water can result in two different soil moisture contents, depending on whether it is wetting or drying or even on its initial moisture content.

As we shall see in the next chapter , we can assume that D is a unique function of Θ and we can obtain soil moisture curves which lie close to experimental ones. With uniqueness assumed, probably the easiest interpretation of D is seen in the equation :

$$\underset{(1)}{\frac{\partial\Theta}{\partial t}} = \frac{\partial}{\partial x} \left(D_{(\Theta)} \cdot \underset{(2)}{\frac{\partial\Theta}{\partial x}}\right)$$

which says that D is a measure of moisture flow (1) under a moisture gradient (2).

The preceding paragraph indicates that D has physical significance. Nevertheless, D must be used with care. For example, in defining D as a factor of proportionality for flow under a moisture gradient, it is tacitly assumed that the porous medium must be homogeneous. That such an assumption is necessary is seen by considering quantities of moist sand and clay in contact in a horizontal tube. The sand and clay could be at the same moisture tension everywhere in the tube, but whould have different moisture contents. Yet, at the clay-sand interface, where there would be a non zero moisture gradient, there would be zero flow. We must not forget that it is basically the gradient ∂/∂x that drives (or pulls) the water in a horizontal soil tube, not ∂Θ/∂x, and that the reason we introduce ∂Θ/∂x and $D_{(\Theta)}$ into the theory is to help solve problems.

3.6.4.1.2. Measurement of diffusivity $D_{(\Theta)}$ (laboratory).

In the derivation of equations (22)and (23) we assumed that D is a unique function of the moisture content Θ of the soil. Equations (22) and (23) have been solved by many authors. Most of the techniques used in solving equations (22) and (23) require values of D for various moisture contents or at least the functional relationship between the diffusivity D and the moisture content Θ of the soil.

$D_{(\Theta)}$ is calculated from a moisture distribution curve plotted from data obtained from the addition of water to horizontal soil columns.

Equation (22) is applied to the horizontal advance of moisture into a horizontal homogeneous long tube of soil to obtain the relation between $D_{(\theta)}$ and the moisture content. The test soils were initially either air-dry or partially moist.

We rewrite the differential equation (22) as :

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left(D_{(\theta)}\frac{\partial\theta}{\partial x}\right)$$

where θ is the moisture content in $cm^3.cm^{-3}$, at a horizontal distance x from an input end, where water at zero head with respect to the level of the tube is applied as fast as the soil will absorb it. Neither the gravitational constant g nor a vertical coordinate z enter in the problem because the soil tube is of small diameter and is horizontal. There are certain other basic physical points that we should remember about equation (22) before solving it mathematically for the diffusivity D.

Equation (22) stems from :

a. Darcy's law.

b. Equation of continuity.

c. the head expression.

d. the diffusivity definition.

The diffusivity in turn is depending on the moisture content θ being a unique function of the tension head h.

The equation of continuity involves differentiations and hence implies that as we pass from volume element to volume element in the soil tube, the moisture movement conditions should vary smoothly. Thus the pore size distribution of the soil particles from volume element to volume element should accordingly also be the same. By volume element we mean a segment of the soil Δx cm long and cross section equal to the inside of the tube, where Δx is say 10 to 100 times the diameter of an average size soil particle or pore. If these conditions are not met, we should not expect equation (22) to be valid. We assume that the conditions are met and we proceed.

With θi and θs being moisture contents, we recognize the initial condition and boundary condition for the moisture flow in the horizontal tube to be :

$$(1)\quad \theta(x,t) = \theta i \quad \text{for } x > 0,\ t = 0 \qquad (24)$$

$$(2)\quad \theta(x,t) = \theta s \quad \text{for } x = 0,\ t > 0 \qquad (25)$$

Condition (1) says that initially our soil column has, for all values of $x > 0$, a constant moisture content θi. Condition (2) implies that we are to apply water at the input end $x = 0$ at time $t = 0$, and at all times $t > 0$; and that we are to apply the water in such a way as to maintain the soil in a very thin layer at the input end of the column, at the constant moisture content $\theta = \theta s$, for all $t \geq 0$. θs is taken as the saturation moisture content.

Equation (22) is a nonlinear partial differential equation and cannot be solved by usual methods.

The Boltzmann transformation is used to obtain equation (22) in the form of an ordinary differential equation.

To make the transformation we let θ be given by :

$$\theta = f(\lambda) \qquad (26)$$

where λ is a function of x and t, defined by :

$$\lambda = x\,t^{-1/2} \qquad (27)$$

Equation (27) is called the Boltzmann transformation.
In view of equation (27), we see that conditions (24) and (25) may be written as :

$$(1)\quad \theta = \theta i \quad \text{for } \lambda = \infty \quad (\lambda \to \infty) \qquad (28)$$

$$(2)\quad \theta = \theta s \quad \text{for } \lambda = 0 \qquad (29)$$

We shall need a third condition. Since the derivation of equation (22) has depended on the assumptions that θ must vary smoothly (be continuous and differentiable) with x and t, we see from equations (26) and (27) that θ must also vary smoothly with λ . Therefore the conditions (28) and (29) imply the further condition :

$$(3)\quad d\theta/d\lambda = 0 \quad \text{for } \theta = \theta i \qquad (30)$$

To help clarify conditions (28), (29) and (30), we present a figure (Figure24).

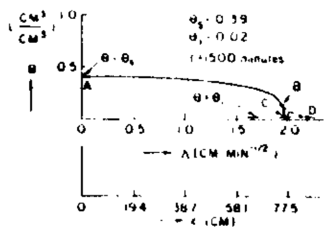


Figure 24 : The curve of θ versus λ for all θi versus x at t = 1.500 min. calculated from equation (21) using the data from Table 1.

Variation of water content θ with distance x for a horizontal soil column of a Hesperia Sandy Loam soil, for a time t = 1500 minutes after water was applied at the input end x = 0 (from Nielsen et al, 1962)

θ	x (cm)	θ	x (cm)	θ	x (cm)
0.02	76.00	0.15	75.25	0.28	69.90
0.03	75.90	0.16	75.18	0.29	68.05
0.04	75.85	0.17	75.05	0.30	66.05
0.05	75.75	0.18	74.90	0.31	63.75
0.06	75.71	0.19	74.75	0.32	61.10
0.07	75.67	0.20	74.65	0.33	58.15
0.08	75.63	0.21	74.45	0.34	54.80
0.09	75.58	0.22	74.15	0.35	51.20
0.10	75.50	0.23	73.80	0.36	47.10
0.11	75.45	0.24	73.40	0.37	41.80
0.12	75.40	0.25	72.80	0.38	32.50
0.13	75.35	0.26	72.08	0.39	0
0.14	75.30	0.27	71.20		

Figure 24 shows an experimentally obtained graph ABC of θ versus λ for water moving into a tube of initially air dry soil where the moisture content θ for graphing the ... was measured at a number of positions along the soil ... at the instant t = 1,500 min. For the graph, we have θ ... the vertical axis and we show two horizontal axes, λ and x related by the Boltzmann transformation $\lambda = x.(1500)^{-1/2}$ which gives x = 38.73λ, as shown.

The curve of θ versus λ (or versus x) is shown in two ways, as ABC and ABD. The end portion BD is theoretical and not observed experimentally. Theoretically, the curve should be ABD with the point D being at λ = ∞ or x = ∞. We see from the figure that for λ → ∞ we have θ = θi as in condition (28). We also see that condition (29), agrees with the figure. Therefore, if we can integrate the differential equation of the shown θ versus λ curve, our problem will be solved. We proceed to obtain the differential equation of the curve and to integrate it.

In view of (27) we may write (28) as :

$$\theta = f\left[\lambda\ (x,t)\right] \tag{31}$$

If we place (31) in (22) we can find an expression for $D_{(\theta)}$. If we use the chain rule for the differentiation of composite functions we find $\partial\theta/\partial t$ as :

$$\frac{\partial\theta}{\partial t} = \frac{\partial\theta}{\partial\lambda} \cdot \frac{\partial\lambda}{\partial t} \tag{32}$$

and $\partial\theta/\partial x$ as :

$$\frac{\partial\theta}{\partial x} = \frac{\partial\theta}{\partial\lambda} \cdot \frac{\partial\lambda}{\partial x} \tag{33}$$

and $\frac{\partial}{\partial x}\ (D_{(\theta)}\frac{d\theta}{\partial x})$ as :

$$\frac{\partial\ (D\ \frac{\partial\theta}{\partial x})}{\partial x} = \frac{\partial\ (D\ \frac{\partial\theta}{\partial\lambda} \cdot \frac{\partial\lambda}{\partial x})}{\partial\lambda} \cdot \frac{\partial\lambda}{\partial x} \tag{34}$$

The total derivatives in (32), (33) and (34) arise because θ is a function of the single variable λ. The variable λ is composed of x and t, which explains the use of partials of λ with respect to x or t, that is $\frac{\partial\lambda}{\partial t}$ and $\frac{\partial\lambda}{\partial x}$.

We can simplify (32), (33) and (34) if we use (27), nl. $\lambda = x.t^{-1/2}$, to find that $\partial\lambda/\partial t$ and $\partial\lambda/\partial x$ can be written as :

$$\frac{\partial\lambda}{\partial t} = -\ 1/2\ x.t^{-3/2} = -\ 1/2\ \lambda.t^{-1} \tag{35}$$

and :

$$\frac{\partial\lambda}{\partial x} = t^{-1/2} \tag{36}$$

On using (35) and (36) in (32) to (34) we have :

$$\frac{\partial\theta}{\partial t} = \frac{\partial\theta}{\partial\lambda} \cdot (-\ \frac{1}{2}\ \frac{\lambda}{t}) \tag{37}$$

and

$$\frac{\partial\theta}{\partial x} = \frac{\partial\theta}{\partial\lambda} \cdot t^{-1/2} \tag{38}$$

and

$$\frac{\partial\ (D\ \frac{\partial\theta}{\partial x})}{\partial x} = \frac{\partial (D\ \frac{\partial\theta}{\partial\lambda}\ .\ t^{-1/2})}{\partial\lambda} \cdot t^{-1/2} \tag{39}$$

and placing (37), (38) and (39) in equation (22), we have :

$$\frac{d\theta}{d\lambda}\ (-\ \frac{\lambda}{2t}) = \frac{d\ (D\ \frac{d\theta}{d\lambda}\ .\ t^{-1/2})}{d\lambda} \cdot t^{-1/2} \tag{40}$$

Equation (40) is then simplified to :

$$-\ \frac{\lambda}{2}\ \frac{d\theta}{d\lambda} = \frac{d\ (D\ \frac{d\theta}{d\lambda})}{d\lambda} \tag{41}$$

On multiplying (41) by $d\lambda$ and integrating both sides from $\theta = \theta i$ to $\theta = \theta x$ (θx being θ at the distance x along the column) we have :

$$-\ \frac{1}{2}\int_{\theta_i}^{\theta_x} \lambda\ d\theta = \int_{\theta_i}^{\theta_x} d\ (D\ \frac{d\theta}{d\lambda}) \tag{42}$$

or

$$-\ \frac{1}{2}\int_{\theta_i}^{\theta_x} \lambda\ d\theta = \left[D_{(\theta_x)}\ (\frac{d\theta}{d\lambda})\right]_{\theta x} - \left[D_{(\theta_i)}\ (\frac{d\theta}{d\lambda})\right]_{\theta i} \tag{43}$$

where $(\frac{d\theta}{d\lambda})_{\theta x}$ and $(\frac{d\theta}{d\lambda})_{\theta i}$ mean $\frac{d\theta}{d\lambda}$ evaluated at $\theta = \theta x$ and θi respectively.

The last term in (43) is zero by (30), nl. $\frac{d\theta}{d\lambda} = 0$ for $\theta = \theta i$.

Therefore (43) becomes :

$$-\frac{1}{2}\int_{\theta_i}^{\theta_x} \lambda \, d\theta = D_{(\theta_x)} \left(\frac{d\theta}{d\lambda}\right)_{\theta_x} \qquad (44)$$

Devision of (44) by $(d\theta/d\lambda)_{\theta x}$ and rearranging yields :

$$D_{(\theta_x)} = \frac{1}{\left(\frac{d\theta}{d\lambda}\right)_{\theta_x}} \left(-\frac{1}{2}\right) \int_{\theta_i}^{\theta_x} \lambda \, d\theta \qquad (45)$$

Since the lower limit in the integral is θi, the equation implies that, if we apply the equation to experimental data, we must integrate completely out to the end of the infinitely long tail at point D in figure24, that is, out to $\lambda = x.t^{-1/2} = (\infty).(t^{-1/2})$, where we put $x = \infty$, because λ must go to infinity for any finite value of t. Thus, the soil tube should theoretically be infinitely long. The Boltzmann transformation is not valid for tubes of length x = L finite. Practically speaking, contribution to the integral will be negligible except at the early part of the tail BD, so a finite length of tube can be used.

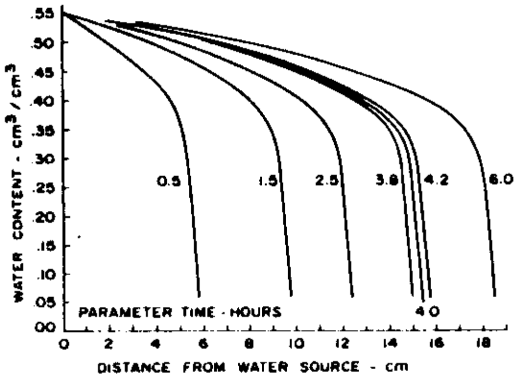
The right side of (45) can be evaluated from experimental data by :

a) Plotting θ versus λ $(=x.t^{-1/2})$ for the water as it enters the soil.

b) Measuring $(d\theta/d\lambda)_{\theta x}$ from the θ versus λ curve.

c) Evaluating the integral $\int_{\theta_i}^{\theta_x} \lambda d\theta$ by an approximate method.

Water content distributions, measured for a Columbia silt loam, using the above method are given in figure 25.



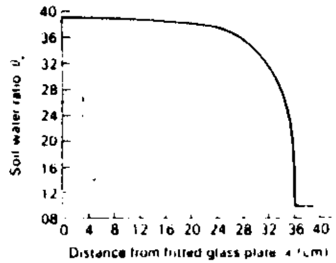


Figure 25: Soil-water distribution curves for Columbia silt loam for boundary conditions (24) and (25).

Figure 26 shows plots of x versus $t^{1/2}$ for various water contents monitored using gamma-ray attenuation. For a given soil-water content, these plots should be straight lines with their slopes being values of $\lambda(\theta)$.

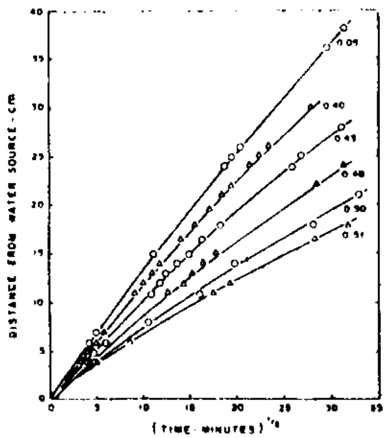
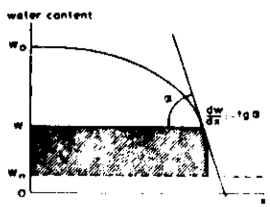


Figure 26 : Distance to the position in the soil column from the water source (x = 0) where the water content is the value indicated on the curves as a function of the square root of time.

The first step (a) is to plot a curve of θ versus λ as we did in figure 24. Step (b) may be done by drawing tangents by eye from the curve (Figure 25).



or it can be done semianalytically from the raw data used in getting the curve. The approximate method for step (c) consists in dividing the area under the curve of θ versus λ into a

finite number of strips and adding up the approximate areas of these strips. The strips are not vertical as we take them in elementary calculus but are horizontal as shown in figure 26 where four strips of widths Δθ and approximate lengths λ_1, λ_2, λ_3 and λ_4 are used to approximate the integral

Figure 26 : Values of θ and λ at different values of x for t = constant.

Whether the strips are vertical or horizontal makes no difference in obtaining the area if it is the whole area under the curve.

The wetting front is at $\lambda = \lambda_0$. From the figure, it is seen that the area under the curve up to the line $\theta = \theta x$ may be approximated by :

$$\int_{\theta_i}^{\theta_x} \lambda \, d\theta = \lambda_0 \, \Delta\theta + \lambda_1 \, \Delta\theta + \lambda_2 \, \Delta\theta + \lambda_3 \, \Delta\theta \qquad (46)$$

In figure 26 we do not need to take the Δθ's all equal. In general the sum formula for the integral, instead of (46), would be :

$$\int_{\theta_i}^{\theta_x} \lambda \, d\theta = \sum_{r=1}^{m} \lambda_r \, \Delta\theta_r \qquad (47)$$

To obtain a working formula to compute $D_{(\theta)}$ versus θ we may now put (47) in (45) and obtain :

$$D_{(\theta_x)} = \frac{1}{\left(\frac{d\theta}{d\lambda}\right)_{\theta_x}} \left(-\frac{1}{2}\right) \sum_{r=1}^{m} \lambda_r \, \Delta\theta_r \qquad (48)$$

which implies that we might obtain graphically by a ruler and the eye the slope $(d\theta/d\lambda)$ from the curve of θ versus λ corresponding to the point $x (= t^{1/2}\lambda)$. However, a semianalytical procedure may be preferred. In figure26, at $\theta x = \theta m = \theta 4$ we see from the geometrie that we may write as an approximation :

$$\left(\frac{d\theta}{d\lambda}\right)_{m=4} = \left(\frac{d\theta}{d\lambda}\right)_{\theta_x} = \frac{\Delta\theta}{\lambda_4 - \lambda_3} = \frac{t^{1/2}\,\Delta\theta}{x_4 - x_3}$$

where in the last equality, because t is constant (= 1,500 min. in our example) we may use equation (27) $(\lambda = x.t^{-1/2})$ to find :

$$\left(\frac{d\theta}{d\lambda}\right)_{\theta_x} = \frac{t^{1/2}\,\Delta\theta}{x_4 - x_3}$$

If m is not equal to 4 but is any integer m, the last equation may be written as :

$$\left(\frac{d\theta}{d\lambda}\right)_{\theta x} = \left(\frac{\Delta\theta}{\Delta\lambda}\right)_m = \frac{t^{1/2}\,\Delta\theta m}{x_m - x_{m-1}} = \frac{t^{1/2}\,\Delta\theta m}{\Delta x_m}, \text{ met } m = 1, 2, 3, \ldots \qquad (49)$$

In (49) Δ_{xm} is negative because we measure $\lambda_m (= x_m . t^{-1/2})$ from right to left in figure .

From the fir[illegible] and last members of (49) we have [illegible] :

$$\left(\frac{d\theta}{d\lambda}\right)_{\theta x} = \frac{t^{1/2}\,\Delta\theta_m}{\Delta x_m} \qquad (50)$$

In (48) we replace $(d\theta/d\lambda)_{\theta x}$ by the right of (50) and obtain :

$$D_{(\theta_x)} = \frac{1}{t^{1/2}\frac{\Delta\theta_m}{\Delta x_m}} \left(-\sum_{r=1}^{m} \frac{1}{2} \lambda_r \, \Delta\theta_r\right) \qquad (51)$$

where $D_{(\theta x)}$ is positive because Δ_{xm} is negative, and where we may cancel $\Delta\theta_r$ and $\Delta\theta_m$ if the $\Delta\theta$'s are all equal. In (51) it appears that $D_{(\theta x)}$ is a function of x. Actually $D_{(\theta x)}$ is a function of θ. The subscript x on θ_x denotes the location x where θ was measured at a certain time t say $t = t_1$. If some other time $t = t_2$ had been chosen to determine values of θ at points x along the tube, the same curve of $D_{(\theta)}$ versus λ should be obtained. That is, the curves of θ versus λ for time t_1 and t_2 should superpose.

The values of θ can be obtained by sectioning the soil colums and weighing the soil in the sections (Figure 27). [illegible] a gamma-ray apparatus is used, both to check on the uniformity of packing of the soils in the soil columns and to measure the moisture content at points along the tube.

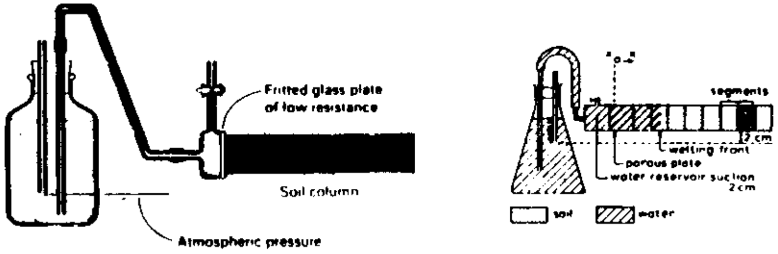


Figure 27 : Sketch of apparatus for obtaining water distribution curves. At time zero the soil column is brought in contact with the fritted glass plate and water enters the soil. The weight pressure potential of the water as it enters the soil column is about -2cm. The initial water ratio of the soil in the column is uniform at θi : the supply tube keeps the water ratio of the soil at the inflow boundary constant at saturation, θs. At a particular time, the soil is cut into sections at the dotted lines, and the water ratio of each section is determined.

The relation of diffusivity D to the moisture content θ is shown in figure 28 . This relation is sometimes expressed in the empirical equation :

$$D_{(\theta)} = \alpha . e^{\beta\theta} \qquad (52)$$

This equation applies only to sections of the curve showing a rise in diffusivity with moisture content. In the very dry range, the diffusivity often indicates an opposite trend, namely, a rise with decreasing soil moisture content. This is apparently due to the contribution of vapor movement. In the very wet range, as the soil approaches complete saturation, the diffusivity becomes indeterminate as it tends to infinity (since $C_{(\theta)}$ tends to zero).

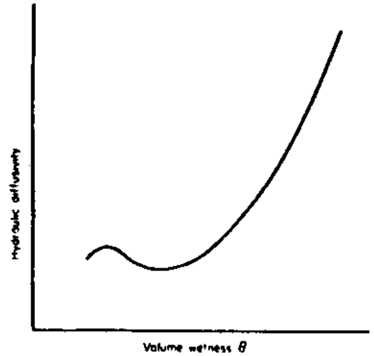


Figure 28 : Relation of diffusivity to soil moisture content.

Soil-water diffusivity is somewhat difficult to visualize physically, but mathematically it is simply the product of the hydraulic conductivity at a given water content and the reciprocal of the slope of the soil-water characteristic curve at that same water content.

$$\text{Hence, } D_{(\theta)} = -k_{(\theta)} . \frac{dh}{d\theta} \qquad (53)$$

When the soil water diffusivity function has been determined, the unsaturated hydraulic conductivity can easily be calculated from equation (53) :

$$k_{(\theta)} = -D_{(\theta)} . \frac{d\theta}{dh} \qquad (54)$$

Resulting curves for $k_{(\theta)}$ and $D_{(\theta)}$ as functions of θ are given in figure 26.

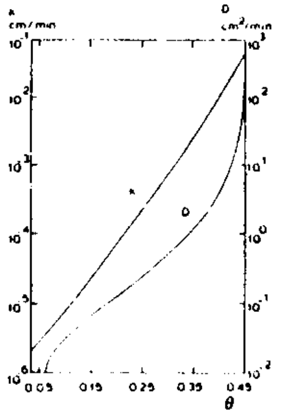


Figure 29. Experimental values of $k_{(\theta)}$ and $D_{(\theta)}$ as functions of θ for a Columbia silt loam.

Equations 42, 28, 29 and 30 can be solved analytically if D is assumed constant. This solution is given in detail in Appendix. The result is:

$$\theta = \theta_i + (\theta_o - \theta_i) \text{ erfc } (\frac{\lambda}{2\sqrt{D}}) \qquad (55)$$

where erfc stands for complementary error function. A plot of this is given in figure 30.

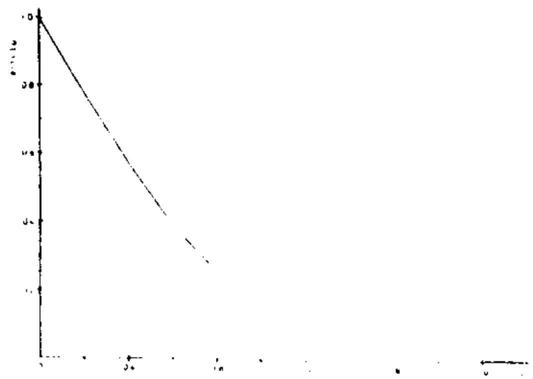


Figure 30. The complementary error function erfc (u) for u ≥ 0.

For a chosen set of values of θ_i, θ_o and D, equation (55) is plotted in figure 31. When this graph is compared with figure 22, it is obvious that this solution, a gradually decreasing volume fraction of water with distance, is far from reality. The reason for this discrepancy is that D is not constant over the range of θ, but varies with a factor of up to about 10^5.

Equations 28, 29, 30 en 45 cannot be solved analytically for variable D. However, various techniques can be used for solving these equations by numerical methods. The result of such calculations, for the same values of θ_i and θ_o but a realistic D-θ, is also plotted in figure 31. This figure shows, as expected, a unique relationship between θ and λ. This implies that $s/\sqrt{t}$ (= λ) is constant for a given θ.

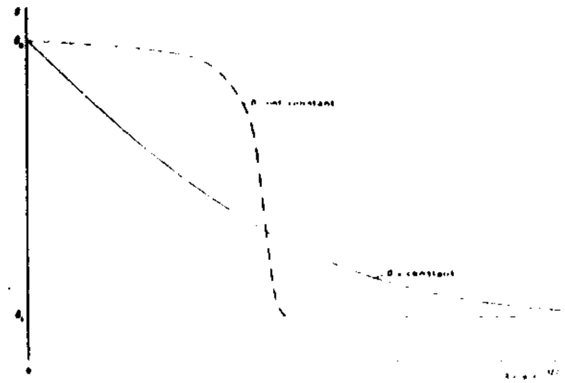


Figure 31. Solutions of equation 22 for D = constant and for D of a soil (D increasing with increasing θ).

Which relationship can be expected between s and t for the visible wetting front and for any other value of θ between θ_i and θ_o?

Since the ratio $s/\sqrt{t}$ is constant for a given value of θ, the distance from the wetting surface to the visible wetting front (or any other value of θ) increases proportional with the square root of time.

Combination of equations:

$$I = \int_{s=0}^{\infty} (\theta - \theta_i)\, ds \quad \text{and } \lambda = s\, t^{-1/2} \text{ gives:}$$

$$I = \int_{s=0}^{\infty} (\theta - \theta_i)\, ds = \int_{\lambda=0}^{\infty} (\theta - \theta_i)\, d\lambda t^{1/2} =$$

$$t^{1/2} \int_{\lambda=0}^{\infty} (\theta - \theta_i)\, d\lambda$$

Introduction of the parameter:

$$S = \int_{\lambda=0}^{\infty} (\theta - \theta_i)\, d\lambda \tag{56}$$

leads to:

$$I = St^{1/2} \tag{57}$$

and consequently,

$$i = \frac{dI}{dt} = 1/2\ St^{-1/2} = \frac{I}{2t} \tag{58}$$

S is called the sorptivity and is a measure for the capacity of a soil to absorb water. It is the cumulative infiltration during the first unit of time. Since

$$S = \int_{\lambda=0}^{\infty} (\theta - \theta_i)\, d\lambda\ ,$$

S can be found by evaluating the area between the θ-axis, the θ_i line and the θ-λ curve. S can also be given by:

$$S = \int_{\theta_i}^{\theta_0} \lambda d\theta$$

Using the product rule $d(\lambda\theta) = \lambda d\theta + \theta d\lambda$ and the boundary conditions:

$\theta = \theta_i$ for $\lambda \to \infty$

$\theta = \theta_0$ for $\lambda = 0$

$$\int_{\lambda=0}^{\infty} (\theta - \theta_i)\, d\lambda = \int_{\lambda=0}^{\infty} d(\theta - \theta_i)\, \lambda - \int_{\theta_0}^{\theta_i} \lambda d\theta =$$

$$(\theta - \theta_i)\lambda \Big|_{\lambda=0}^{\infty} - \int_{\theta_0}^{\theta_i} \lambda d\theta = 0 - 0 - \int_{\theta_0}^{\theta_i} \lambda d\theta = \int_{\theta_i}^{\theta_0} \lambda d\theta$$

Also, from figure 31 it is obvious that

$$\int_{\theta_i}^{\theta_0} \lambda d\theta \quad \text{and} \quad \int_{0}^{\infty} (\theta - \theta_i)\, d\lambda$$

represents the same area.

Would you expect S to be constant for a given soil? S is not constant for a given soil. The θ-λ curve is the solution of equation 11 subject to conditions 23 and 24. Therefore, S depends on θ_i, θ_0 and on the D-θ relationship of the soil. While θ_0 (saturation) and the D-θ relationship normally do not vary for a given soil, S will still depend on θ_i.

For initially dry soil ($\theta_i = 0$) the sorptivity varies from about 5×10^{-5} m s$^{-1/2}$ ($\simeq$ 0.04 cm min$^{-1/2}$) for a heavy clay to about 2×10^{-3} m s$^{-1/2}$ ($\simeq$ 1.5 cm min$^{-1/2}$) for a coarse sand.

P R A C T I C A L   P R O B L E M

a. Calculate the cumulative horizontal infiltration in an initially dry, heavy clay soil during 14 hours

b. Do the same for a coarse sand

ANSWER

a. Using $I = S\sqrt{t}$, we find for heavy clay:

$$I = 5 \times 10^{-5}\ m\ s^{-1/2} \times \sqrt{24 \times 3600\ s} \simeq 1.47 \times 10^{-2}\ m \text{ or } 1.47\ cm$$

b. For coarse sand:

$$I = 2 \times 10^{-3}\ m\ s^{-1/2} \times \sqrt{24 \times 3600\ s} \approx 0.59\ m \text{ of } 59\ cm$$

3.6.4.2. Vertical infiltration.

So far, the infiltration process was described only for cases in which the gravitational potential can be neglected. During vertical infiltration, the influence of gravity becomes more important as time progresses. Then equation 23, together with the initial and boundary conditions of equations 24 and 25, must be solved. A number of different techniques for solving equation 23 are available. A solution obtained by one of these techniques is:

$$s(\theta,t) = a_1 t^{1/2} + a_2 t + a_3 t^{3/2} + a_4 t^2 \ ..... \qquad (59)$$

where a_1, a_2, a_3, a_4 ... are still functions of θ. By means of equation 59 the depth of a given value of θ at time t can be evaluated when a_1, a_2 etc... are known. These coefficients can be evaluated by numerical methods using the relationships D-θ and K-θ.

The cumulative infiltration obtained by the above technique is given by:

$$I = St^{1/2} + At + Bt^{3/2} + Ct^2 + \ ..... \qquad (60)$$

The coefficients of equation 60 are evaluated in the same way as those of equation 59.

The infiltration rate can be derived from equation 60 by differentiating with respect to t (equation 10):

$$i = 1/2\ St^{-1/2} + A + 3/2\ Bt^{1/2} + 2\ Ct + \ .... \qquad (61)$$

Equation 60 and 61 usually are truncated after the first two terms, because for not too large t these series converge rapidly. In that case equations 59 and 60 becomes respectively:

$$I = St^{1/2} + At \qquad (62)$$

and

$$i = 1/2\ St^{-1/2} + A \qquad (63)$$

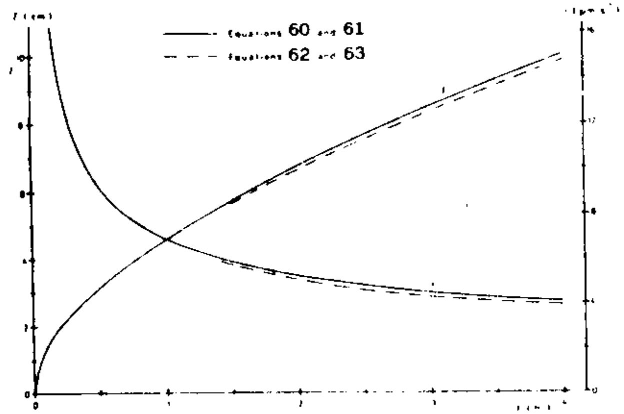
S again the sorptivity defined in equation 56. It is the dominant parameter in the early stage of infiltration. As time progresses, the first term becomes negligible and the importance of A, which represents the main part of the gravitational influence, increases.

PRACTICAL PROBLEM

a. Make a graph of the cumulative infiltration I and the infiltration rate i as a function of time for vertical infiltration during 4 hours in a fine sandy loam for which:
   $S = 7.0 \times 10^{-4}\ m\ s^{-1/2}$
   $A = 1.0 \times 10^{-6}\ m\ s^{-1}$
   $B = 1.0 \times 10^{-9}\ m\ s^{-3/2}$
   Use equations 60 and 61 (first three terms)
b. Give in the same figure the results when equations 62 and 63 are used
c. Compare the results

A N S W E R

a/b.



c. Truncating equations 60 and 61 after the second term introduces an error which increases with time. In this case, the error is still small after 4 hours of infiltration.

4. Determination of the Soil Hydraulic Characteristics in the Field.

4.1. Introduction

Knowledge of the pattern of water movement within the soil profile is essential to the solution of problems involving irrigation, drainage, water conservation, groundwater recharge and pollution, as well as infiltration and runoff control. In recent years, soil physicists have developped extensive mathematical theories to describe soil water movement under different sets of initial and boundary conditions.

Much of the theory now available yet remains to be verified experimentally and applied in practice. However, if the theoretically derived equations of soil physics are to be applied for the description or prediction of actual processes in the field, we must have a way of measuring pertinent soil

parameters on a realistic scale. The pertinent parameters are the hydraulic characteristics of the soil, including the functional relations of hydraulic conductivity or diffusivity and of matric suction to the water content, as well as the spatial and temporal variation of these in the field.

It is inherently unrealistic to try to measure such parameters in the laboratory on discrete and small samples removed from their natural continuum, particularly when such samples are fragmented or otherwise disturbed. Hence it is necessary to devise and test practical methods for measuring bulk soil hydraulic characteristics on a macroscale in situ.

Knowledge of the hydraulic conductivity and soil-water diffusivity at different moisture contents or suctions is generally required before any of the mathematical theories of water flow can be applied in the practice.

Since there is no reliable way to predict these values from more fundamental soil properties, the hydraulic conductivity $K_{(\theta)}$ and soil-water diffusivity $D_{(\theta)}$ must be measured experimentally.

Measurements of hydraulic conductivity during internal drainage in the field are usually based on monitoring the transient flux and potential gradient value within the profile as functions of depth and time.

4.2. Theoretical background

The purpose of these methods is to determine directly on the field the hydraulic conductivity K and the soil-water pressure h as a function of the soil-water content θ by transient analysis of the water content and water head profiles during a drainage experiment.

The main point is to determine simultaneously, and at the same depth, the soil water flux and the gradient of dH/dz.

On a given site the soil is first wet to a given depth (∼ 150 cm). The drainage experiment begins when water disappears from the soil surface, this instant being chosen as the time $t = 0$.

The soil is than covered (plastic + mulch) in order to avoid any flow of water (rainfall/evaporation) through the soil surface ($z = 0$). Changes of water contents and water pressures are measured following a procedure which will be described later using respectively a neutron probe and a series of tensiometers being installed at different depths in the profile from $z = 0$ to $z = 200$ cm.

If we consider a volume of soil of unit area of thickness dz, the equation of continuity says that :

$$\frac{dq}{dz} = - \frac{\partial \theta}{\partial t} \qquad (64)$$

where dq is the difference between the flux entering (q_1) and leaving (q_2) the thin layer of soil dz during the small time interval dt.
Equation (64) can also be written :

$$q_1 - q_2 = \frac{\partial}{\partial t} \int_{z_1}^{z_2} \theta \, dz \qquad (65)$$

This equation, as such, cannot be solved, since we have two unknowns q_1 and q_2.

If, however, we consider a slab of soil limited :

- upwards by the soil surface = 0, where in this experiment q_1 is imposed to be 0,
- downwards by any depth z, z_0, (where the soil is continuously draining), the flux passing through z at any time t is simply

given by :

$$- q_2 = \frac{\partial}{\partial t} \int_0^{z_0} \theta \, dz \quad \text{or}$$

$$q_2 = - \frac{\partial}{\partial t} \int_0^{z} \theta \, dz$$

$$q(z,t) = - \frac{\partial}{\partial t} \int_0^{z} \theta \, dz$$

The integral $\int_0^z \theta \, dz$ represents at any time the volume of water (per unit surface of soil) stored between 0 and depth z.

We will define $S(z,t) = \int_0^z \theta \, dz$ as the STORAGE of water at time t and depth z, and we will determine the flux :

$$\boxed{q = - \frac{\partial S}{\partial t}} \qquad (66)$$

i.e. the slope of the curve S(z,t) at time t.

<u>Note</u> : if the soil drains $\partial S/\partial t$ is negative, and q is positive.

If simultaneously we can determine the profile of head H(z) at the same time from the tensiometer reading the gradient of head dH/dz will simply be given by the slope of this profile at this depth.

Considering Darcy's law $q = - K_{(\theta)} \cdot \frac{dH}{dz}$ the ratio $q/- \frac{dH}{dz}$ between the flux q and the head gradient measured simultaneously at the same time t and the same depth z will give the value of the hydraulic conductivity K corresponding to the water content θ, measured at this time at depth z.

Remembering that the hydraulic head (H) is in general the sum of the soil water pressure head (h), or its negative the suction head, and its gravitational head z :

$$H = h + z$$

Considering the head H (z,t) measured at time t and depth z, the water pressure at this level is given by equation :

$$H = h - z \quad \text{or}$$

$$h = H + z$$

$$\text{p.e.} \quad h = -50 \text{ cm}$$
$$z = 50 \text{ cm}$$
$$H = -50 - 50 = -100 \text{ cm}$$

The relation between the water content θ and the soil water pressure h, measured simultaneously at the same time t and the same depth z, will give one point on the suction curve $h_{(\theta)}$ of the soil.

Using the technique for the same depth at different times, one will obtain the curves $K_{(\theta)}$ and $h_{(\theta)}$ as a function of time for a given depth.

In as much as soil-water diffusivity is the product of hydraulic conductivity, and the reciprocal slope of the soil-water characteristic, $C(\theta) = \frac{d\theta}{dh}$ (cm^{-1}) $\qquad D_{(\theta)} = \frac{K(\theta)}{C(\theta)} = K_{(\theta)} \frac{\partial h}{\partial \theta}$

the equation of Darcy's law can be modofied tc approximate the diffusivity

$$q = \int_0^z \frac{\partial \theta}{\partial t} dz = (K_{(\theta)} \frac{dH}{dz})_z$$

If this equation is simplified by approximating the integral with the product of the soil depth L and the rate of change of the average soil-water content in the profile $\bar{\theta}$ (with the value of the hydraulic gradient retained) it becomes :

$$L \frac{\partial \bar{\theta}}{\partial t} = (K \frac{dH}{dz})_L$$

Assuming that an average soil-water characteristics curve holds for the entire profile, $\frac{\partial \theta}{\partial h} \cdot \frac{\partial h}{\partial t}$ can be substituted for $\frac{\partial \theta}{\partial t}$ and using the relation $D = K . \partial h/\partial \theta$, the equation becomes :

$$L \frac{d\theta}{dh} \cdot \frac{dh}{dt} = K \frac{dH}{dz} \cdot L \frac{\partial h}{\partial t} = D \frac{\partial H}{\partial z}$$

Finally : $D_{(\theta)} = L \frac{\partial h/\partial t}{\partial H/\partial z}$

Hence, the value of D can be calculated for each depth L using only tensiometers. The value $\partial h/\partial t$ is merely the time rate of change of the soil water pressure head h obtained from tensiometer readings for depth L.

4.3. Measuring technique

1. Equipment

- 1 neutron probe, one measurement each 10 cm,
- series of 10 tensiometers installed, if possible, at depths z = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110 and 120 cm from the soil surface, or according to layering of the soil.

It is reminded that if a tensiometer is installed vertically at a depth z, and is equipped with a mercury manometer, the free surface of the mercury being at a level y above the soil surface :

a) the soil water pressure head h (cm water)

$$h = (z + y) - 12.6 \, x \qquad (66)$$

p.e. is y = 23.5 cm above soil surface, then h at 50 cm depth will be (data after 1 day (x = 8.4 cm) and 6 days (x = 9.6)).

h_{1d} = (50 + 23.5) - 12.6, 8.4 = - 32,34 cm water,

h_{6d} = (50 + 23.5) - 12.6, 9.6 = - 47,46 cm water.

b) the hydraulic head H (cm water) :

$$H = h - z$$

$$H_{1d} = -32,34 - 50 = -82,34 \text{ cm water,}$$

$$H_{6d} = -47,46 - 50 = -97,46 \text{ cm water.}$$

2. Measurements

It is recommended to follow with the maximum care the initial stage of drainage. During the first day at least to persons should be on the field, one taking care of neutron measurements, the other taking care of tensiometer readings.

Concerning the neutron measurements, it is advisable to monitor continously changes of water content in the profiles during the first two hours of drainage. The recommended procedure is the following :

With the smallest counting time for a good accuracy, measure for each depth increment (i.e. each 10 cm) the water content at a given time starting from t = 0 with the probe being positioned at z = 10 cm, at the next counting time the probe will be positioned at 20 cm, and so on to z_0.

The series of measurements from z = 10 to z_0 being defined as a cycle, immediately after the end of counting at z_0, the probe will be raised to the standard shield, a standard count will be taken, and a new cycle will be initiated.

For each measurement being taken, the following information should be recorded :

- measuring depth,
- time (from the initiation of drainage),
- number of counts and counting time or count/rate.

After approximately two hours of continuous measurements, ... of measurement will be made every 30 minutes for the following hours.

The interval between cycles will be progressively increased as indicated in the following table.

Time t (hours) from the initiation of drainage	Interval (hr) between cycles
0 - 2 hr	none (continuous monitoring)
2 - 4 hr	0.5 hr
4 - 6 hr	1 hr
6 -12 hr	3 hr
12 -48 hr	12 hr
2 days	24 hr

For the tensiometer readings : during the first period following the initiation of drainage (0 < t < 2 hrs) tensiometers should be recorded each 5 minutes.

Each time one should note (Table 1) :

- tensiometer reference (from 1 to 10...),
- height of mercury,
- time of measurement.

For t > 2 hrs, the same procedure as the one defined for neutron measurements will be used. It is recommended for t > 2 days to take measurement each day, at the same hour.

Table 1-2

EXPERIMENTAL SITE N°

Date:

Neutron moisture meter:

- Height of access tube above soil surface: cm
- Standard radiation counting: - before measurement: cpm
  - after measurement: cpm
  - average (N_s): cpm
- Calibration curves: 1) $y_1 = a_1x_1 + b_1$
  2) $y_2 = a_2x_2 + b_2$
  ⋮
  n) $y_n = a_nx_n + b_n$

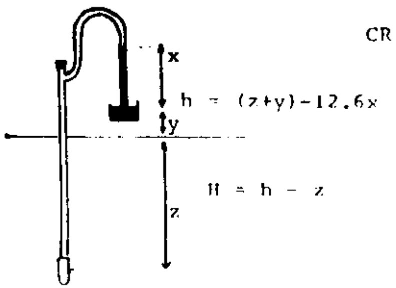
Tensiometer:

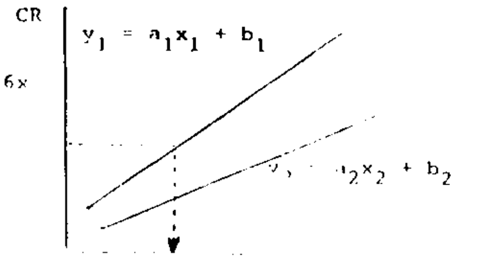
y = height of the mercury container above soil surface: cm

z = depth of tensiometer cup

x = height of the mercury in the manometer

Soil depth (cm)	Tensiometer			Neutron moisture meter		
	x (cm Hg)	h (cm)	H (cm)	count rate N (cpm)	count ratio $\frac{N}{N_s} \times 100$	moisture content (vol %)





4.4. Analysis of data

1. The tables with the data will first be analyzed in order to obtain the following informations :
   for each measuring depth a table giving (Table 2) :
   - the time of measurement t,
   - the water content θ in $cm^3.cm^{-3}$,
   - the soil-water pressure head h,
   - the hydraulic head H.

2. The data will be plotted on a graph in order to obtain at given depths (each 10 cm for the neutron probe or each tensiometer depth) the change of water content or hydraulic head H with time.
   Taking into account the exponential shape of the variation for each variable (θ or/and h or H and depth) two time scales should be used
   - one for the initiation of the drainage : 1 day full scale,
   - one for the second part of the drainage 3 weeks full scale (500 hrs).

   Smooth curves will be drawn (by hand) in order to have a best fit between the measurement points.
   One curve should be drawn for each depth.

3. Using the smooth curves one will determine by interpolation the values of θ and of H at all the measuring depths at the following reference times : t = 0, 0.25, 0.5, 1, 2, 5, 10, 25, 100, 200, 500 hrs.
   Tables of interpolated data will be built in order to obtain :
   - the water content profiles (θ-z),
   - the hydraulic head profiles (H-z),

   at the above mentioned reference times (Table 3).

Time after ponding (hr)	z=10cm		z=20cm		z=30cm		z=40cm		z=50cm		z=60cm		z=70cm		z=80cm	
	θ	H	θ	H	θ	H	θ	H	θ	H	θ	H	θ	H	θ	H
0																
0.25																
0.50																
1																
2																
5																
10																
25																
50																
100																
200																
500																

4. The water content profiles and hydraulic head at the reference times will be plotted :

   Figure : $\theta - z$,

   Figure : $H - z$.

5. The table of interpolated water content will be used to compute $S(z,t)$ using the trapeze rule : "the water content measured" at a given z_i will be affected to a layer of 10 cm surrounding the measuring depth, with an exception for the first depth of measurement (z = 10 cm) where θ (z = 10) will be assumed to represent the mean water content from 0 to 15 cm.

   The storage S will be calculated for the depths of 30, 60, 90 and 120 cm.

   In consequence at a given time t the storage S between 0, and the respectively depths, will be given in mm of water by the formula :

$$S_{30} = (1.5\,\theta_{10} + (\theta_{20})\,1.0 + 0.5\,\theta_{30}) \times 100,$$

$$S_{60} = (1.5\,\theta_{10} + (\theta_{20} + \theta_{30} + \theta_{40} + \theta_{50})\,1.0 + 0.5\,\theta_{60}) \times 100,$$

$$S_{90} = (1.5\,\theta_{10} + (\theta_{20} + \theta_{30} + \theta_{40} + \theta_{50} + \theta_{60} + \theta_{70} + \theta_{80})\,1.0 + 0.5\,\theta_{90}) \times 100,$$

$$S_{120} = (1.5\,\theta_{10} + (\theta_{20} + \theta_{30} + \theta_{40} + \theta_{50} + \theta_{60} + \theta_{70} + \theta_{80} + \theta_{90} + \theta_{100} + \theta_{110})\,1.0 + 0.5\,\theta_{120}) \times 100.$$

6. A table will be built given the value of the storage S, at the reference times, and at the depths 30, 60, 90 and 120 (Table 4).

Time after ponding (hrs)	S_{30}	S_{60}	S_{90}	S_{120}
0				
0.25				
0.50				
1				
2				
5				
10				
25				
50				
100				
200				
500				

7. The curves giving the change of the storage with time at the given depths will be plotted.
At each reference time, and for each depth, the slope taken by hand will give the value of the flux

8. At the same depths (30, 60, 90 and 120) the slope of the hydraulic profiles will be measured at the same times.

9. Finally for each depth (30, 60, 90 and 120) a table will be built, giving the following information (at the reference times) (Table 5).

Soil depth z = cm

Time (t) hr	θ $cm^3.cm^{-3}$	h cm	q mm/hr	dH/dz	K mm/hr
0					
0.25					
0.5					
1					
2					
5					
10					
25					
50					
100					
200					
500	1	2	3	4	5

soil-water characterictic curve or pF curve

10. The K-values were plotted in function of θ ($cm^3.cm^{-3}$) and the equation is calculated

The suction gradient must be taken into account and the hydraulic conductivity is obtained from the ratio of flux to the total hydraulic head gradient (gravitational plus matric). This can be done successively at gradually diminishing water content during drainage, to obtain a series of k versus θ values and thus establish the functional dependence of hydraulic conductivity upon soil moisture content for each layer in the profile.

To apply this method in the field, one must choose a characteristic fallow plot that is large enough so that processes at its center are unaffected by its boundaries. Within this plot, at least one neutron access tube is installed. A series of tensiometers is installed near the access tube, at intervals not exceeding 30 cm. Water is then ponded on the surface and the plot is irrigated long enough so that the entire profile becomes as wet as it can be. In the case of uniform profile, this will mean effective saturation. Tensiometer readings can indicate when steady-state infiltration conditions have been achieved. When the irrigation is deemed sufficient, the plot is covered by a sheet of plastic so as to prevent any water flux across the surface (evaporation or infiltration). As the internal drainage process proceeds, periodic measurements are made of water content and tension throughout the profile. These readings must be taken frequently at first (at least daily) but can be taken at greater time intervals as the internal drainage process shows down.

5. Determination of the infiltration rate.

The best method of measuring intake is to obtain direct measurements by recording the water applied less the water flowing from the field. When direct measurements are not feasible, intake cylinders can be used with reasonable success.

The cylinders should be at least 25 cm in diameter, made of smooth tough steel, strong enough to drive, but thin enough to enter the soil with a minimum of disturbance. Cylinders should be about 30 cm long.

Since approximately five tests should be made in a given location to obtain a representative sample it is well to make five cylinders of somewhat different diameters so that all five can be tested together and thereby be made less bulky for handling.

Care should be taken to place the cylinders in an area representative to the field to be evaluated. Cylinders should be carefully driven into the soil to a depth of about 15 cm. The soil profile should be examined, the moisture estimated or measured, and notes of soil cover and surface condition.

After water penetrates to the bottom of the cylinder, it will begin to spread radially and the rate of intake will change accordingly. When a principal restricting layer does not lie within the depth of penetration of the cylinder, this radial flow will cause considerable change in the intake rate. Buffer ponds can be constructed by forming an earth dike around the cylinder or by driving into the soil a larger diameter cylinder concentric with the intake cylinder. The water levels in both cylinders should be equal and approximately the depth to be expected during the irrigation. When comparative results from different locations are needed, the depth should be essentially the same in all cylinders. Care should be taken not to puddle the soil when water is added to the cylinder or the buffer pond.

The results obtained with cylinders are indicative of the rates to be expected during irrigation provided the surface condition is the same. Closer correspondence is obtained when the soil surface is covered by the irrigation water. Considerable departure usually occurs when the irrigation water is applied by furrows or sprinklers. Hence the cylinders are generally used to obtain an index from which desgin values can be obtained on the basis of local experience.

The cylinder infiltrometer, used in irrigation studies to determine the infiltration rate and cumulative infiltration, can be used at successive depths in a soil pit to study the differences in the hydraulic conductivity of the various layers. The principle of this method is the same as that of the double tube method.

First one infiltrates water around the infiltrometer till the soil is saturated. The infiltrometer is then filled with water, and the rate at which the water level falls is measured. After some time the infiltration rate stabilizes and approximates the hydraulic conductivity K (Figure 32).

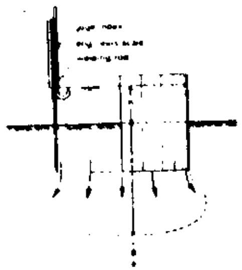


Figure 32 : The infiltrometer.

Calculations

The infiltrometer rate of water in an unsaturated soil measured by a cylinder infiltrometer may be expressed in terms of Darcy's law as :

$$v = K_T \frac{\phi + z + h}{z} \qquad (68)$$

where :

v = infiltration rate (cm/sec),

K_T = hydraulic conductivity of the transmission zone (cm/sec),

ϕ = suction at the bottom of the transmission zone (cm),

z = depth of the transmission zone below the infiltrometer (cm),

h = height of water in the infiltrometer (cm).

The influence of ϕ and h relative to z diminishes as the depth of the transmission zone and the moisture content of the soil increase. So the hydraulic gradient :

$$\left(\frac{\phi + z + h}{z}\right)$$

tends towards 1 in the case of a deep uniform soil profile and the infiltration rate becomes constant, attaining what is known as the basic infiltration rate.
In that case we may write :

$$v = K_T \qquad (69)$$

For wet, medium and heavy textures soils in which the hydraulic conductivity of the transmission zone is approximately the same as in the saturated zone, we get

$$v \simeq K_T \simeq K \qquad (70)$$

When using the infiltrometer for hydraulic conductivity studies - more specificaly for studies on the basic infiltration rate in moist soils- the measurements should extend for ap period long enough to permit a constant infiltration rate to be obtained.

This may take quite some time in dry clay soils, because a decrease in the infiltration rate may be caused by the decrease in the hydraulic gradient as well as by a change in hydraulic conductivity due to swelling. The value obtained in a layered soil only applies to the depth of soil penetrated by the infiltrometer, since lateral flow will occur below if the hydraulic conductivity of the underlying layer is low. If it is possible to determine the distance over which the lateral flow extends in the underlying layers by estimating the change in moisture content, the infiltration rate in the underlying layer can be calculated. This can be done by taking the ratio between the surface of the infiltrometer and the surface over which lateral flow occurs in the underlying layer and multiplying it by the infiltration rate in the infiltrometer. Figure 33 shows an example of lateral flow below an infiltrometer installed at the soil surface to a depth of 5 cm on a silty clay loam with a ploughed layer of 20 to 40 cm. In the situation of the example, the intake rate at a depth of 25 cm would be $(37/77)^2$ times the intake rate near to the bottom of the infiltrometer. At a depth of 65 cm the ratio is $(37/117)^2$.

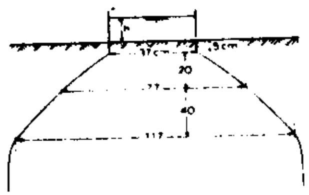


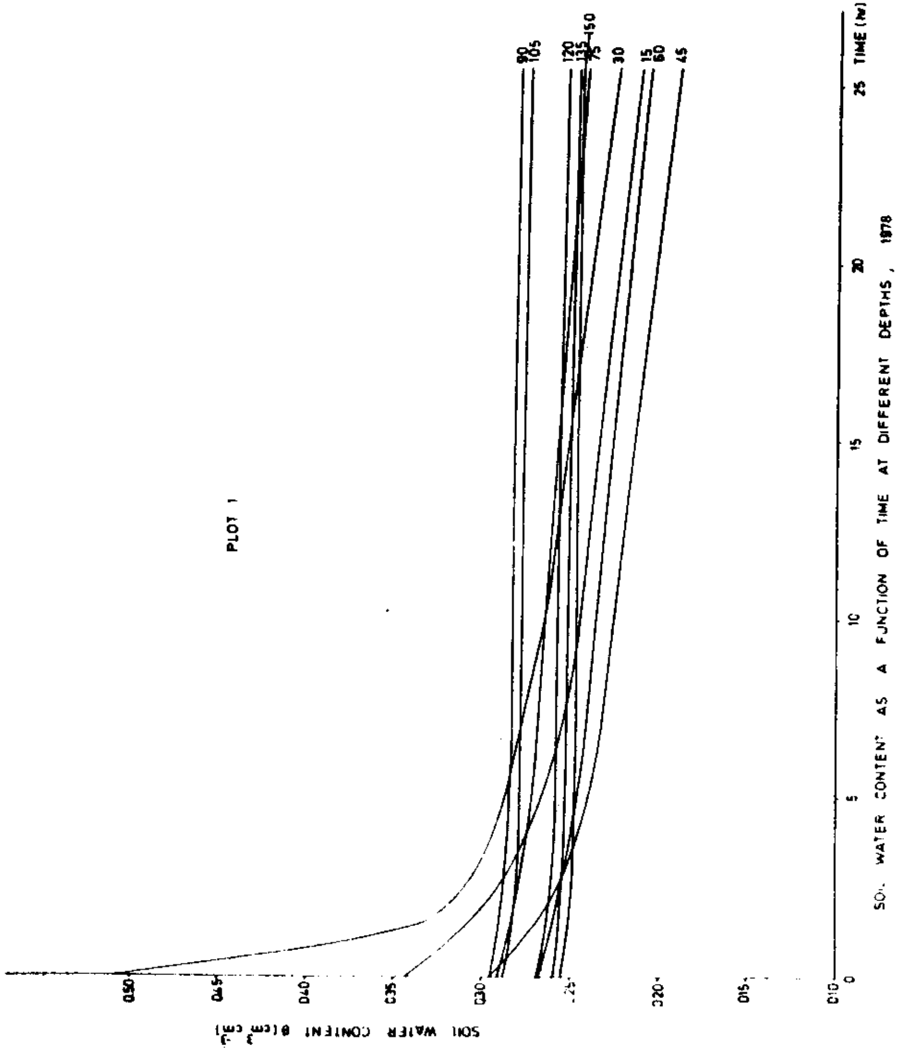
Figure 33 : Lateral flow below infiltrometer.

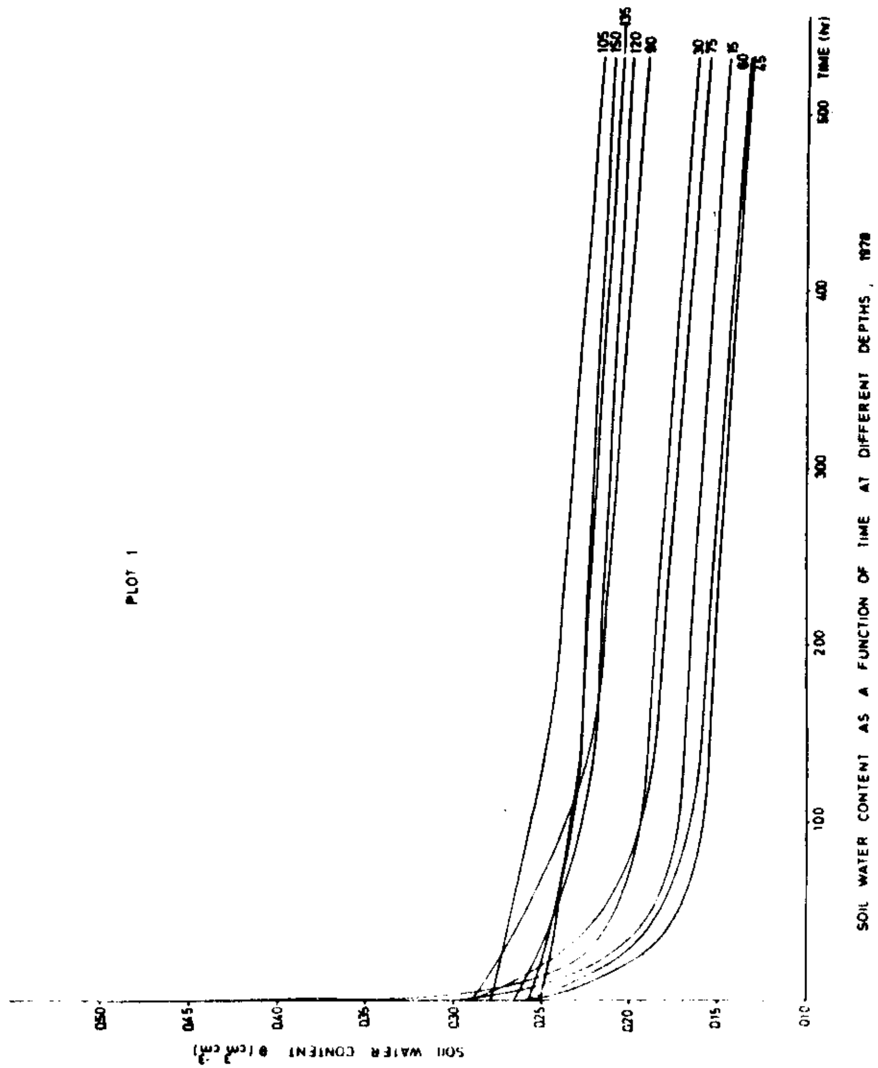
Discussion

The cylinder infiltrometer is suitable for determining the intake characteristics and hydraulic conductivities of irrigated soils. The results of this method are not very accurate, but can be regarded as a fair approximation of the K-value. The method is practical and suitable for large-scale surveys.

6 . Appendix

Results of the $K_{(0)}$ determination in the field following the internal drainage method.


PLOT 1
SOIL WATER CONTENT AS A FUNCTION OF TIME AT DIFFERENT DEPTHS , 1978
SOIL WATER CONTENT θ (cm³ cm⁻³)
TIME (hr)
0
5
10
15
20
25
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
90
105
120
135
150
75
30
15
60
45


PLOT 1
SOIL WATER CONTENT AS A FUNCTION OF TIME AT DIFFERENT DEPTHS , 1978
SOIL WATER CONTENT θ (cm³ cm⁻³)
TIME (hr)
100
200
300
400
500
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
105
150
135
120
90
30
75
15
60
45