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"Gravity Drainage for a Homogeneous and Stratified Profiles"

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Gravity drainage for a homogeneous and stratified profiles.

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Introduction

The Green and Ampt approach is proposed for one dimensional transient flow of water through a porous medium of which part is saturated and part is unsaturated. The model is based upon a theory that implies a sharp front exist between the saturated and unsaturated zones. The model is used to study the gravity drainage problem for a falling water table regime in an initially saturated homogeneous and stratified profiles for both cases zero flux and non zero flux. The formulation the Green and Ampt approach is based on the calculation of the position of draining front with time and the cumulative outflow

time relationship. The predictive capability of the Green and Ampt equation for a homogeneous and a stratified profiles is then tested against results obtained from a computer-based numerical analysis of unsaturated flow equation. The results of the Green and Ampt approach in some cases have shown to agree with the numerical simulation quite satisfactory.

Drainage to a fixed water table

The Green and Ampt (1911) approach has applied to calculate the amount of water drained to a fixed water table. The drainage front has a characteristics value h_f and is separated by sharp front between the saturated and unsaturated zone.

The water table is assumed to be at the base see Fig 1

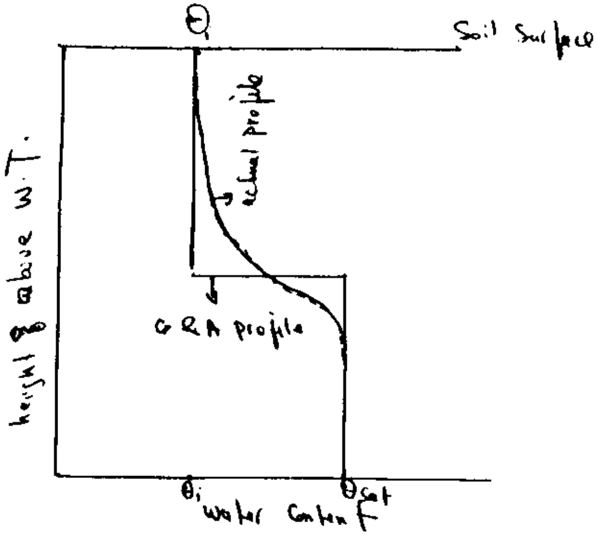

Fig (1)

in case of zero flux residual water content = θ_i
in case of nonzero flux residual water content = the value at the hydraulic conductivity has a value q.

Drainage to a moving water table.

In order to calculate the cumulative outflow drainage to a moving W.T. two types of techniques have been used.

A - homogeneous profile.

1 - numerical solution of unsaturated flow

The general partial differential equation for flow in rigid unsaturated porous media can be derived by combining Darcy's law with the equation of continuity. The equation for vertical flow as

$$c(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left(K(h)\frac{\partial h}{\partial z}\right) + \frac{\partial K(h)}{\partial z} \qquad (1)$$

c(h) specific capacity (cm^{-1})
h pressure head (cm)
z = Cartesian co-ordinates (cm)
K hydraulic conduct

The partial derivations for equation (1) are approximated by the finite difference equation.

The boundary conditions used for simulating gravity drainage to a moving w.t as follows

1- upper boundary

zero surface flux

$$-K(h)\left(\frac{\partial h}{\partial z}+1\right)_{z=0} = 0 \qquad t>0$$

non zero flux

$$-K(h)\left(\frac{\partial h}{\partial z}+1\right)_{z=0} = q \qquad t>0$$

2- lower boundary (w.t)

$$h(z,t)_{z=L+Vt} = 0 \qquad t>0$$

the form of the initial pressure head distribution used in the analysis was assumed to vary linearly from zero at the w.t. to the air entry value at the soil surface.

The numerical data has been tested against experimental data in order to check the accuracy of the numerical analysis. The comparison has shown an excellent agreement between the two sets of h(z) data confirming the predicate capability of the numerical analysis.

2- Green and Ampt approach

The general G&A equation is developed for both cases zero and non zero flux. Parameter used in G & A equation

1) homogeneous profile
2) w.t. positioned at the soil surface and at t=0 sudden drop of w.t. and then it moves down with constant velocity.
3) pressure at the draining front $h_f = -34$ cm
4) K_{sat} (saturated conductivity) = .8 cm/min
5) $\Delta\theta = \theta_{sat} - \theta_i = .238$ cc/cc

see Fig (2)

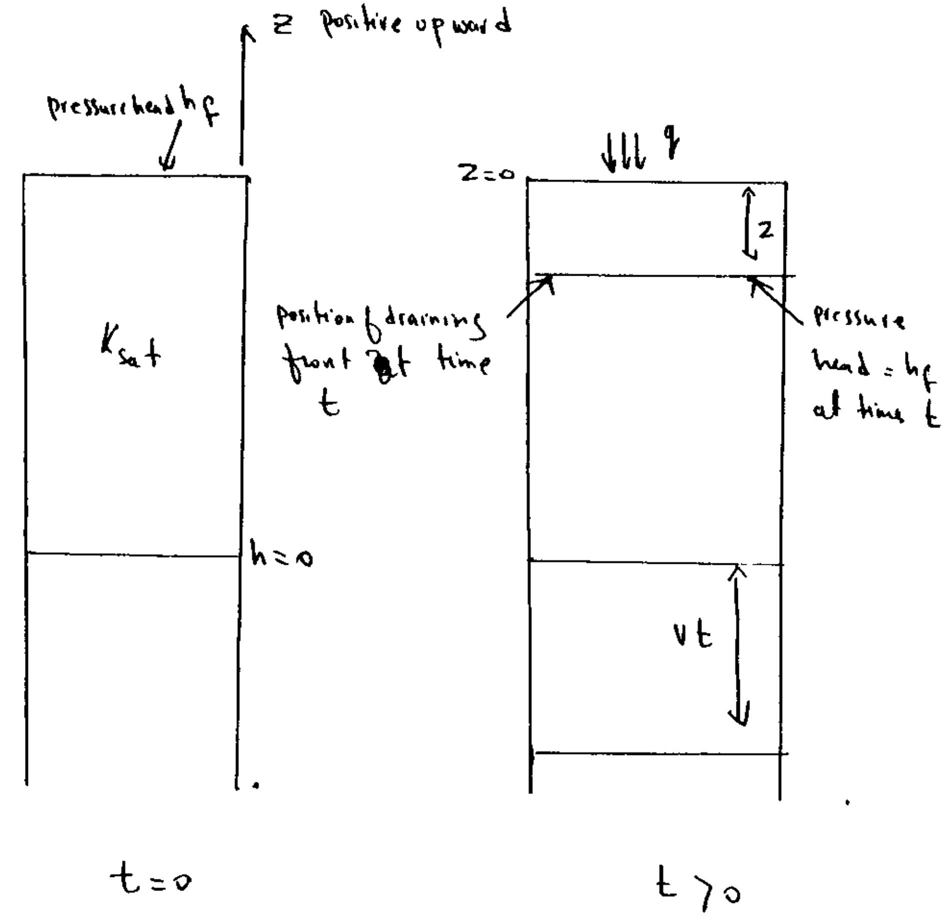

Fig (2) Definition of Parameter used in the Green and Ampt analysis of a falling water table in a homogeneous profile.

Zero flux equation

The differential equation for the downward movement of the sharp front using Darcy's law may be written

$$\Delta\Theta \frac{dz}{dt} = -K_{sat}\left[1 + \frac{h_f}{z-(L+vt)}\right]$$

final solution may be written

$$t = \frac{-z}{\Omega} + \frac{h_f}{(V+\Omega)} \ln\left(1 + \frac{(V+\Omega)(vt-z)}{VL - \Omega h_f + \Omega L}\right)$$

$$\Omega = K_{sat} / \Delta\Theta$$

non zero flux Case

For this Case the differential flow equation becomes

$$\Delta\Theta \frac{dz}{dt} + q = -K_{sat}\left[1 + \frac{h_f}{z-(L+vt)}\right]$$

The Solution may be written

$$t = -\frac{z}{\Omega + W} + \left(\frac{\Omega h_f}{V+\Omega+W}\right) \ln\left[1 + \frac{(V+\Omega+W)(vt-z)}{VL - \Omega h_f + \Omega L + WL}\right]$$

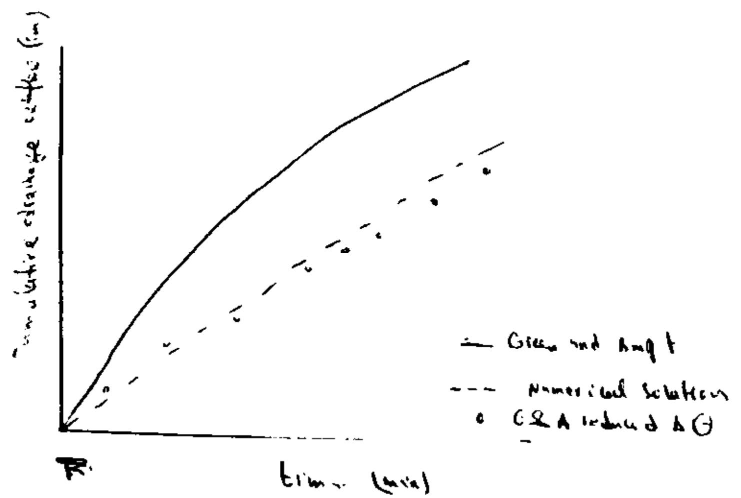

Fig 3 relationship between cumulative drainage outflow and time in a homogeneous column of No 17 Sand with L = -100 cm v = -1 cm/min and q = 0.0 cm/min for both the Green and Ampt and Numerical Solution

The figure shows that the predictive outflow by the Green and Ampt equation is significantly larger than that given by the numerical solution. This due to the fact that the residual Θ value has been used in calculating ΔΘ only occurs in the vicinity of the soil surface rather than through the entire drained profile as the sharp front analysis assumes. It possible to select a smaller value of ΔΘ based on a choosing a Θ value larger than residual Θ value. This is chosen Θ value is considered as an average Θ through the drained profile. As we see there is a significant decrease in the cumulative outflow at any time.

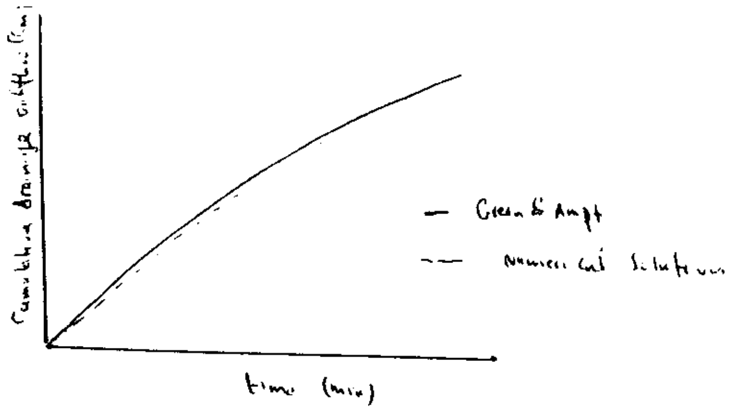

Fig 4 Relation between cumulative drainage outflow and time in a homogeneous column of No 17 Sand with L = -100 cm, v = -2 cm/min and q = -0.1 cm/min for both the Green and Ampt and Numerical Solution.

Conclusion

The Green and Ampt analysis in the case of drainage with no flux on the soil surface has shown variation between the two solutions. This due the nonsharp front of the profiles. This a direct result of the over estimation of the degree of profile drainage and this in turn is due to the residual θ value used here that has been used as the lower bound in calculation Δθ only occurs in the vinicity of the surface rather than through the entire drained profile as the sharp front analysis assume.

B. stratified profile.

1- Numerical solution

one dimensional unsaturated flow equation

$$c(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left(K(h)\frac{\partial h}{\partial z}\right) + \frac{\partial K(h)}{\partial z}$$

the initial condition

$t = 0 \quad z = 0 \quad h = h_f$
$t = 0 \quad z = -L_u \quad h = h_{io}$
$t = 0 \quad z = -L \quad h = 0$

Boundary Condition as described before. numerical solution used here to assess the accuracy of the Green and Ampt equation

2- Materials used

Two types of stratified profile have been used in comparing the derived Green and Ampt equations with numerical analysis.

(13)

i- The first startified sequence

Upper layer No 17 Sand — Soil characteristics:

$K_{sat} = .8$ cm/min
$h_f = -34$ cm
$\Delta\theta = .238$ cc/cc
$z = 50$ cm

Lower layer R 8 A Sand

$K_{sat} = 72$ cm/min
$h_f = -46$ cm
$\Delta\theta = .263$ cc/cc
$z = 50$ cm

ii The second startified sequence

Upper layer No 17 Sand

$K_{sat} = .8$ cm/min
$h_f = -34$ cm
$\Delta\theta = .238$ cc/cc
$z = 95$ cm

lower layer Rubicon Sandy loam

$K_{sat} = .03$ cm/min
$h_f = -66$ cm
$\Delta\theta = .234$ cc/cc

(14)

Green and Ampt equations

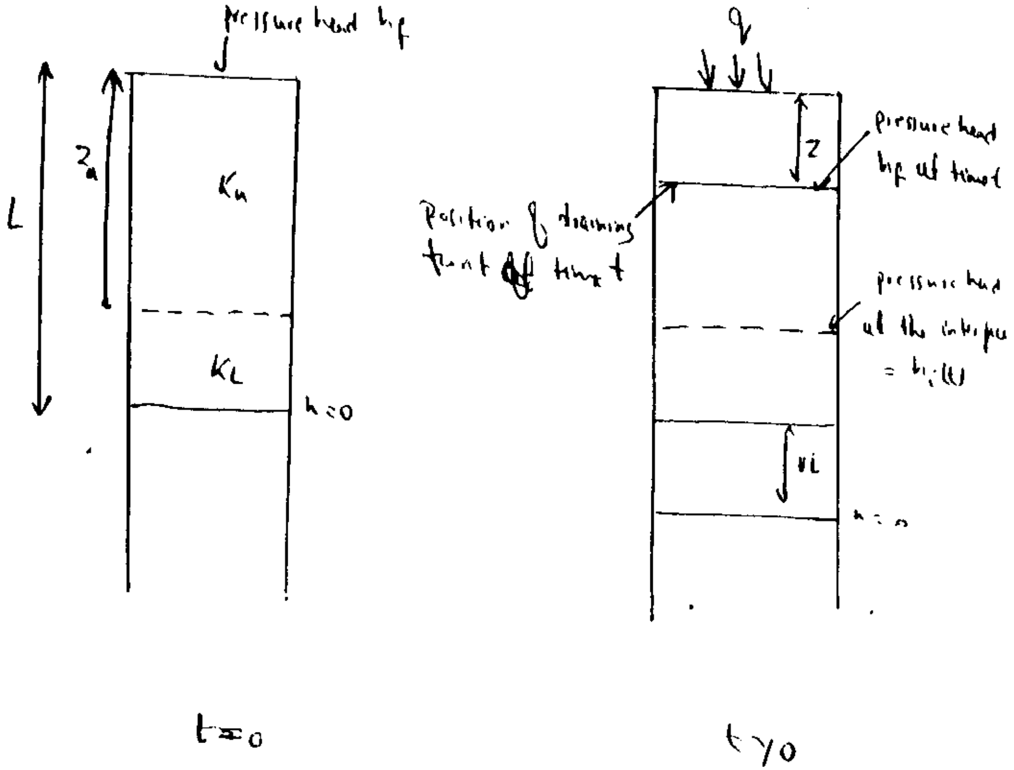

Fig 5 Definition of Parameters in the Green and Ampt analysis of falling water table

$$h_i = \frac{h_f\; K_u(L_u - L) - (z_u - L)(K_u - K_L)}{K_u\;(z_u - L) - K_L\, z_u}$$

$$h_i(t) = \frac{K_u(h_f - z - z_u)(z_u - vt - L) - K_L(z - z_u)(z_u + \ldots)}{K_L(z - z_u) - K_u(z_u - vt - L)}$$

For this case flow equation becomes.

$$\Delta\Theta \frac{dz}{dt} = -K_L \frac{(h_f - z_u) - (vt - L)}{(z_u - vt - L)}$$

Solution

$$\ln\left(\frac{-L}{\varepsilon}\right) = \tfrac{1}{2} \ln \left[-A + (A - B)\, u - \right.$$

equation may seem an awkward equation to use however for a given system, all terms on right hand side are constant except for the parameter μ resulting in a minimum of calculation in arriving at the position of draining front z for a given value of t.

Comparison of G & A and Numerical solutions

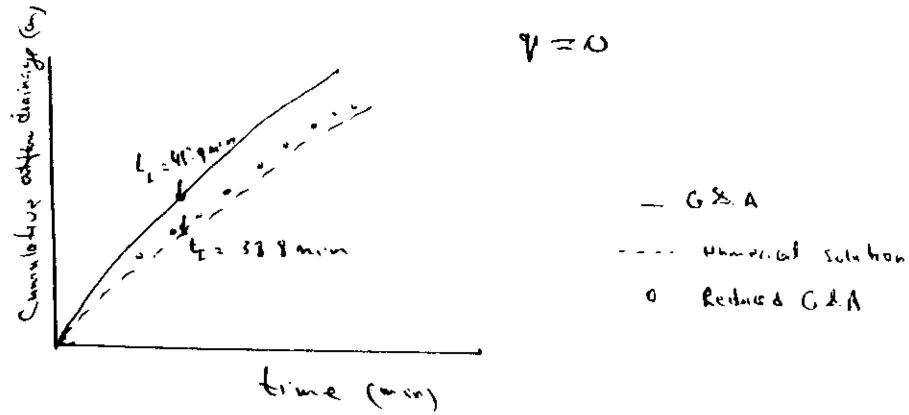

Fig 6 Relationship between Cumulative outflow drainage for a first sequence No. 17 Sand over R&A Sand with L = -100 cm, v = -2 cm/min and q = 0.0 cm/min

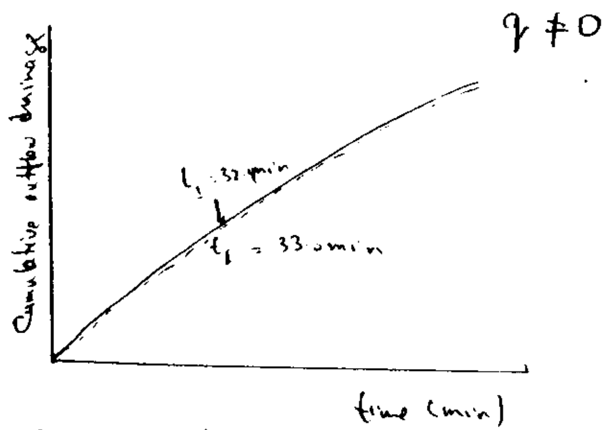

Fig 7 Relationship between Cumulative outflow drainage for a first sequence No. 17 Sand over R&A Sand with L = -100 cm, v = -2 cm/min and q = -1 cm/min

Conclusion

The Green and Ampt analysis is shown to provide good estimates in the case of an applied infiltration flux for the two sequence. The inability of the Green and Ampt to model the physical conditions that occur in the interface region is significant for the No 17 Sand / Rubicon Sandy loam sequence. The position of the Green and Ampt analysis in the case of $q=0$ is less convincing but when reduced $\Delta\theta$ are used this is can provide considerable improvement.

INTEGRATED MODEL OF REGIONAL GROUNDWATER BASIN

In this study the flow system is separated into the unsaturated zone's one dimersional Green and Ampt model for simulating the recharge and the saturated zone's non linear Bonssinesq equation to incorparate the response of water table aquifer to the recharge. This simplified model is succesfully handle the slow response of water table to natural recharge and a good approximation of outflow discharge was compared to the integrated model of Oak Ridge Naitonal (ORN) Laboratory, U.S.A.

INTRODUCTION

Natural recharge refers to the amount of soil moisture recharging the watertable and is the primary mechanism for replenishing the underground water reservoir and subsequent outflow discharge.

The amount of recharge depends mainly on precipitation event and its distribution, soil types, land use and topography of the area.

As a result its magnitude varies with time and space. This phenomenon occurs significantly in a regional groundwater system.

Unsaturated flow case - The Green and Ampt equation

The differential equation for the downwards movement of the sharp draining front may be written

$$\Delta\theta \frac{dz}{dt} = -K_{sat}\left[1 + \frac{h_f}{z - (L + Vt)}\right] \qquad (1)$$

Solution of eq. (1) may be written

$$t = \frac{-z}{\Omega} + \frac{h_f}{(V + \Omega)} \ln\left[1 + \frac{(V + \Omega)(Vt - z)}{VL - \Omega h_f + \Omega L}\right] \qquad (2)$$

Equation can be used to calculate the flux to the water table.

Aquifer response to recharge

The non linear partial differential equation of parabolic type (Bonssinesq 1904) is used to describe the response of water table aquifer to natural recharge and of the form

$$\frac{\partial}{\partial x}\left(h_x \frac{\partial h}{\partial x}\right) + \frac{R}{K}(x,t) = \frac{S}{K}\frac{\partial h}{\partial t} \qquad (3)$$

x is the horizontal coordinate,

t time,

R rate of recharge,

K hydraulic conductivity, and

S specific capacity.

Equation (3) solved using implicit finite defference technique.

Conclusion

The model was simple and had reduced the data input. The solution procedure was direct and had no numerical in stabilities (i.e. time and space could be enlarged) and seemed to have handle the water table response satisfactorily.

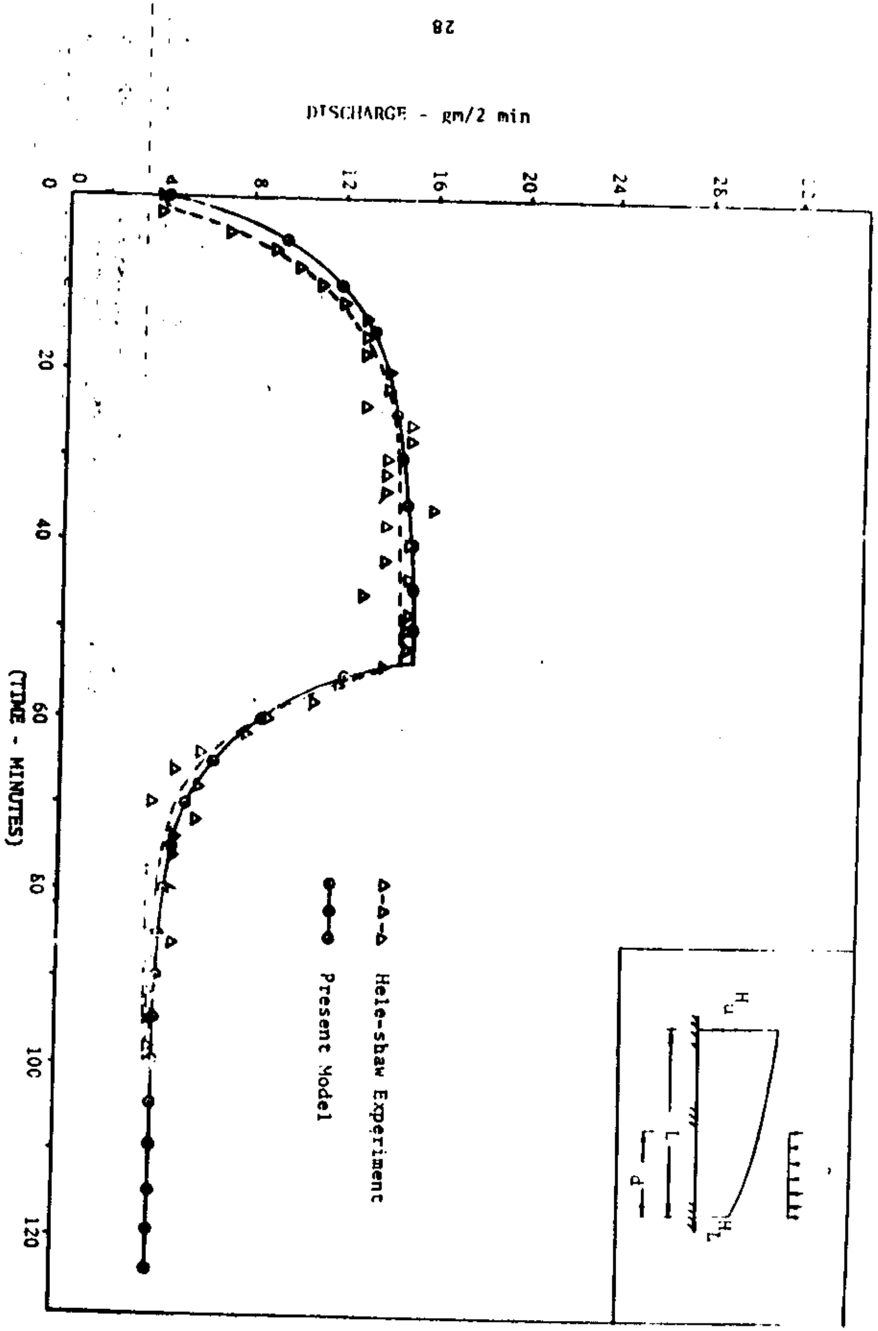

Fig.12. Outflow Hydrograph with Comparison to Hele-shaw Experiment

(H = 15.26" H = 12.24" R = .205"/min d = 56")

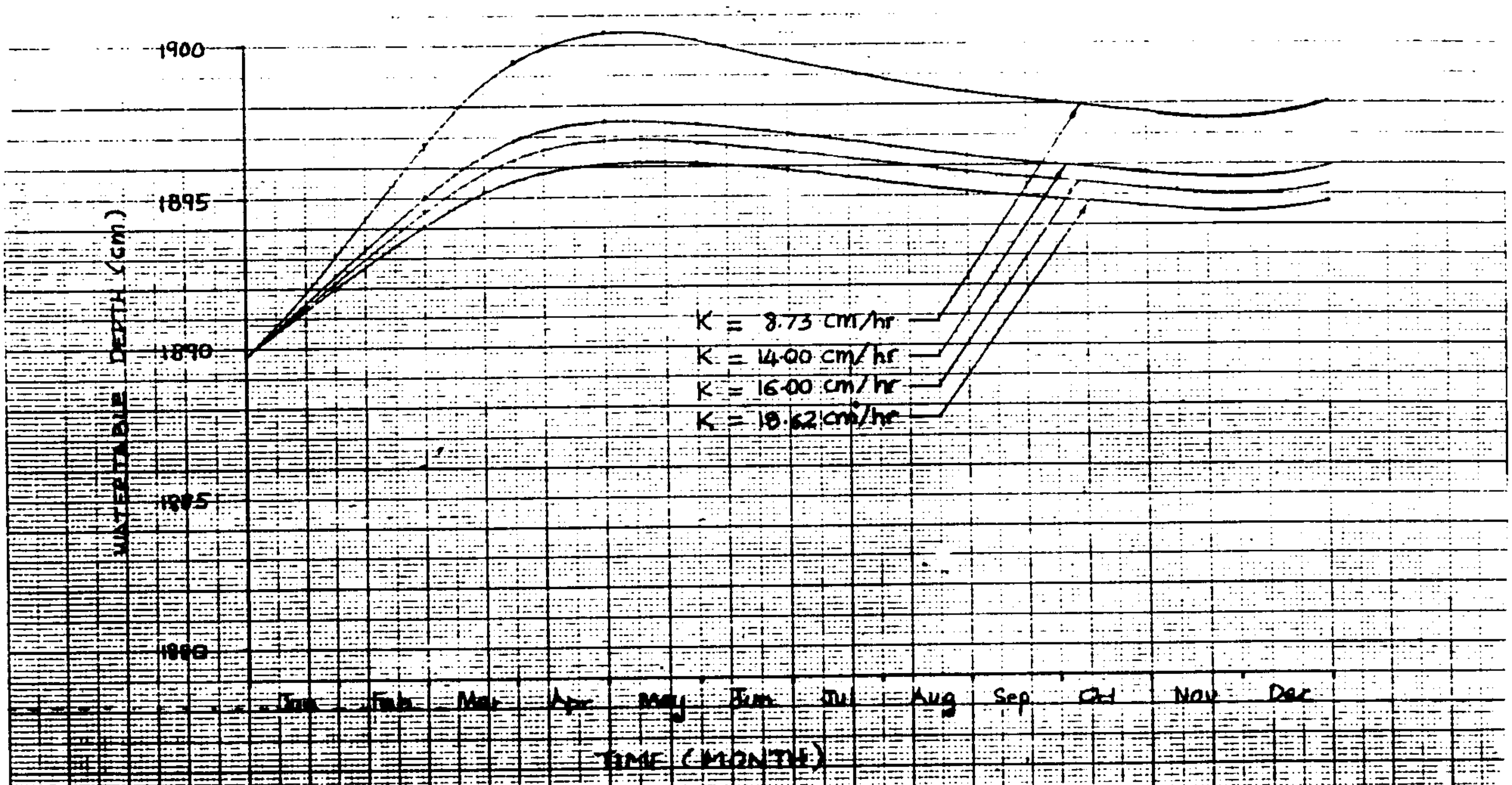

FIG. 17 Watertable Profiles in Response to Different Values of Hydraulic Conductivities at Station 950m (S = .50)