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SECOND AUTUMN WORKSHOP ON
CLOUD PHYSICS AND CLIMATE

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INVESTIGATIONS OF CLOUD DROPLET SPECTRA

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1. OBSERVATIONS OF CLOUD DROPLET SPECTRA.

A cloud droplet spectrum is defined as the size distribution of cloud droplets. There has been a great deal of interest in studies of cloud droplet spectra and other aspects of warm cloud microphysics in recent years. This is in part because the evolution of cloud drops into rain drops via coalescence processes is the dominant mechanism for precipitation formation over a large part of the globe, and also because precipitation formation via ice processes may depend on characteristics of the droplet spectrum (Hallet and Mossop, 1974; Hobbs and Ragno, 1985). Additional interest has arisen from the desire to artificially enhance precipitation and from studies of aerosol and cloud chemistry.

Much effort has been spent on explaining various aspects of the cloud droplet spectrum; e.g. mean radius, droplet spectral width, skewness, possible bimodality and the wide range of measured drop sizes. The description of observed cloud droplet spectra may conveniently be contrasted with the description of cloud droplet spectra calculated with adiabatic cloud models; i.e. models which do not allow for mixing between the cloud and its environment. Fig. 1 shows two cloud drop spectra observed by Warner (1969 a) and Fig. 2 shows a droplet spectrum calculated with an adiabatic parcel model.

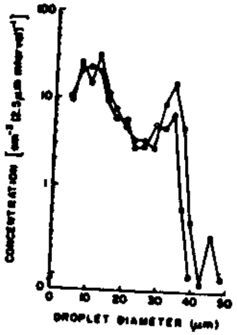

Fig. 1. Two observed cloud droplet spectra taken 100 m apart near the top of a cloud 1400 m deep. The cloud was a marine cumulus observed in Australia. (From Warner, 1969 a).

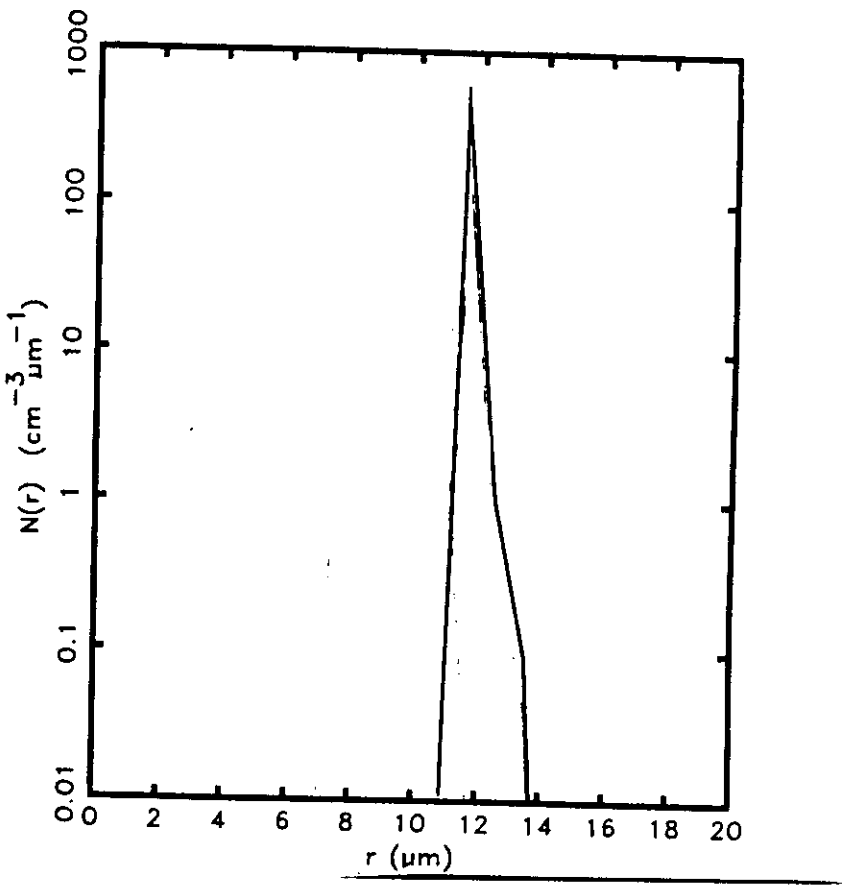

Fig. 2. Cloud droplet spectrum at 2 km above cloud base calculated with an adiabatic parcel model. The spectrum grew on a continental CCN distribution. (From Jensen, 1985).

Although the two spectra shown in Fig. 1 and 2 do not pertain to identical conditions, they do show strikingly different characteristics which can not easily be explained.

The differences to focus on are:

(1) The adiabatic spectrum has a single peak, but the observed spectra often show several peaks; i.e. observed spectra are often bimodal or even multimodal. (Warner, 1969 a; Paluch and Knight, 1984).

(2) The adiabatic spectrum is very narrow, no small droplets exist and the concentration of large drops is very small. The observed droplet spectra are very wide; they contain droplets of all sizes from the smallest to the very largest.

(3) The calculated adiabatic spectrum shows a simple positively skewed distribution. The observed spectra lack this simple characteristic.

The differences may occur due to several reasons, some of which are:

(a) Whereas the model is adiabatic, it is not certain that the observed cloud droplet spectra shown in Fig. 1 were taken in undiluted ("adiabatic") cloud regions. Mixing of cloudy, saturated air with clear, subsaturated air from outside the cloud will evaporate cloud drops. If some cloud drops evaporate more than others, then a wider and possibly bimodal spectrum may arise.

(b) The two spectra shown in Fig. 1 and 2 grew (presumably) on very different CCN spectra. This may explain some differences in the width of cloud spectra, but not as much as shown in the two figures. When using an adiabatic parcel model and a maritime CCN spectrum, a relatively narrow and unimodal spectrum will also result.

(c) The observed spectra may already contain such large drops that coalescence

has begun. The calculated adiabatic spectrum is solely the result of condensation.

(d) When a parcel moves up through cloud base a large number of cloud droplets will nucleate immediately above cloud base. This is in response to a peak in the vapor super-saturation, the size of which is determined in part by the updraft speed. If at some higher level in the cloud the parcel accelerates to obtain a much stronger vertical velocity, then a secondary peak in the super-saturation may occur. If this peak is larger than the one at cloud base, then additional nucleation of droplets may occur. Activation of drops high up in the cloud is often refered to as secondary activation.

(e) Observations of cloud droplet spectra are extremely difficult to perform accurately. Any physical measurement is affected by some uncertainty; in the case of cloud drops sampled from airplane the uncertainty may well be large. The measurements in Fig. 1 were obtained by exposing a sooted glass slide a short time period from an aircraft. The droplets impinge on the soot and the impact creates a "crater", the size of which is related to the drop-size. More modern sensors uses laser beams in which the drops scatter light. Although such instruments are highly sophisticated, they are nevertheless affected by a large number of uncertainties, some of which may distort the spectrum considerably by creating false "tails" on the distribution or even introduce extra peaks. (The differences between the spectra in Fig. 1 and 2 are so large that they can not be explained as resulting from instrumental uncertainties.)

For a number of reasons it is important to have accurate knowledge of details of the droplet spectrum. It is particularly important for investigations of the initial formation of raindrop embryos. It has long been recognized that cloud drops must grow larger than some minimum size before they can sediment and collide efficiently with other cloud drops (Hocking, 1959; Jonas, 1972). Some references repport cases in which warm rain falls from clouds which according to classical theory should not be able to produce precipitation. Mason (1969) thus states:

"Although the computations suggest that rain might develop inside a continental cloud in 50 minutes and in a maritime cloud within 50 min if the average liquid water content were 1 g m^{-3}, showers are in fact sometimes observed to fall from small cumulus clouds only 1-1.5 km deep whose average water contents are probably a good deal less than 1 g m^{-3}, and for which the theory would require considerably longer times than those quoted above."

Apparently these reports are from clouds which were observed at a distance and not explicitly probed with instrumented airplanes.

The processes responsible for the observed shapes of droplet spectra have been the topic of an intense and at times heated debate since the early studies of Zaitsev (1950), Weickman and aufm Kampe (1953) and Squires (1958). Squires found very abrupt changes in the droplet populations during horizontal traverses of cumuli and thus confirmed Warner's (1955) finding that the liquid water content likewise shows very sharp transitions.

Warner (1969a, b) investigated droplet spectra in a number of Australian cumuli and noted that they do not in general posses the positively skewed distributions predicted by the adiabatic parcel models. The dispersion ($\sigma_r/\bar{r}$) was about 0.2 near cloud base as shown theoretically by Twomey (1966), but did not decrease with distance above cloud base as adiabatic parcel models would predict. For some samples the dispersion increased with altitude and this was in part attributed to the more frequent appearance of bimodal droplet spectra with height above cloud base.

The bimodal droplet spectra could not in general be explained by turbulent variations in the updraft speed. Warner also found that small drops are present at all levels, and that these small drops were smaller than classical theories predict. Warner concluded that some process common to all cumuli must be responsible for their formation. Based on the simulatneous occurence of small and large drops, he concluded that the accomodation coefficient for condensation on drops must be between about 0.02 and 0.05. In a later study Warner (1974) used parcel models with and without entrainment to conclude that simple mixing between cloud and environment could not be responsible for the shape of the droplet spectrum.

Three recent theories have been proposed for the formation of the largest drops:

Johnson (1982) proposed that ultra-giant nuclei (radius r greater than 10 μ) consisting primarily of soil dust or sea salt are often present in high enough concentrations to initiate the precipitation process. Johnson suggested that the process occurs readily in maritime locations where sea-salt particles are abundant, but not necessarily in continental situations where large soil dust particles may or may not always be present.

The two other recent theories for the formation of large cloud droplets both invoke the effects of entrainment and mixing rocesses. Based on laboratory experiments by Latham annd Reed (1977) and some time constant arguments, Baker and Latham (1979) and Baker, Corbin and Latham (1980) suggested that inhomogeneities in the mixing process might remove some droplets and leave others unaffected. This inhomogeneous evaporation process would thus reduce the number of droplets both due to total evaporation of some drops and due to the dilution with the entrained air. The remaining drops would therefore be exposed to favoured conditions during subsequent ascent. They could possibly grow to sizes larger than the largest drops in the undiluted ("adiabatic") parcels; i.e. "super-adiabatic" growth would result.

Telford and Chai (1980) suggested that the significant factor in enhancing the size of a few droplets is the repeated vertical cycling of cloudy parcels ("entities") to cloud top with entrainment there resulting in a dilution of the droplet population. As the parcel descends and almost dries out, only the very largest dops will survive and grow at the expense of small drops following mixing with ascending air-streams.

Baker and Latham (1982) used measurements from a cap cloud in Great Britain to show a case where the droplet concentration varied considerably, but the mean radius and droplet spectral shape was almost invariant. These observations were consistent with the Baker and Latham (1979) inhomogeneous mixing model. Paluch and Knight (1984) used observations from cumuli penetrated in eastern Montana to show (i) that a surprisingly large number (in some penetrations all) of the droplet spectra were bimodal or multi-modal, and (ii) that the droplet peak radius (i.e. the radius of the droplet category with most droplets) often was the same in parcels with high and low droplet concentrations. Paluch and Knight suggested that the bimodal droplet spectra might be due to secondary activation and that the constancy in peak radius might be due to the fact that the droplets sampled were averaged over segments of alternating cloudy and clear air.

2. ADIABATIC CLOUD PARCEL MODEL EQUATIONS.

2.1. Simplifying assumptions.

Before proceeding with the equations for growth of droplets and change of thermodynamic parameters, we note that the following approximations have been made:

1. The model is strictly adiabatic. No mixing across the boundaries is allowed for, all condensed drops are assumed to remain in the parcel, and all radiative effects are neglected. Only condensation on droplets is calculated; coalescence is not included. Only warm cloud microphysics is included; the ice-phace has been excluded. The model is Lagrangian.

2. The model is kinematic in that the vertical velocity is assumed constant. Hydrostatic balance is assumed; i.e. pressure perturbation effects are neglected.

3. The equation of state is applied in the following form:

$$p = \rho_a R_d T \tag{1}$$

where p is pressure, ρ_a is the dry air density, R_d is the gas constant for dry air, and T is the temperature. In this formulation the effect of water vapor has been excluded.

4. The water vapor mixing ratio, q_v, is defined as:

$$q_v = \frac{\epsilon e}{p} \tag{2}$$

where ϵ=0.622 and e is the water vapor pressure.

5. The heat capacity of the system of dry air, water vapor, and liquid water has been approxximated as that of just dry air, c_{pd}.

6. The temperature dependence of a number of quantities has been neglected. This holds for the latent heat of vaporization (L_v), the surface tension of water (σ), the thermal conductivity (K), and the diffusivity of water vapor in air (D_v).

7. The solute effect has been described in terms of a Vant Hoff factor, i_v, which is assumed constant.

8. A number of other simplifications have been used in the model. These will be obvious by doing the derivation.

2.2. Model equations.

For cloud droplet radius, r_j, the droplet growth equation is used in the following version (for a derivation see e.g. Rogers (1976)):

$$\frac{dr_j}{dt} = \frac{1}{r_j} \frac{S - 1 - \frac{2\sigma}{\rho_w R_v T r_j} + \frac{3 i_v m_j M_w}{4\pi r_j^3 M_s \rho_w}}{\frac{L_v^2 \rho_w}{K R_v T^2} + \frac{R_v T \rho_w}{D_v e_s}} \tag{3}$$

Here subscript j refers to CCN or droplet class, S is the vapor saturation ratio, σ is the surface tension of water, ρ_w is the density of liquid water, R_v is the gas constant for water vapor, m_j is the CCN mass, M_w and M_s are the molar weights of water and aerosol respectively, and e_s is the saturation water vapor pressure. (A complete symbol list is given in Appendix 1).

The change of water vapor mixing ratio, q_v, is expressed as:

$$\frac{dq_v}{dt} = -\sum_{j=1}^{j_{max}} 4\pi \rho_w N_j r_j^2 \frac{dr_j}{dt} \tag{4}$$

where j_{max} is the total number of aerosol or droplet classes, and N_j is the number of CCN or droplets in the j'th class.

For an adiabatic process, the moist static energy

$$H = c_{pd} T + L_v q_v + gz \tag{5}$$

is approximately constant. Here g is the acceleration due to gravity and z is the vertical coordinate. Hence the change in temperature can be described as:

$$\frac{dT}{dt} = -\frac{L_v}{c_{pd}}\frac{dq_v}{dt} - \frac{g}{c_{pd}}\frac{dz}{dt} \tag{6}$$

The approximate change of air density, ρ_a, is found by differentiating the equation of state with respect to time. By assuming hydrostatic balance, the following expression can be derived:

$$\frac{d\rho_a}{dt} = -\frac{\rho_a}{T}\frac{T}{dt} - \frac{\rho_a g}{R_d T}\frac{dz}{dt} \tag{7}$$

The change of height above cloud base with time, z, is simply given by:

$$\frac{dz}{dt} = w \tag{8}$$

where w is the vertical air velocity.

The model includes a number of additional equations which are not differential equations. Specificly the saturation water vapor pressure, e_s, is found from the Clausius-Clayperon equation, and the saturation ratio, S, is calculated from the known mixing ratio, q_v, and the saturation mixing ratio calculated from e_s.

2.3. Initial conditions.

The model coded for ITCP uses 6 classes of CCN and therefore 6 classes of drops are calculated. The aerosols are assumed to consist of ammonium-bisulphate, NH_4HSO_4, with a Vant Hoff factor, i_v=2. The aerosol size distribution ranges in mass from 7.46×10^{-19} to 1.61×10^{-15} kg.

Additional input parameters are given in the input-file to the model. These are cloud base pressure (900 mb), cloud base temperature (10 °C), and vertical velocity (5 m s^{-1}). The integration time-step is initially given as 0.1 s, but all these parameters may of course be changed to reflect other conditions.

2.4. Tasks for participants in the ICTP course.

The adiabatic parcel model has already been coded up; however, only a "skeleton" version is supplied for the participants. The reasoning behind this is that little would be learned by the participants by supplying them with a complete model, and that it would be too time consuming for everybody if the model should be coded up in its entirety by the ICTP participants during the course. Hence a "skeleton" program is suplied in which the following crucial parts are missing: (i) the calculation of the initial drop-sizes at cloud base, and (ii) the subroutine containing the differential equations; i.e. eqs. (3), (4), (6), (7) and (8).

The code is well documented and consists of three parts: a main program with declarations, file open statements, values of constants, data read statements, initial conditions (some of which should be supplied by you), a main loop with calls to the Runge-Kutta routine (see below) and various print statements.

The Runge-Kutta routine ("subroutine runge" in the code) is a routine for solving first-order, linear differential equations by simple steps forward in time. The Runge-Kutta routine calls the subroutine containing the differential equations ("subroutine diffeq" in the code). In this last subroutine it is your task to code the differential equations.

The task is therefore to (i) write the two missing core parts of the code, (2) to run the program with different integration time-steps in order to investigate stability of the numerical procedure as well as the accuracy of the results, and (iii) to run the program with different meteorological conditions (e.g. updraft speed, cloud base pressure and temperature etc.). Plot on graph paper the following parameters as functions of height: dropsizes, saturation ratio, temperature, water vapor mixing ratio etc. Notice how these vary as the input conditions (updraft speed, cloud base pressure and temperature etc.) are changed.

When writing the code, it is initially preferable to assume that the cloud droplets have their equilibrium size at cloud base. This simplifies the calculation, but it is only rarely the case that droplets are in equilibrium at cloud base. This point and other suggestions for improving the code are given in section 2.5.

2.5. Improvements to the model.

The present adiabatic parcel model is as simple as possible. There are many ways in which it can be improved. Some of these are very worthwhile, whereas others will make little difference.

1. The water vapor mixing ratio, q_v, can be exactly calculated as:

$$q_v = \frac{\epsilon e}{p - e} \tag{9}$$

2. The heat capacity, c_p of the system of dry air, water vapor and liquid water is not just the value for dry air, c_{pd}, but instead:

$$c_p = c_{pd} + q_v c_v + q_l c_w \tag{10}$$

Here q_l is the liquid water mixing ratio, and c_v and c_w are the heat capacities of water vapor and liquid water respectively.

3. In the equation of state, the temperature, T, should be changed to the virtual temperature, T_v.

4. Cloud droplets or rather solution droplets will usually not be in equilibrium at cloud base; this is because the small surface area of droplets right below cloud base. Instead the calculation should be started at a relative humidity of e.g. 98 %. This will lead to considerably smaller droplets growing on the aerosol particles. It introduces considerable numerical problems because the solution of the droplet growth equation requires very small time-steps for values of saturation ratio, S, of 0.98 when normal updraft speeds of a few meters per second occur.

5. The vertical velocity can be changed to reflect the acceleration and decelelartion as a parcel moves up in a cumulus cloud. This can be done either by specifying

w as function of time, or by making the vertical velocity dependent on the parcel buoyancy. In the latter case the rate of change of vertical velocity is given by:

$$\frac{dw}{dt} = \frac{g(T_{vc} - T_{ve})}{T_{ve}} \tag{11}$$

In this equation T_{vc} and T_{ve} represents the virtual temperatures of the cloud and environment resectively. Additional terms could be introduced into eq. (11) by considering drag on the parcel.

The representation of the solute effect may be changed from the vant Hoff factor, i_v, to using chemical potential theory, see Pruppacher and Klett (1981).

6. The CCN spectrum is only defined for a limited size range. Other CCN spectra may be found in e.g. Pruppacher and Klett (1978). Note, however, that the specification of larger and smaller CCN than the ones used in this model may result in numerical difficulties.

7. The effects of inhomogeneities in the condensation process can be included by changing the "vapor diffusion term" (the last in the denominator of eq. (3)) to:

$$\frac{R_v T \rho_w (r_j + \lambda)}{D_v e_s r_j}$$

This change is due to Fukuta and Walter (1971). λ is the condensation length. The effect of this term is to decrease the condensation rate and thus increase the saturation ratio, S.

8. A simple way to abandon the adiabatic principle is to allow for mixing across the parcel boundaries. One way this can be done is by continuous entrainment into the parcel as it ascends. Continuous entrainment may or may not be a good idea; however, it is relatively simple to describe mathematically. The equations for water vapor mixing ratio and temperature would thereby each have one additional term and look like:

$$\frac{dq_v}{dt} = -\sum_{j=1}^{j_{max}} 4\pi\rho_w N_j r_j^2 \frac{dr_j}{dt} + \frac{1}{M}\frac{dM}{dt}(q_{ve} - q_v) \tag{12}$$

In this equation M is the mass of the parcel and q_{ve} is the water vapor mixing ratio in the environment. Additional equations for the change of cloud droplet concentration, N_j, and mass, M, with time would also be needed. The change of mass with time could either be given as constant or e.g. by similarity theory (Morton, Taylor and Turner; 1956).

9. Temperature dependence of L_v, K, D_v, and σ may be included.

10. The ice-phase can be included by defining the number of ice nuclei active as function of temperature, and by having an "ice growth" equation.

11. The vant Hoff factor may be abandoned and the solute effect may instead be described by the chemical potential, see Pruppacher and Klett (1978).

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Symbol list

D_v	Diffusivity of water vapor in air ($m^2\ s^{-1}$)
H	Moist static energy ($J\ kg^{-1}$)
K	Thermal conductivity ($J\ m^{-1}\ s^{-1}\ K^{-1}$)
L_v	Latent heat of vaporisation ($J\ kg^{-1}$)
N_j	CCN or drop concentration in class j (# kg^{-1})
R_d	Gas constant for dry air($m\ s^{-1}\ K^{-1}$)
S	Vapor saturation ratio
T	Temperature (K)
c_p	Heat capacity of dry air, vapor and liquid water ($J\ kg^{-1}\ K^{-1}$)
c_{pd}	Heat capacity of dry air ($J\ kg^{-1}\ K^{-1}$)
c_v	Heat capacity of water vapor ($J\ kg^{-1}\ K^{-1}$)
c_w	Heat capacity of liquid water ($J\ kg^{-1}\ K^{-1}$)
e_s	Saturation water vapor pressure (Pa)
g	Gravity acceleration ($m\ s^{-1}$)
i_v	Vant Hoff's factor
j	Droplet or CCN category
q_l	Liquid water mixing raio ($kg\ kg^{-1}$)
q_v	Water vapor mixing raio ($kg\ kg^{-1}$)
p	Pressure (Pa)
r_j	Droplet radius for class j (m)
t	Time (s)
w	Vertical wind speed ($m\ s^{-1}$)
z	Vertical coordinate (m)
ϵ	Ratio of molar weight of water to that of dry air
ρ_a	Air density ($kg\ m^{-1}$)
ρ_w	Liquid water density ($kg\ m^{-1}$)
σ	Surface tension of water against air ($N\ m^{-1}$)

FINAL VERSION
WITH ENTIRE CODE

```
c      program drop
c
c      lagrangian adiabatic parcel cloud model.
c      calculate the thermodynamic properties and cloud
c      droplet size in a rising airstream.
c
c      -----------------------------------------------------------
c
c      This program was developed for the 1987 International
c      Centre for Theoretical Physics' second college on Cloud
c      Physics and Climate in Trieste, Italy.
c      A set of lecture notes by Jorgen B. Jensen describes
c      the derivation of the model equations.
c
c      For further information, please contact:
c              Dr. Jorgen B. Jensen
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c
c      or:
c              Prof. Marcia B. Baker
c              Geophysics Program, AK-50
c              University of Washington
c              Seattle, Washington 98195
c              U. S. A.
c
c
c      -----------------------------------------------------------
c
c      units: the program is written in MKS with two exceptions:
c             (i) cloud base pressure and temperature are read in
c             in units of (mb) and (degC)
c             (ii) drop radii are calculated in units of (m),
c             but printed out in units of (micron)
c
c      -----------------------------------------------------------
c      variable description:
c
c      arrays:
c        dfdt           : differential coefficients of array f
c        f              : array containing dependent variables
c                         which should be integrated with time
c        f(1) to f(6)   : droplet radii (m) (six classes)
c        f(7)           : water vapor mixing ratio (kg kg-1)
c        f(8)           : temperature (degK)
c        f(9)           : air density (kg m-3)
c        f(10)          : altitude above cloud base (m)
c
c        ccnmass(1) to ccnmass(6) : mass of individual cloud
c                                   condensation nuclei (kg)
c
c        number(1) to number(6) : number of ccn/drops in each
c                                 size class pr mass of air
c                                 (# kg-1)
c
c      normal variables:
c
c        deltat : integration time-step (s)
```

```
c        es     : saturation water vapor pressure (pascal)
c        j      : counter for droplet classes
c        jmax   : number of differential equations to
c                 integrate
c        n      : counter for integration step
c        nmax   : total number of integeration time-steps
c        p      : pressure (pascal)
c        pbase  : pressure at cloud base (first in mb,
c                 then in pascal)
c        qvs    : saturation water vapor mixing ratio (kg kg-1)
c        s      : saturation ratio
c        tbase  : temperature at cloud base (first in degC,
c                 then in degK)
c        term1  : surface term in droplet growth equation
c        term2  : solution term in droplet growth equation
c        term3  : heat conduction term in droplet growth
c                 equation
c        term4  : vapor diffusion term in droplet growth
c                 equation
c        time   : time since integration has started (s)
c        vvel      : vertical air velocity (m s-1)
c
c      physical constants:
c
c        cpd    : dry air heat capacity (j kg-1 degK-1)
c        epsilo : ratio of molar weight of water to
c                 molar weight of dry air
c        g      : gravity acceleration (m s-2)
c        kdiff  : heat conductivity of dry air
c                 (j m-1 s-1 degK-1)
c        lv     : latent heat of vaporization (j kg-1)
c        pi     : pi
c        msalt  : molar weight of salt (here ammonium-
c                 bisulphate) (kg)
c        mwater : molar weight of water (kg)
c        rd     : dry air gas constant (j kg-1 degK-1)
c        rhow   : density of liquid water (kg m-3)
c        rv     : gas constant of water vapor (j kg-1 degK-1)
c        sigma  : surface tension of water against air (n s-1)
c        vdiff  : diffusivity of water vapor in air (m2 s-1)
c        vhoff  : vant hoff factor
c
c
c      system variables for opening files
c
c        file1 : data file
c        file2 : result file
c
c
c      -----------------------------------------------------------
c
c      declaration of variables:

       dimension f(10),ccnmass(6)
       real number(6),kdiff,lv,mwater,msalt
       character*50 file1,file2
c
c      -----------------------------------------------------------
c
c      common blocks:
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```
c
      common /main/ ccnmass,number,vvel,s
      common /const/ cpd,epsilo,g,kdiff,lv,msalt,mwater,pi,
     c               rd,rhow,rv,sigma,vdiff,vhoff
c
c     -------------------------------------------------------
c
c     define constants
c
      data cpd /1.046e3/
      data epsilo /0.622/
      data g /9.8/
      data kdiff /2.43e-2/
      data lv /2.5e6/
      data msalt /0.11511/
      data mwater /0.016/
      data pi /3.1412/
      data rd /2.87e2/
      data rhow /1.e3/
      data rv /4.186e2/
      data sigma /0.07275/
      data vdiff /0.226e-4/
      data vhoff /2./
c
c     -------------------------------------------------------
c
c     open files (standard unix notation):
c
c     data file:
      call getarg(1,file1)
      open(3,file=file1)
      rewind 3
c
c     result file:
      call getarg(2,file2)
      open(4,file=file2)
      rewind 4
c
c
c     -------------------------------------------------------
c
c     read values for physical parameters
c
c     read cloud base pressure (mb) and temperature (degC)
      read(3,*) pbase,tbase
c     convert these to MKSA units
      pbase=pbase*100.
      tbase=tbase+273.16
c
c     read updraft speed (m s-1)
      read(3,*) vvel
c
c     read the integration time-step (s) and the total
c     number of time-steps
      read(3,*) deltat,nmax
c
c     read ccn mass (kg) and number concentration (# kg-1)
c     for 6 classes
      do 10 j=1,6
        read(3,*) ccnmass(j),number(j)
10    continue
```

```
c
c
c
c
c     -------------------------------------------------------
c
c     define all initial values and conditions:
c
c     define all elements of array f
c
c     water vapor mixing ratio (f(7)):
c     first find the saturation vapor pressure at cloud base
      es=6.11e2*exp(lv/rv*(1./273.16-1./tbase))
c     find water vapor mixing ratio
      f(7)=epsilo*es/pbase
c     saturation ratio
      s=1.
c
c     temperature
      f(8)=tbase
c
c     air density
      f(9)=pbase/(rd*tbase)
c
c     height above cloud base
      f(10)=0.
c
c
c
c     .......................................................
c     define initial droplet radii
c     assume that droplet size is in equillibrium
c     exactly at saturation (RH=100 %)
c
      do 100 j=1,6
c       find the surface term in the droplet growth equation
        term1=2.*sigma/(rhow*rv*tbase)
c       find the solution term in the droplet growth equation
        term2=vhoff*ccnmass(j)*mwater/
     c        (msalt*4./3.*pi*rhow)
c       the equillibrium droplet size at saturation is
        f(j)=sqrt(term2/term1)
100   continue
c     .......................................................
c
c     define other initial conditions:
c
c     integration time:
      time=0.
c
c     number of equations to integrate
      jmax=10
c
c     all initial conditions are now defined
c     -------------------------------------------------------
c
c     write the initial conditions out to the result file
c
      write(4,7000)
      write(4,7020) (ccnmass(j),j=1,6)
      write(4,7030) (number(j),j=1,6)
```

```
         write(4,7040) (f(j),j=1,6)
         write(4,7050) f(7)
         write(4,7060) f(8)
         write(4,7070) f(9)
         write(4,7080) f(10)
         write(4,7090) jmax
         write(4,7100) nmax
         write(4,7110) time
         write(4,7120) deltat
         write(4,7130)
         write(4,7140)
c        ------------------------------------------------------------
c
c
c        cycle through the main loop and integrate to find the
c        values of array f in time:
c
         do 2000 n=1,nmax
c
c          print out messages every 10 timesteps to result file
c          (if very close to the cloudbase, print for
c           each timestep)
           if((f(10).le.100.).or.
     c        (f(10).ge.100.and.mod(n-1,10).eq.0)) then
             write(4,8000)
             write(4,8010)
             write(4,8020)
             write(4,8030)
c            write droplet radii in units of micron
             write(4,8040) n,time,(f(j)*1.e6,j=1,6)
             write(4,8050)
             write(4,8060)
             write(4,8070)
c            write mixing ratio in units of g/kg
             write(4,8080) f(7)*1000.,f(8),f(9),f(10),s
           endif
c
c          call the fourth-order runge-kutta integration routine
           call runge(deltat,time,f,jmax)
 2000    continue
c
c        ------------------------------------------------------------
c
c        format section
 7000 format('====================== initial conditions etc.'
     c '  =====================')
 7020 format('  (ccnmass(j),j=1,6)  =',6e12.3,'  (kg)')
 7030 format('  (number(j),j=1,6)   =',6e12.3,'  (# kg-1)')
 7040 format('  drop size (f(1-6))  =',6e12.3,'  (m)')
 7050 format('  mixing ratio        =',e12.3,'  (kg kg-1)')
 7060 format('  temperature         =',f12.2,'  (degK)')
 7070 format('  air density         =',e12.3,'  (kg m-3)')
 7080 format('  height              =',f12.2,'  (m)')
 7090 format('  number of equations to integrate =',i8)
 7100 format('  total number of time-steps       =',i8)
 7110 format('  time                             =',f8.2,
     c '  (s)')
 7120 format('  deltat (time-step increment)     =',f8.2,
     c '(s)')
 7130 format('=================================================')
 7140 format(////'===== model results : ===================')
```

```
 8000 format('===================================================',
     c '==================================')
 8010 format(' step   time ------- drop radii ----------',
     c '------')
 8020 format('     n       t  f(1)  f(2)  f(3)  f(4)  f(5)',
     c '  f(6)')
 8030 format('     #     sec    mu    mu    mu    mu    mu',
     c '     mu')
 8040 format(i5,f7.1,6f6.2)
 8050 format('  mixing ratio   temperature   air density',
     c '         height     saturation ratio')
 8060 format('                   f(7)                f(8)          f(9)',
     c '           f(10)                              s')
 8070 format('                  g kg-1              degK         kg m-3',
     c '                 m                            -')
 8080 format(f14.3,f14.2,f14.3,f14.2,f20.4)
c
c
         stop
         end
c        ===========================================================
c        ===========================================================
c        ===========================================================
c
         subroutine runge(deltat,time,f,jmax)
c
c        standard runge-kutta 4'th order integration routine for
c        integrating a set of linear, first-order differential
c        equations.
c
c        declaration of variables:
c
         dimension f(10),f1(10),f2(10),f3(10)
c
         d2=deltat/2.
         d6=deltat/6.
         call diffeq(f,f1)
         do 10 j=1,jmax
           f2(j)=f(j)+d2*f1(j)
   10    continue
         call diffeq(f2,f3)
         do 20 j=1,jmax
           f1(j)=f1(j)+2.*f3(j)
           f2(j)=f(j)+d2*f3(j)
   20    continue
         call diffeq(f2,f3)
         do 30 j=1,jmax
           f1(j)=f1(j)+2.*f3(j)
           f2(j)=f(j)+deltat*f3(j)
   30    continue
         call diffeq(f2,f3)
         do 40 j=1,jmax
           f(j)=f(j)+d6*(f1(j)+f3(j))
   40    continue
         time=time+deltat
         return
         end
c
c        ===========================================================
c        ===========================================================
c        ===========================================================
```

```
c
      subroutine diffeq(f,dfdt)
c
c     this subroutine contains the differential equations
c
c     --------------------------------------------------------
c
c     content of variables: see main program for most
c     variables; other ones are explained below.
c
c     arrays:
c       dfdt(1) to dfdt(6) : rate of change of droplet radius
c       dfdt(7)            : rate of change of water vapor
c                            mixing ratio
c       dfdt(8)            : rate of change of temperature
c       dfdt(9)            : rate of change of air density
c       dfdt(10)           : rate of change of height
c
c     --------------------------------------------------------
c
c     declaration of variables:
c
      dimension f(10),dfdt(10),ccnmass(6)
      real number(6),kdiff,lv,mwater,msalt
c
c     --------------------------------------------------------
c
c     common blocks:
c
      common /main/ ccnmass,number,vvel,s
      common /const/ cpd,epsilo,g,kdiff,lv,msalt,mwater,pi,
     c               rd,rhow,rv,sigma,vdiff,vhoff
c
c     --------------------------------------------------------
c
c     find the saturation ratio of water vapor:
c     saturated water vapor pressure:
      es=6.11e2*exp(lv/rv*(1./273.16-1./f(8)))
c     find pressure:
      p=f(9)*rd*f(8)
c     saturated water vapor mixing ratio
      qvs=epsilo*es/p
c     find the saturation ratio:
      s=f(7)/qvs
c
c
c     calculate derivatives of array f; i.e. dfdt.
c
c     dropsizes:
      do 10 j=1,6
c       surface term in the droplet growth equation:
        term1=2.*sigma/(rhow*rv*f(8)*f(j))
c       solution term in the droplet growth equation:
        term2=vhoff*ccnmass(j)*mwater/
     c           (msalt*4./3.*pi*rhow*f(j)**3)
c       heat conduction term in denominator of the droplet
c       growth equation:
        term3=lv**2*rhow/(kdiff*rv*f(8)**2)
c       vapor diffusion term in the denominator of the droplet
c       growth equation:
        term4=rv*f(8)*rhow/(vdiff*es)
```

```
c       droplet growth equation:
        dfdt(j)=(1./f(j))*
     c             (s-1-term1+term2)/
     c             (term3+term4)
c       write(0,7010) term1,term2,term3,term4,dfdt(j)
 7010   format(5e12.3)
 10   continue
c
c     change of water vapor mixing ratio due to condensation:
      dfdt(7)=0.
      do 20 j=1,6
        dfdt(7)=dfdt(7)-4.*pi*rhow*number(j)*f(j)**2*dfdt(j)
 20   continue
c
c     change of temperature:
      dfdt(8)=-lv/cpd*dfdt(7)-g/cpd*vvel
c
c     change of air density:
      dfdt(9)=-f(9)*g*vvel/(rd*f(8))-f(9)*dfdt(8)/f(8)
c
c     change of height:
      dfdt(10)=vvel
c
c     --------------------------------------------------------
c
      return
      end
```