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UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

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SECOND AUTUMN WORKSHOP ON

CLOUD PHYSICS AND CLIMATE

(23 November - 18 December 1987)

CLOUD MODELS

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I. Introduction

A cloud model is a set of equations that determines the microphysical, thermodynamic, and dynamic parameters within a cloud as functions of space and time. Models are used in two ways: the first is to test, by comparing model predictions and field observations, our descriptions of cloud processes and our understanding of the underlying physics; the second is to simulate proposed experiments or atmospheric events in order to predict outcomes of situations that we cannot easily observe in the real world.

No single cloud model can provide a realistic description of all cloud parameters as they vary over space and time. Cloud models are always approximations of real cloud behavior, and the proposed use of a model determines which approximations are sufficient. The set of equations used in a particular application is often truncated; no attempt is made to realistically simulate processes that are not of direct interest to that application. In these lectures we will identify the various components and governing equations for several simple cloud models. A list of references given at the end will guide the interested reader to detailed discussions of the topics touched upon here.

Table I shows the variables to be determined by a cloud model.

Table I

Variables		Symbols	Dimensions
Thermodynamic	pressure of cloudy air density of cloudy air temperature of cloudy air	p ρ T	N/m^2 kg/m^3 K or °C
Dynamic	air velocity components	u,v,w	m/s
Water category	specific total water content	Q	kg H_2O/kg cloudy air
	specific humidity	q_v	kg vapor/kg cloudy air
	specific liquid water content	q_l	kg liquid/kg cloudy air
	specific ice content	q_i	kg ice/kg cloudy air
Microphysical	concentration of drops of mass between m_d and $m_d + dm_d$	$n_d(m_d)dm_d$	number/m^3 cloudy air
	concentration of ice particles of mass between m_i and $m_i + dm_i$	$n_i(m_i)dm_i$	number/m^3 cloudy air

We will briefly describe models for each of these sets of variables.

II. Thermodynamic Variables

We assume that both dry air (density ρ_d, pressure p_d) and vapor (density ρ_v, pressure e) behave like ideal gases, so that $q_v = \frac{\rho_v}{(\rho_d+\rho_v)}$, etc., and

$$p = p_d + e : p_d = \rho_d R_d T, \; e = \rho_v R_v T \tag{1}$$

or

$$p \cong \rho R_d T_v \tag{2}$$

where

$$T_v = T\left[1 + q_v\left\{\frac{R_v}{R_d} - 1\right\} - q_i - q_l\right] \tag{3}$$

where R_v and R_d are the gas constants for vapor and dry air. For future reference, we also define the saturation vapor pressures e_{si} and e_{sl}. These are the vapor pressures that characterize air in equilibrium over a flat surface of ice or of water. They are given by the Clausius-Clapeyron equation,

$$\frac{de_{sl}}{dT} = \frac{L_v e_{sl}}{R_v T^2} \tag{4a}$$

$$\frac{de_{si}}{dT} = \frac{L_s e_{si}}{R_v T^2} \tag{4b}$$

where L_s and L_v(J/kg) are the latent heats of sublimation and of evaporation, respectively. (Numerical values for the temperature-dependent latent heats, saturation vapor pressures, and other thermodynamic properties of cloudy air may be readily computed from formulae given in Bolton, 1980.)

Equations (1) to (4) are algebraic equations that link values of several thermodynamic parameters at a given point in space and time. The other variables in Table I are determined by differential equations that yield their values at one point to those at other points. The fundamental form for these equations is the following, letting ξ represent a variable in Table I.

$$\frac{D\xi}{Dt} = \frac{\partial\xi}{\partial t} + \underline{V}\cdot\underline{\nabla}\xi = \text{Source}_{(\xi)} - \text{Sink}_{(\xi)} \tag{5}$$

where vectors are indicated by underlines: $\underline{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ and $\underline{V} = (u, v, w)$. The task of a cloud modeller is to represent the physical processes denoted "sources and sinks" in these conservation equations by realistic, yet manageable mathematical functions of the selected variables.

The equation for ρ is derived by writing Equation (5) in the case where ξ = mass. Let M be the mass of an air parcel of volume $\mathcal{V}$.

Then $M = \rho \mathbf{V}$ and

$$\frac{DM}{Dt} = 0 = \mathbf{V}\frac{D\rho}{Dt} + \rho\frac{D\mathbf{V}}{Dt} = \mathbf{V}\left[\frac{\partial\rho}{\partial t} + \underline{V}\cdot\underline{\nabla}\rho\right] + \rho\mathbf{V}\ \underline{\nabla}\cdot\underline{V}$$

or

$$\frac{\partial\rho}{\partial t} + \underline{\nabla}\cdot(\rho\underline{V}) = 0 \tag{6}$$

For shallow subsonic motions, we can assume ρ = constant, or

$$\underline{\nabla}\cdot\underline{V} = 0 \tag{6'}$$

Furthermore, we usually assume the hydrostatic equation for the pressure, i.e.,

$$\frac{dp}{dz} = -\rho q \tag{7}$$

To derive an equation for the temperature, we first write Equation (5) in the form of a conservation equation for the moist static energy h, defined here as

$$h \equiv L_v q_v - L_f q_i + gz + c_p T$$

L_f(J/kg) is the latent heat of freezing, and C_p(J/kg–°C) is the specific heat of cloudy air. h is approximately conserved in adiabatic processes if $V^2/2 << h$, as we shall assume here. We have

$$\frac{Dh}{Dt} = \text{Source}_{(h)} - \text{Sink}_{(h)} \equiv \dot{H} \tag{8}$$

where $\dot{H}$(W/kg) represents possible external energy sources and sinks such as the radiative flux divergence or precipitation fallout.

The equation for T is found from the definition of h and Equation (8): we have

$$\frac{DT}{Dt} = \frac{gw}{c_p} + \frac{L_f}{c_p}\frac{Dq_i}{Dt} - \frac{L_v}{c_p}\frac{Dq_v}{Dt} + \frac{\dot{H}}{c_p} \qquad (9)$$

III. Dynamic Variables

There are two kinds of models for air motions in atmospheric studies: kinematic models, in which the air motions (and often the temperature) are prescribed as functions of space and time, and dynamic models, in which these are computed. Kinematic models are useful when the dynamics are simple and well understood (as in a steady state orographic, or wave cloud) or in which the links between dynamics and other cloud properties are not of first importance. Kessler's (1969) original parameterized cloud model was kinematic, as were those of Scott and Hobbs (1977) and Rutledge and Hobbs (1983). Kinematic models appear particularly useful in cloud chemistry studies, where the focus is on chemical interactions, and air motions are assumed unaffected by chemistry.

In dynamical models the air velocity components are computed from Equation (5), putting $\xi = \underline{V}$, so that Equation (5) becomes Newton's Second Law. (The sources and sinks of momentum are then the forces.) Ignoring molecular effects, we have

$$\frac{D\underline{V}}{Dt} = -\frac{1}{\rho}\underline{\nabla}p + g\hat{\underline{k}} - 2\underline{\Omega} \times \underline{V} \qquad (10)$$

$\Omega(s^{-1})$ is the angular velocity of the earth; $2\underline{\Omega} \times \underline{V}$ is the 'Coriolis' term, which is important only when the time scales of the phenomena of interest are comparable with or longer than the Coriolis time scale (~ 10^4s in mid-latitudes).

This equation can be simplified in various contexts. We now discuss two very important, one-dimensional cloud models that are often used in investigations of the macroscopic behavior of layer and convective clouds.

A. Stratiform Clouds

In layer clouds the horizontal spatial scale is much larger than the vertical scale, given by the layer depth. Therefore, the variation of base-state cloud properties in the horizontal is usually ignored completely, and the only turbulent fluxes considered are those in the vertical. Layer cloud (or fog) models are written in one, two, or three dimensions.

We shall discuss here only the so-called 'mixed layer' models, which are one-dimensional. For references on these models, see Deardorff (1976), Lilly (1986), Nicholls (1984), and Stage and Businger (1981). We assume that each variable ξ in Equation (5) can be written

$$\xi(x, y, z, t) = \bar{\xi}(z, t) + \xi'(x, y, z, t) \tag{11}$$

where the overbar represents a horizontal average. The quantity $\xi'(x, y, z, t)$ is the local, instantaneous deviation from the horizontal average and

$$\overline{\xi'}(x, y, z, t) = 0$$

Furthermore, we assume

$$|\ \xi'\ | << \bar{\xi}$$

In this case Equation (5) becomes, in the horizontal average,

$$\frac{\partial\bar{\xi}}{\partial t} + \overline{\underline{V} \cdot \underline{\nabla}\xi} = \overline{\text{Source}_\xi} - \overline{\text{Sink}_\xi} \tag{12}$$

The second term is

$$\underset{(a)}{\underline{\bar{V}} \cdot \underline{\nabla}\bar{\xi}} \ + \ \underset{(b)}{\overline{\underline{V}' \cdot \underline{\nabla}\xi'}}$$

(a) represents the effect of the large-scale mean wind $\underline{\bar{V}}$, which advects any anomaly in the mean variable $\nabla\bar{\xi}$, and (b) represents the advection of the local fluctuations, $\underline{\nabla}\xi'$, by the local turbulent velocity $\underline{V}'$. Moreover, we can show that for the layer cloud

$$\underline{\nabla} \cdot \underline{\bar{V}} = \underline{\nabla} \cdot \underline{V}' = 0 \tag{6'}$$

so

$$\underline{V}' \cdot \underline{\nabla}\xi' = \underline{\nabla} \cdot \overline{\underline{V}'\xi'}$$

The term $\underline{V}'\xi'$ is called the turbulent flux of ξ, and we assume further that on horizontal average only the vertical flux is important, so finally, Equation (12) becomes

$$\frac{\partial\bar{\xi}}{\partial t} + \underline{\bar{V}} \cdot \underline{\nabla}\bar{\xi} + \frac{\partial}{\partial z}\left(\overline{w'\xi'}\right) = \overline{\text{Source}_\xi} - \overline{\text{Sink}_\xi} \tag{12'}$$

$$= \frac{D\overline{\xi}}{Dt} + \frac{\partial}{\partial z}\left[\overline{w'\xi'}\right]$$

where $\frac{D}{Dt}$ is the time derivative in a system moving with the mean wind $\underline{\overline{V}}$. Note that if $\overline{\xi}$ remains independent of z, $\overline{w'\xi'}$ is linear in z.

We choose for our analysis the two variables:

$\xi = \Theta_e(K)$, the equivalent potential temperature,

$= \Theta \exp\left[\frac{L q_v}{c_p T}\right]$, and

$\xi = Q\left[\frac{K_g H_2O}{K_g \text{ air}}\right]$, the total water mixing ratio.

We assume $\overline{\Theta}_e(z, t)$ and $\overline{Q}(z, t)$ have the profiles displayed in Figure 1: i.e., they are constant in the boundary layer (as a result of turbulent motions, which eradicate any z-dependence) and they undergo sharp changes at the inversion that caps the layer. We assume the cloud is solid, and we adopt the notation from the Figure.

We can integrate Equation (12′) over the entire mixed layer to find

$$\int_0^{z_B} \frac{D\overline{\xi}}{Dt} dz + \int_0^{z_B} \frac{\partial}{\partial z}\left[\overline{w'\xi'}\right] dz = \int_0^{z_B} \left[\overline{\text{Source}_\xi} - \overline{\text{Sink}_\xi}\right] dz$$

or

$$z_B \frac{D\overline{\xi}}{Dt} + \overline{w'\xi'}\,|_{z_B} - \overline{w'\xi'}\,|_{z=0} = \int_0^{z_B} \left[\overline{\text{Source}_\xi} - \overline{\text{Sink}_\xi}\right] dz$$

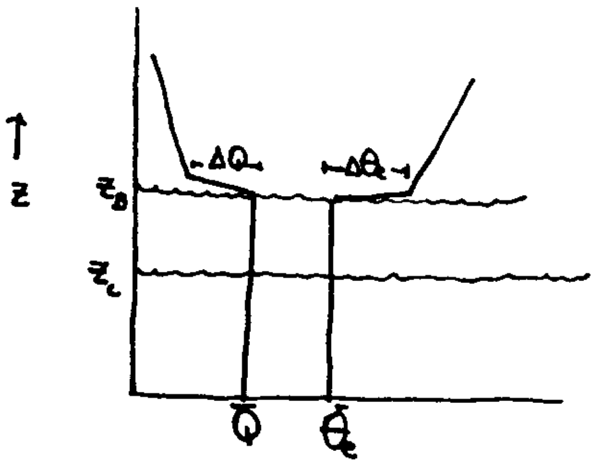
Thus

$$\frac{DQ}{Dt} = \frac{1}{z_B}\left[\overline{w'Q'}\,|_{z=0} - \overline{w'Q'}\,|_{z=z_B}\right] \tag{13a}$$

$$\frac{D\Theta_e}{Dt} = \frac{1}{z_B}\left[\overline{w'\Theta_e'}\,|_{z=0} - \overline{w'\Theta_e'}\,|_{z=z_B} + R\right] \tag{13b}$$

where R(K/s) is the rate of temperature change due to absorption of solar radiation and emission of infrared radiation. These radiative terms are either computed from models (using as inputs the microphysical cloud properties) or they are measured.

Figure 1. An idealised sounding for the cloudtopped boundary layer. Cloudbase is at $z = z_c$ and cloudtop at the inversion, $z = z_B$. Radiative cooling takes place in a very thin layer near cloudtop.



The remaining terms in Equation (13) are the turbulent fluxes of Q and Θ_e at the upper and lower boundaries. These must be parameterised, and the differences among the various mixed layer models lie in the parameterisation of these terms.

In the simplest case, the surface fluxes are expressed in terms of transfer coefficients: that is,

$$\overline{w'\xi'}\,|_o = C_\xi V \left[\xi_o - \xi\right] \tag{14}$$

where V(m/s) is the magnitude of the surface (horizontal) wind and ξ_o is the value of ξ at the surface. The C_ξ are constants known as transfer coefficients, and suggested values may be found in the references cited.

The turbulent fluxes at the upper surfaces are written in terms of an 'entrainment velocity' w_e(m/s); i.e.,

$$\overline{w'\xi'}\,|_{z=z_B} = -\,w_e \Delta\xi \tag{15}$$

where $\Delta\xi \equiv \xi$ (upper air, just above z_B) $-\ \xi$ (mixed layer, just below z_B).

Finally, we need a way of calculating w_e from the layer properties. No completely satisfactory method exists for this at present. Thus, the mixed-layer models must be 'closed' by an additional relationship which sets w_e. This is usually based on the "turbulent kinetic energy budget," where TKE $\equiv \overline{w'^2} + \overline{u'^2} + \overline{v'^2}$.

Schematically,

$$\frac{d}{dt}\int_0^{z_B} \text{TKE}\, dz = P - N - D \tag{16}$$

P is the rate of production of TKE by turbulent motions in which slightly warmer than average air moves up, or slightly cooler than average air moves down, N is the rate of consumption of TKE when slightly $\left.\begin{matrix}\text{warmer}\\ \text{cooler}\end{matrix}\right\}$ than average air moves $\left\{\begin{matrix}\text{down}\\ \text{up}\end{matrix}\right.$, and D is the dissipation of TKE at small scales. It is usually found that the LHS ≈ 0, so Equation (16) becomes

$$P - N = D \tag{16'}$$

Each of the terms in Equation (16') can be calculated in terms of the turbulent fluxes of Θ_e and Q at the layer boundaries and the radiative terms R. The closure assumptions in different models are assumptions about the relationships of these terms to one another. By invoking a closure assumption, we therefore find a relationship linking w_e to the other parameters of the problem, and then we can write (if there is no mean vertical velocity)

$$\frac{Dz_B}{Dt} = w_e \tag{17}$$

As illustration, consider a simple hypothetical case in which the surface fluxes are zero and the radiative cooling occurs in a negligibly thick layer at cloud top. We assume that the entrained air is distributed uniformly within the mixed layer (Stage and Businger, 1981). Then, ignoring radiative warming at cloudbase, we have

$$\overline{w'Q'}(z) = -\, w_e\, \Delta Q\, \frac{z}{z_B} \tag{18a}$$

$$\overline{w'\Theta'_e}\,(z) = \left[-\, w_e\, \Delta\, \Theta_e + R\right]\frac{z}{z_B} \tag{18b}$$

We now calculate P and N in terms of these fluxes. The first step is to note that at height z, the rate of production (or consumption) of TKE by motion of air parcels of different density is $\frac{g\,\overline{w'\rho'}}{\bar{\rho}}$ It can then be shown that

$$\frac{g}{\bar{\rho}}\, \overline{w'\rho'}\,(z) = \alpha \overline{w'Q'}(z) + \beta \overline{w'\Theta'_e}\,(z) \tag{19}$$

That is, the flux of density perturbations is a linear sum of (18a) and (18b): the coefficients α and β are slowly varying functions of temperature and pressure. These functions are different in saturated and in unsaturated air; we will call them α_s and β_s in saturated air and α_u and β_u in unsaturated air. (See Stage and Businger (1981) for their explicit forms.)

Let us assume that the mixtures of cloudy and clear air at cloudtop are lighter than the unmixed cloud, so that work must be done to push them down throughout the mixed layer. Then

$$P = \int_0^{z_c} \beta_u \frac{z}{z_B}\, R dz + \int_{z_c}^{z_B} \beta_s\, \frac{z}{z_B}\; R dz = \frac{\beta_u R z_c^2}{2z_B} + \frac{\beta_s}{2z_B}\, R(z_B^2 - z_c^2) \tag{20}$$

where z_c is the height of cloudbase, and

$$N = -\int_0^{z_c} \left\{ -\alpha_u w_e \Delta Q - \beta_u w_e \Delta\Theta_e \right\} \frac{z}{z_B} dz$$

$$+\int_{z_c}^{z_B} \left\{ -\alpha_s w_e \Delta Q - \beta_s w_e \Delta\Theta_e \right\} \frac{z}{z_B} dz$$

$$= + \left\{ \alpha_u w_e \Delta Q + \beta_u w_e \Delta\Theta_e \right\} \frac{z_c^2}{2z_B}$$

$$+ \left\{ \alpha_s w_e \Delta Q + \beta_s w_e \Delta\Theta_e \right\} \frac{(z_B^2 - z_c^2)}{2z_B} \qquad (21)$$

We now go back to equation (16′) and invoke a closure assumption: in the Stage and Businger (1981) model this is

$$D = (1-A)\, P = P - N \qquad (22)$$

and we use an empirical value $A = 0.2$. Thus $N = 0.2P$, so we have from above

$$w_e = \frac{0.2B \left\{ \beta_u z_c^2 + \beta_s \left[z_B^2 - z_c^2 \right] \right\}}{\left\{ \alpha_u \Delta Q + \beta_u \Delta\Theta_e \right\} z_c^2 + \left\{ \alpha_s \Delta Q + \beta_s \Delta\Theta_e \right\} \left[z_B^2 - z_c^2 \right]} \qquad (23)$$

Equations (13a and b, 23, and 17) constitute a mixed-layer model for the cloud-topped boundary layer, and can be used to predict the thermodynamic and water variables, as well as the inversion height as functions of time.

B. Convective Clouds

In convective clouds the driving force is the generation of density fluctuations which create large up- and down-drafts. These can no longer be treated as small fluctuations on a quiescent state. Moreover, the assumption of infinite horizontally homogeneous layers is, of course, no longer valid. Thus, dynamic modeling of convective clouds requires different approximations than that of stratiform clouds.

Whereas numerical-layer cloud models are usually written in an Eulerian (fixed) coordinate system, which allows the properties of the entire cloud to be calculated at each time step, in convective cloud models it is often desirable to use (Lagrangian) coordinates fixed in a moving air parcel, and to follow the evolution of properties inside that parcel while neglecting the rest of the cloud. The Lagrangian approach is particularly convenient if we can assume (a) the cloud properties at a fixed location are independent of time, or equivalently, (b) that the properties inside the moving parcel change with time only because the parcel location changes. Then $D\xi/Dt = \underline{V}\cdot\underline{\nabla}\xi$ on the LHS of Equation (5). If the parcel path is along the z-axis, then $\frac{D\xi}{Dt} = w\frac{d\xi}{dz}$. Let us consider this simple, often used approximation applied to a shallow, warm, nonprecipitating convective cloud.

Once again, one of the least understood and most important processes to consider is entrainment. Turbulent motions bring about entrainment: the mixing of environmental air into clouds. Entrainment dilutes the cloud and can deform the particle spectra via evaporation.

In a real cloud, there is no artificial boundary separating the cloud from the environment. However, in our simple model we assume that we can define a set volume, called "the cloud," inside which we will solve the model equations. We thus must parameterize the fluxes (entrainment) over the bounding surface of this volume. It is often assumed that convective clouds behave like self-similar plumes or thermals (Morton et al., 1956; Turner, 1962). In this kind of flow the rate of entrainment of environmental fluid into the rising cloud is determined by the assumption that the horizontal velocity at the interface is proportional to the local vertical velocity, w(z) ; i.e.,

$$u_r(z) = \alpha w(z) \tag{24}$$

where u_r is the radial velocity, $\alpha \cong 0.1$ for a plume, and $\alpha = .25$ for a thermal (see Figure 2). This leads to the expressions:

$$\dot{\xi}\Big)_{\substack{\text{entrainment}\\ \text{(plume)}}} = \frac{-2\alpha w}{R}\left[\bar{\xi} - \xi_{\text{environment}}\right]: \tag{25a}$$

$$\dot{\xi}\Big)_{\substack{\text{entrainment}\\ \text{(thermal)}}} = \frac{-3\alpha w}{R}\left[\bar{\xi} - \xi_{\text{env}}\right] \tag{25b}$$

for the plume and thermal, respectively, where $\bar{\xi}$ is the average in-cloud value of any variable obeying a conservation equation (Equation 5), and the radius of the cloud, R, grows linearly with height z. (See the above references for derivations of Equation 25.)

Warner (1970, 1973), Lee and Pruppacher (1977), and many others have used these assumptions. Other authors (Telford, 1966; and Lopez, 1973) have assumed the lateral entrainment velocity is a mean turbulent velocity that can be computed from a conservation equation for turbulent kinetic energy. Note that difficulties with model predictions (to be discussed below) and recent observational studies of entrainment have raised some doubt on the extent to which these model flows are useful analogies. See Simpson (1983) for a review of the successes and failures of plume models in reproducing observed cumulus cloud behavior.

Using the self-similar plume approximation (Eq. 25a) for the entrainment, the cloud model equations become

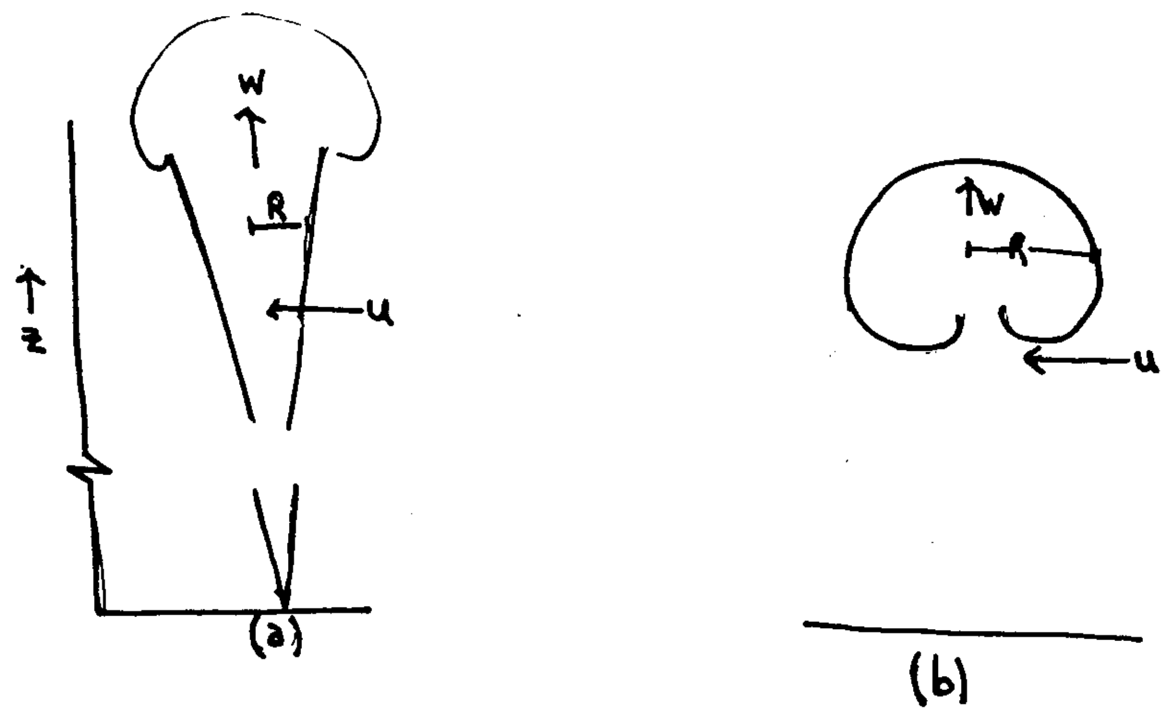
$$\frac{dR}{dz} = \frac{6}{5}\alpha \tag{26}$$

$$\frac{wdw}{dz} = \frac{g(\rho_{env} - \rho)}{\rho_{env}} - \frac{2\alpha w^2}{R} = \frac{g\left[T_{v_{env}} - T_v\right]}{T_{v_{env}}} - \frac{2\alpha w^2}{R} \tag{27}$$

$$\frac{wdQ}{dz} = -\frac{2\alpha w}{R}\,(Q - Q_{env}) \tag{28}$$

$$\frac{wdT}{dz} = -\frac{gw}{c_p} - \frac{L_v}{c_p}\frac{wdq_v}{dz} - \frac{2\alpha w}{R}\left[T - T_{env} + \frac{L}{c_p}\left[q_v - q_{v_{env}}\right]\right] \tag{29}$$

Figure 2. Schematic drawings showing the boundaries of (a) a self-similar plume and (b) a self-similar thermal, both at large distances in a neutral environment from the (point) source. The source is (a) constant in time for the plume; (b) a single pulse for the thermal. There is no generation of buoyancy (i.e., no phase change) and the inflow velocity is proportional to the propagation velocity. With these assumptions the horizontal dimension, R, grows linearly with z in both cases.



where

$$q_v = \frac{R_d}{R_v}\frac{e_{sl}}{p}(T) \tag{30}$$

(see Eq. 4a). We also assume the pressure is given by Equation (7). See Warner (1973) and Lee and Pruppacher (1977) as examples of the use of this model.

If it is necessary to describe phenomena occurring simultaneously in different parts of a cloud, if the time development at fixed locations is important, and/or if the fallout of precipitation through a cloud is of interest, then a Lagrangian model, which follows only one air 'parcel', is inadequate. In this case an Eulerian 1-D, 2-D, or 3-D model must be used. See the reviews by Simpson (1976) and Cotton (1975b) for comparisons of various cumulus models.

IV. Water Category Variables

The aim of parameterized (water continuity) models is to represent the integrated size-dependent microphysical processes in such a way as to preserve their major features without explicitly calculating $n_d(m_d)$ and $n_i(m_i)$. (Note that there are other ways to approximate the size-dependent microphysical processes. See, for example, Yau and Austin, 1977 and Lopez, 1973.) In effect, the models for (warm, nonprecipitating) stratiform and convective clouds described in the previous section were of this type. We made the approximation there that the cloudy air is always exactly saturated; i.e., Equation (30). Thus, given an equation ((13a) or (28)) for the total water Q, we could easily compute the cloud liquid water content without knowing the drop size distribution: $q_l = Q - q_v$. If we add precipitation and ice, we have more water variables and thus we need more equations. There are several parameterization schemes (i.e., sets of equations for these variables) in the literature. In general, these are based on Kessler's original scheme (Kessler, 1969) and differ mainly in their representation of the ice phase processes. The reader is referred to Orville and Kopp (1977), Stephens (1979), and Lin et al. (1983) for details. As an illustration of these models, we briefly describe that of Stephens (1979), as adapted by Taylor and Baker (1985). In this model:

q_c is the specific liquid water content in cloud droplets (those which move with the air) and q_p is the specific

Figure 2. Schematic drawings showing the boundaries of (a) a self-similar plume and (b) a self-similar thermal, both at large distances in a neutral environment from the (point) source. The source is (a) constant in time for the plume; (b) a single pulse for the thermal. There is no generation of buoyancy (i.e., no phase change) and the inflow velocity is proportional to the propagation velocity. With these assumptions the horizontal dimension, R, grows linearly with z in both cases.

liquid water content in precipitation drops. These are usually assumed to be exponentially distributed in radius; i.e., $n_d(r_d) = n_o \exp(-\lambda r_d)$: $q_l = q_c + q_p$

q_s is the specific ice content in snow (low density ice), and q_g is the specific ice content in graupel (high density ice) : $q_i = q_s + q_g$.

Each water content variable obeys an equation of type (5), in which the sources and sinks are due to microphysical transformations, fallout of precipitation, and entrainment. These equations, in a one-dimensional entraining plume model, have the following forms:

$$\frac{Dq_v}{Dt} = -G_c + E_c + E_p - S_{init} - V_{sdep} - V_{gdep} - G_i - \frac{2\alpha}{R}\left[q_v - q_{v_{env}}\right] \quad (31)$$

$$\frac{Dq_c}{Dt} = G_c - E_c - A_{uto} - C_{oll} - S_{melt} - G_{rme} - S_{rme} - C_{sfz} - \frac{2\alpha q_c}{R} \quad (32)$$

$$\frac{Dq_p}{Dt} = - E_c + A_{uto} + C_{oll} + G_{melt} - P_{gcol} - P_{gfz} + P_{flux} - \frac{2\alpha q_p}{R} \quad (33)$$

$$\frac{Dq_s}{Dt} = G_s + S_{init} + V_{sdep} + S_{rme} - S_{gcon} - S_{melt} - C_{sfz} + S_{flux} - \frac{2\alpha q_s}{R} \quad (34)$$

$$\frac{Dq_g}{Dt} = G_{rme} + S_{gcon} + V_{gdep} - G_{melt} + P_{gcol} + P_{gfz} + G_{flux} - \frac{2\alpha q_g}{R} \quad (35)$$

G_c and G_s are condensation onto nonprecipitating liquid and ice, and E_c and E_p are evaporation of cloud and precipitation liquid. Auto is the rate at which cloud liquid is transformed to precipitation liquid by condensation; C_{oll} is the rate of increase of precipitation liquid via collection of cloud water. S_{melt} and G_{melt} are rates of melting of snow and graupel, and S_{rme} and G_{rme} are the rates of riming of snow and graupel. P_{gcol} is the rate of collection of precipitation by graupel. S_{init} is the rate of nucleation of snow crystals, and V_{sdep} is the rate of vapor deposition on ice crystals (assumed to be hexagonal plates). C_{sfz} is the rate of homogeneous freezing of cloud droplets to form snow, and P_{gfz} is the rate of freezing of supercooled precipitation via collision with ice crystals to form graupel. S_{gcon} is the rate of conversion of snow to graupel. The terms P_{flux}, S_{flux}, and G_{flux} are sedimentation of precipitating particles, and the terms in α are entrainment terms. Each of these processes is represented as a semiempirical function of the q's, the temperature T, pressure p, and the air velocity $\underline{V}$. Each water quantity (except q_v) represents an integral over many particles of different sizes, and as such the sources and sinks in the conservation equation represent integrals of the size-dependent processes over the range of sizes of the particles.

To give the flavor of the kinds of approximations involved, let us consider the first few terms. In the Taylor and Baker (1987) parameterization, the net condensation rate is computed as a function of $\Delta q_v(z) \equiv q_v(z+\Delta z) - q_v(z)$ where Δz is the minimum resolvable vertical distance. We have:

$$G_c(z) \equiv \frac{\Delta q_v}{\Delta t},\ E_c(z) = 0 \quad \text{if } \Delta q_v(z) > 0 :$$

$$G_c(z) = 0,\ E_c(z) \equiv \frac{\Delta q_v}{\Delta t} \quad \text{if } \Delta q_v(z) < 0$$

where Δt is the time-step for the computation. Similar simple approximations are applied for the calculation of each of the other parameterized terms in Equations (31)-(35). See also Lord et al. (1984) and Yau (1977) for other representations of these terms.

We note that microphysical processes are important in determining vertical accelerations of cloudy parcels, both because of the drag due to hydrometers and because of the latent heat release during phase change. Moreover, since phase change itself depends on the vertical velocity and acceleration, the dynamics modify the microphysics. Therefore, the use of different microphysical parameterization schemes in numerical models can produce very different microphysical and dynamical results.

V. Microphysical Variables

In a model to be used for examination of size distributions (called 'spectra') of cloud or precipitation particles we need equations for the explicit microphysical variables $n_d(m_d)$ and $n_i(m_i)$ in Table I. We now discuss the appropriate microphysical equations in this case.

A. Warm Clouds

Putting $\xi = n_d(m_d)$ in Equation (5), we can write

$$\frac{Dn_d}{Dt} = \dot{n}_d\Big)_{\text{activation}} + \dot{n}_d\Big)_{\substack{\text{vapor}\\ \text{diffusion}}} + \dot{n}_d\Big)_{\substack{\text{entrain-}\\ \text{ment}}} + \dot{n}_d\Big)_{\text{coalescence}} + \dot{n}_d\Big)_{\text{breakup}} + \dot{n}_d\Big)_{\substack{\text{sedimen-}\\ \text{tation}}} \quad (36)$$

Let us consider each term on the RHS of this equation.

(1) Activation

Let $N_{ccN}(s_l)$ be the concentration of CCN activated in steady state at supersaturation s_l, where

$$s_l \equiv e/e_{sl} - 1 \quad (37)$$

There is experimental evidence (Twomey and Wojciechowski, 1969) that

$$N_{ccN}(s_l) \cong Cs_l k$$

where C and k are constants; k ~ 0.4–0.7 and C depends on time and location. Activation is usually assumed to occur instantaneously as the supersaturation changes and to produce particles of a minimum mass, which we can denote m_1. Similarly, if the saturation is decreasing, droplets with the minimum mass m_1 are assumed to evaporate instantaneously. In other words, in numerical models of activation we usually put

$$\dot{n}_d(m_d)\Big)_{\text{activation}} = 0, \; m_d \neq m_1$$

$$= \frac{Ds_l}{Dt}\frac{\partial N_{ccN}}{\partial s_l}, \; s_l \text{ increasing} \quad (38)$$

$$\dot{n}_d(m_1)\Big)_{\text{activation}} = \frac{-1}{\Delta t} n_d(m_1), \; s_l \text{ decreasing}$$

where Δt is the time-step of the computation. See, for example, Clark (1973) or Takahashi (1978) for details.

(2) Vapor Diffusion

After activation, diffusion of water vapor to and from the surface of a droplet causes its radius $r_d(m_d)$ to change. For $r_d \geq 1\mu m$, the rate of change of the droplet radius is approximately

$$\frac{dr_d}{dt} \simeq \frac{D\rho_{v_s} s_l}{\rho_l r_d} \tag{39}$$

where ρ_l is the density of liquid water, $D(m^2/s)$ is the diffusion coefficient of vapor in air, and ρ_{v_s} the density of saturated vapor at the environmental temperature T. See, for example, Pruppacher and Klett (1978) for details of the derivation of Equation (39).

The concentration of droplets of mass in the interval m_d, $m_d + dm_d$ changes with time due to vapor diffusion as follows:

$$\dot{n}_d(m_d) = -\frac{\partial}{\partial m_d}\left[\frac{dm_d}{dt} n_d(m_d)\right] \tag{40}$$

where

$$\left.\frac{dm_d}{dt}\right)_{\substack{\text{vapor}\\\text{diffusion}}} = \rho_l 4\pi r_d^2 \frac{dr_d}{dt}, \; \frac{dr_d}{dt}$$

given by Equation (39). To compute the RHS of Equation (40) in a numerical model, all droplets are classified into discrete size classes and then the concentration in each size class is evaluated at each time step by considering how many droplets grew into or out of that class.

(3) Entrainment

In a layer cloud, entrainment occurs over the top surface, and, as we have seen (Equation 15), it is usually parameterised by an entrainment velocity. The effect of entrainment is to dilute the cloud with drop-free air. Thus

$$\dot{n}_d(m_d)\Big)_{\text{entrainment}} = -\frac{1}{z_B} w_e \left[\overline{n}_d(m_d) - n_{env}(m_1)\right] \tag{41a}$$

(layer clouds)

where the last term represents entrainment of nuclei from the environment.

In convective clouds, entrainment occurs over the entire surface. In the self-similar plume model (Equation (25a))

$$\dot{n}_d(m_d)\Big)_{\text{entrainment}} = -\frac{2\alpha w}{R} \left[n_d(m_d) - n_{env}(m_1)\right] \tag{41b}$$

(self-similar plume)

For nonprecipitating clouds, when droplet radii remain smaller than $\sim 20\mu m$, the rest of the terms in Equation (36) can be ignored. If, however, it is necessary to consider larger drop sizes, these terms become important.

(4) Coalescence

As droplets grow by condensation (Equation (40)) to sizes at which they begin to fall relative to the air, there are collisions between droplets moving relative to one another. Some of these collisions result in coalescence and the formation of large drops called precipitation, or raindrops. If $A(m_d)$ is the cross-sectional area and $V_t(m_d)$ the terminal fall velocity of a drop of mass m_d, then in time dt a drop of mass m_d encounters and coalesces with all drops of mass m_d' within a volume we define as

$$K(m_d, m_d')dt = (A(m_d) + A(m_d'))\,|V_t(m_d) - V_t(m_d')|\,E(m_d, m_d')dt \tag{42}$$

where the factor $E(m_d, m_d')$ takes into account the deformation of streamlines around the particles and the fact that not all collisions result in coalescence. The rate at which a given particle of mass m_d coalesces with any particle of mass m_d' is thus $n_d(m_d')K(m_d, m_d')dm_d'$.

Then, the mean rate of disappearance via coalescence of drops of mass m_d in the interval $(m_d, m_d + dm_d)$ is $D(m_d) = n_d(m_d)\int_0^\infty K(m_d, m_d')n_d(m_d')dm_d'$. At the same time, new drops are constantly being created in this mass interval by collisions among smaller drops. The mean rate of production by these collisions of droplets in the size range of interest is

$$P(m_d) = \frac{1}{2}\int_0^{m_d} n_d(m_d')n_d(m_d - m_d')K(m_d', m_d - m_d')dm_d'$$

Thus, the fourth term on the RHS of Equation (36) is

$$\left.\dot{n}_d(m_d)\right)_{coalescence} = -D(m_d) + P(m_d) = -n_d(m_d)\int_0^\infty n_d(m_d')K(m_d, m_d')dm_d' + \frac{1}{2}\int_0^{m_d} n_d(m_d')n_d(m_d - m_d')K(m_d', m_d - m_d')dm_d' \quad (43)$$

Equation (43) is the so-called stochastic collection equation (SCE). See Berry and Rhinehart, 1974; Soong and Ogura, 1973; and Pruppacher and Klett, 1978, for detailed discussions of the terms in Equations (42) and (43).

Because of the complexity of the SCE, a simpler approach to the study of coalescence is often adopted when the goal is to roughly determine the size rate of change only the precipitation particles, rather than to develop a time-dependent expression for the entire drop population. This second approach, called the 'continuous collection method,' begins from Equation (40). In the approximation that a few large collector drops of mass m_d fall through a population of much smaller cloud droplets, the instantaneous rate of growth of the large drops is

$$\left.\frac{\partial m_d}{\partial t}\right)_{coalescence} \cong K(m_d, \overline{m}_d')q_l \quad (44)$$

where q_l is the liquid water content in the small droplets and $\bar{m}_d'$ is a mean droplet mass in the cloud.

(5) Breakup

The rate of growth of large droplets is limited by three factors: the rate of supply of liquid water, the breakup of pairs of drops which have recently collided, and the spontaneous breakup of individual drops as they oscillate and deform during fall. The second two processes may be represented by the expression:

$$\dot{n}_d(m_d)\Big)_{\text{breakup}} = -n_d(m_d)\pi(m_d) + \int_{m_d}^{\infty} n_d(m_d')Q_B(m_d', m_d)\pi(m_d')dm_d' \tag{45}$$

where $\pi(m_d)$ is the rate of breakup of a drop of mass m_d and $Q_B(m_d', m_d)$ is the fraction of breakup events of drops of mass m_d' that result in the formation of a daughter drop of mass m_d. The factors Q_B and π are usually taken from a combination of theory and experiment (Srivastava, 1978; Young, 1975).

Note that coalescence and breakup (Equations 43-45) result in the transformation of a large number of cloud droplets into a relatively tiny number of raindrops. To describe this transformation with accuracy requires great computational precision. In a model of drop spectral evolution, any numerical spreading in the drop distribution will result in apparent production of raindrops.

(6) Sedimentation

The rate of change of drop number due to the sedimentation of precipitation-sized particles (r_d - 20 μm) is given by the expression:

$$\dot{n}_d(m_d)\Big)_{\text{sedimentation}} = -\frac{\partial}{\partial z}(V_t(m_d)n_d(m_d)) \tag{46}$$

The terminal velocities are taken from experimental values (see Pruppacher and Klett (1978) for a review of these data).

We have briefly discussed each term in Equation (36), which completely describes the microphysics for clouds which do not rise above the 0°C isotherm. We turn now to an even briefer description of the analogous equation for $n_i(m_i)$.

B. Cold Clouds

The conservation equation for $n_i(m_i)$ which is analogous to Equation (36) for $n_d(m_d)$ is:

$$\frac{Dn_i}{Dt} = \dot{n}_i\Big)_{\text{nucleation}} + \dot{n}_i\Big)_{\substack{\text{vapor}\\\text{diffusion}}} + \dot{n}_i\Big)_{\substack{\text{entrain-}\\\text{ment}}} + \dot{n}_i\Big)_{\text{riming}} + \dot{n}_i\Big)_{\substack{\text{ice-ice}\\\text{collisions}}} + \dot{n}_i\Big)_{\substack{\text{multipli-}\\\text{cation}}} + \dot{n}_i\Big)_{\substack{\text{sedimen-}\\\text{tation}}} \tag{47}$$

The terms in Equation (47) are in general more complicated than their counterparts in Equation (36) because of the complex shapes and density distributions of the solid particles. Moreover, we have fewer observations of glaciation processes than of the corresponding warm cloud processes. Therefore, there have been relatively few attempts to use explicit microphysical models in cold clouds. We shall just touch upon the main features of each term in Equation (47).

(1) Nucleation

It has been customary to assume that the first stage in glaciation is heterogeneous nucleation on ice nuclei (IN) whose concentrations N_{IN} increase exponentially with cooling; (Fletcher, 1962):

$$N_{IN} = A\exp[-\beta T] \tag{48}$$

where T is in °C.

Typical values of the constants are $A \sim 10^{-5}$/liter, $\beta \sim 0.6/°C$. Nucleation is assumed to occur instantaneously as the temperature in a cloudy parcel changes; i.e., in analogy to Equation (38),

$$\dot{n}_i(m_i)\Big)_{\text{nucleation}} = 0, \; m_i \neq m_1$$

$$\dot{n}_i(m_1)\Big)_{\text{nucleation}} = \frac{\partial N_{IN}}{\partial T}\frac{DT}{Dt}, \text{ T decreasing} \tag{49}$$

$$= \frac{-n_i(m_i)}{\Delta t}, \text{ T increasing}$$

where m_1 is the arbitrarily set minimum ice crystal mass and Δt the time step of the model. We note, however, that frequently there are far more ice particles in clouds than can be explained as descendants of the IN whose concentrations are given by Equation (48). The cause of this discrepancy is not yet known, but the use of Equation (48) to predict the concentration of small ice particles appears questionable until more is understood about glaciation; it may be preferable to use empirically determined concentrations of the small crystals. The uncertainties in our present understanding of glaciation limit the confidence to be placed in numerical models of both naturally and induced precipitation development in cold clouds.

(2) Vapor Deposition

For small ice crystals, the most important process of growth or decay is vapor diffusion to and from the particle surface. Because the surface is not spherical, the vapor flux at the surface is not uniform and the growth equation is somewhat more complicated than Equation (39) for spherical droplets. We have:

$$\left.\frac{dm_i}{dt}\right)_{\substack{\text{vapor}\\\text{diffusion}}} \cong \frac{CD\rho_{vs,i}\, s_i f_v}{\rho_i} \tag{50}$$

where $s_i = e/e_{si} - 1$, and C is a shape factor. The shape (growth habit) of small ice crystals depends on temperature and vapor pressure; some shapes and the corresponding expressions for C are given in Pruppacher and Klett (1978).

The 'ventilation factor' f_v is a semiempirical factor representing the effects of particle motion on vapor diffusion to and from its surface. Ventilation is not important (i.e., $f_v \approx 1$) at the small drop sizes ($r_d \lesssim 20\ \mu m$) for which vapor diffusion is the dominant droplet growth mechanism, and thus f_v was put equal to unity in Equation (39). However, ice crystals grow via vapor diffusion to larger sizes, where ventilation is important. f_v may be expressed as a function of particle shape and mass and the viscosity and vapor diffusion coefficients in the air. See Pruppacher and Klett (1978), for a discussion of the relevant experimental and numerical studies of ventilation. The vapor diffusion term in Equation (50) is:

$$\left.\dot{n}_i(m_i)\right)_{\substack{\text{vapor}\\\text{diffusion}}} = -\frac{\partial}{\partial m_i}\left[\left.\frac{dm_i}{dt}\right)_{\substack{\text{vapor}\\\text{diffusion}}} n_i(m_i)\right] \tag{51}$$

Vapor deposition is the most important crystal growth mechanism until crystals are around 150-700 microns in lateral dimensions (the threshold depending on habit), when the large crystals begin to fall relative to the smaller ones and particle-particle collisions ensue.

(3) Entrainment

The comments above on entrainment of liquid particles are relevant also for the ice particles. It has also been recently suggested (Hobbs and Rangno, 1985) that entrainment may play a fundamental role in initiating glaciation by promoting contact nucleation, suggesting that the term $\dot{n}_i)_{\text{entrainment}}$ may be an important one at small sizes.

(4) Collection

Ice particles can collide with other ice crystals to form the low density material called snow, and/or they can collide with supercooled drops to form rimed crystals and eventually graupel and hail. Snow formation is usually represented by a continuous collection model analogous to Equation (44); the relevant fall velocities and collection efficiencies are determined by experimentation (see Pruppacher and Klett, 1978). The rate of growth by riming depends on the rates of shedding and of freezing of any excess water; that is, the efficiency $E(m_i, m_i')$ in the analogue of Equation (42) for ice depends on the liquid water content, the temperatures of the graupel particle and of the environment, the updraft velocity, and the ice concentrations. In general, the continuous collection growth model, i.e., Equation (44), is also used to describe the rate of growth of ice particles by riming, although modified SCE's of the type of Equation (43) have been used (Beheng, 1978). (See Figure 3.)

(5) Breakup/multiplication

The discrepancy between observed numbers of ice particles and observed concentrations of ice nuclei has led to several hypotheses for secondary ice multiplication mechanisms. Among these is the Hallett-Mossop process (Hallett and Mossop, 1974), in which riming particles can give rise to small ice splinters if the riming occurs within a narrow temperature interval (roughly -3°C to -8°C) and if the drop spectrum is sufficiently wide. It has been shown (Chisnell and Latham, 1976) that this process gives rise to a roughly exponential rate of growth of ice crystal concentrations; the rate of production of small crystals is:

$$\dot{n}_i(m_i)\Big)_{\substack{\text{multiplication} \\ \text{(Hallett-Mossop)}}} = p_o n_i(m_i) \tag{52}$$

where the lifetime $1/p_o$ is a complicated function of cloud properties. Ice multiplication is still not sufficiently well understood to allow its detailed description within the framework of a cloud model.

(6) Sedimentation

The convergence of precipitation of ice particles is treated exactly as for liquid precipitation particles (Equation 46). The terminal velocities are taken from experiment (Again, see Pruppacher and Klett (1978) for a review of the experimental literature.)

It is perhaps clear by now that attempts to include all the important microphysical processes within a cloud model are hampered by the large uncertainties in many of the numerical parameters, by the complexity of ice phase processes, and by the wide variety in sizes and types of particles which must be considered, and hence the enormous computing time requirements for a complete model.

III. Access to Numerical Models

This concludes our brief overview of the components of cloud models. We have not dealt at all with many important topics such as modeling radiation, cloud chemistry, or cloud systems, and in the limited space available we have only given superficial descriptions of the more standard cloud model components. The reader interested in using a numerical cloud model should be aware that many of the models discussed here and in the references are coded and stored on magnetic tape at various European and American universities and atmospheric research centers such as NCAR (USA) and the British Meteorological Office (UK). Scientists at these institutions may be contacted for advice on remote access to their computers and programs, as well as for requests for collaborative projects using these facilities.

References

Beheng, K. (1978) *J Atmos Sci 35*, 683.

Berry, E. and R. Rhinehart (1974) *J Atmos Sci 31*, 1814.

Bolton, D. (1980) *Mon Wea Rev 108*, 1046.

Chisnell, R. and J. Latham (1976) *Q J Roy Met Soc 102*, 133.

Clark, T. (1973) *J Atmos Sci 30*, 857.

Cotton, W. (1972) *Mon Wea Rev 100*, 757.

Cotton, W. (1975a) *J Atmos Sci 32*, 548.

Cotton, W. (1975b) *Revs Geophys and Space Phys 13*, 419.

Deardorff, J. W. (1976) *Q J Roy Met Soc 102*, 563.

Dutton, J. and G. Fichtl (1969) *J Atmos Sci 26*, 241.

Fletcher, N. (1962) *Physics of Rain Clouds*, Cambridge University Press, London.

Hallett, J. and S. Mossop (1974) *Nature 249*, 26.

Herman, G. and R. Goody (1976), *J Atmos Sci 33*, 1537.

Hobbs, P. V. and A. Rangno (1985) to be published, *J Atmos Sci*.

Kessler, E. (1969) *Meteorol Mon Amer Meteorol Soc 10*, 84 pp.

Lee, I. and H. Pruppacher (1977) *Pageoph 115*, 523.

Lilly, D. K. (1968) *Q J Roy Met Soc 94*, 292.

Lin, Y. L. et al. (1983) *J Climate and Appl Met 22*, 1065.

Lopez, R. E. (1973) *J Atmos Sci 30*, 1334.

Lord, S. J. et al. (1984) *J Atmos Sci 41*, 1169.

Moeng, C. H. and A. Arakawa (1980) *J Atmos Sci 37*, 2661.

Morton, B., G. I. Taylor and J. S. Turner (1956) *Proc Roy Soc London A 234*, 1.

Ogura, Y. and T. Takahashi (1973) *J Atmos Sci 30*, 262.

Orville, H and F Kopp (1977) *J Atmos Sci 34*, 1596.

Paltridge, G. and C. Platt (1976) *Radiative Processes in Meteorology and Climatology*, Elsevier, 318 pp.

Pruppacher, K. and J. Klett (1978) *Microphysics of Clouds and Precipitation*, D. Reidel, 714 pp.

Rutledge, S. and P. V. Hobbs (1983) *J Atmos Sci 40*, 1185.

Scott, B. and P. V. Hobbs (1977) *J Atmos Sci 34*, 812.

Simpson, J. (1976) in *Advances in Geophysics, 19*, p. 53.

Simpson, J. (1983) in *Mesoscale Meteorology--Theories, Observations and Models*, D. K. Lilly and T. Gal-Chen, ed., D. Reidel.

Soong, S. and Y. Ogura (1973) *J Atmos Sci 30*, 879.

Srivastava, R. (1978) *J Atmos Sci 35*, 108.

Stage, S. and J. Businger (1981) *J Atmos Sci 38*, 2213.

Stephens M. (1979) *Atmos Sci paper #319*, Dept. of Atmos. Sci., CSU, Fort Collins, CO 80523, 122 pp.

Takahashi, T. (1978) *J Atmos Sci 35*, 1549.

Takahashi, T. (1981) *J Atmos Sci 38*, 1991.

Taylor, G. and M. B. Baker (1987) *Mon Wea Rev*.

Telford, J. W. (1966) *J Atmos Sci 23*, 652.

Turner, J. (1962) *J Fluid Mech 13*, 356.

Twomey, S. and T. Wojciechowski (1969) *J Atmos Sci 26*, 684.

Warner, J. (1970) *J Atmos Sci, 27* 1035.

Warner, J. (1973) *J Atmos Sci 30*, 256.

Wilhelmson R., (1974) *J Atmos Sci 31*, 1629.

Wilhelmson R. and Y. Ogora, (1972) *J Atmos Sci 29*, 1295.

Yau, M. (1980) *J Atmos Sci 37*, 488.

Yau, M. and P. Austin (1979) *J Atmos Sci 36*, 655.

Young, K. (1975) *J Atmos Sci 32*, 965.