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(SUMMARIES)

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DYNAMICS OF GRAVITATION

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A non-expert's introduction to the dynamics of the gravitational field. We show first that the non-linear Einstein action is necessarily obtained from the linear massless tensor field as a consequence of Lorentz invariance, universality and consistency. That is, once the tensor nature of gravitation is admitted, the stress tensor of the gravitational field itself must be included as its own source. This leads in one step to the Einstein action, while the requirement that the matter stress tensor is also a source leads to the usual minimal coupling, $I_M(\eta \to g)$, which is the origin of "geometry" for material systems. The above method is identical to that which one would use to derive other non-abelian gauge theories such as Yang-Mills or chirality.

The second topic is the exhibition by hamiltonian methods of the Einstein field as a (self-coupled) massless, spin-2 theory, with two helicity degrees of freedom. Here geometry and field theory both lead to a form of the action characteristic of systems with gauge freedom, namely

$$I = \int d^4x \, [\pi^{ij} \, \dot{g}_{y} - N_\mu \, R^\mu(\pi,g)] \ ,$$

to be compared with the usual parametrized form of dynamics in terms of an arbitrary independent variable τ replacing the time

$$I = \int d\tau \, \left[\sum_{A=1}^{n+1} p_A \frac{dq_A}{d\tau} - N \left\{p_{n+1} + H(p,q)\right\} \right] \ .$$

In each case the "Hamiltonian" vanishes due to covariance under changes of independent variables. Upon "deparametrizing", the number of true field variables is seen to decrease to two pairs of variables, for example the transverse-traceless parts of π^{ij} and g_{ij} appropriate to pure helicity-2 . These variables obey unconstrained P.B. relations. In addition, one obtains the generators of the Poincaré group as functionals of the field variables, these being well defined and invariant with respect to arbitrary co-ordinate transformations maintaining the boundary conditions at infinity that space is flat and co-ordinates minkowskian (only then does energy, etc., make sense anyway). Further, one can establish the positiveness of energy and the forward time-like nature of P_μ. This canonical formulation may be applied to quantum perturbation calculations and lead, for example, to the usual vector particles in closed loops.

Some interrelations with particle theory are discussed, such as the difficulties of higher spin theories interacting with gravitation, and, at the quantum level, the great importance of understanding the contributions not only of the quantized metric modes (the "TT's") but also of the constraint metric components which are functionals of the quantized $T^{0\mu}$ of the system. This is particularly important in assessing the validity of singularity theorems at very high densities in the quantum regime. We mention also the question, to be discussed later, of the discontinuity between massless and massive gravitation, as $m \to 0$.

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