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(SUMMARIES)

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RADIATION, ENERGY AND THE BMS GROUP

R. Penrose

Birkbeck College, London, UK.

To calculate the mass loss due to gravitational radiation from an isolated system, it is necessary, owing to the non-locality of gravitational energy, to perform integrals over hypersurfaces extending all the way to infinity, or over surfaces lying at infinity. This is much facilitated by a device of "making infinity finite" with the introduction of a suitable conformal factor Ω. Accordingly, asymptotically flat space-time acquires a conformal boundary, defined by $\Omega = 0$, consisting of a smooth null hypersurface $\mathscr{I}^-$ representing past null infinity, a smooth null hypersurface $\mathscr{I}^+$ representing future null infinity and three points I^-, I^0 and I^+ (which, however, are singular in most cases) representing, respectively, past timelike, spacelike and future timelike infinities.

The utility of this picture rests on the conformal invariance of null geodesies and of zero rest-mass free fields. The gravitational radiation field is measured by a complex component Ψ_4^0 of the (scaled up) Weyl curvature field on $\mathscr{I}^+$ (outgoing field) or on $\mathscr{I}^-$ (incoming field). Bondi's news function N is a complex component of the Ricci tensor on $\mathscr{I}^\pm$ and turns out to be a time integral of the radiation field Ψ_4^0. The positive-definite quantity $N\bar{N}$ measures the flow of gravitational energy momentum across infinity, according to the Bondi-Sachs mass-loss formula. The total energy momentum at any one retarded time is measured by an integral of the quantity $\sigma^0 N - \Psi_2^0$ over a cross-section Σ of $\mathscr{I}^+$, where Ψ_2^0 is another (scaled) Weyl tensor component at $\mathscr{I}^+$ and where σ^0 is the asymptotic shear of the null (retarded time) hypersurface meeting $\mathscr{I}^+$ in Σ.

The Bondi-Metzner-Sachs group is an asymptotic symmetry group for asymptotically flat space-time. It may be expressed as the group of self-transformations of $\mathscr{I}^+$ preserving a certain ("strong") conformal intrinsic structure. The BMS group is an infinite parameter "Lie" group which contains the translations as a canonically defined normal subgroup. However, it also contains more general transformations known as supertranslations. The homogeneous Lorentz group appears canonically as a factor group of the BMS group by the supertranslation group. Many subgroups of the BMS group can be found which are isomorphic with the Poincaré group, but apparently no one of them can be singled out in a very natural way. Consequently no naturally defined concept of angular momentum for asymptotically flat space-times has yet emerged. The difficulty arises because the asymptotic shear structure σ^0 differs from that of Minkowski space when news function N is present.

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