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(SUMMARIES)

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REPRESENTATIONS OF THE BONDI-METZNER-SACHS GROUP

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In flat space-time, the Poincaré group P plays a fundamental role as the group of symmetries. In a reasonably general curved space-time there is no group of symmetries. However, for a wide class of asymptotically flat space-times, there is an asymptotic symmetry group which is independent of the detailed structure of the metric and which contains but is larger than P. This is the Bondi-Metzner-Sachs group B. [1),2)] Sachs [3)] gave good reasons for believing that the representations of B should be studied in view of their possible role in an S-matrix theory which included gravity, and he found a zero-mass, zero-spin representation of B. Komar [4)] and Newman [5)] emphasised the possible relevance of B to microphysics, and Cantoni [6)-8)] found a class of representations of B with positive mass and ordinary spin. Throughout this period it was felt that, perhaps, the representations of B may be more interesting than those of P. However, the infinite dimensionality of B was an obstacle to further progress, and several attempts to reduce the asymptotic symmetry group to P have been made. However, in my view, none of them has yet succeeded unambiguously and the representations of B still remain an interesting problem. B differs from P in that B contains an infinite-dimensional abelian normal subgroup A (of so-called "supertranslations"). However, A contains a four-dimensional subspace V which is normal in B (the "translations") and, restricting A to V, restricts B to P.

I have found all unitary representations of B induced from connected little groups (in the sense of Mackey [9)]). Recall that P is a semi-direct product of V with the Lorentz group G, and that the little groups of P are SO(3) (for timelike momenta), E(2) (for lightlike momenta) and SL(2,R) (for spacelike momenta). The "spins" of the corresponding particles are given by representation labels for the little groups. For timelike momenta, which have the compact little group SO(3), the spins are discrete, as observed in nature. However, for other momenta, the little groups are non-compact, so that (in general) the spins are continuous. (Apart from a certain subclass of lightlike representations which come from unfaithful representations of E(2) and which are more truly representations of $E(2)/R^2 \approx SO(2)$). Continuous spins are never observed.

If one gives the space A of supertranslations a Hilbert space structure and topology one can proceed rather directly to find representations of B by Mackey theory. (Even though B is not locally compact, there is reason to believe that the induced representations may give all physically significant ones,) I found the surprising result that the only little groups which can appear are all compact, and are e (the identity), SO(2) and SO(3). Hence the only spins allowed by B are discrete spins, in striking contrast to P. The fact that the only types of spin which seem to appear in nature are SO(3) and SO(2) spins may perhaps be explained in this way.*)

This result, unfortunately, is not invariant under changes in the topology of A. For example, if A is given a topology appropriate to a space of C^{∞} functions, non-compact little groups appear also. However, a clear distinction remains between compact and non-compact little groups, namely, the former always correspond to regular functions, but the latter, if they are defined at all (and in some topologies they are not), correspond to "singular functions" or genuine distributions. Thus there is a distinctly different situation from that for P. The representations have been examined in some detail. The SO(3) representations, when restricted to the subgroup P, are irreducible positive-mass P-representations. The others (which may have many values of the mass) contain, for $m^2 > 0$ and $m^2 \leq 0$, respectively, direct sums or direct integrals of different Poincaré spins. (This is shown using the theorem of inducing by stages.) Dr. Michael Crampin and myself have recently been using this picture to find how the Poincaré spins are coupled together by application of proper supertranslations. Also, this provides a direct proof of irreducibility of the representations. The phenomenon of spin mixing referred to here seems to reflect the subtleties which Professor Penrose alluded to in the previous lecture.

In conclusion, I think that B is a fascinating group and its representations may well have more physical significance than those of P. In fact, B should be regarded not as an embarrassment which appears in relativity, which should be explained away or eliminated, but as a positive bonus, which should be studied further.

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*) A preprint of this result is now available and this result, together with some others, will appear in print shortly.

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