IC/71/144

International Atomic Energy Agency

and

United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

# TOPICAL MEETING

# ON GRAVITATION AND FIELD THEORY

13 - 16 July 1971

(SUMMARIES)

MIRAMARE - TRIESTE

November 1971

# QUANTUM COSMOLOGY
(Report on work of Maryland group)

D. Brill

University of Maryland, College Park, Md.20742, USA.

By quantum cosmology I mean the application of the quantum general relativity formalism to closed spaces. Traditionally such spaces have been used to build models because they admit greater symmetry than asymptotically flat spaces, and because the universe in the large indeed appears to be highly symmetric. Here we consider models which are homogeneous but not isotropic ("Mixmaster Universe") [1]. Exact solutions of the classical source-free equations of this type exist and correspond in some sense to the lowest possible excitation of the closed space. We quantize only the lowest degrees of freedom, reducing the infinite-dimensional problem to a three-dimensional one. There is no assurance that this "freezing out" of the higher excitations is a good approximation. Nonetheless the models are useful as an illustration of some of the features of the general quantization scheme.

We introduce a three-dimensional "minisuperspace" whose points are all possible homogeneous geometries (of a given topology) [2]. A classical solution, when sliced into a sequence of homogeneous three-geometries, corresponds to a (one-dimensional) subspace of this space. The law of slicing, i.e. the choice of time co-ordinate t, is arbitrary and corresponds to various parametrizations of the subspace. The wave function depends only on invariants, hence not on t but only on position in minisuperspace. We choose for co-ordinates a function of the total volume and two other variables describing the deviation of the universe from isotropy [1),3)]. We use the one non-trivial constraint ($G_{00} = 0$) to solve for the trace of the extrinsic curvature in terms of the superspace co-ordinates and the remaining extrinsic curvature. When the solution is substituted into the variational principle we obtain an unconstrained system of two degrees of freedom which has Hamiltonian form. The measure of total volume now plays the role of time and the trace of the extrinsic curvature becomes the Hamiltonian. It has the form of a two-dimensional particle motion in a time-dependent potential. By replacing the (superspace) time by a "proper time", one can cast the equations into three-dimensional "invariant" Hamiltonian form, where the additional conjugate variables are the old time and old Hamiltonian, and where the new Hamiltonian is constrained to vanish. It is invariant under Lorentz transformations if one defines a metric in minisuperspace from the kinetic part of the new Hamiltonian. However, the mass and potential part is not positive, so that "particle paths" (i.e. universe histories) can be timelike, null or spacelike in this metric.

A typical path starts in regions of minisuperspace describing vanishing volume (big bang), moves into regions of larger volume and is affected by the effective potential, and moves back into regions of vanishing volume (final collapse). Thus the problem is analogous to a particle-scattering problem, but the particle reverses its "time" direction in minisuperspace (as measured by increasing volume).

The quantization [3] replaces the variables of the new Hamiltonian by the usual operators of the analogous particle theory, resulting in a Klein-Gordon type wave equation. The scattering boundary conditions allow construction of wave packets which approximate the classical path, and the usual conserved current exists. Its "time" component is not positive definite, nor is this required in view of the "time"-reversing nature of the classical motion. The quantum theory with scattering-type boundary conditions does not appear to eliminate in any simple sense the zero-volume singularities of the classical theory.

## REFERENCES

1) O. Heckmann and E. Schücking, "Relativistic cosmology" in Gravitation, ed. L. Witten (Wiley & Sons, New York 1962);
C.W. Misner, J. Math. Phys. 4, 924 (1963);
C.W. Misner, Phys. Rev. 186, 1319 (1969);
V.A. Belinsky and I.M. Khalatnikov, Sov. Phys.-JETP 29, 911 (1969);
C.W. Misner, Phys. Rev. Letters 22, 1071 (1969).

2) B.S. DeWitt, Phys. Rev. 160, 1113 (1967);
J.A. Wheeler, "Superspace and quantum geometrodynamics" in Battelle Rencontres, Eds. C.M. DeWitt and J.A. Wheeler (Benjamin Inc., New York 1968);
B.S. DeWitt, "Spacetime as a sheaf of geodesies in superspace" in Relativity, Eds. Carmeli et al., (Plenum, New York 1970).

3) C.W. Misner, "Classical and quantum dynamics of a closed universe" in Relativity, loc. cit.;
A. Komar, "The quantization programme for general relativity" in Relativity, loc. cit.;
J. Klauder, "Soluble models of quantum gravitation" in Relativity, loc. cit.;
D. Brill and R. Gowdy, Rep. Progr. Phys. 33, 413 (1970);

Unpublished work (University of Maryland preprints):
L. Fishbone, C. Misner and Y. Nutku, "Quantum Cosmology";
K. Jacobs, C. Misner and H. Zapolsky, "Quantum Cosmology" - II;
L. Hughston and K. Jacobs, "More Quantum Cosmology".



9