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COVARIANT QUANTUM GEOMETRODYNAMICS

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With the aid of a compact notation the perturbation theory of the mutual scattering of large-amplitude classical gravitational waves is analysed in terms of tree graphs, and the relation of classical scattering to quantum scattering in the tree approximation is emphasised. The analysis is carried out under the assumption that the waves propagate in a classical background field satisfying Einstein's equations. Because of the existence of the diffeomorphism group as an invariance group for the theory, the second variation, S_2, of the Einstein action has no Green's functions in terms of which one can construct an iterable integral equation for the waves. However, S_2 may be made non-singular by adding to it an operator that depends on two arbitrary two-point functions γ and $\hat{\gamma}$. The Green's functions of the resulting operator may be used immediately to obtain iterative solutions of the finite-amplitude problem. The input function need satisfy only the equation for infinitesimal disturbances. The only effect of changing the γ's is to induce a gauge transformation in the solution.

The classical tree functions, which may be used immediately in the computation of S-matrix amplitudes in the quantum Born approximation, satisfy four invariance laws known as <u>tree theorems</u>: they are invariant under 1) gauge transformations of the background field, 2) gauge transformations of the input (or asymptotic wave) functions, 3) changes in the γ's and 4) changes in the basic field variables of the theory. The full S-matrix, including all closed-loop processes, can in principle be built up from the Born approximation amplitudes via unitarity and causality arguments. An approach to this problem, due to Feynman, is described. Feynman replaces all closed-loop terms with sums over partial mass-shell tree amplitudes by removing from the loops all the "non-causal chains" of the retarded (or advanced) Green's functions and grouping the sums into "baskets " containing <u>complete</u> amplitudes. If one restricts the sums to physical quanta only, then the closed-loop corrections possess all of the invariance properties of the tree amplitudes themselves. These invariance properties may be displayed in an entirely different manner with the aid of fictitious quanta, which enter

into the formalism in a natural way. A functional integral is constructed that generates all the computational rules involving both real and fictitious quanta. The Faddeev-Popov rules in the Landau gauge are shown to emerge as a special case of the rules for arbitrary γ's.

Finally, it is emphasised that no scheme to use gravity as a quantum regulator can hope to be invariant under the diffeomorphism group if it does not sum over all the diagrams in every order of perturbation theory. We are far from having such a scheme today. However, if we are willing (for the present) to accept divergences when the interaction of gravity with itself is studied, then there are several interesting things one can do. Chief among these is the calculation of corrections to Einstein's equations arising from vacuum quadrupole polarization in an attempt to answer the question whether these corrections can halt the formation of singularities. Use of an arbitrary background geometry in the formalism suggests that it may be possible, by application of the well developed theory of Green's functions in curved space-time, to answer this and similar questions in space-times with arbitrary topology.

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