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(SUMMARIES)

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THE FIELD EQUATIONS OF GRAVITATION

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In this seminar we give a reformulation and generalization of Einstein's field equations of gravitation. The main result is given by the following theorem.

Theorem - Let M be a pseudo-riemannian manifold with fundamental tensor g of signature (-,-,-,+). Let T be a symmetric tensor of type (2,0) and R be the Riemann-Christoffel curvature tensor of type (4,0). Define the tensor W of type (4,0) by the equation

$$W = R + g \times_C T \tag{1}$$

where $g \times_C T$ is a tensor of type (4,0) constructed from g and T and having the algebraic properties of the curvature tensor. Then for each $x \in M$, the following conditions are equivalent:

GF-1: W_x regarded as a linear transformation of $\Lambda_x^2(M)$ commutes with the Hodge star operator J on $\Lambda_x^2(M)$,

i.e.,

$$W_x J = J W_x \quad , \tag{2a}$$

or, using the commutator notation

$$[W_x \, , J] = 0 \quad . \tag{2b}$$

GF-2: W_x induces a complex linear transformation of $\Lambda_x^2(M)$ regarded as a complex vector space by using the complex structure defined by J.

GF-3:
$$R^{ij} - \frac{1}{4} R g^{ij} = -T^{ij} + \frac{1}{4} T g^{ij} \tag{3}$$

where a local co-ordinate chart and the induced basis of the tensor algebra is used and R^{ij}, R and T are, respectively, the components of the Ricci tensor, the scalar curvature and the trace of the source tensor.

We say that the triple (M,g,T) is a generalized gravitational field if any one (and therefore, all) of the conditions of the above theorem are satisfied.

Einstein's field equations may be written as

$$R^{ij} - \frac{1}{2} Rg^{ij} = -T^{ij} \tag{4}$$

where the coupling constant accompanying T^{ij} is taken to be unity. Eqs.(4) are easily seen to be a special case of Eqs.(3) corresponding to the case when the scalar curvature is equal to the trace of the source tensor. We observe that Eq.(3) does not lead to any relation between the scalar curvature and the trace of the source tensor, since both sides of Eq.(3) are trace-free.

The new equations may be expressed in terms of gravitational sectional curvature, which generalizes the concept of sectional curvature function to the case when (M,g,T) is a gravitational field.

The theory of classification extends naturally to the new field equations, bringing out the geometric significance of classification.

The conditions GF-1 and GF-2 are suggestive of quantum theoretical formalism.

REFERENCES

1) K.B. Marathe, "Spaces admitting gravitational fields - I: the field equations".

2) K.B. Marathe, "Spaces admitting gravitational fields - II: the problem of classification".

3) K.B. Marathe, "Generalized field equations of gravitation".

(T O A P P E A R)