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(SUMMARIES)

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THE METHOD OF FADDEEV AND POPOV

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In quantizing general relativity one of the basic problems is to make unitarity consistent with general covariance. This problem has been considered by many authors [1)]. One of the simplest approaches is the path-integral formulation given by Faddeev and Popov [2)]. While suffering from the lack of precision which is common to all path-integral formulations, this method has great flexibility on a formal level and is well adapted to the treatment of gauge conditions and the formulation of Feynman rules in particular.

The starting point of the Faddeev-Popov formulation is the assumption that the Green's functions of general relativity can be represented by path-integrals of the form

$$\langle T\, g_{\kappa\lambda}(x_1)\, g_{\mu\nu}(x_2) \ldots \rangle = \int (dg)\, g_{\kappa\lambda}(x_1)\, g_{\mu\nu}(x_2) \ldots e^{\frac{i}{\hbar}\mathcal{S}(g)}$$

where the action functional consists of two parts. It is the sum of the classical (Einstein) part, $\mathcal{S}_E(g)$, and a supplementary, gauge-determining part, $\mathcal{S}_\Phi(g)$. While $\mathcal{S}_E$ is invariant under the group of general co-ordinate transformations, $x^\mu \to \Omega^\mu(x)$, the supplementary action, $\mathcal{S}_\Phi$, is not. On the other hand, $\mathcal{S}_\Phi$ must satisfy a "normalizing condition" - identically in $g_{\mu\nu}$ -

$$1 = \int d(\Omega)\, e^{\frac{i}{\hbar}\mathcal{S}_\Phi(g^\Omega)}$$

where $g^\Omega_{\mu\nu}$ denotes the transform of $g_{\mu\nu}$ and the integral extends in some sense over the space of functions $\Omega^\mu(x)$ on which $(d\Omega)$ is an invariant measure. The integrals over both $g_{\mu\nu}(x)$ and $\Omega^\mu(x)$ have not as yet been given precise meaning. The need to give a rigorous sense to them is at the very heart of the quantization problem. Until this has been achieved it will not be possible to use the path-integral method to treat the deeper problems which arise from factor-ordering ambiguities and from the constraints of general relativity.

The path-integrals can be given at least a formal sense through perturbation methods. Both of the integrals above can be expanded in powers of the newtonian constant. Of course such expansions suffer from the usual non-renormalizability of the operator formulations. It is encouraging, however, to find that the analogous expansions for the Green's functions of Yang-Mills theory in the radiation gauge correspond precisely to those obtained by the traditional method of canonical quantization and are therefore at least formally unitary.

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