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MASSIVE GRAVITATION AND ITS MASSLESS LIMIT

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We discuss the problems of "massive gravitation" and of its infinite range limit.

The crucial difficulty lies in the fact that a finite range affects not only the denominators of propagators ($\square \rightarrow \square - m^2$) but also the numerators (helicity), a circumstance absent in scalar Yukawa models. In massive electrodynamics, Schrödinger long ago pointed out the physical desirability of a smooth limit from $2s + 1 = 3$ degrees of freedom to the two pure helicity-1 components of the $m \equiv 0$ Maxwell field. The zero helicity parts do indeed decouple from the current in the limit there. [We point out, however, that the stress tensor of the longitudinal photons does not decouple from gravitation!]

For spin-2 the situation is much worse. First, even the (ghost-free Pauli-Fierz) linearized approximation leads to a discretely different matter-matter interaction from that of linearized $m \equiv 0$ theory, with a consequent light bending $3/4$ of the Einstein value, however small the mass. What happens is that the zero helicity modes (unlike the helicity-one part) do not decouple from the (trace of the) stress tensor and lead to an effective additional scalar interaction.

In the full theory (and perhaps even in linearized theory with self-interactions) things are even more startling. There are six rather than $2s + 1 = 5$ degrees of freedom and there are strong indications that the system no longer has positive energy (ghosts). That the presence of six degrees of freedom is a consequence solely of Lorentz invariance (mass of course breaks co-ordinate invariance just as it breaks gauge invariance in electrodynamics) can be seen directly in the canonical analysis. What occurs is that the old Lagrange multipliers (N, N_i) of general relativity (see my other summary) change their role because the mass term necessarily contains terms at least quadratic in them. This contrasts with the linear case where N_i enters quadratically, but N only linearly, in the Pauli-Fierz mass term. In the full theory, however, the only quantity linear in N is the

cosmological factor $\sqrt{-g}$ which preserves co-ordinate invariance and is not a mass term. As a result, the equations obtained by varying the N_μ become determining equations for them, rather than 4 constraints among the (π^{ij}, g_{ij}) at $m \equiv 0$, and the latter are now six unconstrained degrees of freedom. It is not even known whether the $m \rightarrow 0$ limit exists at all here. Thus general relativity stands as an isolated theory with respect to theories with arbitrarily large (e.g. \gg Hubble radius) range. Some of the consequences for strong gravitation are also mentioned, among them the persistence of the extra degree of freedom there.

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