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(SUMMARIES)

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PROPERTIES OF f-g THEORY IN THE CASE $g = \eta$

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A. We have studied three different properties of f-g theory [1], in the case where $g_{\mu\nu} = \eta_{\mu\nu}$ and where $f_{\mu\nu}$ is the only non-trivial field, by considering the first-order action principle:

$$I(f,\eta) = \int \frac{f^{\mu\nu}}{\sqrt{-f}} R_{\mu\nu}(\gamma)\, d^4x - \frac{1}{4} m^2 \int (f^{\mu\nu} - \eta^{\mu\nu})(f^{\alpha\beta} - \eta^{\alpha\beta}) \eta_{\mu[\alpha}\eta_{\nu]\beta} \frac{d^4x}{\sqrt{-f}} \qquad (1)$$

and the field equations obtained upon independent variations of $\gamma^{\alpha}_{\mu\nu}$ and $f^{\mu\nu}$ $(f^{\mu\nu}\tilde{f}_{\nu\alpha} = \delta^{\mu}_{\alpha})$

$$\gamma^{\alpha}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} (\tilde{f}); \quad G_{\mu\nu}(\gamma) = \frac{1}{2} m^2\, S_{\mu\nu}(f,\eta) \quad .$$

Besides this set, there is also a constraint equation which is, in this case, the conservation law $\nabla^{\nu}_{f} S_{\mu\nu} = 0$.

B. Defining the Σ-intrinsic quantities $(\Sigma \equiv x^0 = \text{const})$ in a similar way to the gravitational case [2]

$$f_{ij} \equiv \tilde{f}_{ij}; \quad M^0_i = -f_{0i}; \quad M \equiv (-f^{00})^{-\frac{1}{2}} \qquad (2a)$$

$$\psi^{ij} \equiv \frac{1}{\sqrt{-f}} (\gamma^{0}_{pq} - f_{pq}\, \gamma^{0}_{\ell m}\, {}^3f^{\ell m})\, {}^3f^{ip}\, {}^3f^{iq}; \quad {}^3f^{ip} f_{pj} = \delta^{j}_{j} \qquad (2b)$$

we can write the action (1) in the form (having already substituted M_i):

$$I(f_{ij}, M, \eta, \psi^{ij}) = \int \{\psi^{ij}\, \partial_0 f_{ij} - [MA(\psi^{ij}, f_{\ell m}) + B(f_{\ell m})\, M^{-1}]\}\, d^4x \ . \qquad (3)$$

The value of M is determined by $\delta I/\delta M = 0$; in this case this is equivalent to finding the stationary point of $z(M) \equiv MA + M^{-1}B$, which is a real quantity, if, and only if, $AB > 0$. On the other hand, as A and B are functionals of the Cauchy data (ψ^{ij}, f_{pq}), for the case [2]:

$$\psi^{ij} = 0; \quad f_{ij}^{\,0} = \delta_{ij} + \varphi_{ij}^{TT} + \varphi_{i,j}^{T} + \varphi_{,i,j}^{L}; \quad |\varphi_{ij}| << 1 \qquad (4a)$$

we have that $A \cong -\frac{1}{2} m^2 \Delta \varphi^L$, $B \cong \frac{1}{2} m^2 \Delta \varphi^L$ and consequently $AB < 0$.

C. Assuming the existence of an exact solution $\tilde{\mathring{f}}_{\mu\nu}$ of the field equations, we can analyse the behaviour of a small excitation $h_{\mu\nu}$: $f_{\mu\nu} = \tilde{\mathring{f}}_{\mu\nu} + h_{\mu\nu}$, regarding the excitation as a field which propagates in the background riemannian variety determined by the metric tensor f°. It turns out that the spin-0 part (h^{α}_{α}) of the excitation $h_{\mu\nu}$ is a dynamical variable which obeys the equation (keeping terms up to first order in both $(\tilde{\mathring{f}}_{\mu\nu} - \eta_{\mu\nu})$ and $h_{\alpha\beta}$)

$$\left[\left(\eta^{\alpha\beta} - \mathring{f}^{\alpha\beta}\right) \partial_{\alpha} \partial_{\beta} - m^2\right] h^{\sigma}_{\sigma} =$$

$$= \frac{7}{3} m^2 \left(\eta^{\alpha\beta} - \mathring{f}^{\alpha\beta}\right) h^{t}_{\alpha\beta} + \frac{1}{3} \left[6\partial^{\alpha} \mathring{f}^{\mu\beta} - 5\partial^{\mu} \mathring{f}^{\alpha\beta}\right] \partial_{\mu} h^{t}_{\alpha\beta} \qquad (4b)$$

where $h^{t}_{\alpha\beta} \equiv h_{\alpha\beta} - \frac{1}{4} h^{\sigma}_{\sigma} \mathring{f}_{\alpha\beta}$. [3] That $(\eta^{00} - \mathring{f}^{00})$ does not vanish can be seen by looking either at the static case where $(\eta^{00} - f^{00} = (4\pi)^{-1} \cdot g \cdot r^{-1} \cdot e^{-mr}$ or at a plane wave with its 4-momentum p_{μ} non-collinear with the x^0-axis). Therefore, we see that the number of independent variables in this specific case of f-g theory $(g = \eta)$ is six.

D. Using the mixed form of the field equations $G^{\mu}_{\nu}(\gamma) = \frac{1}{2} m^2 S^{\mu}_{\nu}$, we can study the behaviour of a spherically symmetric solution close to the two points of physical relevance: $r = 0$ and $r = \infty$. The form of a spherically symmetric solution is

$$f^0_0 = D = 1-u(r) = x^{\delta} \sum_{n=0}^{\infty} d_n x^n \quad ;$$

$$f^r_r = A = 1-w(r) = x^{\alpha} \sum_{n=0}^{\infty} a_n x^n \quad ;$$

$$f^{\theta}_{\theta} = f^{\varphi}_{\varphi} = B = 1-v(r) = x^{\beta} \sum_{n=0}^{\infty} b_n x^n \quad , \qquad (5)$$

and the leading asymptotic behaviour (which corresponds to the linearized Pauli-Fierz theory) is

$$u_1(r) = -a\, e^{-rm}\, (rm)^{-1} \quad ;$$

$$v_1(r) = \frac{1}{2}\, a\, e^{-rm} \left\{ (rm)^{-1} + (rm)^{-2} + (rm)^{-3} \right\} \quad ;$$

$$w_1(r) = -a\, e^{-rm}\, [(rm)^{-2} + (rm)^{-3}] \qquad (6a)$$

(where $4\pi a = gm$ when the static source is $S_{ij} = g\,[\delta_{ij} - \Delta^{-1}\,\partial_i \partial_j]\,\delta_3(\vec{x})$).

Going a step further, it is also possible to see that, because of the non-linear terms, a more accurate asymptotic behaviour is given by

$$u_2(r) = u_1(r) - \frac{1}{3}\, a^2\, e^{-2(mr)}\, (rm)^{-2} \quad ; \quad v_2 = v_1 + \frac{1}{6}\, a^2\, e^{-2mr}\, (mr)^{-2} \quad ;$$

$$w_2 = w_1 + \frac{5}{4}\, a^2\, e^{-2mr}\, (mr)^{-2}. \qquad (6b)$$

At $r = 0$, if we assume that we can expand as we indicated in the right-hand sides of Eqs.(5), we found that the leading terms of the expansions (5) are

$$A[r = 0] = -\frac{1}{14}(rm)^{-2} \quad ; \quad B[r = 0] = -\frac{1}{2}\,(rm)^{-2} \quad ; \quad D(r = 0) = \frac{1}{2}\,(rm)^{-2} \qquad (7)$$

which do not depend upon any physical constant besides m. We note that this property is due to the existence of the massive term and not to the non-linear character of f-g theory.

REFERENCES

1) C.J. Isham, Abdus Salam and J. Strathdee, Phys. Rev. D3, 867 (1971).

2) R. Arnowitt, S. Deser and C.Misner, in Gravitation , ed. L. Witten (Wiley & Sons, New York 1962).

3) C. Aragone and S. Deser, Nuovo Cimento 3A, 709 (1971).