

# INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

INFORMAL MEETING

ON

NEURAL NETWORKS

24 - 26 July 1972

(SUMMARIES)



**INTERNATIONAL  
ATOMIC ENERGY  
AGENCY**



**UNITED NATIONS  
EDUCATIONAL,  
SCIENTIFIC  
AND CULTURAL  
ORGANIZATION**

1972 MIRAMARE-TRIESTE

IC/72/83

International Atomic Energy Agency  
and  
United Nations Educational Scientific and Cultural Organization  
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

I N F O R M A L   M E E T I N G

O N

N E U R A L   N E T W O R K S

24 - 26 July 1972

(SUMMARIES)

MIRAMARE - TRIESTE

August 1972

Please note that copies of papers referred to may be obtained direct from the authors and not from the ICTP.

## MATHEMATICAL LEARNING MODELS AND NEURAL NETWORKS

E. Pfaffelhuber and P.S. Damle

Institute for Information Sciences, University of Tübingen, Fed. Rep. Germany.

Mathematical learning models <sup>1, 2)</sup>, as used in experimental psychology, attempt to describe and predict the learning behavior of humans and animals from a phenomenological viewpoint. It should in principle be possible to derive these models from a microscopic approach, using basic principles of the dynamics of neurons and synapses (in the same way as phenomenological thermodynamics should be derivable from quantum statistics). The two main mathematical learning models describing experimental data sufficiently well are the linear models and the stimulus sampling models.

Linear models describe the temporal change, due to reinforcement, of the system's probability  $q_n(r/s)$  of reacting with response  $r$  to stimulus  $s$  at the  $n^{\text{th}}$  trial by a linear difference equation, which in the simplest case reads:

$$q_{n+1}(r/s) - q_n(r/s) = \beta (\chi_{n+1}(r) - q_n(r/s)) .$$

The inhomogeneous "force" term  $\chi_n(r)$  equals 1 (0) if response  $r$  was (was not) reinforced at trial  $n$ , and  $\beta$  is the learning parameter. For continuous reinforcement the familiar exponentially saturating learning curve results,  $1/\beta$  being the corresponding time constant.

While the linear models assume a gradual formation of a stimulus-response association, the stimulus sampling model, in its simplest form, the so-called one-element or all-or-none model, postulates that a single reinforcement produces either complete learning or no learning

at all. It thus describes the consecutive states of the system by a Markov chain, a switching from the unconditioned state  $q(r/s)=0$  to the conditioned state  $q(r/s)=1$  occurring, due to reinforcement, with a certain transition probability. To describe more complex learning data, one assumes a number  $N$  of individual all-or-none models, each learning independently from all others the association between a particular element of the total stimulus presented and the response  $r$ . The system's overall response probability is then obtained by sampling the elements and building a weighted average of their individual response probabilities.

If very many elements are sampled it follows from the law of large numbers that with a high probability the time behavior of the response probability  $q_n(r/s)$  in the stimulus sampling model will closely resemble the time course as predicted by the linear model, provided the transition probability equals the learning parameter  $\beta$ . Thus the linear model can be considered as a limiting case of the stimulus sampling model, and hence the latter, and in particular its basic unit, the all-or-none model, seems to be of more fundamental importance.

The simplest way to generate this basic unit on a neural basis is to assume a neuronal pathway from a stimulus neuron  $St$ , sensitive to the presented stimulus, via a memory cell  $M$  to a response neuron  $Rs$ , which is connected to the motor (or other) output, such that the  $St$ - $M$  synapse possesses two (or more) stable states, a transition from one to the other occurring with a certain transition probability due to the arrival of signals from a reinforcing neuron  $Rf$ , excitable by the presented reinforcement. It appears that in the octopus' visual lobes these cells have been in part identified<sup>3)</sup>. The postulated synaptic-switching mechanism gives, for suitable parameters of the synaptic states and for low noise level, a good approximation to the mathematical one-element model, and is in accordance with the derepressor hypothesis<sup>4)</sup>. In order that the approach is meaningful, however, we must assume that the time required for stabilization of the synaptic states is smaller than or at most at the order of the time distance between consecutive learning trials.

The step to the general N-element stimulus sampling model is made, analogous to the mathematical model, by assuming N independent synaptic switching mechanisms for N different memory and response cells, the overall response probability being taken proportional to the number of response cells firing. For large N the time behavior of the response probability will closely resemble the linear model, the deviations decreasing with increasing time.

Assuming instead a large number of synapses at one memory cell, as is true in mammalian brains<sup>5)</sup>, and using a time continuous version of Caianiello's neuronal equations<sup>6)</sup>, it follows from a statistical analysis with the help of the central limit theorem<sup>7)</sup> that the time course of the response probability is approximately given by  $1/2 \operatorname{erfc}(A_n/\sqrt{B_n})$  where  $A_n$ ,  $B_n$  are exponentially saturating functions of the trial number n. The result is a typically S-shaped curve, which may also approach a step function for synaptic parameters which still seem to be within the known experimental data<sup>5, 8)</sup>. In the latter case a memory cell with many inputs would represent a more complex neural representation of the abstract all-or-none model.

The mathematical analysis based on a probabilistic synaptic switching mechanism can be extended to include forgetting behavior, inhibitory synapses to describe the closing of pathways<sup>3)</sup>, probability learning<sup>1)</sup>, and more layer series connections of memory cells. It is hoped that the synaptic transition probability which can in principle be inferred from experimental data of learning experiments, if the neuronal topology is known, gives hints as to the underlying chemical or other mechanism of synaptic conditioning.

## REFERENCES

- 1) see R. C. Atkinson, G.H. Bower, and E.J. Crothers, An Introduction to Mathematical Learning Theory, Wiley, 1967; and the literature cited there
- 2) E. Pfaffelhuber, Intern. J. Neuroscience 3, 83 (1972)
- 3) J.Z. Young, The Memory System of the Brain, Oxford University Press, 1966, p. 39 ff
- 4) E.R. John, Mechanisms of Memory, Acad. Press, 1967, p. 131
- 5) J. Doran, Machine Intelligence 6, 207 (1971)
- 6) E.R. Caianiello, A. de Luca, and L.M. Ricciardi, Kybernetik 4, 10 (1967); and the literature cited there
- 7) A. de Luca and L.M. Ricciardi, J. Theor. Biol. 14, 206 (1967)
- 8) B. Katz, Nerve, Muscle, and Synapse, McGraw-Hill, 1966