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(SUMMARIES)



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## COMPLEX SYSTEMS IN TERMS OF COMPARISON AND TOLERANCE RELATIONS

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The role errors play in the performance of complex systems, e.g. in the performance of neural networks, is analysed within the framework of the theory of relational structures <sup>1),2)</sup>. An error is viewed not as an extraneous and misdirected or misdirecting accident but as an essential part in the performance of a complex system and the capacity to control and correct errors is considered essential for systems which exhibit some kind of natural or artificial intelligence. However, the operation, e.g. of a noisy neural network whose components are subject to errors, is often too complicated to be economically expressed by dynamical equations. With any dynamical equation missing, the possibility still remains of comparing the systems operation with that of an error-free reference net whose operation (though not its dynamics) is known or prescribed. This comparative analysis is formalised by means of relational structures. In fact, a conventional function-type system description appears in many instances not to be adequate when dealing with qualitative aspects of complex forms of organization.

It is shown how any deterministic or non-deterministic network of formal neurons is described within the relational framework. The nets operation is, essentially, given by two relations, viz., its next-state relation and its output relation. Also, the state and input errors of a neural net are best represented by certain relations, e.g. state transition errors are given by binary relations on the state set of the net. From the relational point of view, physical causes of errors are irrelevant.

Various notions of masking permanent and temporary state transition errors through the net's organization are analysed and illustrative examples are discussed.

A single error in the performance of a complex neural network can cause the net to be in the wrong state forever. Fortunately, not all errors have this effect. Some errors can persist only for a bounded time. Some, although they could persist forever, can be corrected through the action of certain appropriate input sequences. Those input sequences constitute the

reliable language of a neural network. An effective procedure is given by which the reliable language of a neural network is determined. This algorithm is based on a comparison of the operation of the faulty net with that of its reference net. The reliable languages are tools important for careful routing of a net in the presence of state transition errors from a faulty state into its correct state.

Moreover, by means of the reliable language of a neural network its state set is endowed with a structure (similar to a topology) given by a family of binary relations called failure tolerances. Two states  $q$  and  $q'$  are within failure tolerance  $\tau_\ell$  if state  $q$  can be reached from state  $q'$  (and vice versa) through the action of a reliable input sequence containing less than  $\ell$  inputs.

Obviously, any error relation which is contained in a failure tolerance can be corrected. The notion of a  $\tau_\ell$ -continuous next-state relation is introduced and utilized for an analysis of those networks ( $\ell$ -nets) which have high error correction capacities. Finally, an analysis of state transition errors is given in terms of failure tolerances. State error relations are characterized by two parameters: (a) the length of the shortest input sequence needed in order to bring the faulty and the correct state of the net within failure tolerance and (b) the "diameter"  $\ell$  of this tolerance. An algorithm is given by which these two parameters of an arbitrary temporary or permanent error relation can be determined. Implementations for the organization of a noisy neural network whose error relation is bounded by the above parameters are discussed.

#### REFERENCES

- 1) T. Jónsson, Topics in Universal Algebra, Springer Lecture Notes in Mathematics, 1972.
- 2) H. Böhring, Endliche Automaten I/II, BI Mannheim 1969.