

# INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

INFORMAL MEETING

ON

NEURAL NETWORKS

24 - 26 July 1972

(SUMMARIES)



**INTERNATIONAL  
ATOMIC ENERGY  
AGENCY**



**UNITED NATIONS  
EDUCATIONAL,  
SCIENTIFIC  
AND CULTURAL  
ORGANIZATION**

1972 MIRAMARE-TRIESTE

IC/72/83

International Atomic Energy Agency  
and  
United Nations Educational Scientific and Cultural Organization  
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

I N F O R M A L M E E T I N G

O N

N E U R A L N E T W O R K S

24 - 26 July 1972

(SUMMARIES)

MIRAMARE - TRIESTE

August 1972

Please note that copies of papers referred to may be obtained direct from the authors and not from the ICTP.

### Part III: KINETIC APPROACH

F. Ventriglia

Laboratorio di Cibernetica, Arco Felice, Napoli, Italy.

Models have traditionally played a significant role in the study of neural systems. In view of the complexity of the system which it attempts to describe (system with a large number of elements which have a singular functional behavior and a very entangled net of connections), this is not at all surprising. As usual these models will be tremendously simplified versions of the actual situations. Several models, deterministic or statistical, of neural systems have been formulated<sup>(1-8)</sup>, and all of them, although apparently more or less different, are based upon the same assumption, partially demonstrated by electrophysiological evidence, that significant parameters in the description of the neural system are the activity states of its neurons and the values of their thresholds. In deterministic models the magnitude of the coupling coefficients of the neurons is a significant parameter, while in statistical models it is the average number of input to the neurons. Some models consider only systems with excitatory neurons. It is perhaps well to stress that although the models are mathematically simpler, the actual discussion is still quite complicated. The complete formal solution of many of the models referred to does not yet exist. In accordance with the preceding remarks we have introduced a new statistical model of neural systems, which for its simplicity, is quite suitable for study with statistical mechanic techniques. In a first approach, excitatory neurons only are considered in the model. The model can usefully be visualized as a system composed of two different kinds of particles without mass in a finite volume of 3-D space:  $N$  particles, neurons,  $I$  particles, impulses. The  $N$  particles are in fixed positions, while the  $I$  particles are free. An internal parameter  $E$  is associated with each  $N$  particle; it is quantized and can only assume the values  $E = 0, 1, \dots, S$ . The

"gas" of the I particles does not have a fixed number of elements. In fact, during their motion the I particles collide with the N particles and are absorbed instantaneously. This absorption produces a transition in the internal parameter  $E$  of the N particle receiving the shock:  $\ell \rightarrow \ell + 1$ . Moreover, after some collisions, each N particle reaches the inner state S and produces a bundle of new I particles, while  $E$  returns instantaneously to the zero value. In this model we neither consider  $I \rightarrow I$  collisions nor introduce interaction forces; therefore the only mechanism which changes the velocities of the I particles is the mechanism of creation and destruction. Because the duration of  $I \rightarrow N$  collisions is infinitesimal, only binary collisions occur. It is assumed that, for a macroscopic description of the neural system, it suffices to consider only the distributions  $f(\bar{r}, \bar{v}, t)$  and  $\varphi(\ell, \bar{r}, t)$  where:

$f(\bar{r}, \bar{v}, t) d\bar{r} d\bar{v}$  = probable number of impulses in the positional range  $(\bar{r}, d\bar{r})$  and velocity range  $(\bar{v}, d\bar{v})$  at time  $t$ ;

and

$\varphi(\ell, \bar{r}, t) d\bar{r}$  = probable number of neurons in positional range  $(\bar{r}, d\bar{r})$  in the inner state  $E = \ell$  at time  $t$ .

Later on, making use of the old Boltzmann method for the study of dilute gases there have been constructed the "kinetic equations" of the neural system, i. e. the equation for the rate of change of the distributions  $f$  and  $\varphi$  with the time.

In constructing such equations, for including the possibility of existence of zones of the neural system where the axodendritic termination are more

dense and of others where they are less dense, there has been introduced a sort of cross-section  $\sigma$  which is a smooth real function of the position:  $\sigma \equiv \sigma(\bar{r})$ . Moreover, the axonic structure has also been taken into account by working on the distribution of the velocities of the  $I$  new impulses emitted by the neurons which are in the inner state  $S$ . Finally, we want to point out that the "kinetic equations" are coupled differential equations; thus, for obtaining the distributions  $f$  and  $\varphi$  which describe the model, such equations must be solved simultaneously.

#### REFERENCES

- 1) McCulloch, W. S. and W. Pitts: Bull. Math. Biophys. 5, 115 (1943)
- 2) Rapoport A., : Bull. Math. Biophys. 12, 109 (1950)
- 3) Caianiello, E. R.: J. Theor. Biol. 1, 204 (1961)
- 4) Allanson, J. T.: In Information Theory (Third London Symposium) Butterworth and Co. Ltd. London 303(1956.)
- 5) Beurle, R. L.: Phil. Trans. Roy. Soc. London Sec B 240, 55 (1956)
- 6) Farley, B.G. and W. A. Clark: In Information Theory (Fourth London Symposium) C. Cherry, ed. Butterworth and Co. Ltd. London 242 (1961)
- 7) Hart, E. M., J. Csermely, B. Beek and R. D. Lindsay: J. Theor. Biol. 26, 93 (1970)
- 8) Wilson, H. R. and J. D. Cowan: Bioph. J. 12, 1 (1972)