THE CLASSICAL ELECTRON

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A B S T R A C T

Taking the self force of an electron as zero, the Lorentz-Dirac equation is modified to a new form. It is found that the stability of an electron against the repulsive action of its parts can be explained and the difficult situations like run-away solutions and pre-acceleration can be avoided. Also the procedure of mass renormalization is no longer necessary.

1. INTRODUCTION

The difficulties of the classical theory of electron is known since the beginning of the theory. The first model of the electron was that of a rigid sphere with spherically symmetric charge distribution. This model was studied in detail by Abraham (Abraham.1903). This theory is inconsistent with Lorentz invariance.

One important feature of the theory of electron was the exact evaluation of the force that an electron exerts on itself. This force arises because according to coulomb's law each part of the charged sphere repels all other parts. This self force was first obtained by Lorentz(Rohrlich, 1965). The electron theory based on these equations faced several difficulties.

1) The equation is not Newtonian and it contains structure dependent terms and even if structure dependent terms are dropped for a point electron, it is a third-order differential equation of motion for the position instead of a second order equation.

2) Even if one tries to eliminate the structure-dependent terms, the self-energy(W self) diverges.

Moreover problems arose when one tried to make a relativistic theory from non-relativistic Abraham - Lorentz theory of the electron. For every free particle, the momentum

$$\tilde{P} = \frac{E}{c^2}\,\tilde{V} \qquad (1)$$

Numerous attempts have been made for the modification of this unsuccessful theory(Mie, 1912; Born, 1943; Weyl, 1918; Wessel, 1934; Fokker, 1929).

Dirac(Dirac, 1938) gave an equation which is relativistic generalization of Lorentz equation when applied to a point electron. The electromagnetic mass, m_{elm} is infinite in the limit $R \to 0$ but it is lumped into m_0 to yield the observed mass m, ignoring its divergent nature. (m_0 is the mass of the particle without charge)

The Lorentz - Dirac equation is not without difficulties. The appearance of a third derivative of position does not allow solutions in terms of Newtonian initial condition of position and velocity. A set of solutions results and all but one solutions are meaningless. They give the electron a velocity which increases asymptotically (as $t \to \infty$) to the velocity of light, even when ther is no applied forces. These run-away solutions are the set backs of Lorentz-Dirac equation. Another difficulty of the Lorentz-Dirac equation is that it violates causatity over very short time interva e.g. an acceleration occurs prior to the applied force. Such difficulties made Dirac's theory unacceptable.

After the point charge theory failed, attempts were made to consider a finite, extended charge distribution. They were persued by Bopp and his co-workers(Bopp,1940,1943). But such theo were arbitrary because the form factor cannot be determined from experiment. A theory of this type was given by Prigogine and Heni (Prigogine et. al. 1962) but charge structure of the particle is now dependent on its motion and has no absolute meaning even in th co-moving system.

$\neq 0$ for an extended electron.

In both the cases we have

$$\frac{2}{3}\frac{e^2}{c^3}\dot{a} = 0 \qquad (10)$$

because O(R) being of the order of radius of the electron O(R) $\neq$
The relativistic generalization of this term is given by the fo
vector Γ^μ and is known as the Abraham four vector of radi
reaction.

The Lorentz-Dirac equation is

$$m a^\mu = \frac{e}{c} F_{in}^{\mu\nu} v_\nu + F_{ext}^\mu + \Gamma^\mu \qquad (11$$

where

$$\Gamma^\mu = \frac{2}{3}\frac{e^2}{c^3}\left(\dot{a}^\mu - \frac{1}{c^2} a^\lambda a_\lambda v^\mu\right) \qquad (1$$

m and e are the observed mass and charge of the point electron
v^μ the four velocity, a^μ the four acceleration and τ the
proper time. $F_{in}^{\mu\nu}$ is the incident radiation field and F_{ext}^μ
is the external forces with which the electron may interact.

In our model of electron we have $\Gamma^\mu = 0$ and the
Lorentz-Dirac equation reduces to

$$m a^\mu = \frac{e}{c} F_{in}^{\mu\nu} v_\nu + F_{ext}^\mu$$

where m is the observed mass of the electron.

Now if a charged particle is placed in external field
or incident radiation field it accelerates and an acceleratec
charge units radiation(Lorentz invariant criterion). Thus in
the Lorentz-Dirac equation, we have to write a term which gi
the radiation from the accelerated charge. We do not assume

that radiation is due to an extra force which is another aspect of the electromagnetic field $F^{\mu\nu}_{in}$ (Mo et al.1971). Instead we assume that the radiation emitted is due to external force or incident radiation and vanishes as soon as these are absent and hence posing no problem of solution in the absence of external forces or incident radiation. The equation(13) can now be written in the following form

$$m a^{\mu} = \frac{e}{c} F^{\mu\nu}_{in} v_{\nu} + F^{\mu}_{ext} - \frac{2}{3}\frac{e^2}{c^5} a^{\lambda} a_{\lambda} v^{\mu}$$

The last term is the radiation reaction force $-\frac{1}{c^2} R v^{\mu}$ where

$$R = \frac{2}{3}\frac{e^2}{c^3} a^{\lambda} a_{\lambda}$$ (

In the absence of any external force and incident radiation, the right hand side of equation (14) is zero and we have $a_{\mu} = 0$. Thus there is no run-away solution.

The equation(14) is a local equation and hence no pre-acceleration. There is no infinite self-mass and hence no mass renormalization is required.

3. Summary and Conclusions.

It is often assumed that the electron consists of a bare particle of mass m_0 and a Coulomb field around it. Sin[ce] the bare particle and its Coulomb field always move together

$$m = m_{bare} + m_{coulomb}$$

The quantity m coulomb is the mass equivalent of the electro static self-energy. Since this quantity diverges for a poin[t] charge, the corresponding m bare would have to be $-\infty$ so th[at] m has the observed value. This is the usual mass renormaliz procedure. In our theory $m_{\text{Coulomb}} = 0$ (because charges of one part of an electron do not act on another part), hence $m = m_{bare}$.

Since both m_{bare} and m_{Coulomb} are un-determined, we can distribute m into m bare and m coulomb, such that $m_{bare} = m$ and m coulomb $= 0$. Thus it is highly satisfactory that no relation of the type(16) needs to be specified and no mass renormalization is required. We take m as the physical mass of the electron and do not divide it into parts which cannot be experimentally ascertained.

The equation(14) differs from the Lorentz-Dirac equation by the term $\frac{2}{3}\frac{e^2}{c^3}\dot{a}^\mu$ which is called the Schott term. Our theory do not contain this term. There are some disadvantages of having this term. The physical interpretation of this term is obscure. It is responsible for non-local time dependence of the equation of motion. Let us define

$$K^\mu(\tau) = F^\mu_{in} + F^\mu_{ext} - \frac{2}{3}\frac{e^2}{c^5} a^\lambda a_\lambda v^\mu$$

For a classical system, K^μ changes very little over a time interval τ_0 so that in good approximation(Rohrlich,1965, pp. 150)

$$m a^\mu(\tau) = K^\mu(\tau + \xi\tau_0) = K^\mu(\tau)$$

This local theory obtained from Lorentz-Dirac equation coincides with our equation of motion(14).

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