MODEL OF A CLASSICAL PARTICLE WITH DIRECT SELF-INTERACTION[*]

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ABSTRACT

A Lorentz-covariant model of a classical stable
particle is presented. The model consists of a point-like
charged superluminal 'object' interacting with itself on the
light-cone through a Fokker-type direct interaction. The total
system (i.e. 'object' + 'self-interaction') has the characteristics
of a classical particle. It has a positive rest mass, a time-
like constant linear momentum and a conserved angular momentum
which is composed of 'orbital' and 'internal' parts. No arbitrary
agent is introduced to explain the stability of the particle
and no radiation is associated with the free system. All physical
quantities are finite without renormalisation.

1. Introduction

Fokker-type action principles for describing elementary particle self-interactions have recently been proposed by the late A. Schild[1].

In this paper, we construct a Lorentz-covariant model for a classical one-particle system with an action-at-a-distance-type self-interaction. The precise meaning of these words becomes clear below.

The motivation for this work was to construct a model which could describe the existence of stable charged particles on a classical level.[*]

It is well known that if one tries to imagine a classical model for a charged particle one runs into difficulties of infinite self-fields and self-energies as long as the particle is assumed to be point-like. For a model with a finite extension of charge and mass one has difficulties in explaining the stability of the object unless arbitrary assumptions are made about the internal properties of the charge distribution. All this is well known and documented[2].

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[*] We do not propose to defend here the relevance and the usefulness of a classical model. This has been done very persuasivel and often enough in the past[2].

The model suggested here avoids these difficulties and shows some attractive and novel features éven in the simple approximation which is used below.

The basic ideas of the model are as follows.

Consider a point-like object interacting with itself. The interaction does not involve any field but is described by a dynamical action-at-a-distance principle. From the variation of the action, one obtains equations of motion for the object. The invariance of the action determines conserved quantities for the whole system. Poincaré invariance in particular determines the conservation of a momentum four-vector and an angular momentum antisymmetric tensor. These conservation laws refer to the total system of the 'object' plus the 'self-interaction'. We refer to the total system as particle and distinguish it from the self-interacting object.

One can now ask the question whether a reasonable model could be found for the above-described dynamical situation such that the following simple demands are met: (a) The conserved quantities should behave like the classical momentum and angular momentum of a free particle. That is, the four momentum be a time-like constant four-vector. The system should have a positive rest-mass and possess an angular momentum which has an orbital and an internal part, and which is conserved. (b) There should be no radiation associated with the motion of the free particle.

(c) The stability of the charged particle should be a consequence of the electromagnetic self-interaction only. (d) There should not be infinite self-energies and/or self-fields associated with the particle.

It is shown below that one can construct a simple model of a particle as a self-interacting system satisfying the above requirements.

The essentially new idea here is that one may account for all the above features of a classical particle if the 'object' is allowed to have unorthodox properties. This does not contradict ordinary physics as long as the object is not directly observable. This is in fact the case with the model described below.

There are two unorthodox properties associated with our object. Both are needed to satisfy (c) above. If the self-interaction is to be electromagnetic in nature, it must propagate on the null-cone. Simple geometry shows that an object may interact with itself on the null-cone if the orbit of the object is at least partly space-like. Special relativity demands then that the object always moves on a space-like orbit, i.e. it always moves faster than the speed of light. Our solution as given below yields for the orbit a space-like circle with a time-like axis.

The second unorthodox property associated with the object is needed for the requirement that the orbit and hence the system be stationary. An object which interacts with itself via

electromagnetic action must experience an attractive action

from itself from different times (i.e. from its past and

future) in order to have a stable non-expanding orbit.   This

can happen, as shown below, only if a certain parameter intrinsic

to the object and having the dimension of mass is negative

(the object obviously cannot have a rest-mass).

The time-symmetric character of the self-interaction

assures that there is no radiation.   Moreover the time-symmetric

interaction makes the system stationary.

There are no infinite energies or fields associated

with the particle.   The point-like character of the object

produces an infinite constant in the action only.   This constant

plays no role in the physics of the situation.   The model also

provides an invariant finite size for the particle.

Section II describes and discusses the self-action-at-

a-distance principle.   Section III contains the form of the

conserved quantities.   Section IV shows that the model as described

above solves the dynamical problem.

2. The self-action-at-a-distance principle

Consider a point-like object in Minkowski space* whose dynamics are governed by the following variational principle expressing 'self-action-at-a-distance'

(1)
$$\lim_{\tau_2, \tau_1 \to \infty} \delta S(\tau_2, \tau_1) = 0$$

with

$$S(\tau_2, \tau_1) \doteq \frac{1}{2} \int_{\tau_i}^{\tau_2} \int d\tau d\tau' \Lambda(X - X', \dot{X}, \dot{X}', \tau - \tau')$$

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*   Throughout this paper tensor indices will be completely omitted. A symbol will represent a scalar, a vector, or more generally a tensor according to its definition. Thus a vector x^μ will be denoted by x, a tensor $A_{\mu\nu}$ by A, etc. Scalar products like $a^\mu b_\mu$ will be indicated by $a \cdot b$, squares $a \cdot a$ will be denoted by a^2 and antisymmetric tensor products $a_\mu b_\nu - a_\nu b_\mu$ by $a \wedge b$. In products like $A \cdot a$, where A is a tensor of rank 2, and a is a vector, a saturates the index that is closer to it, i.e. $A \cdot a$ is the vector $A^\mu_{\ \nu} a^\nu$ while $a \cdot A$ stands for $a^\nu A_\nu^{\ \mu}$. Since throughout this paper we do not use tensors of rank higher than 2, this notation will not give rise to any ambiguity. The metric tensor is taken with signature -2, the speed of light is set equal to unity.

m_o here is a real constant with the dimension of a mass, while

q is the interaction constant. It is to be noted that, since

both X and X′ describe the same trajectory, the variation δ

acts on both variables, namely $\delta = \delta_X + \delta_X{}'$. Due to the symmetry

of Λ with respect to X, X′, and their time derivatives, it

follows that $\delta_X = \delta_X{}'$; and consequently, $\delta = 2\delta_X$.

Consider now a point X on the object's trajectory,

and the null-cone generated at that point. The cone will inter-

sect the object's orbit, both in the future and in the past of X,

an odd number of times; in the simplest case just once, as

shown in figure 1. If we fix X then the values of the other

variable which make Λ_1 non-zero are the points of intersection

and X. In other words, the argument of the delta function

in the self-interaction term is either a null-vector or is identi-

cally zero. This suggests that we split the self-interaction

part of the action

$$S_1(\tau_2, \tau_1) \doteq \frac{1}{2} q^2 \int_{\tau_1}^{\tau_2}\!\!\int d\tau d\tau' \; \dot{X}\cdot\dot{X}' \; \delta[(X - X')^2],$$

into two parts.

The first part we define as

$$S_{local}(\tau_2, \tau_1) \doteq q^2 \int_{\tau_1}^{\tau_2} d\tau \int_{\tau-\Delta\tau_1}^{\tau+\Delta\tau_2} d\tau' \dot{X}\cdot\dot{X}' \; \delta[(X - X')^2]$$

where $\Delta\tau_1$ and $\Delta\tau_2$ are positive constants such that in the time intervals given by the above limits $(X - X')^2 \neq 0$ for $X \neq X'$.[*]

The other part is then the difference between $S_1(\tau_2, \tau_1)$ and $S_{local}(\tau_2, \tau_1)$. S_{local} thus defined contains one zero of the argument of the delta function, namely the one where $X \equiv X'$. This is a part of the action which generates an infinite term. It is easy to see however, that $S_{local}(\tau_2, \tau_1)$ is merely an infinite constant and as $\delta S_{local} = 0$ it does not contribute to the equations of motion or to the particle energy momentum.

Consider $S_{local}(\tau_1, \tau_2)$ after the integration with respect to τ' has been performed

$$S_{local}(\tau_2, \tau_1) = \frac{1}{4} q^2 \int_{\tau_1}^{\tau_2} d\tau \lim_{\Delta\tau \to 0} \dot{X}^2 \left(\frac{1}{\dot{X} \cdot \epsilon^+} - \frac{1}{\dot{X} \cdot \epsilon^-} \right)$$

$$\epsilon^{\pm} \doteq X(\tau \pm \Delta\tau) - X(\tau).$$

The limit is taken along the trajectory; hence we can define a scalar ϵ such that $\dot{X} \cdot \epsilon^{\pm} = \pm \epsilon \dot{X}^2$, and write

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[*]   One notices that this term never occurs in the usual formulation of action-at-a-distance theories. These terms are excluded by the very definition of the action-at-distance treatment of a many body system. In this model of self-interaction we have to include it.

$$S_{local}(\tau_2, \tau_1) = \frac{1}{2} q^2 \int_{\tau_1}^{\tau_2} d\tau \; \underset{\epsilon \to 0}{\ell im} \; \frac{1}{\epsilon} \quad .$$

The integral is clearly path-independent. Consequently, $S_{local}(\tau_2, \tau_1)$ is not affected by an infinitesimal variation of the path of integration while keeping the limits of integration fixed, i.e. $\delta S_{local}(\tau_2, \tau_1) \equiv 0$. The local self-action does not play any role at all in the equations of motion which are divergence-free.

The interaction part of the action principle then reads

$$(3) \qquad S_1(\tau_2, \tau_1) = \frac{1}{2} q^2 \underset{\tau_1}{\oint\!\oint}^{\tau_2} d\tau d\tau' \; \dot{X} \cdot \dot{X}' \; \delta[(X - X')^2]$$

where the star signifies that the above discussed 'local' term need not further be considered in the integral.

From eq. (1) we obtain the equations of motion

$$\int_{-\infty}^{+\infty} d\tau' \left(\frac{\partial \Lambda}{\partial X} - \frac{d}{d\tau} \; \frac{\partial \Lambda}{\partial \dot{X}} \right) = 0$$

or

$$(4) \qquad m_o \frac{d}{d\tau} \frac{\dot{X}}{(-\dot{X}^2)^{\frac{1}{2}}} = q \; F \cdot \dot{X}$$

with

(5) $\qquad F = + \dfrac{\partial}{\partial X} \wedge A$.

where again

(6) $\qquad A = q \displaystyle\oint_{-\infty}^{+\infty} d\tau' \ \delta[(X - X')^2]\dot{X}'$

and

$$F = \frac{1}{2}(F_{ret} + F_{adv}) \quad ,$$

with

(7) $\qquad F \begin{smallmatrix} ret \\ adv \end{smallmatrix} = \pm q\sum_{i} \left\{ \dfrac{1}{[\dot{X}_i \cdot (X_i - X)]^3} \ \{\dot{X}_i \cdot (X_i - X)\ddot{X}_i \wedge (X_i - X) - \right.$

$$\left. -[\dot{X}_i^2 + \ddot{X}_i \cdot (X_i - X)] \ \dot{X}_i \wedge (X_i - X)\} \right\} \begin{smallmatrix} ret \\ adv \end{smallmatrix} \quad .$$

The summation in the definition of $F_{ret}(F_{adv})$ is extended to all the intersections of the trajectory with the past (future) light-cone generated at X .

3. Conserved quantities

The action $S(\tau_2, \tau_1)$ is invariant under transformations of the Poincaré group. As a result, the following ten conservation laws follow[3]

$$(8) \quad P = \int\limits_{-\infty}^{\infty} d\tau' \, \frac{\partial}{\partial \dot{X}} \Lambda(X-X', \, \dot{X}, \, \dot{X}', \, \tau-\tau') +$$

$$+\frac{1}{2}(\int\limits_{\tau-\infty}^{\infty\tau}\!\!\! - \int\limits_{-\infty\tau}^{\tau\infty}\!\!\!) \, d\tau' d\tau'' \, \frac{\partial}{\partial(X'-X'')} \, \Lambda(X'-X'', \, \dot{X}', \, \dot{X}'', \, \tau'-\tau'') = \text{const}$$

$$J = \int\limits_{-\infty}^{\infty} d\tau' X_\Lambda \frac{\partial}{\partial \dot{X}} \, \Lambda(X-X', \, \dot{X}, \dot{X}', \, \tau-\tau') +$$

$$+\frac{1}{2}(\int\limits_{\tau-\infty}^{\infty\tau}\!\!\! - \int\limits_{-\infty\tau}^{\tau\infty}\!\!\!) d\tau' d\tau'' \, [X'_\Lambda \frac{\partial}{\partial(X'-X'')} + \dot{X}'_\Lambda \frac{\partial}{\partial \dot{X}}]\Lambda(X'-X'', \dot{X}', \dot{X}'', \tau'-\tau'') = \text{const}$$

The substitution of the expression (2) for Λ into P yields

$$(9) \quad P = \frac{m_0 \dot{X}}{(-\dot{X}^2)^{\frac{1}{2}}} + q^2 \int\limits_{-\infty}^{\infty} d\tau' \, \dot{X}' \, \delta[(X-X')^2] +$$

$$+q^2(\int\limits_{\tau-\infty}^{\infty\tau}\!\!\! - \int\limits_{-\infty\tau}^{\tau\infty}\!\!\!) d\tau' d\tau'' \, (X'-X'')\dot{X}' \cdot \dot{X}'' \, \delta'[(X'-X'')^2],$$

where $\delta'(X'-X'')^2$ means the derivative of $\delta[(X'-X'')^2]$ with respect to its argument, and equals $\dfrac{1}{2\dot{X}'' \cdot (X''-X')} \dfrac{d}{d\tau''} \delta[(X'-X'')^2]$. Consequently (9) reads

$$
(10) \qquad P = \frac{m_o \dot{X}}{(-\dot{X}^2)^{\frac{1}{2}}} + q^2 \int_{-\infty}^{\infty} d\tau' \dot{X}' \, \delta[(X-X')^2] +
$$

$$
+\frac{1}{2}q^2 \left(\int\!\!\!\int_{\tau-\infty} - \int\!\!\!\int_{-\infty\tau} \right) d\tau' d\tau''(X'-X'') \frac{\dot{X}' \cdot \dot{X}''}{\dot{X}'' \cdot (X''-X')} \frac{d}{d\tau''} \delta[(X'-X'')^2].
$$

Integration by parts then gives

$$
(11) \qquad P = \frac{m_o \dot{X}}{(-\dot{X}^2)^{\frac{1}{2}}} + q^2 \int_{-\infty}^{\infty} d\tau' \left[\dot{X}' + (X'-X) \frac{\dot{X} \cdot \dot{X}'}{\dot{X} \cdot (X-X')} \right] \delta[(X-X')^2] -
$$

$$
-\frac{1}{2}q^2 \left(\int\!\!\!\int_{\tau-\infty} - \int\!\!\!\int_{-\infty\tau} \right) d\tau' d\tau'' \frac{d}{d\tau''} \left[(X'-X'') \frac{\dot{X}' \cdot \dot{X}''}{\dot{X}'' \cdot (X''-X')} \right] \delta[(X'-X'')^2].
$$

By using the same procedure, we obtain for the angular momentum J the following expression

$$
(12) \qquad J = \frac{m_o X_\wedge \dot{X}}{(-\dot{X}^2)^{\frac{1}{2}}} + q^2 \int_{-\infty}^{\infty} d\tau' X_\wedge \left[\dot{X}' + \frac{X'}{2} \frac{\dot{X} \cdot \dot{X}'}{\dot{X} \cdot (X-X')} \right] \delta[(X-X')^2] +
$$

$$
+\frac{1}{2}q^2 \left(\int\!\!\!\int_{\tau-\infty} - \int\!\!\!\int_{-\infty\tau} \right) d\tau' d\tau'' \left\{ \dot{X}'_\wedge \dot{X}'' + \frac{d}{d\tau''} \left[X'_\wedge X'' \frac{\dot{X}' \cdot \dot{X}''}{\dot{X}'' \cdot (X''-X')} \right] \delta[(X'-X'')^2] \right.
$$

4. The description of the solution

We assume that the orbit of the object is a cylindrical helix with a time-like axis. Any vector X on this helix may be split into two components

$$(13) \qquad X(\tau, \tilde{\tau}) = x(\tau) + \rho(\tilde{\tau})$$

where $x(\tau)$ is time-like and assumed to be parametized by what is called the ordinary 'proper time' τ which is defined as $d\tau^2 = dx^2$. Consequently

$$(14) \qquad \dot{x}^2 = 1$$

and, as seen below, a solution exists for which

$$(15) \qquad \ddot{x} = 0.$$

ρ in general is parametized by some other parameter $\tilde{\tau}$. We may, however, express the total motion in terms of τ only. Consider the point $x(\tau)$ and the hyperplane which passes through it and is orthogonal to $\dot{x}(\tau)$. The hyperplane intersects the curve X at only one point, because the motion is a helix with axis parallel to $\dot{x}$. Therefore, we can establish a one-to-one correspondence $\tilde{\tau}(\tau)$ between τ and $\tilde{\tau}$ and

write X and ρ in terms of the proper time only

$$X(\tau) \doteq X[\tau, \tilde{\tau}(\tau)] \ , \quad \rho(\tau) = \rho[\tilde{\tau}(\tau)]$$

and hence

(16) $X(\tau) = x(\tau) + \rho(\tau).$

The vector ρ thus defined lies on the hyperplane and, therefore, obeys the equation

(17) $\dot{x}(\tau) \cdot \rho(\tau) = 0$.

Since the orbit $X(\tau)$ is a helix $\rho(\tau)$ describes a plane circular orbit. It, therefore, satisfies the following equations

(18) $\ddot{\rho} + \omega^2_{o}\rho = 0$

(19) $\rho \cdot \dot{\rho} = 0.$

In (18) ω_{o} is the frequency of the rotation. As the motion is circular ω_{o} may be expressed by the invariants r_{o} and v_{o}

(20) $\qquad r_o \omega_o = v_o$

(21) $\qquad v_o^2 = -\dot{\rho}^2$

and

(22) $\qquad r_o^2 = -\rho^2$

the invariant constant system radius. In our approximation $v_o^2 = \text{const}$ and so is $\dot{X}^2$

(23) $\qquad \dot{X}^2 = 1 + 2\dot{\rho} \cdot \dot{x} + \dot{\rho}^2 = 1 - v_o^2.$

It is convenient to redefine the 'mass parameter' characteristic to the object

$$m = m_o \Big/ \sqrt{-\dot{X}^2} = \frac{m_o}{\sqrt{v^2 - 1}}$$

and this parameter is used below.

We show now that the kinematics described above constitutes a solution to the dynamical problem.

The following exact expressions are used to connect x' and ρ' at τ' with their values at τ:

(24) $x' = x + (\tau'-\tau)\dot{x}$

$$\rho' = \rho \cos[\omega_o(\tau'-\tau)] + \frac{1}{\omega_o}\dot{\rho} \sin[\omega_o(\tau'-\tau)] .$$

Substituting (16) and (24) into (7) and (7) into (4), we obtain using eqs. (17)-(19)

$$0 = \frac{d}{d\tau} m\dot{X} - qF \cdot \dot{X} = \{-m\omega_o^2 -$$

$$- q^2 \sum_i \frac{1}{(s_i - r_o v_o \xi_1)^3} [(1-\eta_i)(1+v_o^4) -$$

$$- \omega_o \xi_i s_i(1+v_o^2) + v_o^2 \xi_i^2]\} \rho \doteq$$

$$\doteq \{-m\omega_o^2 + \frac{q^2}{r_o} \sum_i f(s_i)\} \rho$$

where

$$s_i \doteq \tau_i - \tau, \quad \xi_i \doteq \sin \omega_o s_i, \quad \eta_i \doteq \cos \omega_o s_i.$$

One notes that v_o is not an independent variable here as the s_i's determine it from the equation

$$s_i^2 = 2r_o^2(1-\eta_i)$$

which in turn expresses that the interaction is on the light-cone, i.e. that $(X-X')^2 = 0$. From eq. (25) it follows that

$$(26) \qquad m = \frac{q^2}{r_o \omega_o^2} \sum_i f(s_i).$$

As defined above, the summation in this equation extends to all the intersections of the trajectory with the past and future light-cones.

The simplest is the case when the trajectory intersects each light-cone only once. The next simplest one is when the intersection occurs at three points and so on. For the first two cases, one obtains by explicit calculation that[*]

$$(27) \qquad m < 0 .$$

One notices that eq. (26) establishes a connection between m, r_o and v_o. Also from eq. (26) $m = \text{const}$ as the s_i's are constants by construction

Eqs. (24) and (26) are sufficient to calculate the conserved momentum and angular momentum of the particle using

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[*] The calculations become rather involved for the terms with more intersections. We believe, however, that (27) will be valid for all the terms.

expressions (10) and (11). Substituting into them the expressions

for x' and ρ' from eq. (24) one obtains for the momentum

$$(28) \qquad P = \{m - v_o^2 \frac{q^2}{\omega_o^2 r_o} \sum_i f(s_i)\} \dot{x} +$$

$$+ \{m - \frac{q^2}{r_o \omega_o^2} \sum_i f(s_i)\} \dot{\rho}$$

The expression for the mass as derived from the

equations of motion in eq. (26) now yields the desired results

in (28)

$$(29) \qquad P = M\dot{x}$$

where

$$(30) \qquad M = m(1-v_o^2) > 0$$

is the positive rest-mass of the particle. The conserved

momentum is a time-like constant as required.

Similarly one obtains for the angular momentum after

some algebra

$$(31) \qquad J = x_\wedge P + \varkappa \, \rho_\wedge \dot{\rho} + \{m - \frac{q^2}{r_o \omega_o^2} \sum_i f(s_i)\}(x_\wedge \dot{\rho} + \rho_\wedge \dot{x}),$$

where

$$(32) \qquad \varkappa = m + \frac{q^2}{r_o} \sum_i g(s_i)$$

with

$$(33) \qquad g(s_i) = [\frac{1}{2}(1+2\eta_i-3\eta_i^2) - v_o\xi_i s_i(1+\eta_i) +$$

$$+ (1-\eta_i + \frac{1}{2}\xi_i^2\eta_i)v_o^2]r_o^3 / (s_i-r_o v_o\xi_i)^3 .$$

The use of eq. (26) in the expression (31) again yields the desired result for the angular momentum

$$(34) \qquad J = x \wedge P + \varkappa \rho \wedge \dot{\rho} ,$$

where $\varkappa$ is a constant and the orbital and 'spin' angular momenta are conserved separately.

This is then the last step in proving that the model solves the dynamical problem and the particle has all the properties which are listed in the first section.

To finish this note we may consider some numerical examples. We emphasize again that in our opinion it would be naïve to consider this very simple free-particle model as 'realistic'. Numerical examples nevertheless may give some feeling about the physical nature of this model.

Eq. (26) gives a connection between m, r_o, v_o, and q. Eqs. (28) and (31) connect these with the observable or 'dressed' mass M and with the 'spin' angular momentum $S = \varkappa \rho_\wedge \dot{\rho}$. The simplest approach to obtain numbers is to take M and q to be the electron mass and charge. We may then calculate r_o, v_o, S and μ the magnetic moment of the particle due to the object. Some examples are given below.

For the state where the object's orbit intersects the light-cone once

$$m = -299 m_e, \quad r_o = 3.5 \times 10^{-12} \text{cm.}, \quad v_o = 1.002c, \quad S = -0.091 \hbar \quad \mu = 0.091 \mu_B$$

$$m = -0.050 m_e, \quad r_o = 6.9 \times 10^{-13} \text{cm.}, \quad v_o = 4.6c, \quad S = 1.0 \times 10^{-3} \hbar, \quad \mu = 0.082 \mu_B$$

and for three intersections

$$m = -0.017 m_e, \quad r_o = 1.0 \times 10^{-13} \text{cm.}, \quad v_o = 7.7c, \quad S = 6.0 \times 10^{-4} \hbar, \quad \mu = 0.21 \mu_B$$

where the minus sign in S indicates that the gyrogmagnetic ratio is negative.

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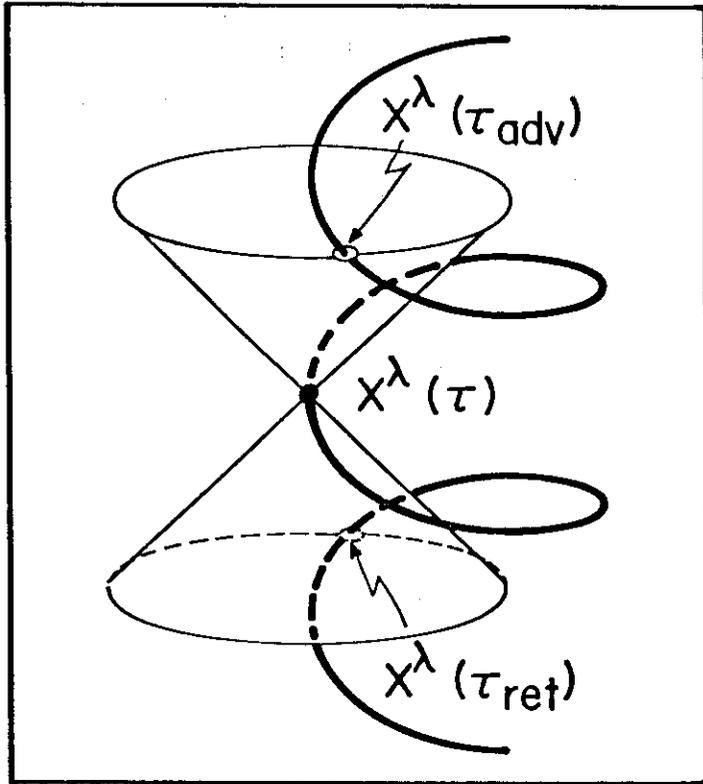
FIGURE 1

where $X \doteq X(\tau)$, $X' \doteq X(\tau')$ and τ is some suitable invariant parameter. To be varied are the vectors X and X' and it is assumed that the variations are zero at and outside of the boundaries. The limits with respect to τ_1 and τ_2 are taken after the variation has been performed. Here

(2) $\qquad \Lambda = \Lambda_o + \Lambda_1$

with

$$\Lambda_o = -2m_o(-\dot{X} \cdot \dot{X}')^{1/2} \; \delta[(\tau - \tau')]$$

$$\Lambda_1 = q^2 \dot{X} \cdot \dot{X}' \quad \delta[(X - X')^2]$$

called 'inertial' and 'self-interaction' terms. The first term is clearly identical to the ordinary inertial term in relativistic dynamics. It is, however, preferable to write it as above when establishing invariance properties, conservation laws, etc.[*] The action is then invariant under the change of the parameter.

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[*] Another symmetric way to write the integrand is

$\Lambda = \dot{X} \cdot G \dot{X}'$ with $G = 2 \, m_o \; \delta[(\tau - \tau')(-\dot{X} \cdot \dot{X}')^{1/2}] + q^2 \; \delta[(X - X')^2]$.