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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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SECOND SCHOOL ON ADVANCED TECHNIQUES
IN COMPUTATIONAL PHYSICS
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The REDUCE Program for Algebraic Computation

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THE REDUCE PROGRAM

FOR ALGEBRAIC COMPUTATION

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The RAND Corporation

REDUCE IS ...

- A symbolic and algebraic computation system
- Useful for general purpose algebraic calculations in science and engineering
- Available at many sites
  (Over 1000 copies worldwide)
- Widely referenced
  (Over five hundred papers)

DESIGN GOALS

- Portable
- Interactive
- Moderate size working set
- Large program library
- Modular construction
- Clear algorithmic specification
- Standardized language

DISTINCTIVE FEATURES

- Available on a wide range of computers
- Available worldwide
- Widely referenced
- Distributed support
- Source system
- Active development

EXPRESSION SYNTAX

- "Natural" notation for arithmetic expressions
- Obvious extensions for algebraic constructs (DF, INT, SUB ...)
- Combining forms for looping and conditional testing (FOR, IF, REPEAT, WHILE ...)
- Additional data structures (arrays, matrices ...)

SIMPLIFICATION CONTROL

- Functions vs switches
- Limited dependence on declarations
- Internal control vs output control
- Heuristic pattern-directed vs algorithmic

A REDUCE SAMPLER

In the following sample run comments are represented in italics, user typed input is represented by the bold typeface and computer output is represented by the regular typeface.

Integer arithmetic is exact

A := (FOR I := 1:50 PRODUCT I);

```
A := 30414093201713378043612608166064768844377641568960512000000000000
```

and so is rational arithmetic.

A/2^60;

```
 216105129892080882169214875191192738017616943359375
-----------------------------------------------------
                         8192
```

Floating point numbers can also be as accurate as you wish.

ON BIGFLOAT, NUMVAL;
PRECISION 25;

```
25
```

This looks like an integer ...

E(PI*SQRT(163));**

```
26253 74126 40768 744.0
```

... but higher precision shows the truth.

PRECISION 35;

```
35
```

E(PI*SQRT(163));**

```
26253 74126 40768 743.9 99999 99999 92500 7
```

Polynomials can be factored.

ON FACTOR;
(X^20-1);

```
  8    6    4    2          4    3    2                4    3    2
(X  - X  + X  - X  + 1)*(X  + X  + X  + X + 1)*(X  - X  + X  - X + 1)*
  2
(X  + 1)*(X + 1)*(X - 1)
```

OFF FACTOR;

Differential and Integral calculus examples.

INT(1/(X^8-1),X);

```
                        SQRT(2) + 2*X                           SQRT(2) - 2*X
 - (2*SQRT(2)*ATAN(---------------) - 2*SQRT(2)*ATAN(---------------)
                           SQRT(2)                                  SQRT(2)
                                                        2
    + 4*ATAN(X) + SQRT(2)*LOG(SQRT(2)*X + X  + 1) - SQRT(2)*LOG( -
                      2
    SQRT(2)*X + X  + 1) - 2*LOG(X - 1) + 2*LOG(X + 1))/16
```

DF(WS,X);

```
    1
--------
  8
 X  - 1
```

Other examples ... ...

```
DF( (X*(COS(LOG(X)) + SIN(LOG(X))))/2, X);
COS(LOG(X))
INT(COS(LOG(X)),X);
 X*(COS(LOG(X)) + SIN(LOG(X)))
-------------------------------
               2
INT(1/COS(X),X);
              X                     X
 - (LOG(TAN(---) - 1) - LOG(TAN(---) + 1))
              2                     2
INT((A+B*X)^N,X);
            N
 (A + B*X) *(A + B*X)
---------------------
      B*(N + 1)
```

... again using the variable ws.

```
DF(EXP EXP EXP EXP X,X);
    X
   E      X
  E      E     X
 E    + E   + E  + X
E

INT(WS,X);
    X
   E
  E
 E
E
```

REDUCE knows about sine, cosine, exponentials, and logarithms, and many other functions. If that is not enough users can define their own functions.

```
OPERATOR MYFN;
MYFN(X)+Y;
MYFN(X) + Y
```

You can give rules for differentiation for example

```
FOR ALL X LET DF(MYFN(X),X) = EXP(X^2);
```

and now use it to simplify a second derivative,

```
DF(MYFN(X)^3,X,2);
    2                          2
   X                          X
6*E  *MYFN(X)*(MYFN(X)*X + E   )
```

and some integrals.

```
INT(X*MYFN(X),X);
                                 2
              2                 X
 2*MYFN(X)*X  + MYFN(X) - E  *X
--------------------------------
               4
```

```
                 5       4        3        2
                X  - 19*X  + 143*X  - 533*X  + 984*X - 720
MAT(3,3) := ------------------------------------------
                                  4
```

1/mata might be useful in a FORTRAN program.

```
ON FORT;
WS;

      MAT(1,1)=(X**5-11.*X**4+47.*X**3-97.*X**2+96.*X-36.)/
     . 4.
      MAT(1,2)=-(X**5-13.*X**4+65.*X**3-155.*X**2+174.*X-
     . 72.)/2.
      MAT(1,3)=(X**5-15.*X**4+85.*X**3-225.*X**2+274.*X-
     . 120.)/4.
      MAT(2,1)=-(X**5-13.*X**4+65.*X**3-155.*X**2+174.*X-
     . 72.)/2.
      MAT(2,2)=X**5-15.*X**4+88.*X**3-252.*X**2+352.*X-192.
      MAT(2,3)=-(X**5-17.*X**4+113.*X**3-367.*X**2+582.*X-
     . 360.)/2.
      MAT(3,1)=(X**5-15.*X**4+85.*X**3-225.*X**2+274.*X-
     . 120.)/4.
      MAT(3,2)=-(X**5-17.*X**4+113.*X**3-367.*X**2+582.*X-
     . 360.)/2.
      MAT(3,3)=(X**5-19.*X**4+143.*X**3-533.*X**2+984.*X-
     . 720.)/4.

OFF FORT;
```

Matrix eigenvalues ...

```
MATRIX RZ;
RZ := MAT( (2,-1,1), (0,1,1), (-1,1,1))$
MATEIGEN(RZ,ETA);

{{ETA - 1,
  MAT(1,1) := ARBCOMPLEX(1)
  MAT(2,1) := ARBCOMPLEX(1)
  MAT(3,1) := 0
  },
 {ETA - 2,
  MAT(1,1) := 0
  MAT(2,1) := ARBCOMPLEX(2)
  MAT(3,1) := ARBCOMPLEX(2)
  }}
```

Complex Numbers.

```
factorize(A**2+1);

  2
{A  + 1}

ON COMPLEX;
factorize(A**2+1);

{A - I,A + I}
```

Solving a system of regular linear equations.

```
SOLVE({2*X+Y+3*Z-9, X-2*Y+Z+2, 3*X+2*Y+2*Z-7},{X,Y,Z});

{{X=-1,Y=2,Z=3}}
```

Solution of non-linear equations not ready yet.

```
SOLVE({X^2 + Y^2, 2*X + 1},{X});

***** NON LINEAR EQUATION SOLVING NOT YET IMPLEMENTED
```

REDUCE can handle matric es as well.

```
MATRIX RX(3,3), RY(3,3), MATA(3,3);
RX := MAT( ( 1,         0,         0),
           ( 0,  COS(TH), SIN(TH)),
           ( 0, -SIN(TH), COS(TH)) )$
RY := MAT( ( COS(PHI), 0, -SIN(PHI)),
           (        0, 1,         0),
           ( SIN(PHI), 0,  COS(PHI)) )$
RX*RY;
```

```
MAT(1,1) := COS(PHI)
MAT(1,2) := 0
MAT(1,3) :=  - SIN(PHI)
MAT(2,1) := SIN(TH)*SIN(PHI)
MAT(2,2) := COS(TH)
MAT(2,3) := COS(PHI)*SIN(TH)
MAT(3,1) := COS(TH)*SIN(PHI)
MAT(3,2) :=  - SIN(TH)
MAT(3,3) := COS(TH)*COS(PHI)
```

Inversion of Hilbert matrices.

```
MATA := MAT( (1/(X-1), 1/(X-2), 1/(X-3)),
             (1/(X-2), 1/(X-3), 1/(X-4)),
             (1/(X-3), 1/(X-4), 1/(X-5)) )$
1/MATA;
```

$$MAT(1,1) := \frac{X^5 - 11X^4 + 47X^3 - 97X^2 + 96X - 36}{4}$$

$$MAT(1,2) := -\frac{X^5 - 13X^4 + 65X^3 - 155X^2 + 174X - 72}{2}$$

$$MAT(1,3) := \frac{X^5 - 15X^4 + 85X^3 - 225X^2 + 274X - 120}{4}$$

$$MAT(2,1) := -\frac{X^5 - 13X^4 + 65X^3 - 155X^2 + 174X - 72}{2}$$

$$MAT(2,2) := X^5 - 15X^4 + 88X^3 - 252X^2 + 352X - 192$$

$$MAT(2,3) := -\frac{X^5 - 17X^4 + 113X^3 - 367X^2 + 582X - 360}{2}$$

$$MAT(3,1) := \frac{X^5 - 15X^4 + 85X^3 - 225X^2 + 274X - 120}{4}$$

$$MAT(3,2) := -\frac{X^5 - 17X^4 + 113X^3 - 367X^2 + 582X - 360}{2}$$

Table of REDUCE Timings

Machine	CPU Time (seconds)	Wall Clock (seconds)	Region Size (megabytes)	Total Size (megabytes)
Amdahl 470 V7/A	10.3	11.9	0.7	8.0
Amdahl 470 V8	7.2	179.6	0.7	12.1
Apollo DN 300	78.7	108.5	2.0	1.4
Apollo DN 320	80.4	121.0	2.0	1.4
Apollo DN 400	125.5	189.0	2.0	1.0
Apollo DN 600	89.9	110.0	2.0	2.0
CDC Cyber 170/825	106.8	106.8	0.6	*
DEC 1099	17.7	240.0	1.0	4.5
DEC 2020	122.8	192.0	0.4	1.1
DEC 2060	22.5	125.0	0.7	5.1
DEC KL-10	24.6	*	*	0.6
DEC KL-1091	44.1	234.0	*	*
DEC VAX 11/750	78.7	132.6	1.3	1.3
DEC VAX 11/780	50.3	60.1	2.6	5.5
Facom M-180N	21.3	25.6	0.5	8.0
Facom M-200	7.2	8.2	1.3	7.0
Facom M-380	3.8	25.0	2.4	32.0
Facom M-382	3.6	4.1	4.0	64.0
Hewlett-Packard 9836	65.3	65.3	4.4	4.4
Hitachi M-200H	6.5	8.4	0.7	0.8
Hitachi M-280H	4.0	5.0	2.9	32.0
Hitachi S-810	2.8	4.5	1.5	1.5
IBM 3031	40.7	48.7	0.5	5.4
IBM 3033	9.6	11.4	1.1	13.1
IBM 3081	6.6	8.3	1.2	24.0
IBM 3083	7.6	8.2	2.0	9.0
IBM 3084	5.4	6.3	0.7	32.0
IBM 370 158	49.9	58.0	2.0	6.0
IBM 4341 Model 1	52.0	60.9	1.0	1.7
IBM 4341 Model 2	30.1	45.7	0.9	7.2
MRP ES-1060	68.4	113.4	0.3	*
NAS AS6	18.2	*	4.0	6.0
NAS NAS7000	23.4	*	*	*
Robotron ES-1040	149.2	175.5	0.5	0.5
Sage IV	224.8	224.8	1.0	1.0
Siemens 7865	19.4	21.9	1.0	*
Siemens 7890	3.8	4.2	0.9	14.0
SML Darksur	227.9	227.9	1.7	1.7
Symbolics 3600	45.0	45.0	160.0	6.0
Xerox Dolphin	322.0	322.0	1.2	1.2

REDUCE IMPLEMENTATIONS

- Cray 1, X-MP
- IBM 3090, 3080, 4300, 370, 360 and clones
- DECSystem 10 and 20 series
- DEC VAX series
- CDC Cyber series
- Many MC68000 systems
- IBM PC and clones

COMMON LISP VERSION

- Data General
- HP 9000/300 series
- IBM PC/RT
- Prime
- Symbolics
- Sun

WHAT'S NEW IN REDUCE 3.3?

- Algorithmic improvements
- Bug fixes
- New LIST data structure

```
solve({2x1+3x2=y1,3x1+x2=y2},{x1,x2});
```

$$\{\{X1=-\frac{Y1-3*Y2}{7},X2=\frac{3*Y1-2*Y2}{7}\}\}$$

REDUCE 3.3 CONTRIBUTED PACKAGES

- Algebraic function integration
- Algebraic numbers
- Exterior calculus computations
- Groebner basis computations
- Improved FORTRAN output
- Symmetries of partial differential equations

SYSTEM SPECIFIC FACILITIES

- Graphics
- Interactive User Interface
- On-line Help