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SECOND SCHOOL ON ADVANCED TECHNIQUES
IN COMPUTATIONAL PHYSICS
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LECTURE NOTES

by

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ALGORITHMS

Background and References

An Algorithm is a method for solving a particular problem or class of problems, which is suitable for implementation in the form of a computer program. Ideally the algorithm should be independent of the particular computer to be used.

Algorithms have been developed for a wide range of problems : numerical, sorting and merging, pattern recognition, data storage and retrieval, graphics, computational geometry, robotics, artificial intelligence, etc., etc.

For any algorithm we need to know under what circumstances it can be guaranteed to do what it is supposed to do, and, in particular will it always detect invalid data and will it always terminate. We also need to know the time and/or space requirements of the algorithm (and any special requirements, e.g. two disc drives).

In these lectures an introduction to the theory of algorithms, illustrated by examples, will be given and some time will be devoted to "the most challenging problem facing computer scientists today" - the design of algorithms for parallel computers.

REFERENCES

1. Knuth, D.E. — The Art of Computer Programming, Vol. 1: Fundamental Algorithms, Addison-Wesley, 1968.

2. Knuth, D.E. — The Art of Computer Programming, Vol.2: Semi-numerical Algorithms, Addison-Wesley, 198l (2nd edition).

3. Knuth, D.E. — The Art of Computer Programming, Vol. 3: Sorting and Searching, Addison Wesley, 1975.

4. Sedgewick,R. — Algorithms, Addison-Wesley, 1983.

APPLICATION OF COMPUTERS TO MATHEMATICS

Computers offer mathematicians opportunities and challenges without parallel in the history of mathematics.

Opportunities: To exploit the power of computers:

(i) To make new discoveries in mathematics;

(ii) to improve mathematical publication, communication and information retrieval;

(iii) in the teaching of mathematics itself;

(iv) (longer term) in automatic proof verification and (very long term) automatic theorem proving.

CHALLENGES:

(i) To develop new methods (algorithms) which are efficient on computers, and particularly on systems of parallel computers, for the solution of a wide range of problems.

(ii) To re-examine old or neglected areas of mathematics to see if we can use the computer to shed new light on old problems.

(iii) To invent new branches of mathematics.

Conversely a knowledge of certain branches of mathematics can be of immense benefit in increasing the efficiency of programs when applied to

problems involving discrete variables. The theory of numbers and the subject of Fractals will be used to illustrate these varous points.

REFERENCES

Papers relating to the use of computers in mathematical research will be found in journals such as Mathematics of Computation (U.S), and the Bulletin of the Institute of Mathematics and its Applications (U.K.).

There are many good books on the Theory of Numbers, including

G.H.Hardy and E.M.Wright " An introduction to the theory of numbers", Oxford University Press (4th edition, 1959).

An excellent all-round introduction to most branches of Number Theory. Long or difficult proofs (e.g. the Prime Number theorem) are not given.

W.J.LeVeque "Topics in Number Theory" Vols I and II;Addison-Wesley,1956.

Vol. I deals with the more elementary topics, roughly corresponding to those covered in these lectures.

Vol. II covers more advanced topics including a proof of the Prime Number Theorem, and Roth's Theorem (1955) on rational approximation to algebraic numbers.

Both volumes are well-written and are very nice to read.

H. Davenport "The Higher Arithmetic", Hutchinson (1952).

A beautifully written little book that goes deeper than might appear at first sight.

B.W.Jones "The theory of numbers'; Holt, Reinhart and Winston (1961).

J. Hunter "Number Theory"; Oliver and Boyd (1964).

Both can be recommended as introductory texts.

Oystein Ore: "Number Theory and its History"; McGraw-Hill (1948).

A very readable account from a historical point of view.

Two books on Fractals:

B. Mandelbrot "The fractal geometry of nature" Freeman (San Francisco), 1982.

H-O.Peitgen and P.H.Richter "The beauty of fractals,"(Springer-Verlag), 1986.

INTEGRAL EQUATIONS

SOME COMMENTS AND REFERENCES

A University-level course on Integral Equations would occupy at least 20 lectures and so in 2 lectures it is possible only to give an introduction to the subject, an idea of some of the methods of solution and a short list of books which can be recommended for those who wish to pursue the subject further.

Although the research literature on Integral Equations is fairly substantial, though not comparable to that on Differential Equations, there are relatively few books devoted exclusively to them. The person who is meeting the subject for the first time might however find it useful to look at some of the general books on Numerical Analysis which contain a chapter on Integral Equations; a few are given below, but there are many others.

REFERENCES

Specialised books

Baker, C.T.H. (1977). The numerical treatment of integral equations; Oxford University Press. [A large and comprehensive work, over 1000 pages, essential for researchers in the subject.]

Buckner, H.F. (1952) Die praktische Behandlung von Integralgleichungen; Springer-Verlag, Berlin. [A relatively early, but standard, work in German.]

Delves, L.M. and Walsh, J. (1974) Numerical solution of integral equations; Oxford University Press. [A good introduction to the principal problems and methods with some applications and theoretical background.]

Hochstadt, H. (1973) Integral Equations; Wiley, New York. [Well illustrated by examples and very readable.]

General books on Numerical Analysis

Churchhouse, R.F. (ed) (1981) Handbook of Applicable Mathematics, Vol. 3 (Numerical Analysis), Wiley.

Collatz, L. (1966) Functional Analysis and Numerical Analysis; Academic Press, New York.

Froberg, C-E (1985) Numerical Mathematics, Theory and Computer Applications, Benjamin/Cummings, Menlo Park, California.

Todd, J.(ed)(1962) Survey of Numerical Analysis, McGraw-Hill, New York