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QUANTUM NOISE EFFECTS IN SINGLE-MODE INJECTION LASERS

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Recently, a great, increasingly growing interest has arisen in the field of optical coherent communication systems because of two main factors: first, the great development of the technology of semiconductor materials which has made commercially available injection lasers capable of stable operation in a single longitudinal and transverse mode; second, the well acquired capability of producing single-mode fibers with very low attenuation losses.

Coherent communication systems via optical fibers, both of the homodyne or heterodyne type, present, in fact, several advantages with respect to direct detection systems such as, for example, the ease of obtaining wavelength multiplexing with closely spaced channels, the possibility of employing optical amplifiers, a greater receiver sensitivity with a gain in the range of 10-20 dB depending on the adopted modulation system. For the above mentioned reasons, one immediately understands why a detailed knowledge of the noise properties of single-mode semiconductor lasers is absolutely needed, since these devices represent the most important part of the systems due to the possibility of employing them as transmitters, modulators, local oscillators and/or optical amplifiers. In fact, a detailed knowledge of the power spectral density of noise and, in particular, of the noise associated with the fluctuations of the instantaneous frequency of the emitted radiation, makes it possible to determine the correct shape and width of the emission line of the source, which is the most important characteristic of the device both from physical and applicative points of view.

To take into account the amplitude and frequency fluctuations of the laser field, the electric component of the radiation from a single-mode laser is written

$$\mathcal{E}(t) = E(t)\exp(i\,\omega(t)t) = (E_o + e(t))\exp(i(\omega_o t + \phi(t))) \quad (1).$$

Here $e(t) \ll E_o$, the deviation from the mean amplitude E_o, and $\phi(t)$, the random phase of the field such that $\dot{\phi}(t)=\omega(t)-\omega_o$ represents the deviation of the instantaneous frequency $\omega(t)$ from its mean value ω_o, are real quantities which give rise to amplitude and phase noise respectively.

In semiconductor lasers, which are characterized by a low Q cavity value, noise has to be actually associated with quantum processes, that is the spontaneous emissions of photons into the lasing modes, while in other kinds of lasers (solid state, gas, dye lasers) the most important causes of noise must be seeked for in the mechanical and thermal fluctuations of the cavity length. This can be immediately recognized by the following example. According to the Shalow-Townes' theory, the width, at half height, of the emission line of a single mode laser, evaluated taking into account only spontaneous emission processes, is given by

$$\Delta\nu_{sp} = \frac{h\omega_o \Delta\nu_c^2}{P} \qquad (2),$$

where $\Delta\nu_c$ represents the linewidth of the passive cavity and P is the emitted power. For an He-Ne laser with P=1mW, $\omega_o \cong 3\cdot 10^{15}$ rad·s^{-1}, $\Delta\nu_c \cong 10^5$ s^{-1}, we have $\Delta\nu_{sp} \cong 10^{-3}$ s^{-1} ; in the case of an AlGaAs laser with P=1mW, $\omega_o \cong 2\cdot 10^{15}$ rad s^{-1}, $\Delta\nu_c \cong 10^{11}$ s^{-1}, so that $\Delta\nu_{sp} \cong 10^7$ s^{-1} (the great difference of $\Delta\nu_c$ for the two lasers is due to the substantial differences in their mirror reflectivities and cavity lengths). Now, because the processes other than spontaneous emissions cause a further broadening of the line which can be easily kept as low as 10^4 s^{-1}, one immediately understands why in laser diodes only quantum noise must be pratically accounted for.

Intensity noise in semiconductor lasers has been fully analyzed both from an experimental and a theoretical point of view since many years[1,2]; for what concerns phase and frequency noise, however, only in these last three years some of their peculiar characteristics have been observed and explained. The starting point was given by an experiment performed by Fleming and Mooradian[3]; measuring the emission line of an AlGaAs injection laser they found, according to the Shalow-Townes' theory, it to be Lorentzian in shape, with a full width at half intensity which varied inversely with output power P, but about 50 times greater, at 300 K, than expected. The explanation of this unexpected fact is due to Henry[4]. He underlines as every spontaneous emission process gives rise to a total change in the phase of the field which is the sum of two different contributions: the first is the one considered usually, associated with the random phase of the spontaneously emitted photon, while the second has to be attributed to the change in the field intensity, subsequent to any emission process, which determines a variation of the refractive index of the cavity. Considering the two above contributions, Henry was able to state that the emission line had to be Lorentzian in form but that its width had to be enhanced by a factor $(1+\alpha^2)$, when compared to that predicted by conventional theories; α, the so-called "linewidth enhancement factor", is the ratio between the derivatives of the real and imaginary part of the refraqtive-index in the active region with respect to the carrier density, and, while being negligible in other kinds of lasers, is greater than unity in injection lasers as to substantially determine their emission line width (values in the range 2-6 have been found, or estimated, depending on the composition and the physical structure of the laser itself[4-6]). This last fact can be understood considering that semiconductor lasers act as detuned cavity oscillators[4].

The above theory, however, even if accounts for the excessive anomalous broadening, is valid only when an equilibrium or, to be more precise, a quasi-equilibrium condition exists. In fact, it takes into account only changes of the instantaneous phase of the electromagnetic field over time intervals, that in injection lasers are of the order of 1 ns, during which the relaxation oscillations undergone by the system after any spontaneous emission process have completely died out. That is equivalent to consider in the power spectrum of $\dot{\phi}(t)$, the instantaneous frequency deviation of the field, only that region of frequencies $\Omega < \tau_r$, τ_r being the relaxation oscillation damping time. In order to have a more complete information about the shape of the emission line, one must also take into account the effect of the relaxation oscillations consequent to any spontaneous emission event. In fact, one expects some form of correlation in the laser noise over time intervals shorter than τ_r, and hence a substantial difference in the shape of the power spectrum of the instantaneous frequency deviation at $\Omega > \tau_r$ with respect to the completely flat behaviour one would have if the emission lineshape were fully Lorentzian.

Phase noise measurements in the high frequency region have been performed using the experimental set-up shown in fig.1[7,8].

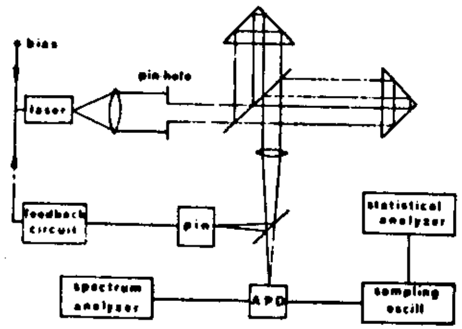

Fig.1-The experimental set-up.

It consists essentially of a Michelson interferometer whose mirrors

are substituted with two right angle prisms in order to avoid undesired reflections which could induce instabilities in the source, a single-mode semiconductor laser stabilized in temperature within 0.1 °C. The output from the interferometer is spatially filtered and detected by a fast APD whose signal is sent to a spectrum analyzer or to a sampling oscilloscope followed by a statistical analyzer. In so doing, one can obtain both the power spectral density and the probability density function of the signal from the APD. A slow feedback circuit, driven by a fraction of the light output from the interferometer detected by a p-i-n photodiode, is used to keep the mean wavelength of the laser radiation locked to the interferometer.
It is easy to see that if the interferometer is adjusted in quadrature, that is $\omega_0\tau = \pi/2$, where τ is the time delay between the beams travelling in its two arms, and τ is small enough that $|\Delta_\tau\phi(t)| = |\phi(t+\tau) - \phi(t)| \ll \pi/2$, the power spectra $S_{\Delta_\tau\phi}(\Omega)$ and $S_{\dot\phi}(\Omega)$, associated to the phase shift $\Delta_\tau\phi(t)$ and to the instantaneous frequency deviation $\dot\phi(t)$ respectively, are related to the power spectrum $S_i(\Omega)$ of the signal from the APD by the relation

$$S_{\Delta_\tau\phi}(\Omega) = \frac{S_{\dot\phi}(\Omega)}{\Omega^2}(1-\cos\Omega\tau) = S_i(\Omega) \,/\, (i_1^2 R/4) \qquad (3).$$

Here R is the input impedance of the spectrum analyzer and i_1 is the maximum excursion of the fringe pattern revealed from the APD.

Figure 2 shows $S_{\dot\phi}(\Omega)$ for different lasers measured at different operating conditions. At low frequencies these spectra exhibit a flat behaviour just as predicted by conventional theories; in the high frequency region, however, they show the presence of a sharp resonant peak at nearly the same frequency of the already known peak in the power spectrum of intensity noise [1,2] (for completeness, fig.3 shows the measured power spectra of intensity noise of two lasers of fig.2). This fact suggests that the above peak in $S_{\dot\phi}(\Omega)$ must be related to the change of the refractive-index of the active medium consequent to the spontaneous emission processes, and its most important consequence is that the emission lineshape is not Lorentzian. In fact, being $\dot\phi(t)$ a stationary ergodic normally distributed process [8,9], the lineshape can be shown to be

$$S_E(\omega) \propto \int_{-\infty}^{+\infty} \exp(i(\omega-\omega_0)\tau)\,\exp(-\sigma^2(\tau)/2)\,d\tau \qquad (4),$$

where

$$\sigma^2(\tau) = \langle(\Delta_\tau\phi(t))^2\rangle = (2/\pi)\int_0^{+\infty} S_{\dot\phi}(\Omega)(1-\cos\Omega\tau)/\Omega^2\,d\Omega \qquad (5)$$

is the variance of $\Delta_\tau\phi(t)$ which, in turn, is a zero mean, ergodic and normally distributed process, too [8,9]. The emission lineshape would be Lorentzian only in the case in which $\sigma^2(\tau) \propto |\tau|$. Figure 4 shows $\sigma^2(\tau)$ evaluated through eq.(5) for two different lasers; as it results immediately, $\sigma^2(\tau)$ is proportional to $|\tau|$ only at values of $|\tau|$ high enough that the relaxation oscillations after any spontaneous

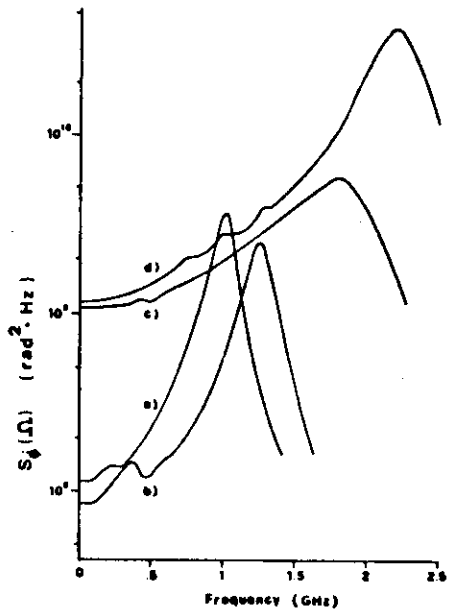

Fig.2- Two-sided spectral density $S_{\dot\phi}(\Omega)$ vs. frequency. Curve a and b : ITTLS7709 laser at P=4mW and 7mW respectively. Curve c: RCA C86014 at P=1.6mW. Curve d : Laser Diode LCW10 at P=3mW.

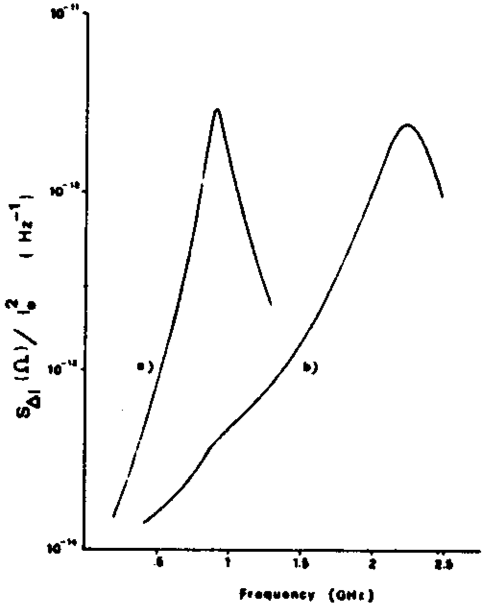

Fig.3- Two-sided normalized intensity-noise spectra $S_I(\Omega)$ vs. frequency. Curve b : Laser Diode LCW10 at P=3mW. Curve a : ITTLS7709 at P=4mW.

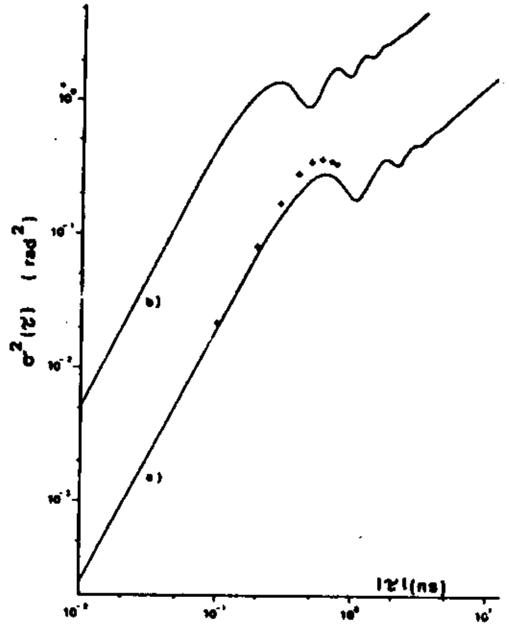

Fig.4- Variance $\sigma^2(\tau)$ versus τ for the ITTLS7709 laser (curve a) and Laser Diode LCW10 laser operating in the same conditions of fig.3.

emission has died out. At low values of $|\tau|$, the curves show at first a quadratic dependence on $|\tau|$ followed by an oscillatory one; this is due to the mentioned relaxation phenomena of the cavity refractive-index which alter the spectrum of the noise and introduce a non zero correlation time in the phase fluctuations. Crosses in fig.4 represent values of $\sigma^2(\tau)$ derived directly from measurements of the probability density function of $\Delta_\tau\phi(t)$ obtained through the statistical analyzer. For all the employed lasers the distribution of $\Delta_\tau\phi(t)$ turned out to be Gaussian.

Figure 5a shows the normalized lineshape of one of the employed lasers, evaluated through eq.(4) by using the measured $S_{\dot\phi}(\Omega)$, while fig.5b shows the lineshape of the same laser obtained directly by means of a Fabry-Perot scanning interferometer. The lineshape deviates from a Lorentzian form because of the appearance of the two satellite peaks, due to the presence of the oscillating part in $\sigma^2(\tau)$, not predicted by conventional theories.

Recently, papers from different authors, which face the problem of phase and frequency noise from a theoretical point of view, have been published, giving results which fully explain all the experimental results reported above [11-14]. These theoretical approaches hinge

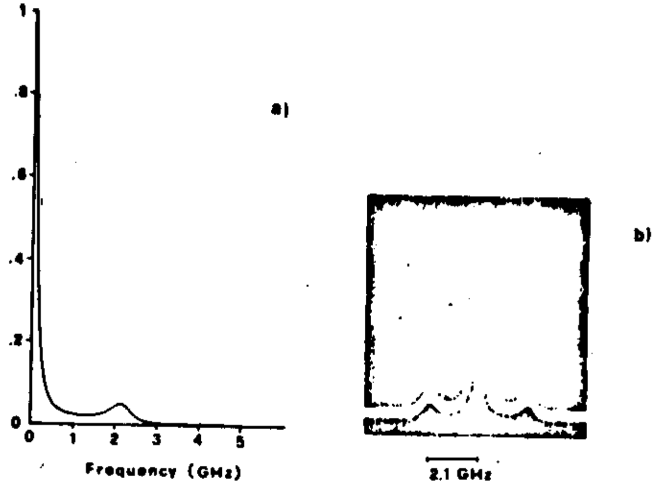

Fig.5- Emission lineshape of the Laser Diode LCW10 at P=3mW : a) computed through the measured values of $S_{\dot\phi}(\Omega)$; b) measured with a Fabry-Perot interferometer.

upon the use of a set of equations which describe the time evolution of the electric field of the cavity mode [15] and of the injected carrier density [16], together with eq.(1). Every spontaneous emission process gives rise to small deviations of the amplitude and phase of the field and of the carrier density around their stationary values, so that a first order approximation is sufficient to describe accurately the time evolution of the physical system after any perturbation has taken place. The corresponding equations, written in terms of the new variables $N(t)=n(t)\Gamma V$ and $I(t)=|\mathcal{E}(t)|^2 V$, which represent respectively the number of carriers and the number of photons belonging to the mode in the cavity (V is the mode volume, Γ the filling factor and $n(t)$ the density of the injected carriers), read

$$\begin{cases} \Delta\dot{I}(t) = (G_n I_o/\ \Gamma V)\ \Delta N(t) & (6a) \\ \dot\phi(t) = (G_n\ \alpha/\ 2\Gamma V\ )\Delta N(t) & (6b) \\ \Delta\dot{N}(t) = -\gamma_e\Delta N(t) - \Gamma G_o\Delta I(t) & (6c), \end{cases}$$

where G_o and I_o are the stationary values of gain and number of photons, G_n the derivative of gain with respect to the carrier density, $\gamma_e=\gamma+(G_n I_o/V)$, γ being the inverse of the spontaneous lifetime of the excited carriers and $\alpha=-(2\omega_o\eta_n v_g/G_n c)$ is the linewidth enhancement factor (η_n being the derivative of the cavity refractive-index η with respect to n and v_g the group velocity of the light).

The set of equations (6) shows how $\Delta I(t)$, $\Delta N(t)$ and $\dot\phi(t)$, the deviations of the number of photons and excited carriers, and of the

the frequency from their stationary values, respectively, are coupled among themselves. In order to describe noise in the system under study, we must add to eqs.(6) Langevin terms, which take into account the randomness of the spontaneous emission events. For eqs.(6a) and (6b) these terms can be modeled as

$$\begin{cases} F_{\Delta I}(t) = \sum_i (2I_o^{1/2}\cos\theta_i + 1)\,\delta(t - t_i) & (7a) \\ F_{\phi}(t) = \sum_i I_o^{-1/2}\sin\theta_i\,\delta(t - t_i) & (7b), \end{cases}$$

where θ_i is the phase of the i-th photon, emitted at a time t_i, relative to the phase of the field just before t_i, and $\delta(t)$, the delta function, accounts for the discontinuous changes in intensity and phase during spontaneous emission events.
The Langevin term for eq.(6c) is

$$F_{\Delta'}(t) = -\sum_i \delta(t - t_i) \qquad (7c),$$

because any emission causes a unitary decrease in the number of carriers. Because the complete set of equations, that is eqs.(6) to which the corresponding Langevin terms have been added, is linear, their solutions, which can be found by means of standard mathematical techniques, will be expressed as the sum of the responces of the unperturbed system to any i-th term in eqs.(7). Furthermore, since the phase θ_i is independent from the time t_i of the emission and uniformly distributed, the autocorrelation functions, and hence the power spectral densities of $\Delta I(t)$, $\Delta N(t)$, and $\dot{\phi}(t)$ can be simply derived [13]. The power spectra of intensity and frequency noise read

$$S_{\Delta I}(\Omega) = R_s \frac{((G_nI_o/\Gamma V)(G_nI_o/\Gamma V - 2\gamma_e) + (2I_o+1)(\gamma_e^2 + \Omega^2))}{((\Omega_R^2 - \Omega^2)^2 + \gamma_e^2\,\Omega^2)} \qquad (8)$$

$$S_{\dot{\phi}}(\Omega) = \frac{R_s}{2I_o}\left(\frac{\alpha^4\,\Omega_R^4\,(2I_o + 1 + \Omega^2/(\Gamma G_o)^2)}{2I_o((\Omega_R^2 - \Omega^2)^2 + \gamma_e^2\,\Omega^2)} + 1\right) \qquad (9),$$

where R_s is the average rate of spontaneous emissions in the mode and $\Omega_R = (G_nG_oI_o/V)^{1/2}$.
Equation (8), which, as stressed before, was previously derived [1,2], and (9) represent the complete solution of noise problem in single-mode semiconductor lasers. The unitary term into the brackets in eq.(9) is the term usually considered and is due to the randomness of the relative phase of any spontaneously emitted photon. The other term, which is proportional to α^2, and therefore strictly bound to the refractive-index changes induced by carrier density fluctuations, is responsible both for the appearance of the satellite peaks and for the excess broadening of the emission line. At low frequencies, in fact, that is when $\Omega \ll \Omega_R$, the term into brackets is well approximated by $(1+\alpha^2)$, in agreement with Henry's results [4]. It must be stressed that an increase in the emitted power, and then in I_o, determines both a reduction of low frequency noise content proportional to I_o^{-1} and an increase in the frequency of the resonant peak proportional to $I_o^{1/2}$. Figures 6a and 6b show $S_{\dot{\phi}}(\Omega)$ and $S_{\Delta I}(\Omega)/I_o^2$ for an ITTLS7709 gain guided laser operating at λ=865.5nm with P=4mW ; the continuous curves are computed through eqs.(8) and (9) choosing appropriate values for the physical parameters [13], and the dashed ones are obtained experimentally.

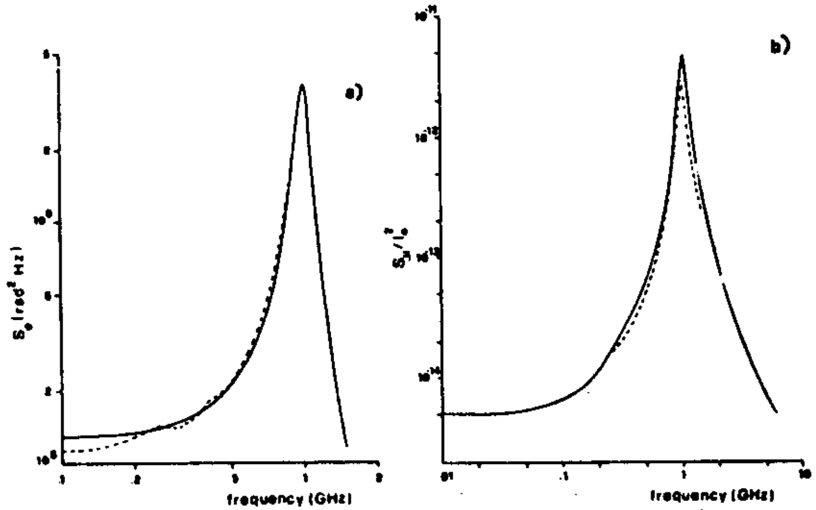

Fig.6- Two sided normalized power spectral densities of frequency (a) and intensity noise (b) for an ITTLS7709 laser emitting, at λ=865.5nm, a power P=4mW. The continuous curves are theoretical, while the dashed ones are experimental.

Substituting the right side of eq.(9) into eq.(5), one can directly evaluate $\sigma^2(\tau)$ [17]. The complete resulting expression, which is rather long and involved, can be substantially simplified as follows

$$\sigma^2(\tau) = R_s/(2I_o)\left((1+\alpha^2)|\tau| + \alpha^2\left(1/\gamma_e - \exp(-\gamma_e|\tau|/2)\left(\cos\Omega_o\tau/\gamma_e + 3\sin\Omega_o|\tau|/(2\Omega_o)\right)\right)\right) \qquad (10),$$

where $\Omega_0=(4\Omega_R^2-\gamma_e^2)^{1/2}/2$ is the relaxation oscillation frequency. Equation (10) can be simply shown to account for the behaviour of $\sigma^2(\tau)$ as a function of $|\tau|$, as previously described. Equivalent expressions for $\sigma^2(\tau)$ have been found by other authors[12,14]. Substituting eq.(10) into eq.(4) allows to evaluate the normalized lineshape function which reads

$$G(\omega)=\left(1-\frac{R_s\alpha^2}{2I_0\gamma_e}\right)\frac{2\,\Delta\Omega_1}{(\omega-\omega_0)^2+\Delta\Omega_1^2}+$$

$$\frac{R_s}{4I_0}\left(\frac{2/\gamma_e(\,\Delta\Omega_1+\gamma_e/2)+3/\Omega_0(\omega-\omega_0+\Omega_0)}{(\omega-\omega_0+\Omega_0)^2+(\,\Delta\Omega_1+\gamma_e/2)^2}+\right.$$

$$\left.\frac{2/\gamma_e(\,\Delta\Omega_1+\gamma_e/2)+3/\Omega_0(\omega-\omega_0-\Omega_0)}{(\omega-\omega_0-\Omega_0)^2+(\,\Delta\Omega_1+\gamma_e/2)^2}\right) \qquad (11),$$

where $\Delta\Omega_1=R_s(1+\alpha^2)/(2I_0)$.

By inspecting eq.(11), it is possible to recognize the characteristic Lorentzian behaviour of the central lobe (which is essentially due to the flat low-frequency content of $S_{\dot\phi}(\Omega)$), and the presence of the sideband peaks shifted in frequency by an amount Ω_0, in agreement with the experimental observations.

As a conclusion, it is worth while to stress that the presented results, besides showing and explaining some interesting physical phenomena such as the excessive linewidth and the presence of satellite peaks in the emission line of single-mode injection lasers, have an immediate pratical application in the design of optical coherent communication systems due to the strong dependence of their performances (for example, in such systems the error probability is an increasing function of $\sigma^2(\tau)$[18]) on the characteristics of the above mentioned devices.

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