INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

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WINTER COLLEGE ON
LASER PHYSICS: SEMICONDUCTOR LASERS
AND INTEGRATED OPTICS

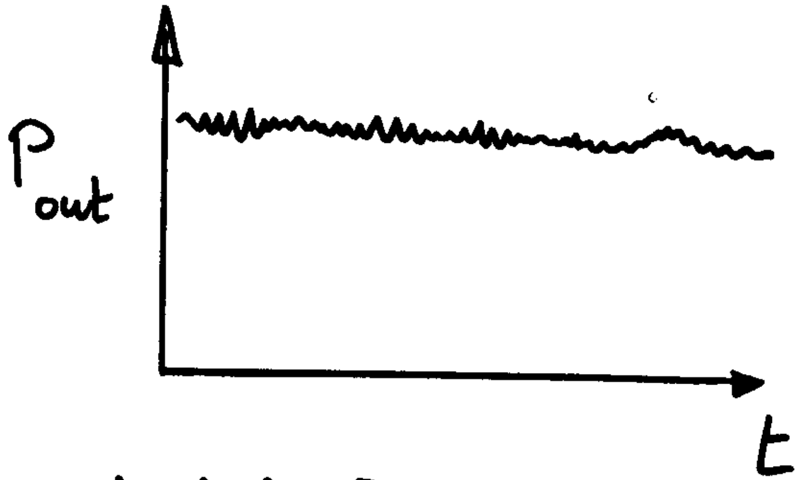
(22 February - 11 March 1988)

COHERENCE & NOISE PROPERTIES OF LASER DIODES (II)

P. Spano
Fondazione Ugo Bordoni
Roma, Italy

Intensity Noise

- 1 Total Intensity Noise

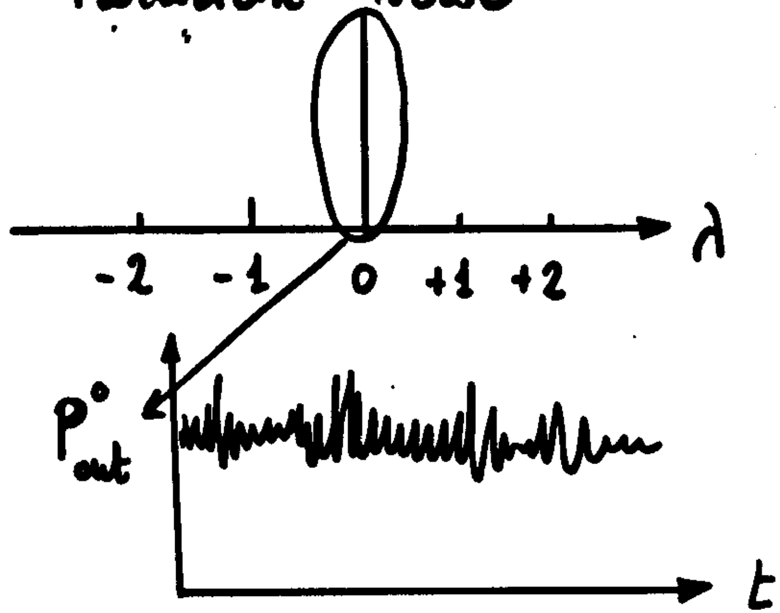


Important in Intensity modulated communication systems

Causes

Amplification of quantum fluctuations of electron and photon population in the resonator.

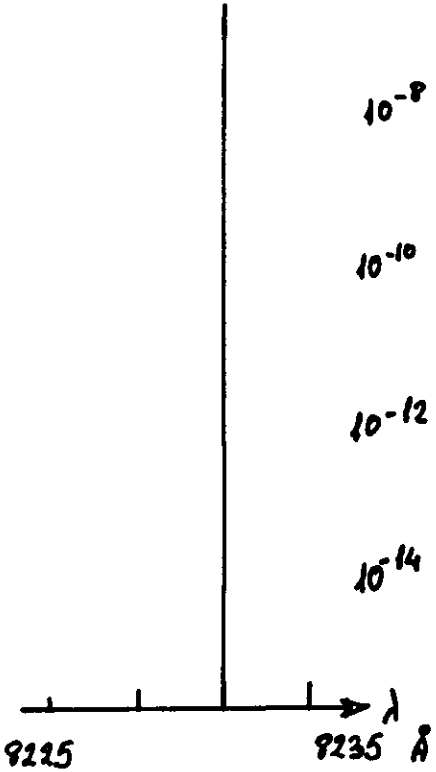
-2 Partition Noise

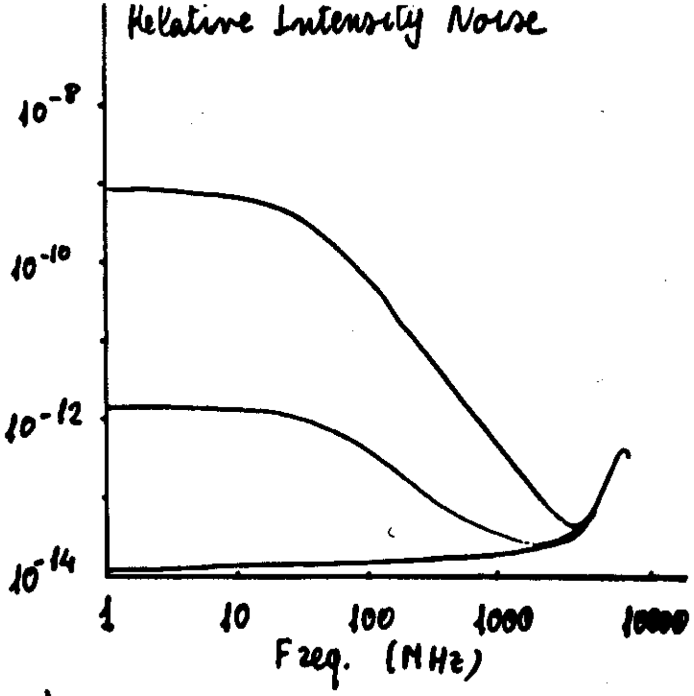


Important in high frequency Intensity modulated communication systems (due to chromatic dispersion in fibers)

Causes

Quantum fluctuations, again.

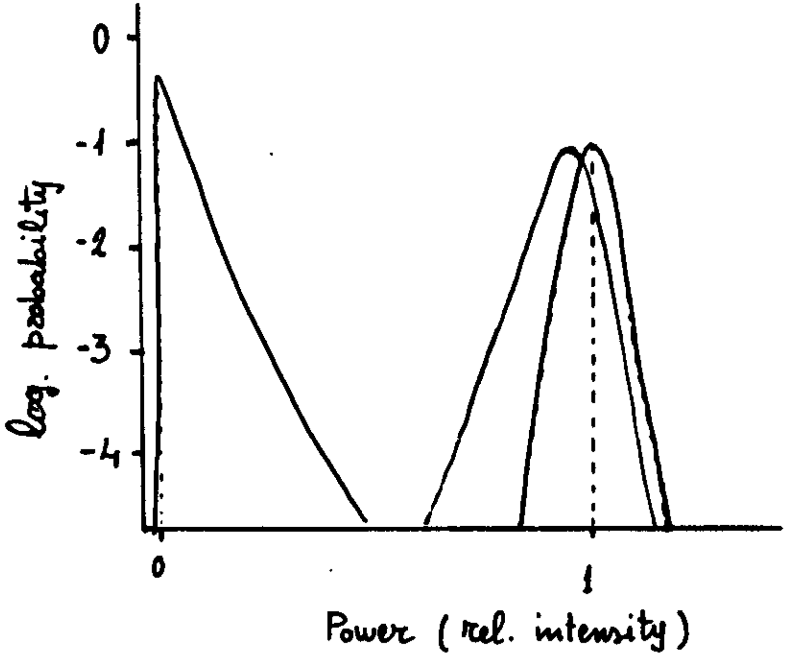




-3 Phase or frequency noise

$$\mathcal{E}(t) = E_0 \exp[i(\omega_0 t + \phi)]$$

$$\omega_{inst} = \omega_0 + \dot{\phi}$$



Important in coherent communication systems

Causes

Thermal and mechanical deformation of the cavity. (extrinsic noise)

Spontaneous emission of photons with random phase into coherent field (intrinsic noise)

SLM-LD can be employed as:

- Transmitters
- Modulators
- Local oscillators
- Optical amplifiers

In all these applications phase-noise determines a degradation of the system performances.

(Kikuchi K. et al. J. Lightwave Tech. 1984)

- General Introduction to the formalism

- Experimental observations

- Theory

- Methods useful for reduction of phase-noise.

Introduction to the formalism.

Phase-noise can be characterized by

1) Power Spectrum of the field

=

lineshape

$$S_{\mathcal{E}}(\omega) = \int_{-\infty}^{+\infty} R_{\mathcal{E}}(\tau)\, e^{-i\omega\tau}\, d\tau$$

2) Power spectrum of instantaneous frequency $\dot{\phi}$

$$S_{\dot{\phi}}(\Omega) = \int_{-\infty}^{+\infty} R_{\dot{\phi}}(\tau)\, e^{-i\Omega\tau}\, d\tau$$

3) Variance of the phase difference

$$\sigma^2_{\Delta_\tau\phi}(\tau) = \langle [\phi(t)-\phi(t-\tau)]^2 \rangle - \langle \phi(t)-\phi(t-\tau) \rangle^2$$

Power spectrum of the field

$\Longleftrightarrow$ LINESHAPE

$$S_{\mathcal{E}}(\omega) = \int_{-\infty}^{+\infty} R_{\mathcal{E}}(\tau)\, e^{-i\omega\tau}\, d\tau$$

$$R_{\mathcal{E}}(\tau) = \langle \mathcal{E}(t+\tau)\, \mathcal{E}^*(t) \rangle \simeq$$

$$E_0^2\, e^{i\omega_0\tau} \langle e^{i\Delta_\tau\phi(t)} \rangle =$$

$$= E_0^2\, e^{i\omega_0\tau} \left\langle \exp\left[i \int_t^{t+\tau} \dot{\phi}(t')\, dt' \right] \right\rangle$$

$$\Delta_\tau\phi(t) = \phi(t+\tau) - \phi(t)$$

$\dot{\phi}(t)$: normally distributed ergodic stationary process

$$R_{\mathcal{E}}(\tau) = E_0^2 e^{i\omega_0\tau} \exp\left\{-\frac{1}{2}\int_0^\tau dt' \int_0^\tau dt'' \langle\dot\phi(t')\dot\phi(t'')\rangle\right\} = \sigma^2_{\Delta_\tau\phi}(\tau) = \sigma^2(\tau) =$$

$$= E_0^2 e^{i\omega_0\tau} \exp\left\{-\frac{1}{2}\,\sigma^2(\tau)\right\}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{\Delta_\tau\phi}(\Omega)\, d\Omega =$$

$$= \frac{1}{\pi}\int_{\infty}^{\infty} S_{\dot\phi}(\Omega)\,(1-\cos\Omega\tau)\,/\Omega^2 \; d\Omega$$

$$\sigma^2(\tau) = \langle[\Delta_\tau\phi(t)]^2\rangle - [\langle\Delta_\tau\phi(t)\rangle]^2 =$$

$$= \langle[\Delta_\tau\phi(t)]^2\rangle$$

$$S_{\mathcal{E}}(\omega) = E_0^2 \int_{-\infty}^{+\infty} e^{-i(\omega-\omega_0)\tau}\, e^{-\frac{1}{2}\sigma^2(\tau)}\, d\tau$$

$$\Delta \nu_L (\text{spont.}) = \frac{\hbar \omega_0 (\Delta \nu_c)^2}{P}$$

He-Ne laser:

$P = 1$ mW, $\omega_0 = 3 \times 10^{15}$ rad/sec

$$\Delta \nu_c \simeq 10^5 \ s^{-1}$$

$$\underline{\Delta \nu_L \simeq 10^{-3} \ s^{-1}}$$

Ga Al As laser:

$P = 1$ mW, $\omega_0 \simeq 2 \times 10^{15}$ rad/sec

$$\Delta \nu_c \simeq 10^{11} \ s^{-1}$$

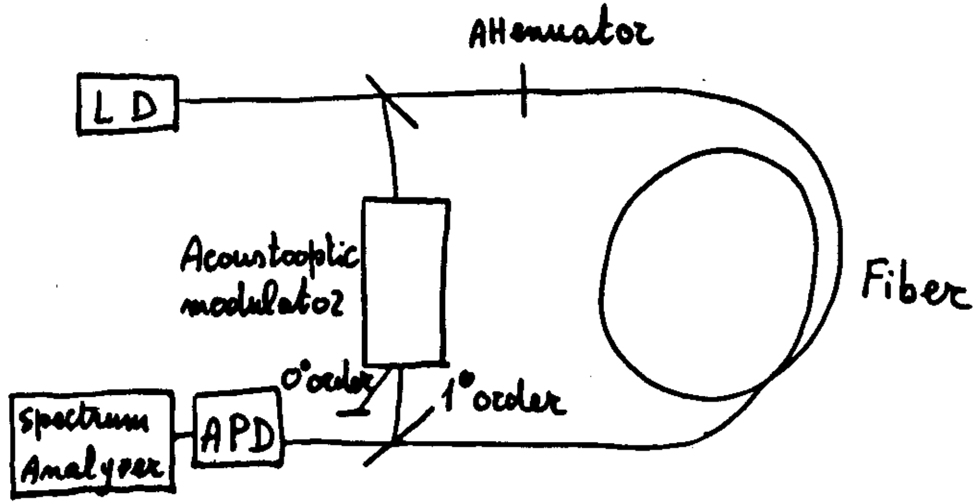
$$\underline{\Delta \nu_L \simeq 10^{7} \ s^{-1}}$$

Experimental results

1) Measurement of the linewidth - lineshape

Fabry - Perot interferometer ($\Delta \nu_L \gtrsim 10$ MHz)

Self - heterodyne technique ($\Delta \nu_L < 100$ KHz, <u>only</u> linewidth)



<u>If</u> the emission lineshape is Lorentzian

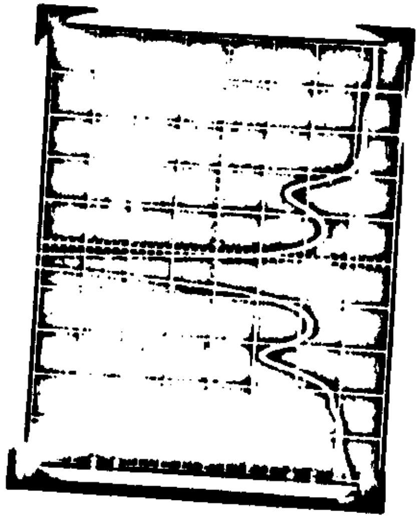
$$S_{\mathcal{E}}(\omega) = \frac{\delta\omega}{(\omega - \omega_0)^2 + (\delta\omega/2)^2} \quad \text{and } \tau_2 - \tau_1 > 2\pi/\delta\omega$$

$\Rightarrow$ signal displayed Lorentzian and FWHM $= 2\,\delta\omega$

Results

- Linewidth about 20 – 100 times larger than predicted
(Fleming, Mooradian - Appl. Phys. Lett. 1981)

- Lineshape not Lorentzian:
Presence of satellite peaks at a frequency difference from the center of the spectrum in the range $0.5 \div 5$ GHz.

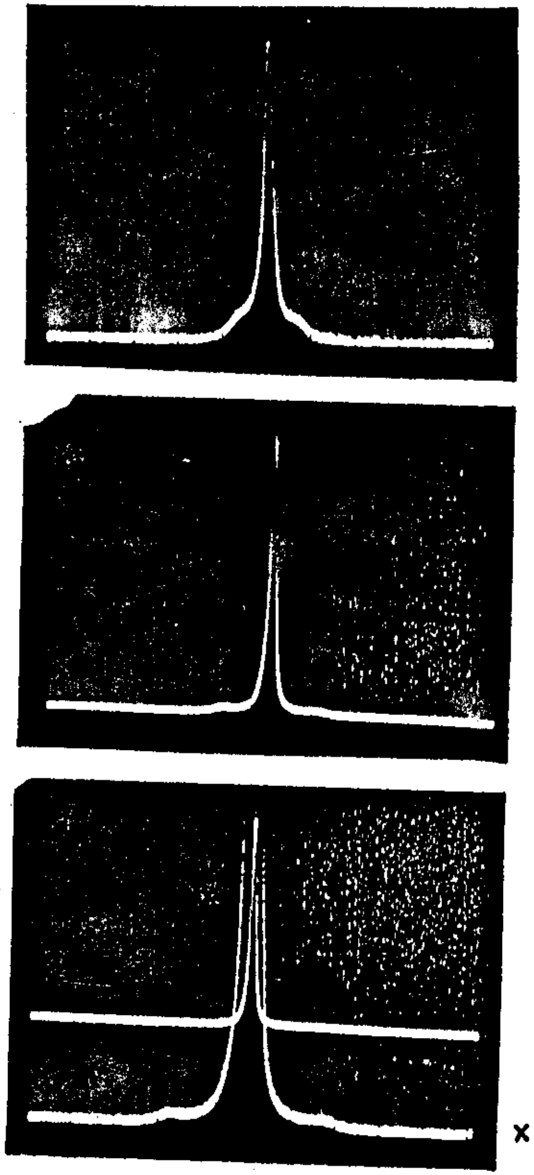


1GHz/div

Laser Diode P. 3mW

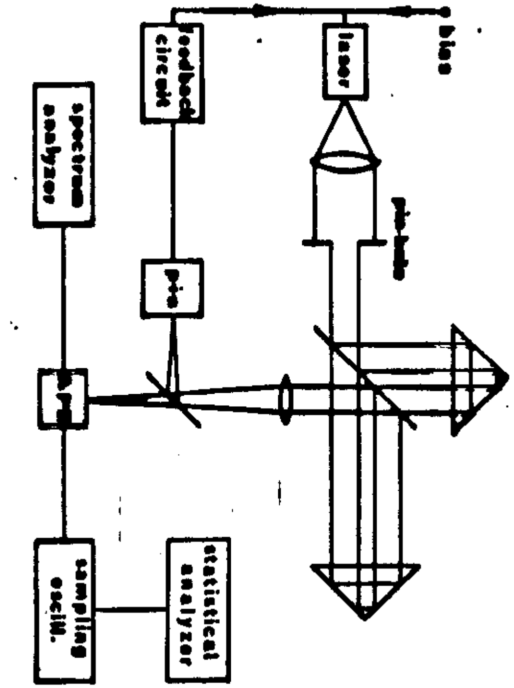
InGaAsP DFB laser

$I_{th} = 52\,mA$



2) Measurement of $S_i(\Omega)$

Unbalanced Michelson interferometer



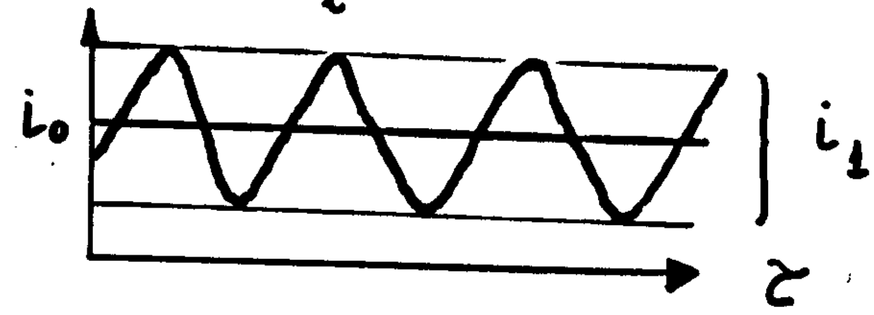
(Piazzolla et al. Appl. Phys. Lett. 1982)

Output current from the APD

$$i(t) \propto \left| \mathcal{E}(t) + a\, \mathcal{E}(t-\tau) \right|^2$$

a = ratio of attenuation of the two fields

$$i(t) = i_0 + \frac{i_1}{2} \cos\left[\phi(t) - \phi(t-\tau) - \omega_0 \tau\right]$$



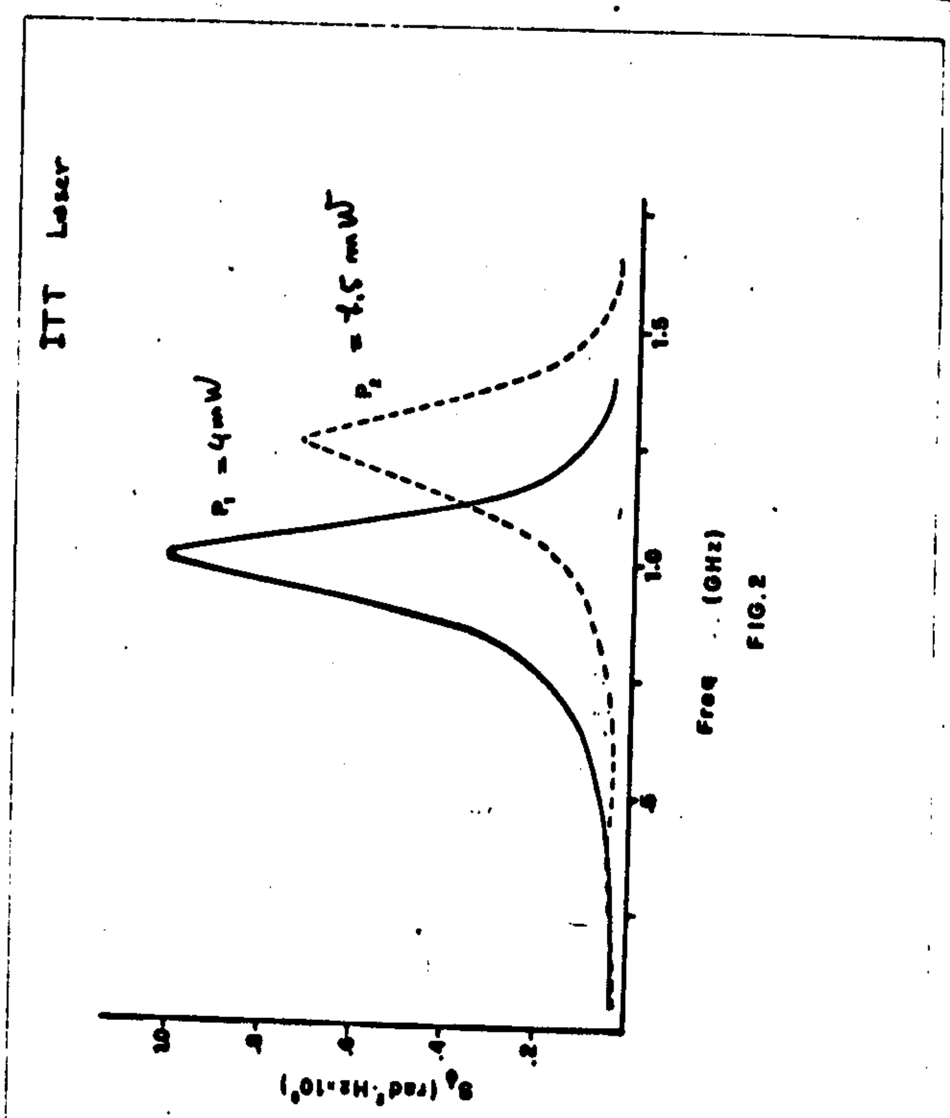
if $\omega_0 \tau = \dfrac{\pi}{2}$ and

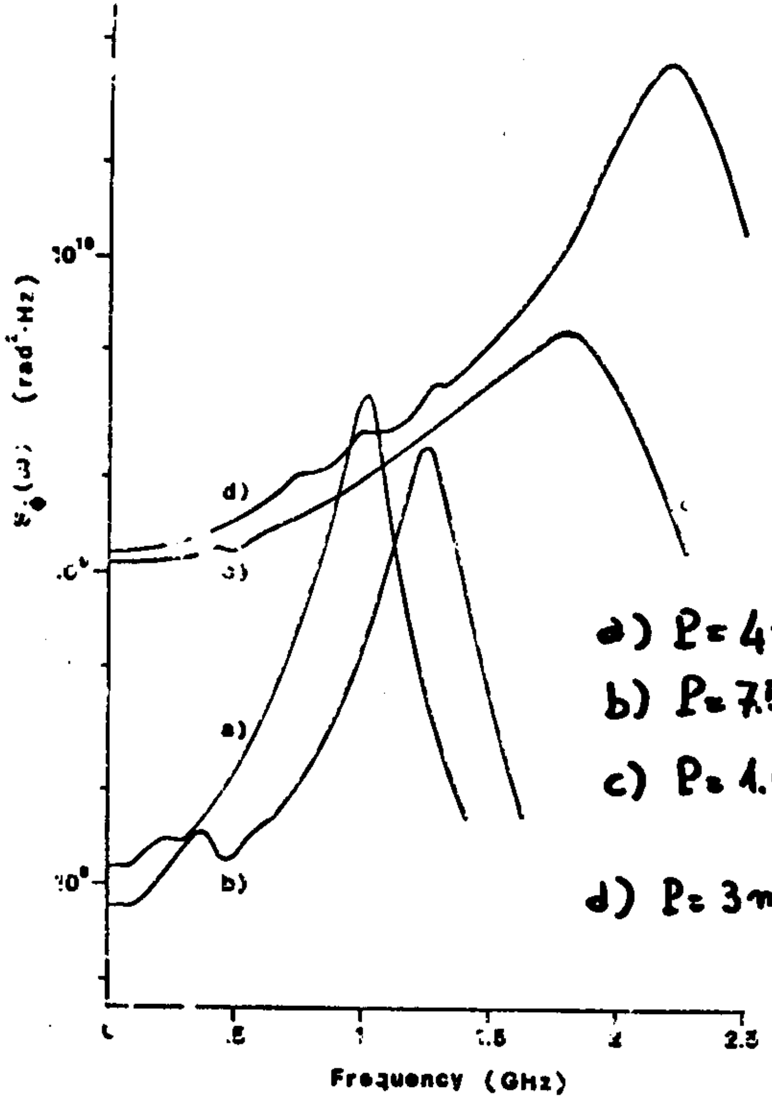
$$\left| \phi(t) - \phi(t-\tau) \right| \ll \pi/2$$

$$i(t) \simeq i_0 + \frac{i_1}{2}\left[\phi(t) - \phi(t-\tau)\right] = i_0 + \frac{i_1}{2} \Delta_\tau \phi$$

$$S_i(\Omega) = \frac{i_1^2}{4} R \, S_{\Delta_\tau \phi}(\Omega) =$$

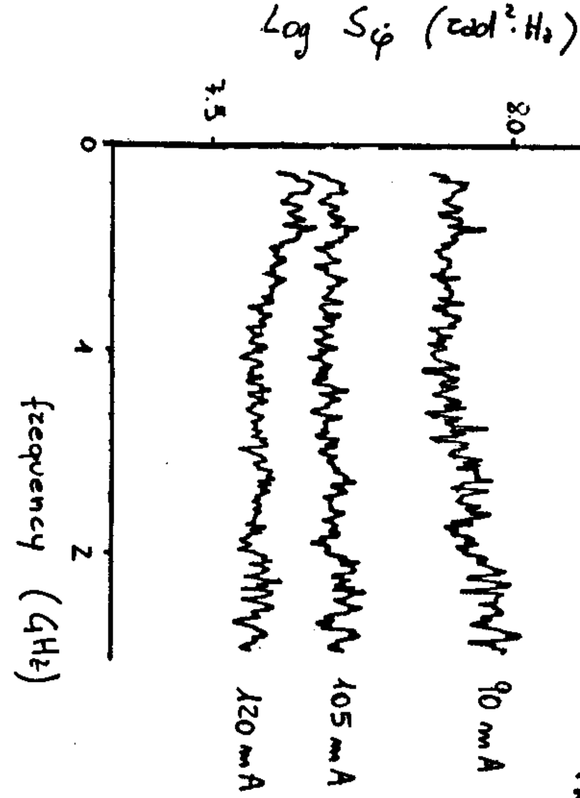
$$= \frac{i_1^2}{\cdot} R \, S_{\dot{\phi}}(\Omega)(1 - \cos \Omega\tau)/\Omega^2$$





a) P = 4 mW } ITT
b) P = 7.5 mW } g.g.
c) P = 1.6 mW RCA (CDH)
d) P = 3 mW Laser Diode g.g.

(TKACH and CHRAPLYVY J.Q.E. 1986)

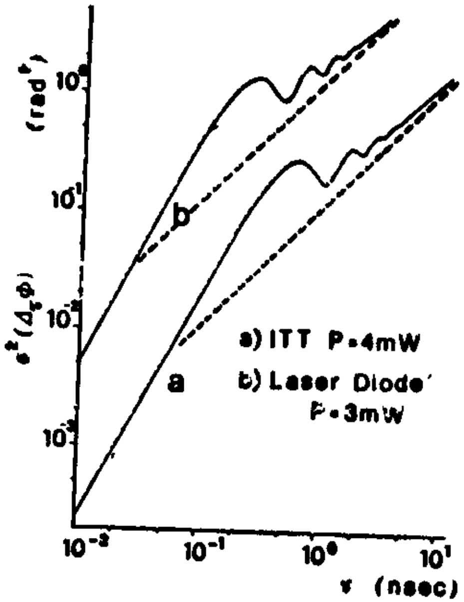


DFB InGaAsP [redacted]

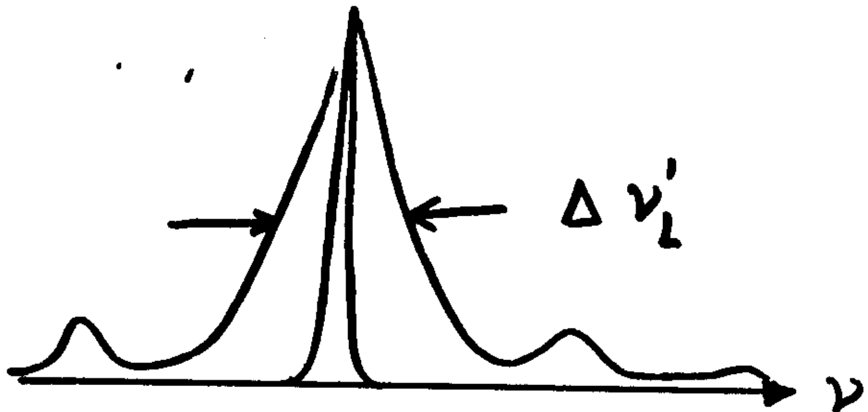
I_{th} = 70 mA

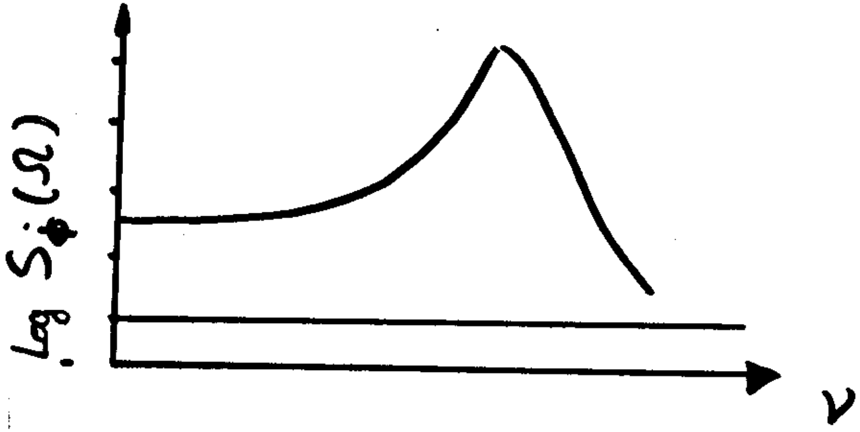
3) Measurement of $\sigma_{\Delta\phi}(\tau)$

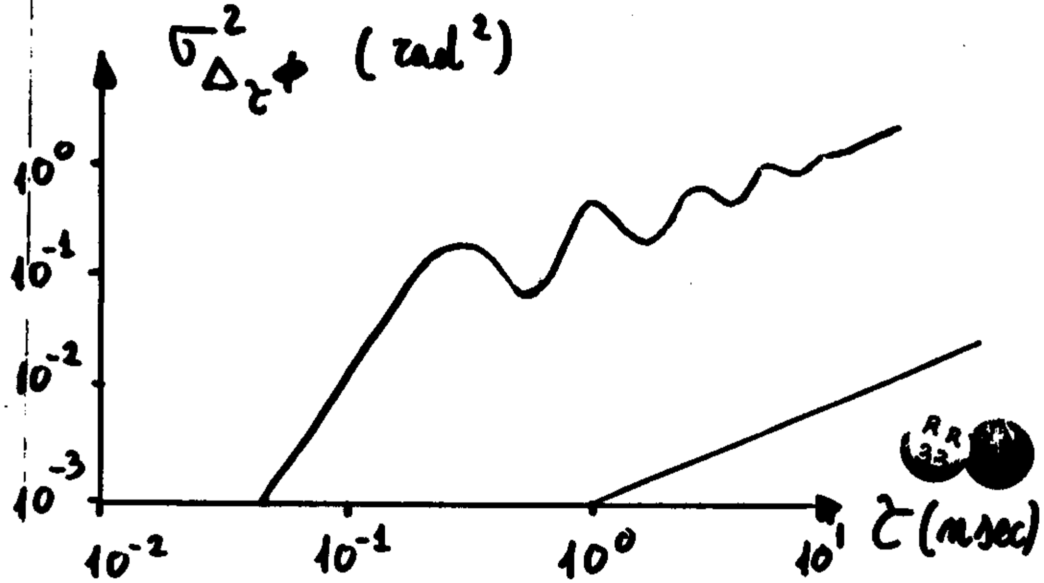
Unbalanced Michelson Interferometer + statistical analyzer



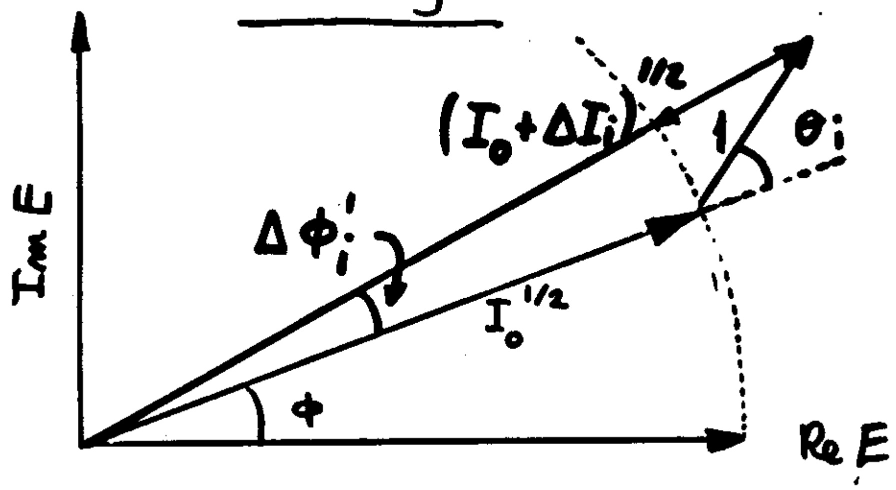
$$\sigma^2(\tau) = \frac{2}{\pi}\int_0^{\infty} \frac{S_{\dot\phi}(\omega)}{\omega^2}(1-\cos\omega\tau)\,d\omega$$







Theory



$$E = I_0^{1/2}\, e^{i\phi}$$

$I_0 = E\,E^* =$ average number of photons in the cavity

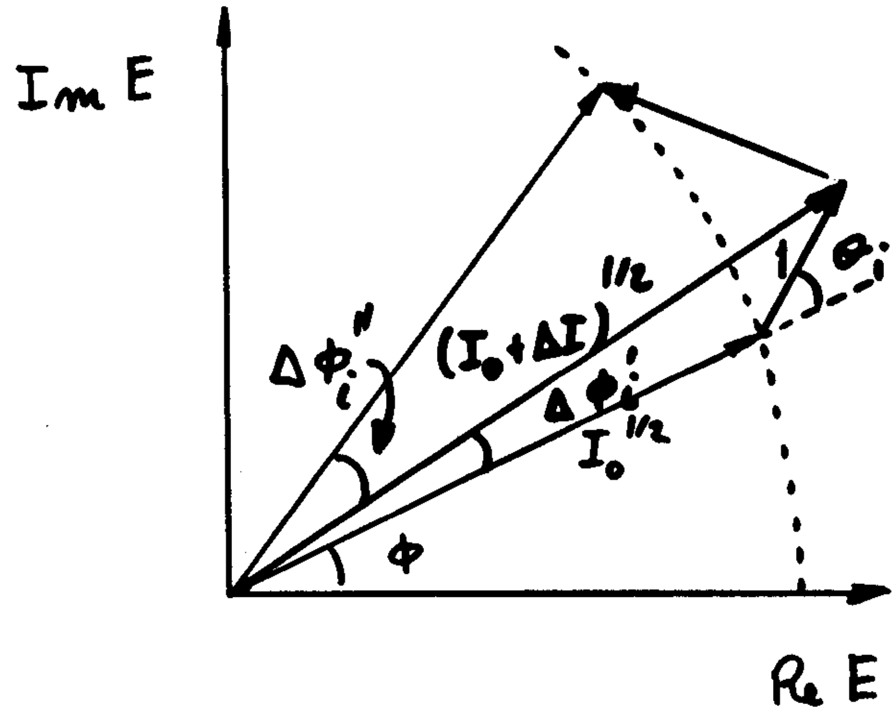
Effect of the emission of the i-th photon spontaneously emitted in the mode.

$$\Delta E_i = e^{i(\phi+\theta_i)}$$

$$\Delta I_i = |E+\Delta E_i|^2 - |E|^2 = 1 + 2 I_0^{1/2}\cos\theta_i$$

$$\Delta\phi'_i = I_0^{-1/2}\sin\theta_i$$

What happens if the refractive index η is dependent on the field intensity?



$$\Delta\phi = \Delta\phi'_i + \Delta\phi''_i$$

(Henry J. Q. E. 1982)

$$\eta = \eta' + i\,\eta''$$

$$\alpha = \Delta\eta' / \Delta\eta''$$

$$\Delta I \Rightarrow \Delta n \Rightarrow \Delta\eta'' \Rightarrow \Delta\eta'$$

$$\Downarrow$$

$$\Delta\phi_i'' = \frac{\alpha}{2I_0}\left(1 + 2\,I_0^{1/2}\cos\theta_i\right)$$

$$\Delta\phi_i = \Delta\phi_i' + \Delta\phi_i'' =$$

$$= \frac{\alpha}{2I_0} + \frac{1}{I_0^{1/2}}\left[\sin\theta_i - \alpha\cos\theta_i\right]$$

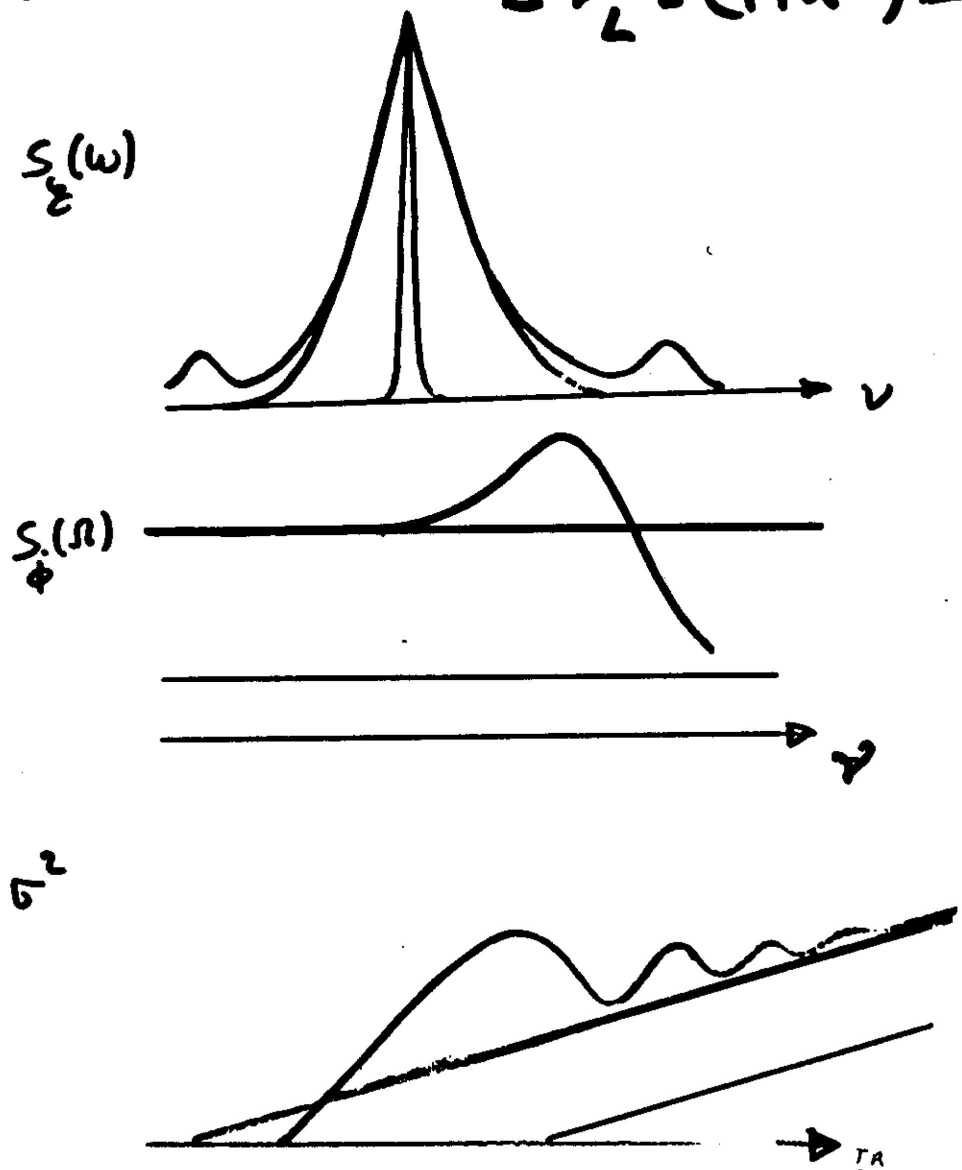
R_S = average number of spontaneous emission rate in the lasing mode

$N = R_S\tau$ = number of spontaneous emission events in a time interval τ

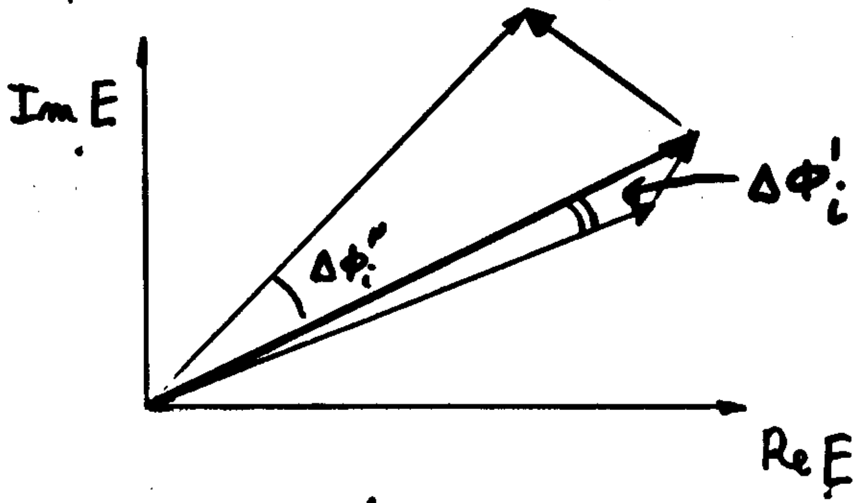
$$\sigma^2(\tau) = \langle\Delta\phi_i^2\rangle - \langle\Delta\phi_i\rangle^2 =$$

$$= \frac{R_S\,\tau}{2\,I_0}\left(1 + \alpha^2\right)$$

This variance linear in τ corresponds to an emission lineshape Lorentzian, with a width $\Delta\nu'_L = (1+\alpha^2)\Delta\nu_L$



Why the simple model does not explain the presence of satellite peaks?



$$\Delta\Phi = \Delta\Phi'_i + \Delta\Phi''_i$$

$\boxed{\Delta\Phi'' = \text{phase shift after relaxation}}$

$\boxed{\tau > \tau_r}$

If $\tau > \tau_r$ $\Phi(t)$ and $\Phi(t-\tau)$ are incorrelated, $\Phi(t)$ executes a Brownian motion and

$$\sigma^2(\tau) \propto \tau$$

$\tau < \tau_r$

We can expect

- Correlation between $\phi(t)$ and $\phi(t-\tau)$

- Characteristics of correlation affected by oscillation in relaxation phenomenon

$$\begin{cases} \dfrac{d}{dt}\left\{E(t)e^{i\omega_0 t}\right\} = \left\{i\omega_N(n) + \dfrac{1}{2}\left[G(n)-\Gamma_0\right]\right\} E(t)e^{i\omega_0 t} \\ \dfrac{d}{dt}\, n(t) = -\gamma\, n(t) - G(n)\,|E(t)|^2 + P \end{cases}$$

$\omega_N(n) = \dfrac{N\pi c}{\eta l}$: resonant frequency of the N-th diode cavity longitudinal mode

$G(n)$: gain coefficient

Γ_0 : cavity loss

γ : inverse of the spontaneous lifetime of the excited carriers

$P = \dfrac{j}{ed}$: number injection rate per unit volume of the excited carriers

(Schimpe, Harth - Electron. Lett. 1983
Vahala, Yariv J. Q. E. 1983
Spano, Piazzolla, Tamburrini J. Q. E 1983)

The stationary solution (i.e. $E(t) = E_0$) gives (1^{st} eq.)

$$\begin{cases} G(n_0) = G_0 = \Gamma_0 \\ \omega_N(n_0) = \omega_0 \end{cases}$$

n_0 : stationary value of the carrier density (obtained through 2^{nd} eq.)

Small variations $\Delta n(t) = n(t) - n_0$ will change the values of the gain and of the refractive index. In a first order approximation

$$\begin{cases} G = G_0 + G_n \Delta n + G_I \Delta I \\ \eta = \eta_0 + \eta_n \Delta n \end{cases}$$

$$\boxed{G_n = \left[\frac{\partial}{\partial n} G(n)\right]_{n=n_0}} \quad \boxed{\eta_n = \left[\frac{d}{dn}\eta(n)\right]_{n=n_0}}$$

$$\boxed{\eta_0 = \eta(n_0)} \quad \boxed{G_I = \frac{\partial}{\partial I} G}$$

$$E(t) = \left[E_0 + e(t)\right] e^{i\phi(t)}$$

Substituting into the 1^{st} eq. of the previous system gives

$$\begin{cases} \dot{e}(t) = \frac{1}{2}\left[G - \Gamma_0\right]\left[E_0 + e(t)\right] \\ \dot{\phi}(t) = \omega_N(n) - \omega_0 \end{cases}$$

Let us introduce the new variables

$N = nV$: the number of carriers

$$I(t) = V|E(t)|^2 \cong V\left[E_0^2 + 2e(t)E_0\right] =$$

$= I_0 + \Delta I(t)$: the number of photons belonging to the mode in the cavity

V : mode volume

filling factor $= 1$

Within a first approximation

$$\boxed{\omega_N(n) = \omega_0 - \frac{\omega_0}{\eta_0}\left[\eta_n \Delta n + \left.\frac{\partial \eta}{\partial \omega}\right|_{\omega=\omega_0} \dot{\phi}(t)\right]}$$

so that

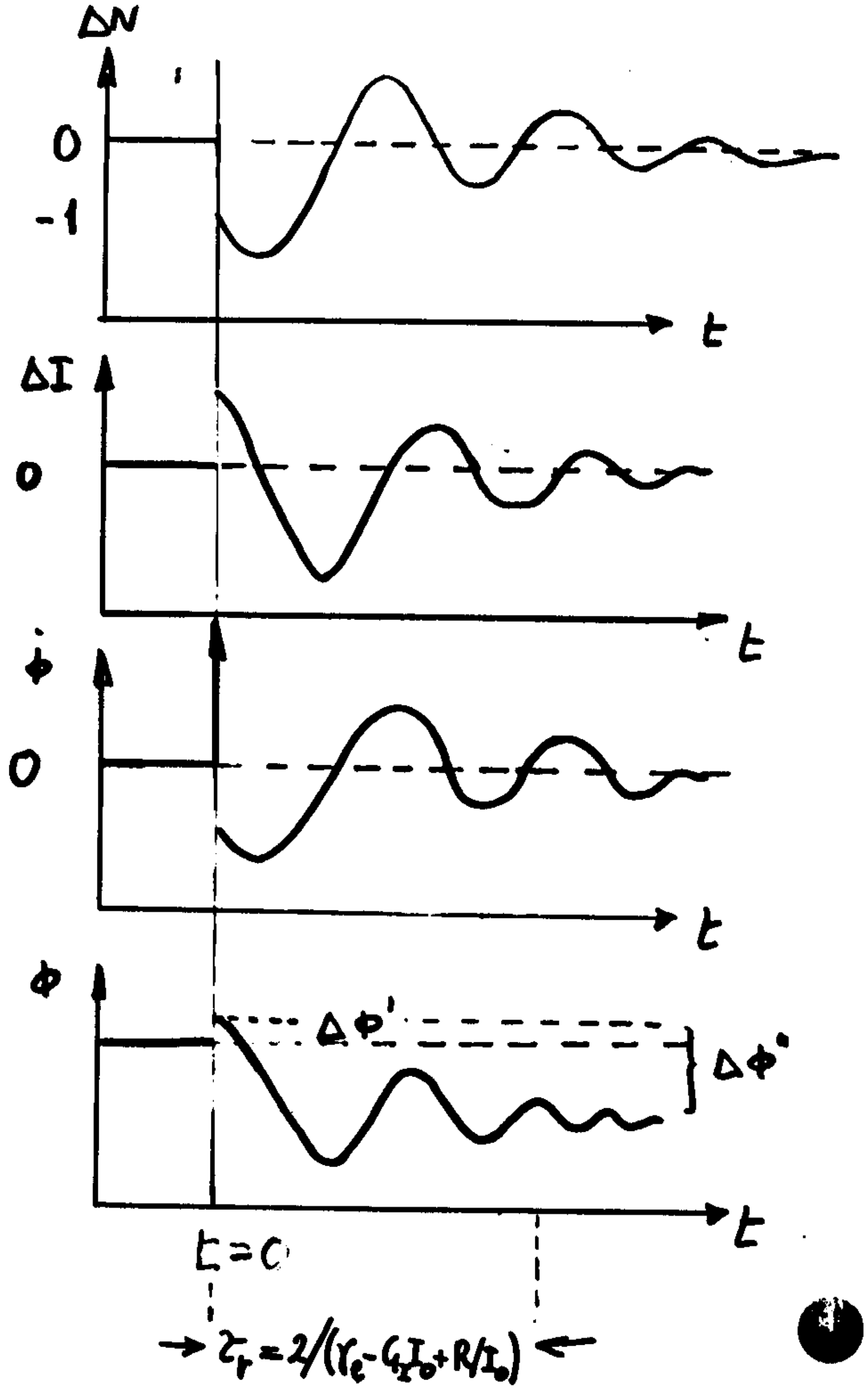
$$\begin{cases} \Delta \dot{I}(t) = \dfrac{G_n I_0}{V} \Delta N(t) + \left(G_I I_0 - \dfrac{R}{I_0}\right)\Delta I \\ \dot{\phi}(t) = \dfrac{G_n}{2V}\alpha\, \Delta N(t) \\ \Delta \dot{N}(t) = -\gamma_e \Delta N(t) - \left(G_0 + G_I I_0\right)\Delta I \end{cases}$$

$$\boxed{\alpha = -\frac{2\omega_0 \eta_n v_g}{G_n c}}$$

R = rate of spont. emission into the mode

$$\boxed{\gamma_e = \gamma + \frac{G_n I_0}{V}}$$

$$\boxed{v_g = \frac{c}{\left[\eta_0 + \left.\dfrac{\partial \eta}{\partial \omega}\right|_{\omega=\omega_0} \omega_0\right]}}$$



In order to describe noise in the system under study we must add Langevin terms which take into account the randomness of the spontaneous emission events

$$\begin{cases} \Delta\dot{I}(t) = \dfrac{G_n I_0}{V}\,\Delta N(t) + F_{\Delta I}(t) \\ \dot{\phi}(t) = \dfrac{G_n \alpha}{2V}\,\Delta N(t) + F_{\Delta\phi}(t) \\ \Delta\dot{N}(t) = -\gamma_e\,\Delta N(t) - G_0\,\Delta I(t) + F_{\Delta N}(t) \end{cases}$$

$$F_{\Delta I}(t) = \sum_i \left(2 I_0^{1/2}\cos\theta_i + 1\right)\delta(t-t_i)$$

$$F_{\Delta\phi}(t) = \sum_i I_0^{-1/2}\sin\theta_i\;\delta(t-t_i)$$

$$F_{\Delta N}(t) = -\sum_i \delta(t-t_i)$$

From the complete set of equations it is possible to evaluate the autocorrelation function of the variables $\dot{\phi}, \Delta I, \Delta N$.

The Fourier transform of these functions gives the power spectral densities.

For instance when $G_I = 0$

$$\Downarrow$$

$$S_{\dot{\phi}}(\Omega) = \frac{R_s}{2I_0}\left\{\frac{\alpha^2\Omega_R^4}{2I_0}\,\frac{\left(2I_0+1+\dfrac{\Omega^2}{G_0^2}\right)}{\left[(\Omega_R^2-\Omega^2)^2+\gamma_e^2\Omega^2\right]} + 1\right\}$$

At $\Omega = 0$ we have

$$S_{\dot{\phi}}(0) = \frac{R_s}{2I_0}(1+\alpha^2)$$

P.N.

P.N.

ITT laser

$P = 4\,mW$

$\lambda = 865\,mm$

$I_0 = 4.5 \cdot 10^5$

$V = 2.2 \cdot 10^{-9}\,cm^{-3}$

$R_s = 1.4 \cdot 10^{12}\,s^{-1}$

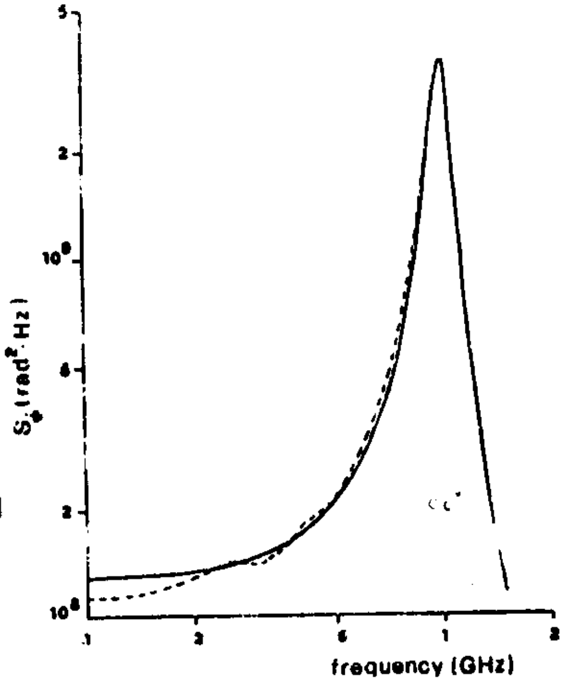
$G_n = 8.3 \cdot 10^{-7}\,cm^{-3}\,s^{-1}$

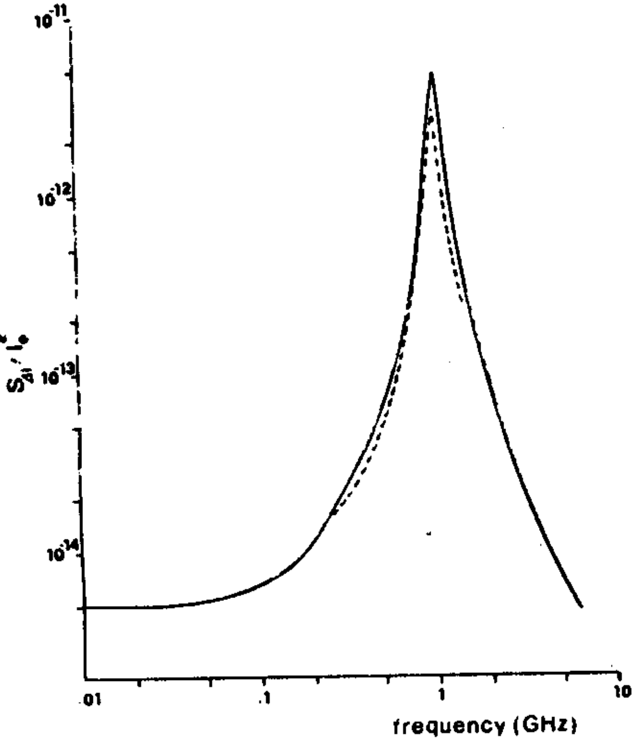
$G_0 = 2.5 \cdot 10^{11}\,s^{-1}$

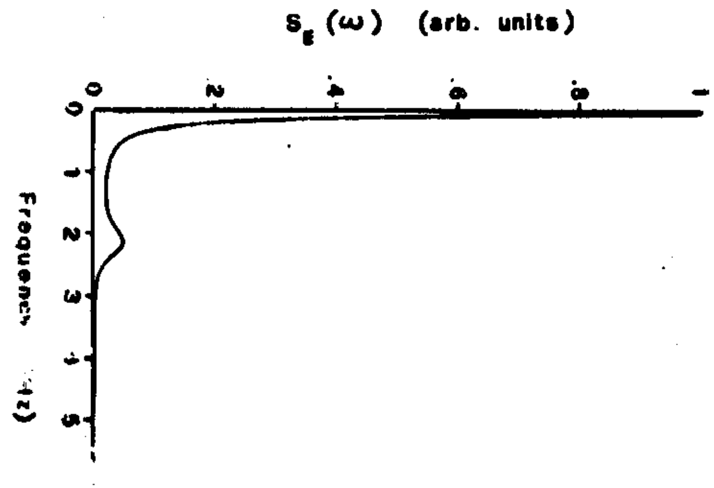
$\gamma_e \simeq 10^9\,s^{-1}$

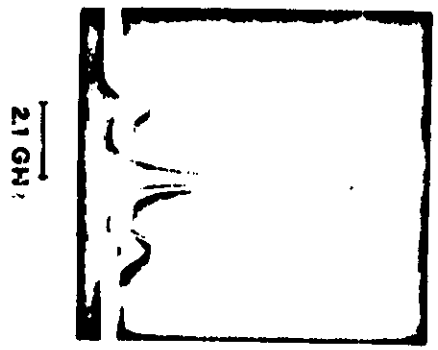
$\alpha \simeq 9$

$\nu_R = \frac{\Omega_R}{2\pi} = 1.02\,GHz$



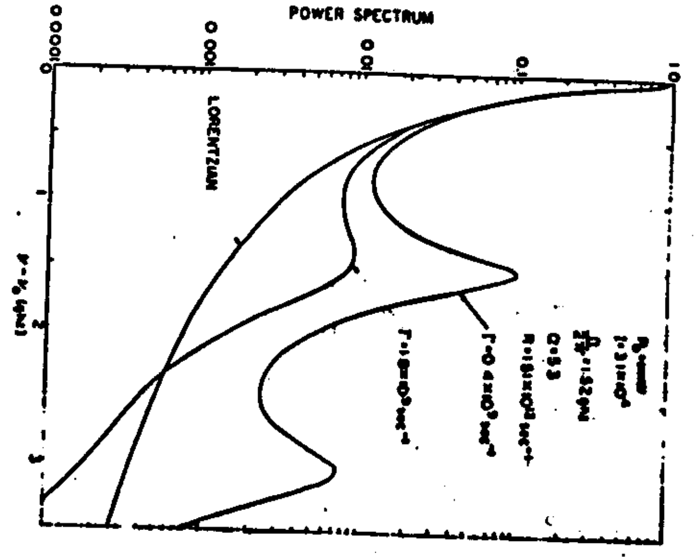


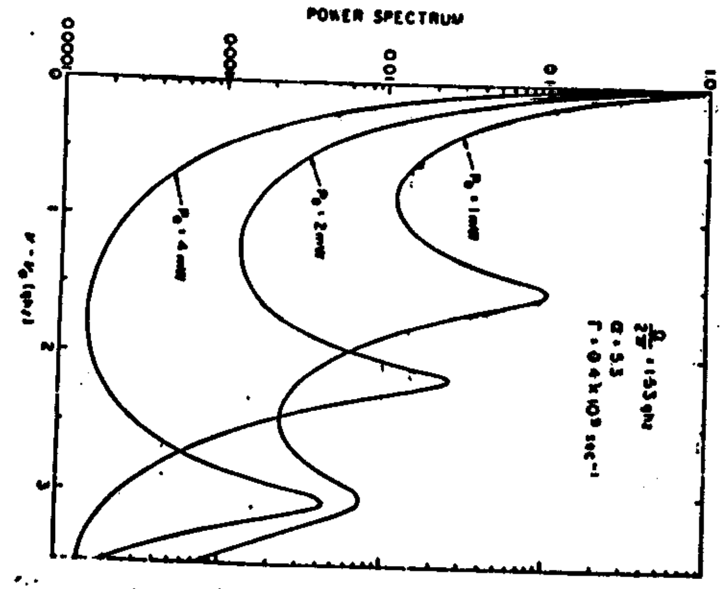


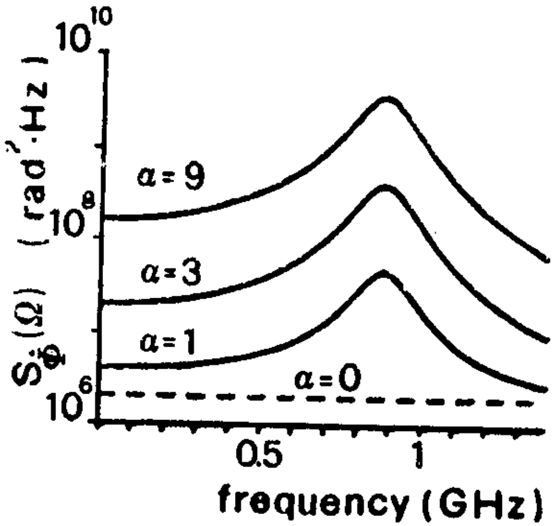


Laser Diode

$P = 3\,mW$

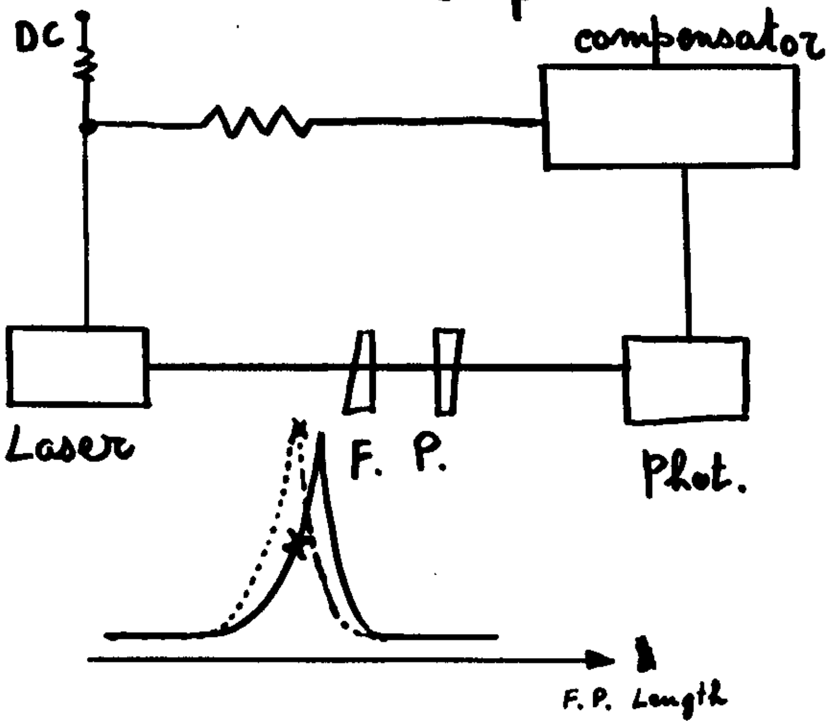

POWER SPECTRUM
LORENTZIAN

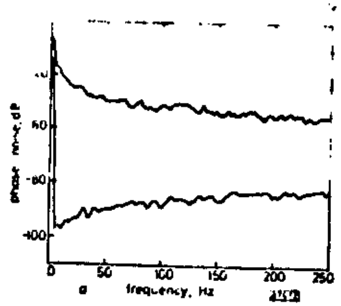

POWER SPECTRUM


10^{10}
10^{8}
10^{6}
$S_{\dot\phi}(\Omega)$ $(rad^2 \cdot Hz)$
$\alpha = 9$
$\alpha = 3$
$\alpha = 1$
$\alpha = 0$
0.5
1
frequency (GHz)

Linewidth reduction

- 1) Electrical loop





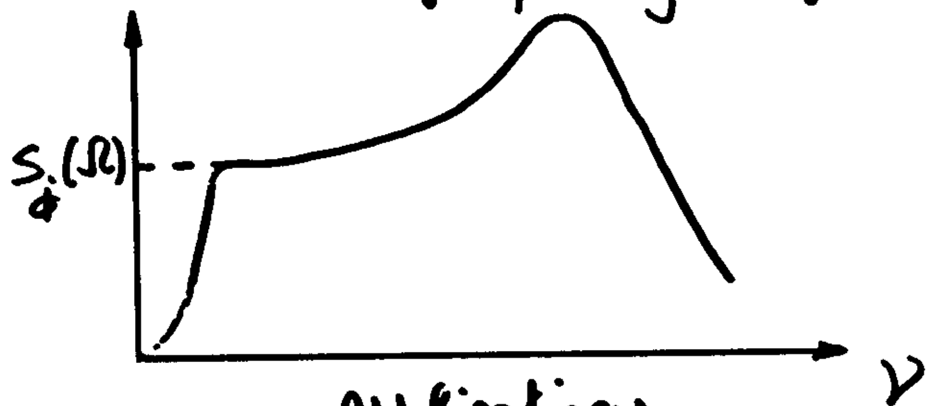
(Dandridge, Tveten.. Electron. Lett. 1981)

Advantages

- Good stability.
- High reduction of $S_{\dot\phi}(\Omega)$.
- [redacted] easy to make.

Disadvantages

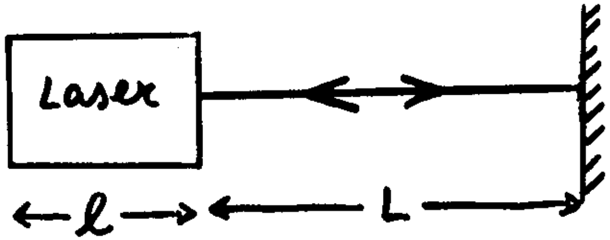
- Reduction of $S_{\dot\phi}(\Omega)$ only in the low frequency region.



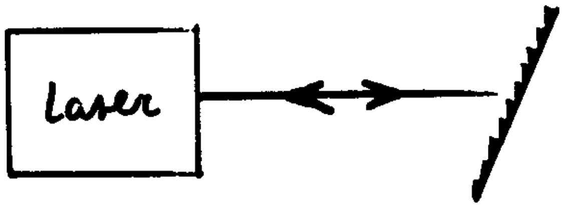
Applications

- Optical sensors

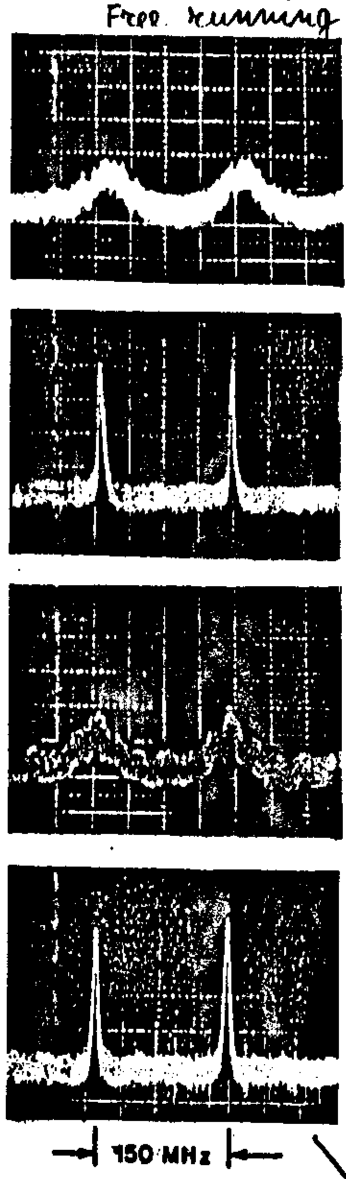
2) External Cavity

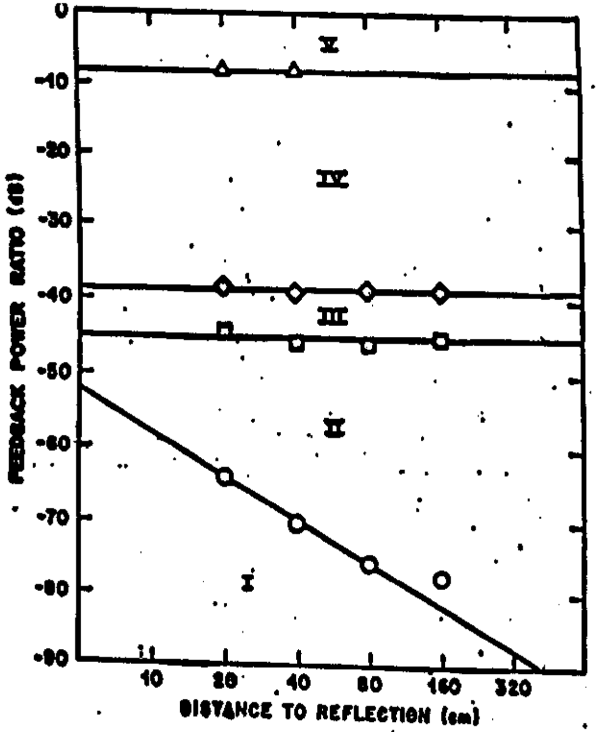


Mirror



Grating





I) reduction or broadening of the emission linewidth depending on the phase of the reinjected field

II) reduction of the linewidth or splitting in two lines

III) reduction but with high sensitivity to external unwanted reflections

IV) broadening of the linewidth

V) very sensitive reduction of the linewidth

(TKACH et al. JQE 1986)

2 cases

- Laser A.R. coated and strong feedback
- Laser without A.R. coating low feedback

Laser A.R. coated, strong feedback

(Mooradian J QE, 1981; Mattews et al Electron. Lett. 85)

- Reduction of α of the order of

$\ell/(\ell+L)$

Experimental results available only for the linewidth value

$$\Delta\nu_L \simeq \begin{matrix} 20\ \text{KHz (short term)} \\ 150\ \text{MHz (long term)} \end{matrix}$$

<u>Advantages</u>

- Very good short term linewidth

<u>Disadvantages</u>

- Mechanical complexity
- Mechanical and thermal noise

$$\frac{d}{dt}\left\{E(t)e^{i\omega_0 t}\right\} = \left\{i\omega_N(n) + \frac{1}{2}\left[G(n)-\Gamma_0\right]\right\}E(t)e^{i\omega_0 t} + K\,E(t-\tau)\,e^{i\omega_0(t-\tau)}$$

$$\frac{d}{dt}n(t) = -\gamma n - G(n)\,|E(t)|^2 + P$$

$$K = \frac{c}{2\eta\ell}(1-R_2)\sqrt{\frac{R_3}{R_2}}$$

τ : round-trip time of the ext. cavity

Now, the stationary solution ($E(t)=E_0$) gives

$$\begin{cases} G(n_0) \equiv G_0 = \Gamma_0 - 2K\cos\omega_0\tau \\ \omega_N(n_0) = \omega_0 + K\sin\omega_0\tau \end{cases}$$

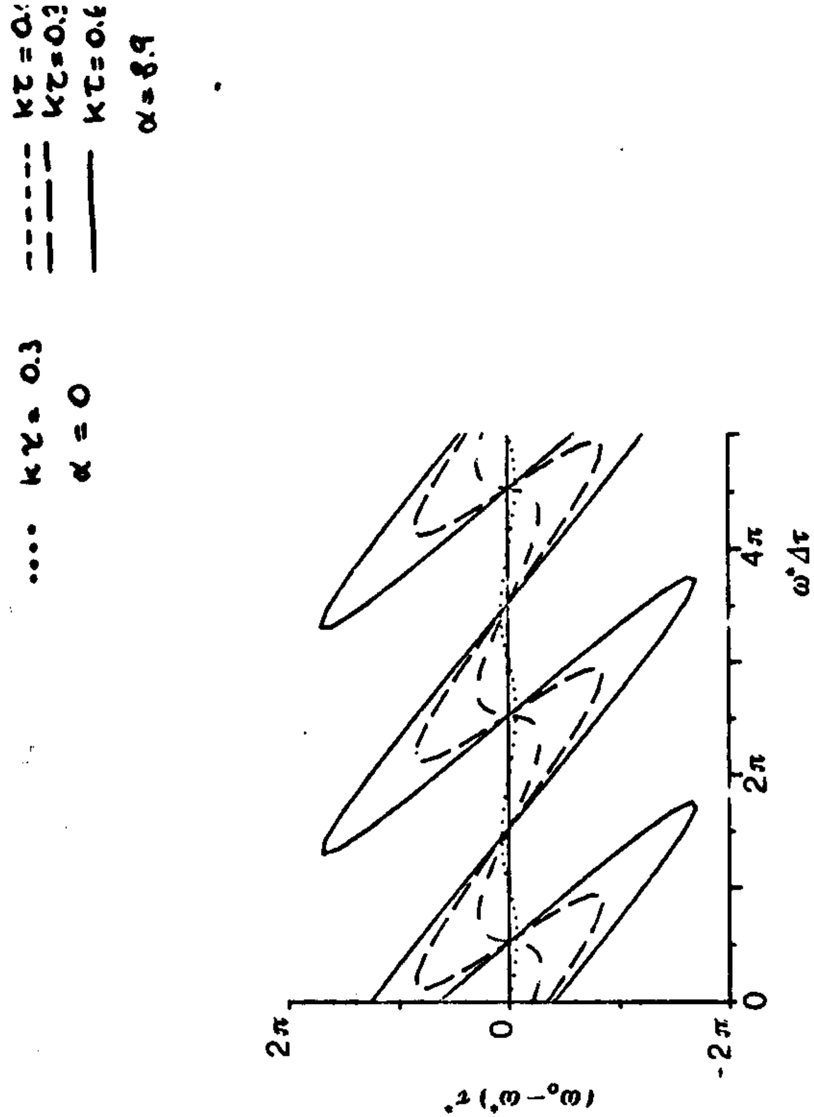
$\omega_0\tau$: phase of the reinjected field choosing the length of the external cavity in such a way that

$$|\Delta_\tau \phi(t)| = |\phi(t) - \phi(t-\tau)| \ll \pi/2$$

we have

$$\begin{cases} \Delta \dot{I}(t) = -2kI_0 \Delta_\tau \phi(t) \sin\Psi \\ \qquad -k\left[\Delta I(t) - \Delta I(t-\tau)\right]\cos\Psi \\ \qquad + \left(G_n I_0 / V\right)\Delta N(t) + F_{\Delta I}(t) \\ \dot{\phi}(t) = -k\,\Delta_\tau \phi(t)\cos\Psi + \left(\alpha G_n / 2V\right)\Delta N(t) \\ \qquad + \frac{k}{2I_0}\left[\Delta I(t) - \Delta I(t-\tau)\right] + F_{\Delta\phi}(t) \\ \Delta \dot{N}(t) = -\gamma_e \Delta N(t) - G_0 \Delta I(t) + F_{\Delta N}(t) \end{cases}$$

$$\| \Psi = \omega_0 \tau$$



Two values of k

$$\boxed{k_1 = \left[(\alpha^2+1)^{1/2}\tau\right]^{-1}}$$

$$\boxed{k_2 \cong \frac{3}{2}\pi k_1}$$

can be found such that

a) when $K < k_1$ the operating regime is strictly single-mode

b) when $k_1 < K < k_2$ the laser can oscillate on a single mode or not depending on τ

c) when $K > k_2$ the regime is multimode whatever the value of τ

Analysis of the results

The lineshape of a single mode injection laser is determined by the form of $S_{\dot\phi}(\Omega)$. In particular, to determine the linewidth $S_{\dot\phi}(0)$ is important. Were the lineshape Lorentzian

$$\boxed{\Delta\nu = \frac{S_{\dot\phi}(0)}{2\pi}}$$

$$S_{\dot\phi}(0) = \frac{R_s}{4I_0^2}\left\{\frac{\alpha^2(2I_0+1)+2I_0}{\left[1+K\tau(\cos\psi-\alpha\sin\psi)\right]^2}\right\}$$

$$\boxed{\psi = \omega_0\tau}$$

The minimum of $S_{\dot\phi}(0)$ is attained at the value

$$\Psi_{min} = -\operatorname{tg}^{-1}(\alpha)$$

$$\frac{3\pi}{2} \le \Psi_{min} \le 2\pi$$

- Reduction of the quantity $S_{\dot\phi}(0)$ depending on $K\tau$

- Maximum reduction of $S_{\dot\phi}(0)$ in a single mode operation equal to $[1+3\pi/2]^{-2}$ obtainable a value $K\tau = \dfrac{3\pi}{2[\alpha^2+1]}$

- Shape of $S_{\dot\phi}(\Omega)$ strongly dependent on τ at the same value of $K\tau$

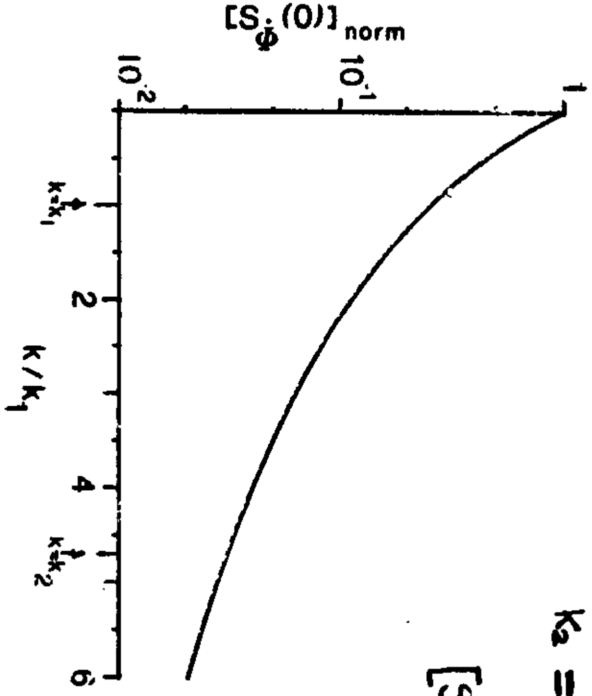


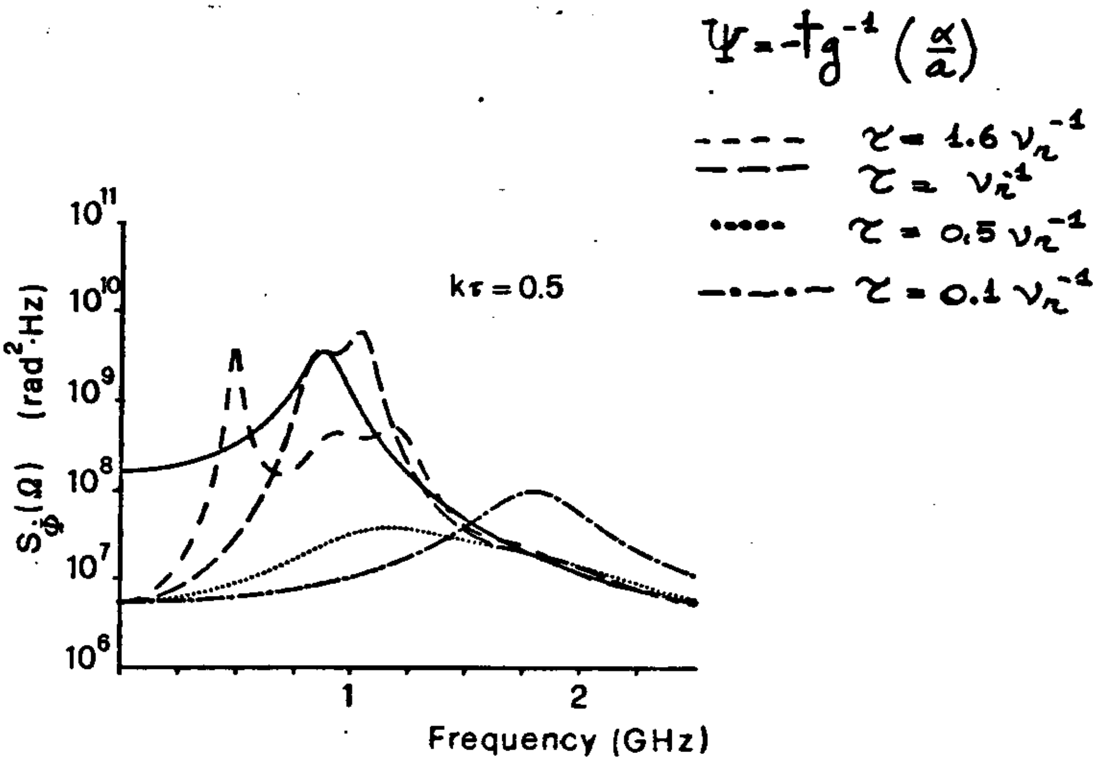
fig. 5

$$[S_{\dot\phi}(0)]_{norm} = \left[1+K\tau(1+\alpha^2)^{1/2}\right]^{-2}$$

$$K_2 = \frac{3}{2}\pi\left[\tau(1+\alpha^2)^{1/2}\right]^{-1}$$

$$[S_{\dot\phi}(0)]_{norm} = \left(1+\frac{3\pi}{2}\right)^{-2} = \cong 35^{-1}$$

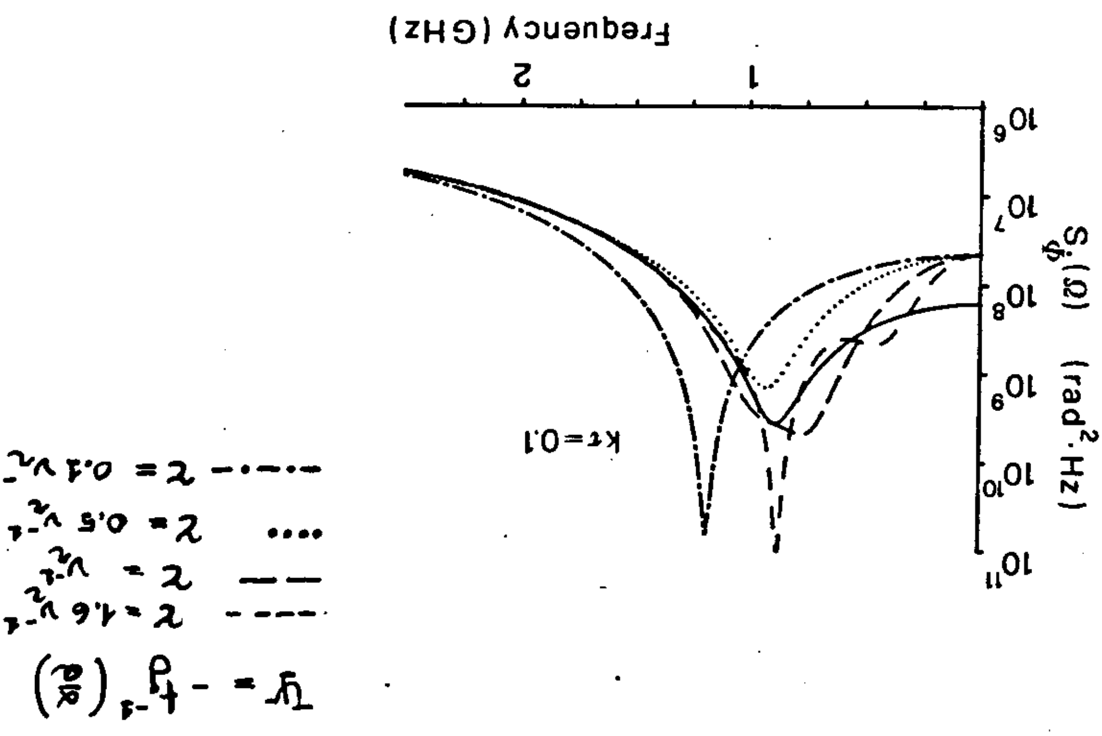
at $K=K_2$



$\Psi = -tg^{-1}\left(\frac{\alpha}{a}\right)$
$\tau = 1.6\,\nu_r^{-1}$
$\tau = \nu_r^{-1}$
$\tau = 0.5\,\nu_r^{-1}$
$\tau = 0.1\,\nu_r^{-1}$
$S_{\dot\Phi}(\Omega)$ ($rad^2 \cdot Hz$)
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
$k\tau = 0.5$
1
2
Frequency (GHz)

fig. 6b

fig. 6a



Frequency (GHz)
2
1
10^{6}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
$S_{\dot\Phi}(\Omega)$ ($rad^2 \cdot Hz$)
$k\tau = 0.1$
$\tau = 0.1\,\nu_r^{-1}$
$\tau = 0.5\,\nu_r^{-1}$
$\tau = \nu_r^{-1}$
$\tau = 1.6\,\nu_r^{-1}$
$\Psi = -tg^{-1}\left(\frac{\alpha}{a}\right)$

Advantages of the method

- Simplicity

Disadvantages

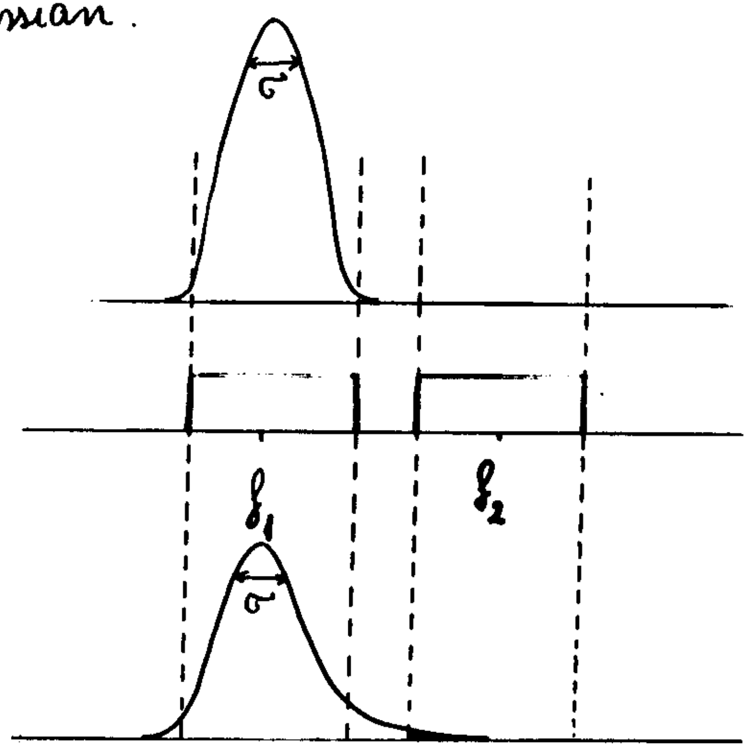
- Noise Reduction too much sensitive to variations of the phase of the reinjected field

$\Downarrow$

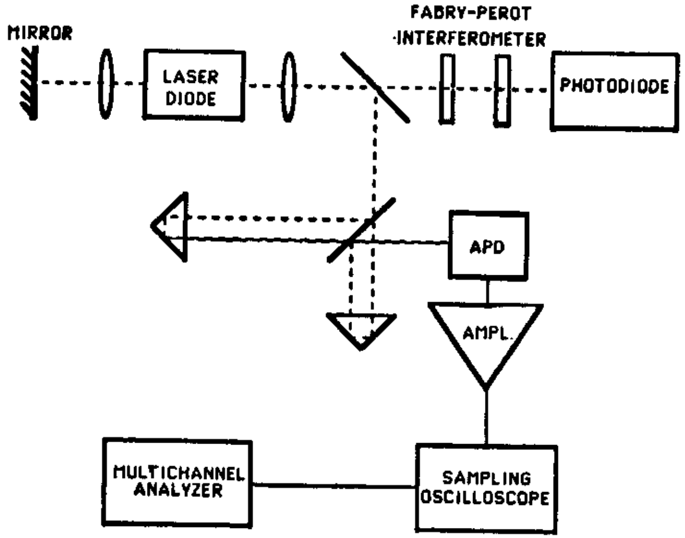
Mechanical and Thermal noise

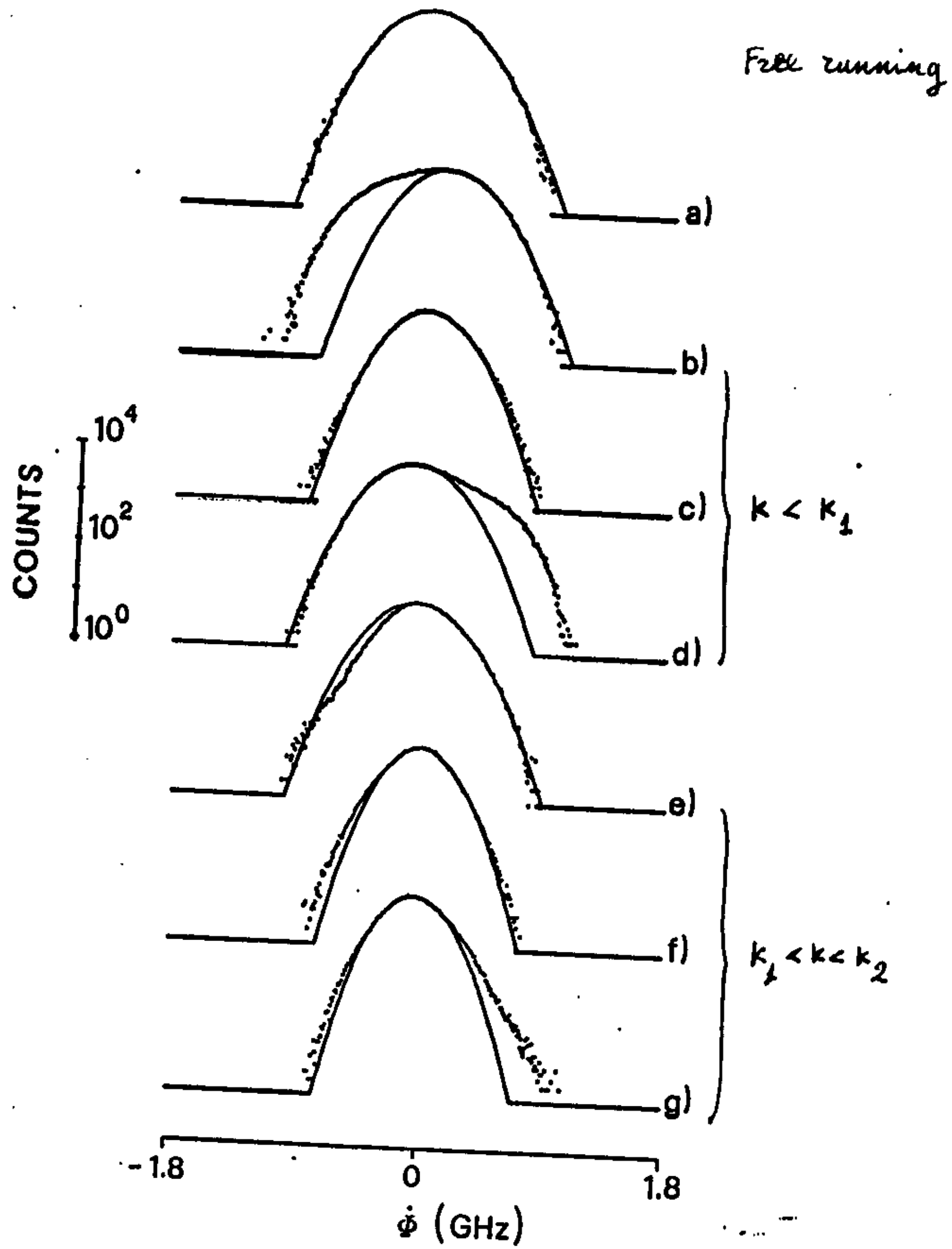
Untill now we considered only the linewidth or equivalent quantities to characterize phase noise.

In coherent optical systems and in other applications one usually assumes the statistics of phase noise to be Gaussian.



Experimental set up



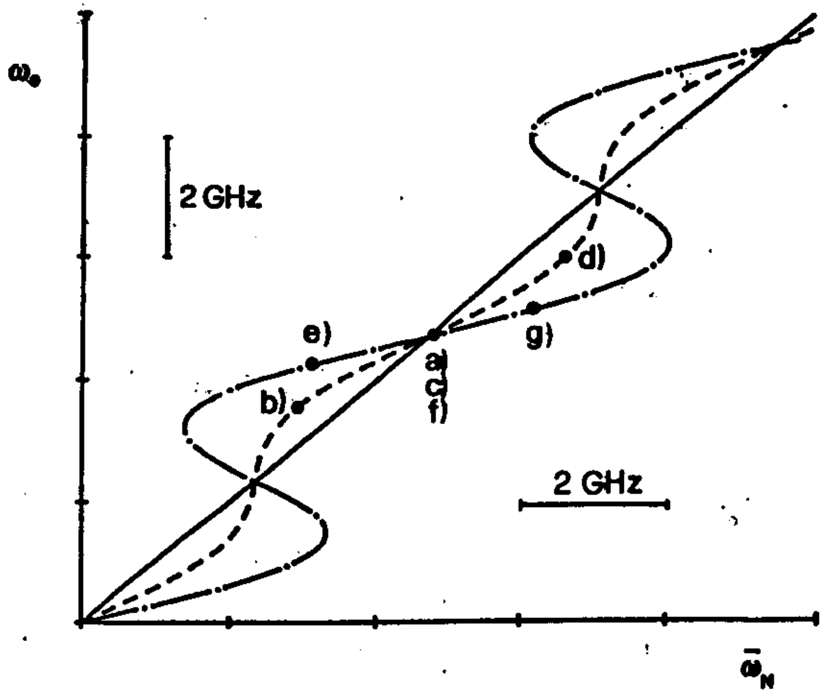


If $\omega_0 \tau = \frac{\pi}{2}$, $|\Delta_\tau \phi| \ll \pi$ and

$\tau < \frac{1}{\Omega_{max}}$

$$\Delta_\tau \phi(t) = \int_{t-\tau/2}^{t+\tau/2} \dot\phi(t')\,dt' \simeq \dot\phi(t)\,\tau$$

so that one can measure the statistical distribution of $\dot\phi(t)$ by measuring the statistical distribution of $\Delta_\tau \phi$.



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